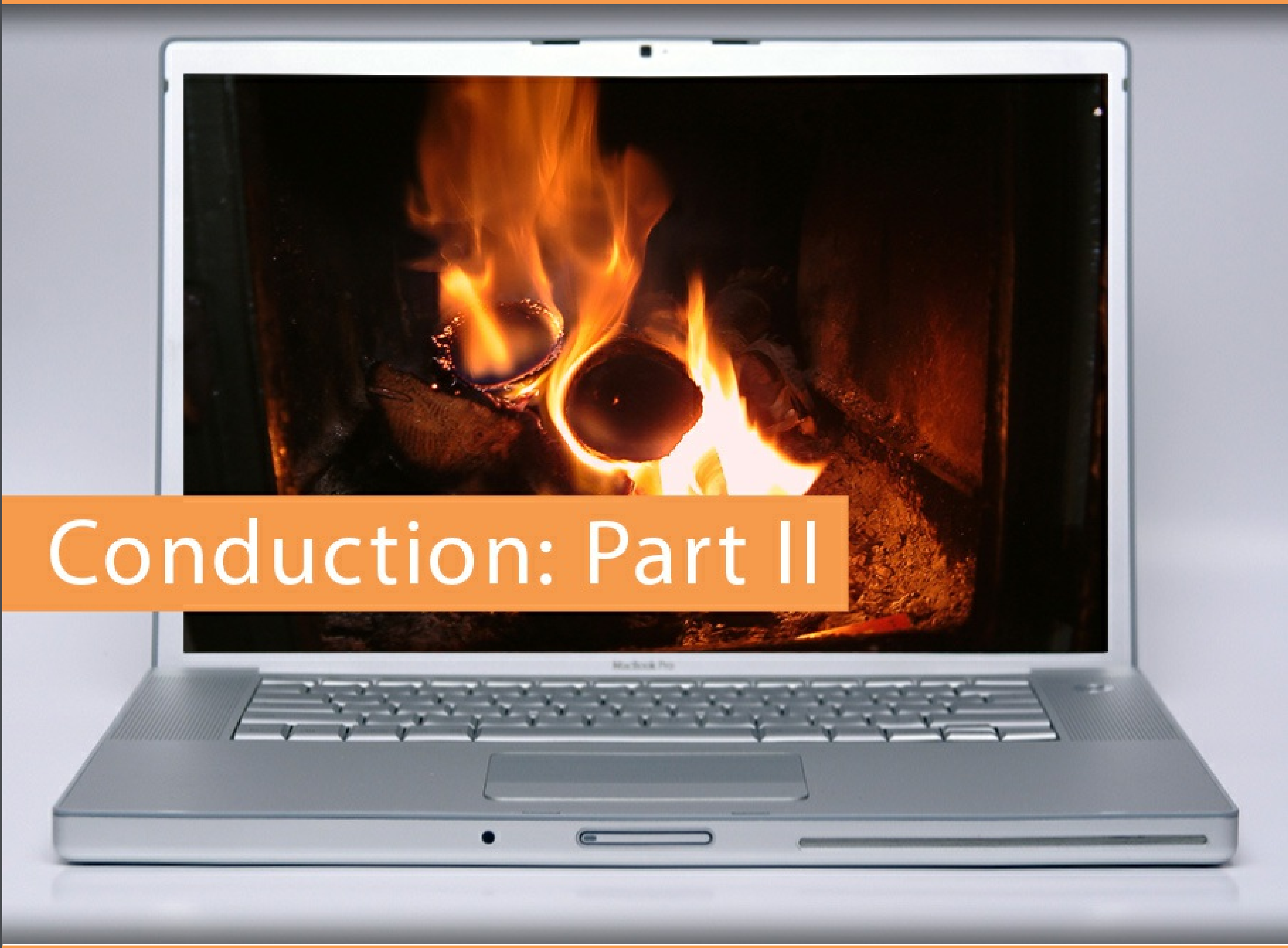
Software Solutions to Problems on Heat Transfer

Conduction: Part II

Dr. M. Thirumaleshwar



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Conduction – Part II

Fins, 1D conduction with heat generation, 2D conduction & Transient conduction

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1E Heat transfer with Fins

Learning objectives:

1. Fins are generally used to enhance the heat transfer from a given surface. Addition of fins can increase the heat transfer from the surface by several folds. Therefore Fins are used very widely in practice:
2. Typical application areas of Fins are:
   - Radiators for automobiles
   - Air-cooling of cylinder heads of Internal Combustion engines (e.g. scooters, motor cycles, aircraft engines etc.), air compressors etc.
   - Economizers of steam power plants
   - Heat exchangers of a wide variety, used in different industries
   - Cooling of electric motors, transformers etc.
   - Cooling of electronic equipments, chips, I.C. boards etc.
   - Fin theory is also used to estimate error in temperature measurement while using thermometers or thermocouples.
3. We consider fins of uniform cross-section with the following boundary conditions:
   a) Infinitely long fin
   b) Fin of finite length with insulated end
   c) Fin of finite length losing heat from its end by convection
   d) Fin of finite length with specified temperatures at its ends
4. Problems are solved for the various types of fins mentioned above using Mathcad, EES, or EXCEL. Many University problems are also solved. So, studying these problems should clarify the solution techniques adopted in solving such problems.

Fin Formulae:

Temperature distribution and heat transfer rate for fins of uniform cross-section

$\theta(x) = (T(x) - T_a)$, $m = \sqrt{\{h.P / (k.A_c)\}}$

Case	Tip condition (x = L)	Temperature distribution, { θ(x)! θ$_o$ }	Heat transfer rate, Q_{fin}
1	Infinitely long $L \to \infty$:, $\theta(L) = 0$	$\frac{\theta(x)}{\theta_o} = \exp(-m \cdot x)$	$Q_{fin} = k \cdot A_c \cdot m \cdot \theta_o$
2	Insulated at the tip $(d\theta/dx)\vert_{x=L} = 0$	$\frac{\theta(x)}{\theta_o} = \frac{\cosh(m \cdot (L-x))}{\cosh(m \cdot L)}$	$Q_{fin} = k \cdot A_c \cdot m \cdot \theta_o \cdot \tanh(m \cdot L)$
3	Convection from tip $-k.(d\theta/dx)\vert_{x=L} = h.\theta(L)$	$\frac{\theta(x)}{\theta_o} = \frac{\cosh(m \cdot (L-x)) + \frac{h}{m \cdot k} \cdot \sinh(m \cdot (L-x))}{\cosh(m \cdot L) + \frac{h}{m \cdot k} \cdot \sinh(m \cdot L)}$	$Q_{fin} = k \cdot A_c \cdot m \cdot \theta_o \cdot \frac{\left(\tanh(m \cdot L) + \frac{h}{m \cdot k}\right)}{\left(1 + \frac{h}{m \cdot k} \cdot \tanh(m \cdot L)\right)}$
4(a)	Prescribed temp. at the tip, $\theta(L) = \theta_L$	$\theta(x) = \frac{\theta_1 \cdot \sinh(m \cdot (L-x)) + \theta_2 \cdot \sinh(m \cdot x)}{\sinh(m \cdot L)}$	$Q_{fin} = k \cdot A_c \cdot m \cdot (\theta_1 + \theta_2) \cdot \left(\frac{\cosh(m \cdot L) - 1}{\sinh(m \cdot L)}\right)$
4(b)	When temp. at both ends are equal, T1=T2 or, $\theta_1 = \theta_2$	$\theta(x) = \frac{\theta_1 \cdot \sinh(m \cdot (L-x)) + \theta_1 \cdot \sinh(m \cdot x)}{\sinh(m \cdot L)}$ Min. temp. is given by: $\theta_{min} = \frac{2 \cdot \theta_1 \cdot \sinh\left(\frac{m \cdot L}{2}\right)}{\sinh(m \cdot L)}$	$Q_{fin} = k \cdot A_c \cdot m \cdot (2 \cdot \theta_1) \cdot \left(\frac{\cosh(m \cdot L) - 1}{\sinh(m \cdot L)}\right)$

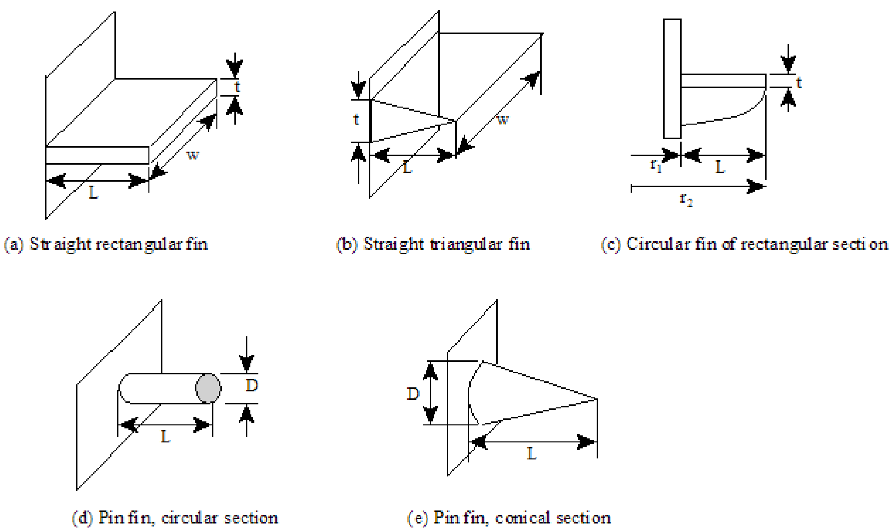

Fig.6.9 Typical fins: (a) and (d) of uniform cross-section, and (b), (c) and (e): of nonuniform cross-section

Table 1E.1

Fin efficiency (η_f) for a few fin shapes

A_c = area of crosssection, A_f = total fin surface area, L_c = corrected length, P = perimeter of fin section, h = heat tr. coeff., $m = \sqrt{h.P / (k.A_c)}$

Sl.No.	Description	Parameters	Fin efficiency (η_f)
1	Straight fin of rectangular section. See Fig. (6.9, a):	$A_f = 2 \cdot w \cdot L_c$ $L_c = L + \frac{t}{2}$ $m = \sqrt{\frac{2 \cdot h}{k \cdot t}}$ ...thin fins, w>>t	$\eta_f = \frac{\tanh(m \cdot L_c)}{m \cdot L_c}$
2	Straight fin of triangular section See Fig. (6.9, b)	$A_f = 2 \cdot w \cdot \left[L^2 + \left(\frac{t}{2}\right)^2\right]^{\frac{1}{2}}$ $m = \sqrt{\frac{2 \cdot h}{k \cdot t}}$	$\eta_f = \frac{1}{m \cdot L} \cdot \frac{I_1(2 \cdot m \cdot L)}{I_0(2 \cdot m \cdot L)}$
3	Circular fin of rect. section See Fig. (6.9, c)	$A_f = 2 \cdot \pi \cdot \left(r_{2c}^2 - r_1^2\right)$ $r_{2c} = r_2 + \frac{t}{2}$ $m = \sqrt{\frac{2 \cdot h}{k \cdot t}}$	$\eta_f = C_2 \cdot \left[\frac{\left(K_1(m \cdot r_1) \cdot I_1(m \cdot r_{2c}) - I_1(m \cdot r_1) \cdot K_1(m \cdot r_{2c})\right)}{\left(I_0(m \cdot r_1) \cdot K_1(m \cdot r_{2c}) + K_0(m \cdot r_1) \cdot I_1(m \cdot r_{2c})\right)}\right]$ $C2 = \frac{\left(\frac{2 \cdot r_1}{m}\right)}{\left(r_{2c}^2 - r_1^2\right)}$
4	Pin fin, circular section See Fig. (6.9, d)	$A_f = \pi \cdot D \cdot L_c$ $L_c = L + \frac{D}{4}$ $m = \sqrt{\frac{4 \cdot h}{k \cdot D}}$	$\eta_f = \frac{\tanh(m \cdot L_c)}{m \cdot L_c}$
5	Pin fin, conical section See Fig. (6.9, e)	$A_f = \frac{\pi \cdot D}{2} \cdot \left[L^2 + \left(\frac{D}{2}\right)^2\right]^{\frac{1}{2}}$ $m = \sqrt{\frac{4 \cdot h}{k \cdot D}}$	$\eta_f = \frac{2}{m \cdot L} \cdot \frac{I_2(2 \cdot m \cdot L)}{I_1(2 \cdot m \cdot L)}$

Table 1E.2

Note: In the above Table:

I_o = modified zero order Bessel function of first kind
K_o = modified zero order Bessel function of second kind
I_1 = modified first order Bessel function of first kind
K_1 = modified first order Bessel function of second kind

==

"**Prob. 1E.1**. The handle of a ladle used for pouring molten metal at 327 C is 30 cm long and is made of 2.5 cm × 1.5 cm mild steel bar stock (k = 43 W/m.K). In order to reduce the grip temp it is proposed to make a hollow handle of mild steel plate of 0.15 cm thick to the same rectangular shape. If the surface heat transfer coeff. is 14.5 W/m^2.K and the ambient temp is 27 C, estimate the reduction in the temp of the grip. Neglect the heat transfer from the inner surface of the hollow shape. [VTU – VI Sem. B.E. – Dec. 2010]:"

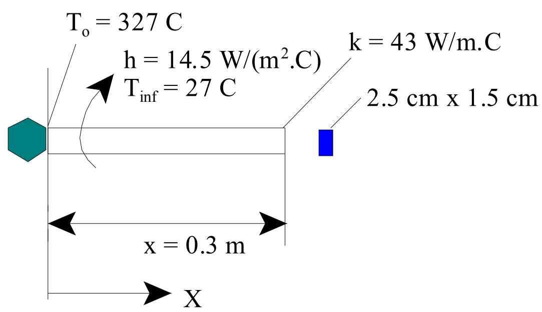

Fig.Prob.1E.1

EES Solution:

"Data:"

T_0 = 327 [C]
x = 0.3 [m]
A_1 = 0.025*0.015 [m^2] "…cross sectional area of the handle in the first case when bar stock is used"
k = 43 [W/m-C]
P_1 = 2*(0.025 + 0.015) [m] "…perimeter of ladle"
A_2 = P_1*0.0015 [m^2]"…cross sectional area of the handle in the second case when plate is used"
h = 14.5 [W/m^2-C]
T_inf = 27 [C]

"Calculations:"

m_1 = sqrt((h*P_1)/(k*A_1)) "[1/m]…fin parameter in first case"

"Using the formula for a long fin:"

(T_1 -T_inf)/(T_0 – T_inf) = exp(-m_1*x) "finds T_1, temp at the end of handle in first case"
m_2 = sqrt((h*P_1)/(k*A_2)) "[1/m]…fin parameter in second case"
(T_2 -T_inf)/(T_0 – T_inf) = exp(-m_2*x) "finds T_2, temp at the end of handle in second case"

"Therefore reduction in temp:"

Temp_reduction = T_1 – T_2 "[C]"

Results:

Unit Settings: SI C kPa kJ mass deg

A_1 = 0.000375 [m^2]	A_2 = 0.00012 [m^2]	h = 14.5 [W/m^2-C]
k = 43 [W/m-C]	m_1 = 8.482 [m^{-1}]	m_2 = 14.99 [m^{-1}]
P_1 = 0.08 [m]	$Temp_{reduction}$ = 20.21 [C]	T_0 = 327 [C]
T_1 = 50.55 [C]	T_2 = 30.34 [C]	T_{inf} = 27 [C]
x = 0.3 [m]		

Thus:

T_1 = 50.55 C … temp at the end of handle, in the first case … Ans.
T_2 = 30.34 C … temp at the end of handle, in the second case …. Ans.
Therefore, temp reduction = 20.21 C ….Ans.

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"**Prob.1E.2**. One end of a long rod of 1 cm dia is maintained at a temp of 500 C by placing it in a furnace. The rod is exposed to air at 30 C with a heat transfer coeff of 35 W/m^2.K. The temp measured at a distance of 78.6 mm was 147 C. Determine the thermal conductivity of the material. [VTU – VI Sem. B.E. – May/June 2006]:"

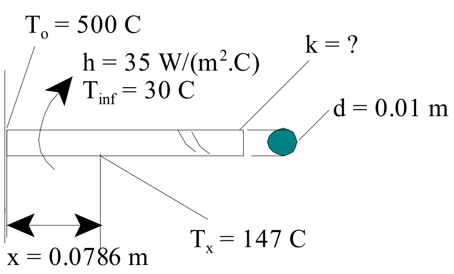

Fig.Prob.1E.2

EES Solution:

"Data:"

d = 0.01[m]

T_0 = 500 [C]

T_inf = 30 [C]

h = 35 [W/(m^2. K)]

x = 0.0786 [m]
T_x = 147 [C]

"Calculations:"

A_c = pi*d^2/4 "[m^2]… area of cross-section"
P = pi*d "[m] … perimeter"
m = sqrt((h*P)/(k*A_c)) "[1/m]…. fin parameter … this eqn determines k"

(T_x – T_inf)/(T_0 – T_inf) = exp(-m*x) "….temp distribution for a long fin… determines m"

Results:

Unit Settings: SI C kPa kJ mass deg

$A_c = 0.00007854$ [m^2]	$d = 0.01$ [m]	$h = 35$ [W/(m^2·K)]
$k = 44.73$ [W/(m-C)]	$m = 17.69$ [m^{-1}]	$P = 0.03142$ [m]
$T_0 = 500$ [C]	$T_{inf} = 30$ [C]	$T_x = 147$ [C]
$x = 0.0786$ [m]		

Thus:

k = 44.73 W/m.C…thermal cond. of rod .. Ans.

===

"**Prob. 1E.3**.The Aluminium square fins (0.5 mm × 0.5 mm), 10 mm long, are provided on the surface of a semiconductor electronic device to carry 1 W of energy generated. The temp of the surface of the device should not exceed 80 C, when the surrounding temp is 40 C. k for Al = 200 W/m.C, h = 15 W/m^2.C. Determine the number of fins required to carry out the above duty. Neglect the heat loss from the end of the fin. [VTU – VII Sem. B.E. – May 2007]:"

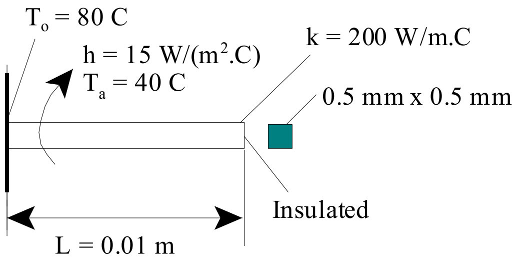

Fig.Prob.1E.3

EES Solution:

"Data:"

A_c = 0.5 * 0.5 * 10^(-6) " [m^2]…cross-sectional area of fin"
P = 2 * (0.5 + 0.5) * 10^(-3) „ [m] ….perimeter"
L = 0.01[m]
Q_tot = 1[W]
T_0 = 80[C]
T_a = 40[C]
k = 200[W/m-C]
h = 15[W/m^2-C]

"Calculations:"

"**It is a fin insulated at its end**…by data; So, use the relevant formula (see the Table)

Calculate the heat transferred from one fin; then, knowing the total heat to be transferred, one can calculate the no. of fins required."

m = sqrt((h * P)/(k * A_c)) "[1/m]…fin parameter"

Q_fin = k * A_c * m * (T_0 – T_a) * tanh(m*L) "[W] …. heat transferred by single fin, insulated at its tip"

"Therefore, no. of fins required:"

N = ceil(Q_tot/Q_fin) " no. of fins rounded off to the next higher integer value"

Results:

Unit Settings: SI C kPa kJ mass deg

A_c = 2.500E-07	h = 15 [W/m^2-C]	k = 200 [W/m-C]	L = 0.01 [m]
m = 24.49 [1/m]	N = 85 [-]	P = 0.002 [m]	Q_{fin} = 0.01177 [W]
Q_{tot} = 1 [W]	T_0 = 80 [C]	T_a = 40 [C]	

Thus:

N = 85 …. No. of fins required to dissipate 1 W .. Ans.

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"**Prob. 1E.4.** A casing of electric motor is an approx. cylinder of 250 mm dia and 500 mm long. There are 30 equi-spaced longitudinal fins of thickness 5 mm and height 25 mm on the periphery of the casing. If the casing temp is 56 C and ambient temp is 26 C, determine the heat dissipation from the casing body. Neglect the circular plane surface on either side. Take h = 25 W/m^2.C and k_fin = 30 W/m.C [VTU – VI Sem. B.E. – Jan.–Feb. 2004]:"

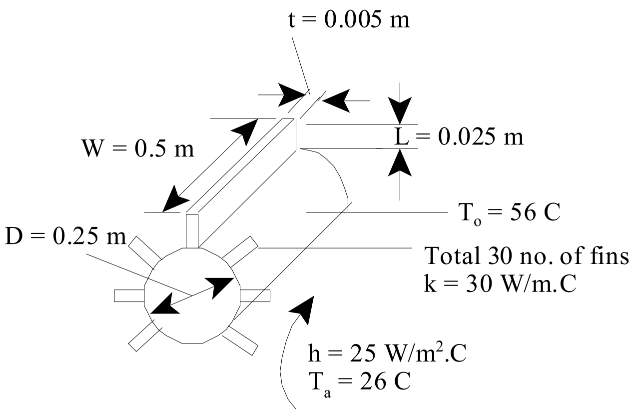

Fig.Prob.1E.4

EES Solution:

"Data:"

D = 0.25 [m] "..dia of cyl."
W = 0.5 [m] "..width of fins = length of cyl."
N_fins = 30 [-]
t = 0.005 [m] "…thickness of fins"
L = 0.025 [m] ".length of fins"
T_0 = 56 [C]
T_a = 26 [C]
h = 25 [W/m^2-C]
k = 30 [W/m-C]

"Calculations:"

"Note that area of the tip of the fin is not negligible; therefore, tip can not be considered as insulated. So, this is a problem of a fin with convection off its ends.

Also, the heat transfer from the un-finned area (i.e. prime area) should be included while calculating the total heat transfer;"

"Note from the Table 1E.1 that the formula for heat transfer from a fin with convection from its tip is quite complicated.

So, generally, the method followed is to use the formula for a fin with insulated tip, but the taking a 'corrected length 'L_c' instead of the given length L of the fin.

L_c is given by L_c = (L + t/2) for a rectangulat fin of thickness t, and L_c = (L + r/2) for a circular fin of radius r:"

"Therefore:"

L_c = L + t/2 "[m]…corrected length of fin"
P = 2 * (W + t) "[m]…perimeter"
A_c = W * t "[m^2]…area of cross-section of fin"
m = sqrt((h*P)/(k*A_c)) "[1/m]….fin parameter"
Q_perfin = k*A_c*m*(T_0 – T_a) * tanh(m*L_c) "[W]….heat transfer per fin"
Q_fins = Q_perfin * N_fins "[W]…total heat transfer for N fins"
A_unfin = (pi*D – N_fins*t) * W "[m^2]…un-finned area on the cyl. surface"
Q_unfin = h*A_unfin * (T_0 – T_a) "[W]…heat transfer from un-finned area"

Q_tot = Q_fins + Q_unfin "[W]… total heat transfer"

"Now, calculate with the exact formula, anyway, to verify:"

Q_perfin_exact = k*A_c*m*(T_0 – T_a) * (tanh(m*L) + h/ (m*k))/(1+(h/(m*k))*tanh(m*L)) "[W]…heat transfer per fin, using the exact formula for a fin with convection off its end"

Results:

Unit Settings: SI C kPa kJ mass deg

A_c = 0.0025 [m^2]	A_{unfin} = 0.3177 [m^2]	D = 0.25 [m]
h = 25 [W/m^2-C]	k = 30 [W/m-C]	L = 0.025 [m]
L_c = 0.0275 [m]	m = 18.35 [m^-1]	N_{fins} = 30 [-]
P = 1.01 [m]	Q_{fins} = 576.8 [W]	Q_{perfin} = 19.23 [W]
$Q_{perfin,exact}$ = 19.21 [W]	Q_{tot} = 815.1 [W]	Q_{unfin} = 238.3 [W]
t = 0.005 [m]	T_0 = 56 [C]	T_a = 26 [C]
W = 0.5 [m]		

Thus:

Q_fins = 576.8 W …. Heat transfer from 30 fins
Q_unfin = 238.3 W …. Heat transfer from un-finned cylinder body
Q_tot = 815.1 W ….total heat transfer ….Ans.

Note that Q_unfin is considerable as compared to the heat transferred from 30 fins.
Also, using simpler approx. formula gives almost the same value(19.23 W) for Q_perfin as compared to the complicated, exact formula 19.21 W).

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"**Prob. 1E.5.** A rod (k = 200 W/m.K), 5 mm in dia and 5 cm long has its one end maintained at 100 C. The surface of the rod is exposed to ambient air at 25 C with convection heat transfer coeff of 100 W/m^2.K. Assuming the other end to be insulated, determine:

1) temp of the rod at 20 mm distance from the end at 200 C,
2) heat dissipation rate from the surface of the rod, and
3) fin effectiveness [VTU – VI Sem. B.E. – May–June 2010]:"

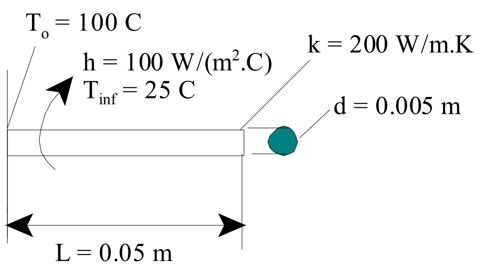


Fig. Prob.1E.5

Mathcad Solution:

Data:

$k := 200$ W/m.K $\quad d := 0.005$ m $\quad T_{inf} := 25$ C $\quad h := 100$ W/m^2.K

$T_0 := 100$ C $\quad L := 0.05$ m

Calculations:

$P := \pi \cdot d \qquad P = 0.016$ m.... perimeter

$A_c := \frac{\pi \cdot d^2}{4} \qquad A_c = 1.963 \cdot 10^{-5}$ m^2 .. area of cross-section of fin

$m := \sqrt{\frac{h \cdot P}{k \cdot A_c}}$ i.e. $m = 20$ 1/m ..... fin parameter

Temperature distribn:

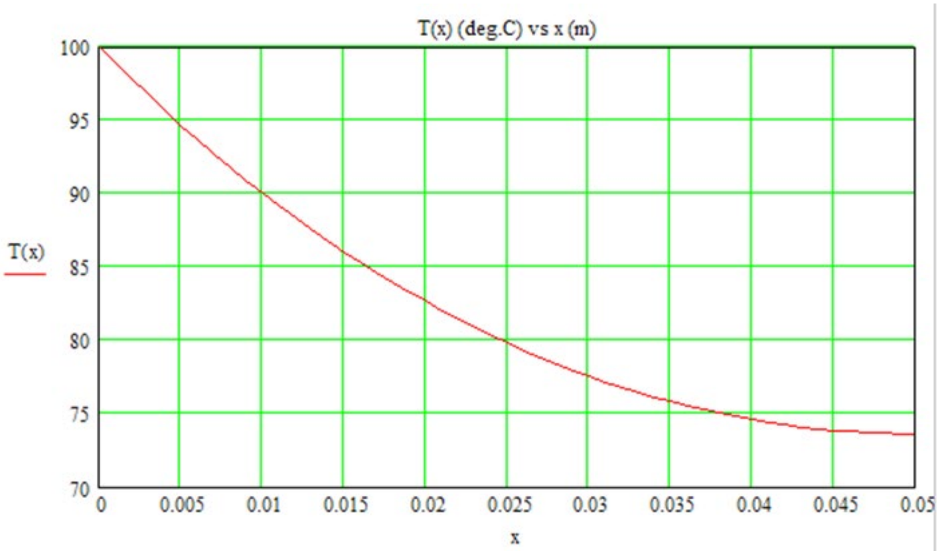
$$\frac{T - T_{inf}}{T_0 - T_{inf}} = \frac{\cosh(m \cdot (L - x))}{\cosh(m \cdot L)}$$ ...for a fin with insulated end

i.e. $$T(x) := (T_0 - T_{inf}) \cdot \frac{\cosh(m \cdot (L - x))}{\cosh(m \cdot L)} + T_{inf}$$

$T(0.02) = 82.618$ **C....temp at 20 mm distance from the end at 100 C...Ans.**

Draw the temp distribution in the fin:

$x := 0, 0.001 .. 0.05$ ...define a range variable x, from 0 to 0.05, with an increment of 0.001 m



Heat flow Q:

$Q := k \cdot A_c \cdot m \cdot (T_0 - T_{inf})$ W ...heat transfer for a fin with insulated end.

i.e. $Q = 5.89$ **W.....Ans.**

Effectiveness:

$Q_{nofin} := h \cdot A_c \cdot (T_0 - T_{inf})$ ...Q when there is no fin... consider only the base area Ac of fin

i.e. $Q_{nofin} = 0.147$ W

And: $\varepsilon := \frac{Q}{Q_{nofin}}$ $\varepsilon = 40$ **.....effectiveness....Ans.**

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"Prob. 1E.6 In a conductivity measuring experiment, two identical long rods are used. One rod is made of Aluminium (k = 200 W/m.K). The other rod is a specimen rod. One end of both rods are fixed to a wall at 100 C, while the other end is suspended in air at 25 C. The steady temp at the same distance along the rods were measured and found to be 75 C on Aluminium rod and 60 C on specimen rod. Find k of the specimen. [VTU – VI Sem. B.E. – Jan.–Feb. 2003]"

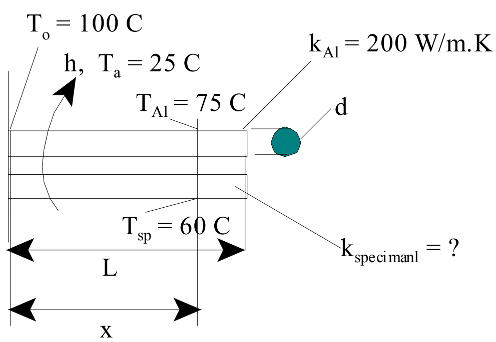


Fig. Prob.1E.6

Mathcad Solution:

Data:

$k_{Al} := 200$ W/m.K $T_0 := 100$ C $T_a := 25$ C

$T_{Al} := 75$ C ... temp of Al rod at a distance x

$T_{specimen} := 60$ C ... temp of specimen rod at a distance x

Calculations:

Let the distance at which the temperature are measured on both the rods be x.

Taking the rods as long fins:

$$\frac{T_{Al} - T_a}{T_0 - T_a} = \exp(-m_{Al} \cdot x)$$ ...temp profile for Al rod, where m is the fin parameter for Al

$$\frac{T_{specimen} - T_a}{T_0 - T_a} = \exp(-m_{specimen} \cdot x)$$ ...temp profile for specimen, m is fin parameter for specimen

Taking logarithms for both the equations and dividing:

$$\frac{m_{Al}}{(m_{specimen})} = \frac{\ln\left(\frac{T_{Al} - T_a}{T_0 - T_a}\right)}{\ln\left(\frac{T_{specimen} - T_a}{T_0 - T_a}\right)}$$ ...eqn (1)

i.e. $$\frac{m_{Al}}{(m_{specimen})} = 0.532$$

Now, fin parameter m for the rods is given by:

$$m_{Al} = \sqrt{\frac{h \cdot P}{k_{Al} \cdot A_c}}$$ where h, P, Ac are the same for both the rods

and,

$$m_{specimen} = \sqrt{\frac{h \cdot P}{k_{specimen} \cdot A_c}}$$

Therefore, eqn (1) becomes:

$$\frac{\sqrt{\frac{h \cdot P}{k_{Al} \cdot A_c}}}{\sqrt{\frac{h \cdot P}{k_{specimen} \cdot A_c}}} = 0.532$$

i.e. $$\sqrt{\frac{k_{specimen}}{k_{Al}}} = 0.532$$

==

"**Prob. 1E.7.** A very long 25 mm dia copper rod (k = 380 W/m.C) extends horizontally from a wall maintained at 120 C. Temp of ambient air is 25 C and h = 9 W/m^2.C. (i) Determine the heat loss (ii) how long the rod should be to be considered as infinite?"

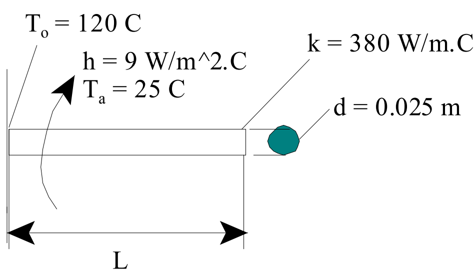


Fig. Prob. 1E.7

EES Solution:

"Data:"

d = 0.025[m]
k = 380 [W/m-C]
T_0 = 120 [C]
T_a = 25 [C]
h = 9 [W/m^2-C]

"Calculations:"

P = pi * d "[m]…perimeter"

A_c = pi * d^2/4 "[m^2] … area of cross-section of fin"

m = sqrt((h * P)/(k * A_c)) "[1/m] … fin parameter"

"For a very long (i.e. infinitely long fin) fin:

heat transfer Q is given by: "

Q = k * A_c * m * (T_0 – T_a) "[W] …. heat transfer for a very long fin"

"Now, for a fin with insulated end:

Q_ins = k * A_c * m * (T_0 – T_a) * tanh (m * L) …[W] … for a fin with insulated end.

Comparing the expressions for Q for the insulated and very long fins, it can be seen that the fin with insulated end approaches a very long fin (i.e. infinitely long fin) when tanh(m L) approaches the value of 1.

Let us plot tanh(mL) against mL and see the nature of variation:"

{
{mL = 2} "…commented out to draw the plot of tanh(mL) vs mL."

tanh(mL) = y

"It is seen from the plot that the graph (see below) approaches the value of 1 asymptotically.

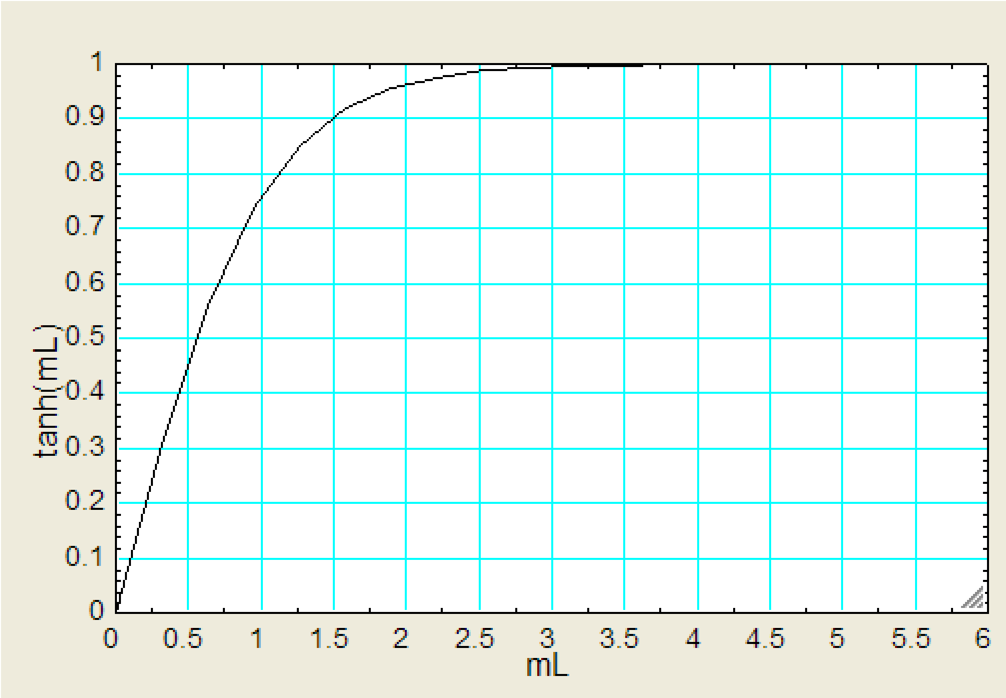
So, let us say that the fin can be considered as infinitely long if tanh (mL) is equal to 0.99. Then, the corresponding value of mL is:"

}

tanh(mL_inf) = 0.99 "…determines the value of mL for an infinitely long fin"

Results:

Plot of tanh (mL) vs mL:



Unit Settings: SI C kPa kJ mass deg

A_c = 0.0004909 [m^2]	d = 0.025 [m]	h = 9 [W/m^2-C]
k = 380 [W/m-C]	L_{inf} = 1.267 [m]	m = 1.947 [1/m]
mL_{inf} = 2.647 [-]	P = 0.07854 [m]	Q = 34.5 [W]
T_0 = 120 [C]	T_a = 25 [C]	

Thus:

Q = 34.5 W …. Heat transfer for a very long fin …. Ans.

mL_inf = 2.467 … value of mL required if the fin has to be considered as infinitely long…..

i.e. L_inf = 1.267 m ….. length required for the fin to be considered as infinitely long … Ans.

===

Prob. 1E.8. Thin fins of brass whose k = 75 W/m.K are welded longitudinally on a 5 cm dia brass cylinder which stands vertically and is surrounded by air at 20 C. The heat transfer coeff from metal surface to air is 17 W/m^2.K. If 16 uniformly spaced fins are used, each 0.8 mm thick, and extending 1.25 cm from the cylinder, what is the rate of heat transfer from the cylinder per metre length to the air when the cylinder surface is maintained at 150 C? [VTU – VI Sem. B.E. – June–July 2011]:"

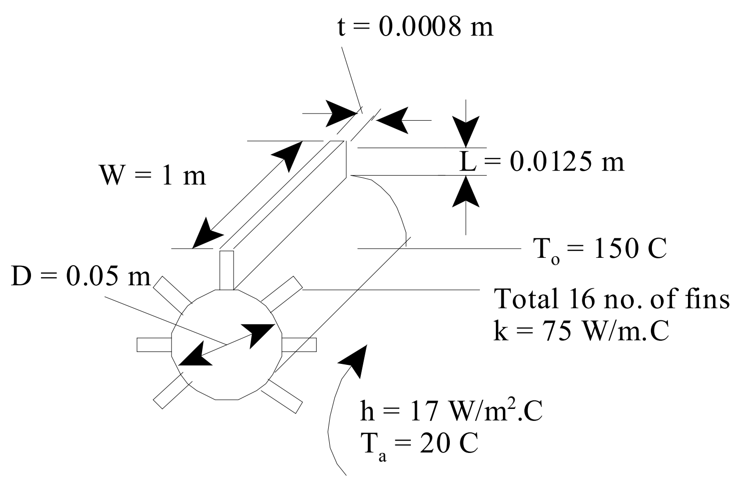


Fig.Prob.1E.8

Mathcad Solution:

Data:

$k := 75$ W/m.K $D := 0.05$ m $T_a := 20$ C $h := 17$ W/m^2.K

$N := 16$ ...no. of fins $t := 0.0008$ m....thickness $L := 0.0125$ m .. length of fin

$W := 1$ m... width of fin = length of cylinder $T_0 := 150$ C

Calculations:

$P := 2 \cdot (W + t)$ $P = 2.002$ m.... perimeter

$A_c := t \cdot W$ $A_c = 8 \cdot 10^{-4}$ m^2... area of cross-section of fin

$L_c := L + \frac{t}{2}$ $L_c = 0.013$ m.... corrected length

$m := \sqrt{\frac{h \cdot P}{k \cdot A_c}}$ $m = 23.814$ 1/m .... fin parameter

Note: Longitudinal fins on cylinder…so, it is a fin with convection off its end.

We use the simpler formula of that for a fin with insulated end, but with the 'corrected length':

$Qperfin := k \cdot A_c \cdot m \cdot (T_0 - T_a) \cdot \tanh(m \cdot L_c)$ ....Note: corrected length Lc is used

i.e. $Qperfin = 55.334$ W

So, for 16 fins: $Q_{fins} := Qperfin \cdot N$ $Q_{fins} = 885.34$ W

For un-finned area of cylinder:

$A_{unfin} := (\pi \cdot D - N \cdot t) \cdot W$ $A_{unfin} = 0.144$ m^2

$Q_{unfin} := h \cdot A_{unfin} \cdot (T_0 - T_a)$ $Q_{unfin} = 318.858$ W

Total heat transfer:

$$Q_{tot} := Q_{fins} + Q_{unfin}$$

$$Q_{tot} = 1.204 \cdot 10^3 \quad \text{W.....Ans.}$$

==

"Prob. 1E.9. A carbon steel (k = 54 W/m.C) rod with a cross-section of an equilateral triangle (each side 5 mm) is 80 mm long. It is attached to a plane wall maintained at a temp of 400 C. The surrounding environment is at 50 C and unit surface conductance is 90 W/m^2.C. Compute the heat dissipated by the rod, assuming that the tip is insulated. [VTU – VI Sem. B.E. – June 2012]:"

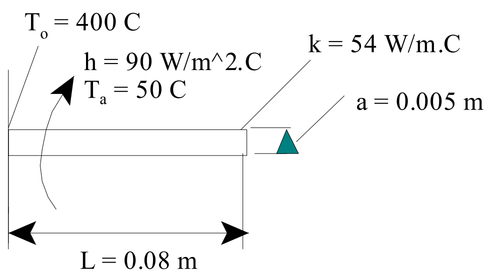


Fig.Prob.1E.9

EES Solution:

"Data:"

a = 0.005[m]"…side of equilateral triangle"
L = 0.08[m]
k = 54 [W/m-C]
T_0 = 400 [C]
T_a = 50 [C]
h = 90 [W/m^2-C]

"Calculations:"

P = 3 * a "[m]…perimeter"

A_c = 0.5 * a * sqrt(3 * a^2 /4) "[m^2] … area of cross-section of triangular fin (= (1/2) * base * height)"

m = sqrt((h * P)/(k * A_c)) "[1/m] … fin parameter"

"Now, for a fin with insulated end:"

Q_ins_end = k * A_c * m * (T_0 – T_a) * tanh (m * L) "[W] … for a fin with insulated end."

"In addition:"

"Now, for an infinitely long fin:"

Q_long = k * A_c * m * (T_0 – T_a) "[W] … for an infinitely long fin."

"And, for a fin with convection off its end:"

Q_conv_end = k * A_c * m * (T_0 – T_a) * ((tanh(m*L) + h/(m*k)) / (1 + h * tanh (m*L)/(m*k))) "[W] … for a fin with convection off its end"

Results:

Unit Settings: SI C kPa kJ mass deg

a = 0.005 [m]	A_c = 0.00001083 [m^2]	h = 90 [W/m^2-C]
k = 54 [W/m-C]	L = 0.08 [m]	m = 48.06 [1/m]
P = 0.015 [m]	$Q_{conv,end}$ = 9.824 [W]	$Q_{ins,end}$ = 9.823 [W]
Q_{long} = 9.832 [W]	T_0 = 400 [C]	T_a = 50 [C]

Thus:

Q_ins_end = 9.823 W …. heat transfer from the fin with insulated end…. Ans.

Q_long = 9.832 W … heat transfer from the infinitely long fin …. Ans.

Q_conv_end = 9.824 W …. heat transfer from the fin with convection off its end…. Ans.

==

"**Prob. 1E.10.** Consider an alloyed Aluminium rectangular fin (k = 180 W/m.C) with L = 10 mm, t = 1 mm, and width W = 1 m. Its base temp = 100 C. The surrounding environment is at 25 C and h = 100 W/m^2.C. Compute the heat dissipated by the fin, fin efficiency, fin effectiveness, tip temperature and thermal resistance for:

1) A fin with the tip insulated
2) A fin with convection off its end, and
3) An infinitely long fin (Ref: [3])"

Additionally: Find the effect of variation of h (from h = 10 W/m^2.K to 1000 W/m^2.K) on Q for fins of Aluminium (k = 1800) and also for also fins of Stainless steel (k = 15 W/m.C):

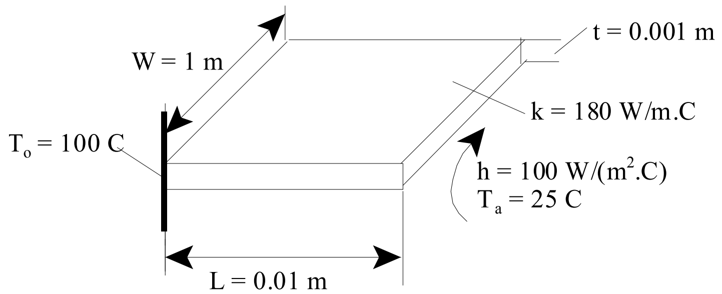


Fig.Prob.1E.10

EES Solution:

"Data:"

W = 1[m]

L = 0.01[m]

t = 0.001[m]

k = 180 [W/m-C]

T_0 = 100 [C]

T_a = 25 [C]

{h = 100 [W/m^2-C]} "….commented out to plot graphs

"Calculations:"

P = 2 * (W + t) "[m]…perimeter"

A_c = W * t "[m^2] … area of cross-section of triangular fin (= (1/2) * base * height)"

m = sqrt((h * P)/(k * A_c)) "[1/m] … fin parameter"

"Now, for a fin with insulated end:"

Q_ins_end = k * A_c * m * (T_0 – T_a) * tanh (m * L) "[W] … for a fin with insulated end."
eta_f_ins_end = tanh (m * L)/(m * L) "…fin efficieny"
Tiptemp_ins_end = T_a + (T_0 – T_a)/cosh(m * L) "[C] .. tip temp."
epsilon_ins_end = Q_ins_end/(h * A_c * (T_0 – T_a)) "….fin effectiveness"
R_f_ins_end = (T_0 – T_a)/ Q_ins_end "[C/W]… fin thermal resistance"

"And, for an infinitely long fin:"

Q_long = k * A_c * m * (T_0 – T_a) "[W] … for an infinitely long fin."
eta_f_long = 1/(m * L) "…fin efficieny"
Tiptemp_long = T_a "[C] .. tip temp…. is equal to ambient temp for infinitely long fin"
epsilon_long = Q_long/(h * A_c * (T_0 – T_a)) "….fin effectiveness"
R_f_long = (T_0 – T_a)/ Q_long "[C/W]… fin thermal resistance"

"Also, for a fin with convection off its end:"

Q_conv_end = k * A_c * m * (T_0 – T_a) * ((tanh(m*L) + h/(m*k)) / (1 + h * tanh (m*L)/(m*k))) "[W] … for a fin with convection off its end"
eta_f_conv_end = Q_conv_end/(h * (P * L + A_c) * (T_0 – T_a)) "…fin efficieny"
Tiptemp_conv_end = T_a + (T_0 – T_a) / (cosh(m * L) + h * sinh(m * L)/(m*k)) "[C] .. tip temp."
epsilon_conv_end = Q_conv_end/(h * A_c * (T_0 – T_a)) "….fin effectiveness"
R_f_conv_end = (T_0 – T_a)/ Q_conv_end "[C/W]… fin thermal resistance"

Results:

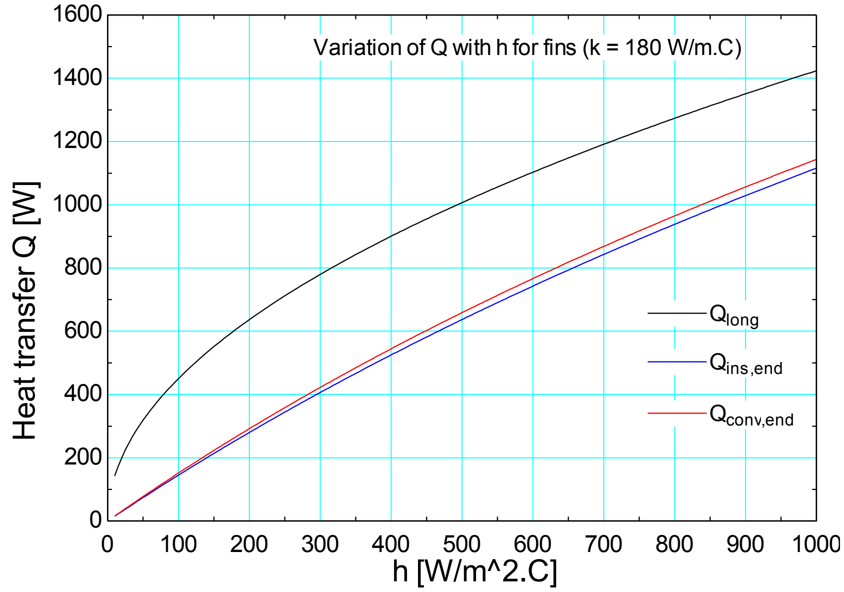
Unit Settings: SI C kPa kJ mass deg

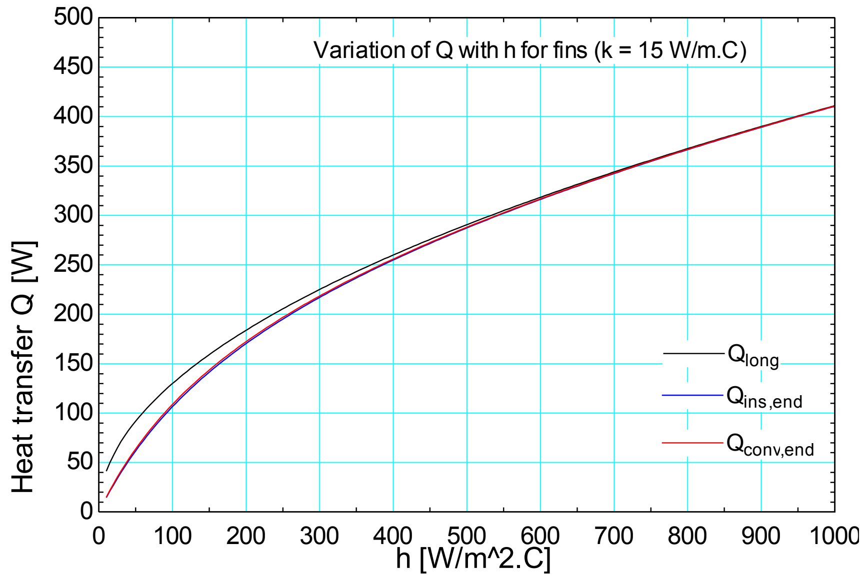
A_c = 0.001 [m^2]	$\varepsilon_{conv,end}$ = 20.2 [-]	$\varepsilon_{ins,end}$ = 19.31 [-]
ε_{long} = 60.03 [-]	$\eta_{f,conv,end}$ = 0.961 [-]	$\eta_{f,ins,end}$ = 0.9645 [-]
$\eta_{f,long}$ = 2.999E-12 [-]	h = 100 [W/m^2-C]	k = 180 [W/m-C]
L = 0.01 [m]	m = 33.35 [1/m]	P = 2.002 [m]
$Q_{conv,end}$ = 151.5 [W]	$Q_{ins,end}$ = 144.8 [W]	Q_{long} = 450.2 [W]
$R_{f,conv,end}$ = 0.495 [C/W]	$R_{f,ins,end}$ = 0.5179 [C/W]	$R_{f,long}$ = 0.1666 [C/W]
t = 0.001 [m]	$Tiptemp_{conv,end}$ = 95.64 [C]	$Tiptemp_{ins,end}$ = 96.01 [C]
$Tiptemp_{long}$ = 25 [C]	T_0 = 100 [C]	T_a = 25 [C]
W = 1 [m]		

Thus:

1) For a fin with the tip insulated: Q = 144.8 W, fin effcy, eta_f = 0.9645, fin effectiveness, epsilon = 19.31, Tip temp = 96.01 C and fin thermal resistance = 0.5179 C/W. **….Ans.**
2) For a fin with convection off its end: Q = 151.5 W, fin effcy, eta_f = 0.961, fin effectiveness, epsilon = 20.2, Tip temp = 95.64 C and fin thermal resistance = 0.495 C/W. **….Ans.**
3) For an infinitely long fin: Q = 450.2 W, fin effcy, eta_f =2.999E-12 = zero, fin effectiveness, epsilon = 60.03, Tip temp = 25 C, i.e. ambient temp, and fin thermal resistance = 0.1666 C/W. **….Ans.**

Then, draw the graphs:





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Prob. 1E.11. An iron bar 15 mm in dia spans the distance between 2 plates, 50 cm apart. Air at 25C flows in the space between the plates resulting in heat transfer coeff of 15 W/m^2.K. Calculate the heat transfer and temp at the middle of the bar if the plates are maintained at 125 C each. For iron, k = 45 W/m.K. [M.U.]

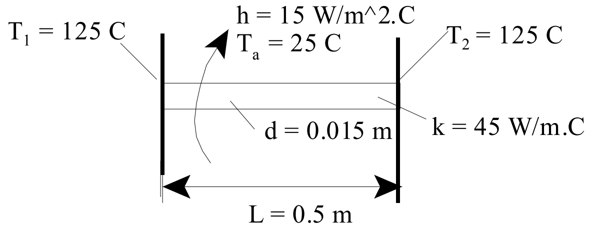


Fig.Prob.1E.11

Mathcad Solution:

Data:

$T1 := 125$ C $T2 := 125$ C $T_a := 25$ C

$h := 15$ W/m^2.K $d := 0.015$ m $k := 45$ W/m.K $L := 0.5$ m

Calculations:

$P := \pi \cdot d$ i.e. $P = 0.047$ m....perimeter

$A := \frac{\pi \cdot d^2}{4}$ $A = 1.767 \cdot 10^{-4}$ m^2 ... area of cross-section of fin

$m := \sqrt{\frac{h \cdot P}{k \cdot A}}$ $m = 5.774$ 1/m ... fin parameter

There is a ready formula available in Table 1E.1 for fin with specified temps at the ends.

However, let us first work out this problem from fundamentals, and then verify the result with the formula from the Table:

Governing differential equation for a fin is:

$$\frac{d^2}{dx^2}\theta - m^2 \cdot \theta = 0$$ where θ is 'excess temp.' = (T - Ta)

For a fin the general solution for temp distribution along x is given in either of the following two equivalent forms:

$\theta(x) = C1 \cdot \exp(m \cdot x) + C2 \cdot \exp(-m \cdot x)$ ....eqn. (1)...where, excess temp $\theta(x) = T(x) - T_a$

$\theta(x) = A \cdot \cosh(m \cdot x) + B \cdot \sinh(m \cdot x)$ ....eqn. (2)...where, excess temp $\theta(x) = T(x) - T_a$

In the above eqns C1, C2, A and B are four separate constants.

In this problem, let use the solution given by eqn.(1):

Define:

$\theta_1 := T1 - T_a$ i.e. $\theta_1 = 100$ C

$\theta_2 := T2 - T_a$ i.e. $\theta_2 = 100$ C

B.C's: At the left end: x = 0 , θ = θ1, and

At the right end: x = L, θ = θ2

Applying the B,C's to eqn. (1), we get the two constants C1 and C2:

From B.C.(i): $\theta_1 = C1 + C2$

From B.C.(ii): $\theta_2 = C1 \cdot \exp(m \cdot L) + C2 \cdot \exp(-m \cdot L)$

Then:

$$C1 := \frac{\theta_2 - \theta_1 \cdot \exp(-m \cdot L)}{\exp(m \cdot L) - \exp(-m \cdot L)}$$ i.e. $C1 = 0.889$

$C2 := \theta_1 - C1$ i.e. $C2 = 99.111$

Therefore: the temp distribution is given by:

$\theta(x) := C1 \cdot \exp(m \cdot x) + C2 \cdot \exp(-m \cdot x)$ .....eqn.(1)

Now, temp at mid-point of rod:

Put x = 0.25 in eqn.(1):

i.e. $\theta(0.25) = 18.772$ ...Excess temp at mid-point

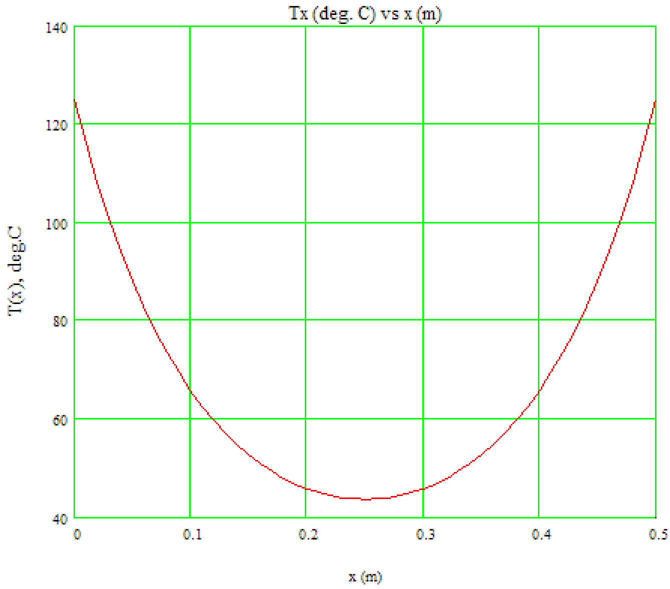
i.e. $T_{mid} := \theta(0.25) + T_a$

i.e. $T_{mid} = 43.772$ **C..Temp. at mid point .... Ans.**

Draw the temp profile:

$x := 0, 0.01 .. 0.5$ ....define a range variable x from 0 to 0.5, with an increment of 0.01 m

$\theta(x) := C1 \cdot \exp(m \cdot x) + C2 \cdot \exp(-m \cdot x)$ ...gives T(x) - T_a



It is clear that the min. temp occurs at the mid-point.

To calculate the heat transfer:

Find out the heat transferred at the left and right ends, using Fourier's law:

Define: $\theta'(x) := \frac{d}{dx}\theta(x)$ ...first derivative of θ w.r.t. x

At x = 0 and x = 0.5 m:

$\theta'(0) = -926.048$ $\theta'(0.5) = 926.048$

Therefore, by Fourier's law:

$Q_{left} := -k \cdot A \cdot \theta'(0)$

i.e. $Q_{left} = 7.364$ W....Heat tr. from left end

And,

$Q_{right} := -k \cdot A \cdot \theta'(0.5)$

i.e. $Q_{right} = -7.364$ W....Heat tr. from right end, -ve since flow from right to left

$Q_{total} := |Q_{left}| + |Q_{right}|$

i.e. $Q_{total} = 14.728$ **W... Total heat tr. from the rod....Ans.**

Verify by finding the heat lost by convection from the surface of fin:

$\theta(x) := C1 \cdot \exp(m \cdot x) + C2 \cdot \exp(-m \cdot x)$

$$Q := h \cdot P \cdot \int_0^{0.5} \theta(x)\,dx$$

$Q = 14.728$ **W.... verified.**

Now, apply the direct formulas from Table 1E.1 and verify the results obtained:

Temp distribution:

$$\theta(x) := \frac{\theta_1 \cdot \sinh(m \cdot (L - x)) + \theta_1 \cdot \sinh(m \cdot x)}{\sinh(m \cdot L)}$$

Therefore, at mid-point, i.e. at x = 0.25 m:

i.e. $\theta(0.25) = 18.772$ **C... verified with result obtained earlier.**

Heat transfer:

$$Q := k \cdot A \cdot m \cdot \left(2 \cdot \theta_1\right) \cdot \frac{\cosh(m \cdot L) - 1}{\sinh(m \cdot L)}$$

i.e. $Q = 14.728$ W.... **verified with result obtained earlier.**

Another *easier* way of solving this problem is as follows:

We observe from the temp profile that a minimum occurs at the mid-point. i.e. dT/dx = 0 at x = 0.25 m.
So, the rod can be considered as two separate fins, each insulated at its end.
Calculate the heat transferred from each fin and add them up to get total heat transferred from the rod to the ambient.

$Q_{insulatedfin} := k \cdot A \cdot m \cdot \left(T1 - T_a\right) \cdot \tanh(m \cdot L)$ W ... heat transfer from one insulated fin

i.e. $Q_{insulatedfin} = 7.496$ W

Therefore, heat transferred from the rod:

$Q_{total} := 2 \cdot Q_{insulatedfin}$ i.e. $Q_{total} = 14.992$ **W ....almost same as earlier value.**

Effect of h on Q and Temp profile:

It is obvious that heat transferred from the fin to ambient will depend on h. Also, h will affect the temp. profile in the fin.

Let us use the direct formulas from the Table to write T(x) and Q as functions of h and then plot the graphs:

We have:

$$m(h) := \sqrt{\frac{h \cdot P}{k \cdot A}}$$ ...fin parameter m is written as a function of h

i.e. $m(h) = 9.428$ 1/m ... fin parameter

Temp distribution:

$$\theta(x,h) := \frac{\theta_1 \cdot \sinh(m(h) \cdot (L - x)) + \theta_1 \cdot \sinh(m(h) \cdot x)}{\sinh(m(h) \cdot L)}$$ ...θ is written as function of x and h

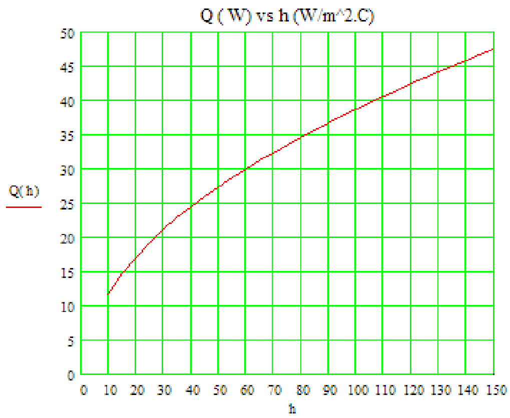
Heat transfer:

$$Q(h) := k \cdot A \cdot m(h) \cdot (2 \cdot \theta_1) \cdot \frac{\cosh(m(h) \cdot L) - 1}{\sinh(m(h) \cdot L)}$$ ....Q is written a a function of h

To plot Q as a function of h:

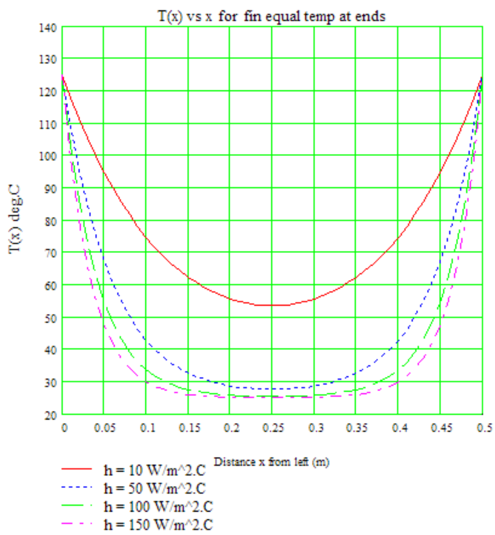
Let h vary from 0 to 150 W/m^2.C:

h := 10, 15 .. 150 ....define a range variable h, from h = 10 to 150 W/m^2.C



To plot T(x) as a function of x for different values of h:

x := 0, 0.01 .. 0.5 ....define a range variable x, from x = 0 to 0.5 m, with an increment of 0.01



It is noted that Q increases as h increases.

Also, the temp profile becomes lower and flatter at the centre as h increases i.e. as h increases larger region in the centre of the rod attains lower temp.

==

"Prob. 1E.12. A bar of square cross-section 20 mm × 20 mm, and of length 100 mm connects two metal surfaces. One surface is maintained at 200 C and the other is at 50 C. The bar is made of steel with k = 60 W/m.K. The surroundings are at 20 C with the surface heat transfer coeff of 10 W/m^2.K. Derive an equation for the temp distribution along the length of the bar and hence calculate the total heat flow rate from the bar to the surroundings. [VTU – VI Sem. B.E. – July–Aug. 2004]:"

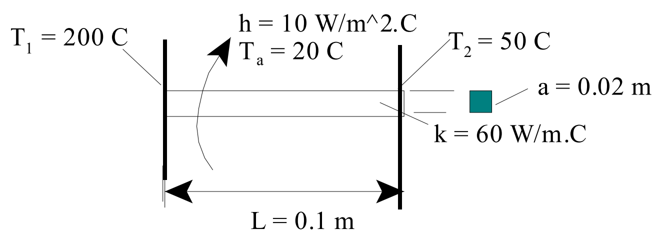


Fig.Prob.1E.12

Mathcad Solution:

Data:

$T1 := 200$ C $T2 := 50$ C $T_a := 20$ C

$h := 10$ W/m^2.K $k := 60$ W/m.K $L := 0.1$ m side of square bar: $a := 0.02$ m

Calculations:

$P := 4 \cdot a$ i.e. $P = 0.08$ m....perimeter

$A_c := a^2$ $A_c = 4 \cdot 10^{-4}$ m^2 ... area of cross-section of rod

$m := \sqrt{\frac{h \cdot P}{k \cdot A_c}}$ $m = 5.774$ 1/m ... fin parameter

There is a ready formula available in Table 1E.1 for *fin with specified temps at the ends.*

However, let us first work out this problem from fundamentals, and then verify the result with the formula from the Table:

Governing differential equation for a fin is:

$$\frac{d^2}{dx^2}\theta - m^2 \cdot \theta = 0$$ where θ is 'excess temp.' = (T - Ta)

For a fin the general solution for temp distribution along x is given in either of the following two equivalent forms:

$\theta(x) = C1 \cdot \exp(m \cdot x) + C2 \cdot \exp(-m \cdot x)$ ....eqn. (1)...where, excess temp $\theta(x) = T(x) - T_a$

$\theta(x) = A \cdot \cosh(m \cdot x) + B \cdot \sinh(m \cdot x)$ ....eqn. (2)...where, excess temp $\theta(x) = T(x) - T_a$

In the above eqns C1, C2, A and B are four separate constants.

In this problem, let use the solution given by eqn.(2):

Define:

$\theta_1 := T1 - T_a$ i.e. $\theta_1 = 180$ C

$\theta_2 := T2 - T_a$ i.e. $\theta_2 = 30$ C

B.C's: (i) At the left end: x = 0 , θ = θ1, and

(ii) At the right end: x = L, θ = θ2

Applying the B.C's to eqn. (2), we get the two constants A and B :

From B.C.(i): $\theta_1 = A$

From B.C.(ii): $\theta_2 = A \cdot \cosh(m \cdot L) + B \cdot \sinh(m \cdot L)$

i.e. $\theta_2 = \theta_1 \cdot \cosh(m \cdot L) + B \cdot \sinh(m \cdot L)$

i.e. $B = \dfrac{\theta_2 - \theta_1 \cdot \cosh(m \cdot L)}{\sinh(m \cdot L)}$

Then:

$A := \theta_1$ i.e. $A = 180$

$B := \dfrac{\theta_2 - \theta_1 \cdot \cosh(m \cdot L)}{\sinh(m \cdot L)}$ i.e. $B = -296.481$

Now, temp at mid-point of rod:

Put x = 0.05 in eqn.(2):

i.e. $\theta(0.05) = 100.772$ ...Excess temp at mid-point

i.e. $T_{mid} := \theta(0.05) + T_a$

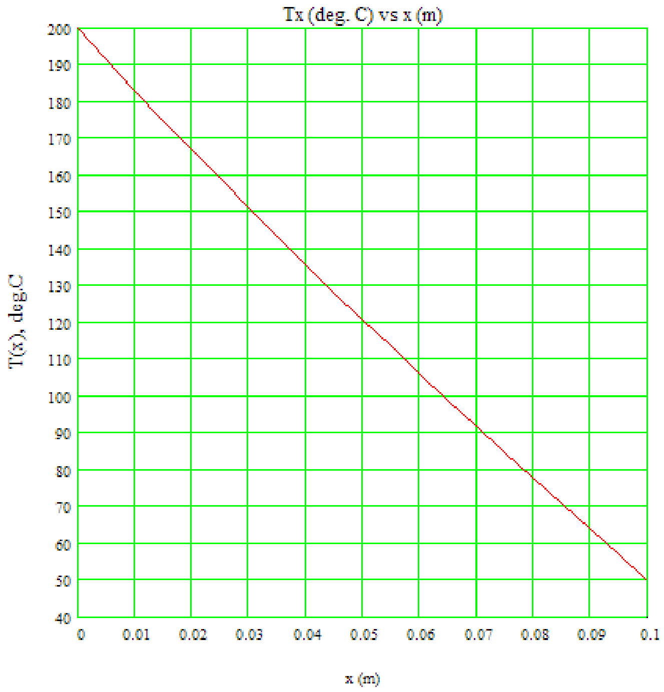
i.e. $T_{mid} = 120.772$ **C..Temp. at mid point .**

Draw the temp profile:

$x := 0, 0.005 .. 0.1$ ....define a range variable x from 0 to 0.1 m, with an increment of 0.005 m

$\theta(x) := A \cdot \cosh(m \cdot x) + B \cdot \sinh(m \cdot x)$ ...gives T(x) - T_a

Therefore, T(x) = θ(x) + Ta:



To calculate the heat transfer:

Find out the heat transferred at the left and right ends, using Fourier's law:

Define: $\theta'(x) := \frac{d}{dx}\theta(x)$ ...first derivative of θ w.r.t. x

At x = 0 and x = 0.1 m:

$\theta'(0) = -1.712 \cdot 10^3$ $\theta'(0.1) = -1.371 \cdot 10^3$

Therefore, by Fourier's law:

$Q_{left} := -k \cdot A_c \cdot \theta'(0)$

i.e. $Q_{left} = 41.082$ W....Heat tr. from left end

And,

$Q_{right} := -k \cdot A_c \cdot \theta'(0.1)$

i.e. $Q_{right} = 32.907$ W....Heat tr. from right end

It is observed that Q_left enters the rod from left and Q_right leaves to the right plate. So, the difference is dissipated from the surface of the rod to ambient. i.e.

$$Q_{finsurface} := Q_{left} - Q_{right}$$

i.e. $Q_{finsurface} = 8.174$ **W... heat tr. from the rod surface to ambient....Ans.**

Verify by finding the heat lost by convection from the surface of fin:

$$\theta(x) := A \cdot \cosh(m \cdot x) + B \cdot \sinh(m \cdot x)$$

$$Q := h \cdot P \cdot \int_0^{0.1} \theta(x)\,dx$$

$Q = 8.174$ **W.... verified.**

Now, apply the direct formulas from Table 1E.1 and verify the results obtained:

Temp distribution:

$$\theta(x) := \frac{\theta_1 \cdot \sinh(m \cdot (L - x)) + \theta_2 \cdot \sinh(m \cdot x)}{\sinh(m \cdot L)}$$

Therefore, at mid-point, i.e. at x = 0.05 m:

i.e. $\theta(0.05) = 100.772$ **C... verified with result obtained earlier.**

Heat transfer:

$$Q := k \cdot A_c \cdot m \cdot (\theta_1 + \theta_2) \cdot \frac{\cosh(m \cdot L) - 1}{\sinh(m \cdot L)}$$

i.e. $Q = 8.174$ W.... **verified with result obtained earlier.**

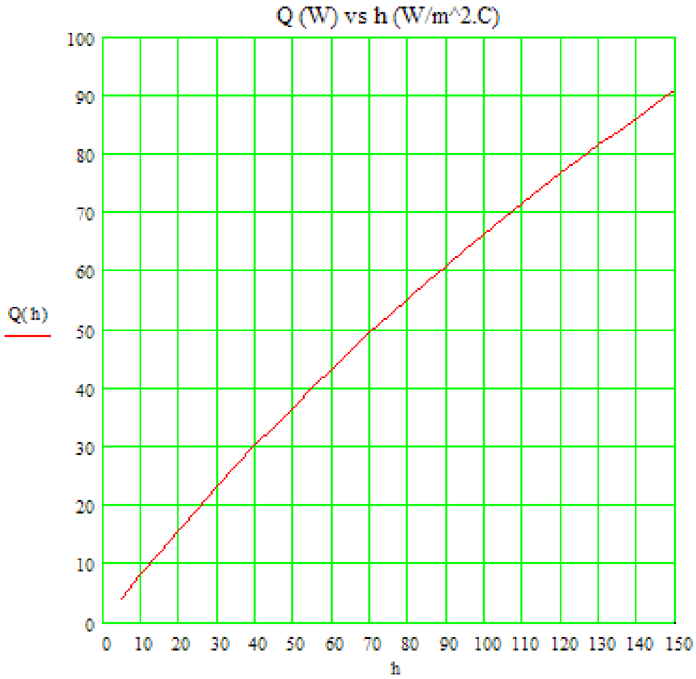
To plot Q as a function of h:

$$m(h) := \sqrt{\frac{h \cdot P}{k \cdot A_c}}$$ 1/m ... fin parameter defined as a function of h

$$Q(h) := k \cdot A_c \cdot m(h) \cdot (\theta_1 + \theta_2) \cdot \frac{\cosh(m(h) \cdot L) - 1}{\sinh(m(h) \cdot L)}$$ ...Q defined as a function of h

Now, plot the graph:

h := 5, 10.. 150 ...define a range variable h from h = 5 to 150 W/m^2.C



==

"**Prob. 1E.13.** A rod of copper of k = 116 W/m.K and 1.25 cm in dia spans the distance between two parallel plates 15 cm apart. Air flows in the space between the parallel plates, providing h = 58 W/m^2.K at the surface of the rod. The surface temp of the plates exceeds that of air by 40 C. What is the temp in excess at the centre of the rod over the temp of air? Determine the rate of heat transfer. Derive the eqns used. [VTU – M.Tech. – June–July 2009]:"

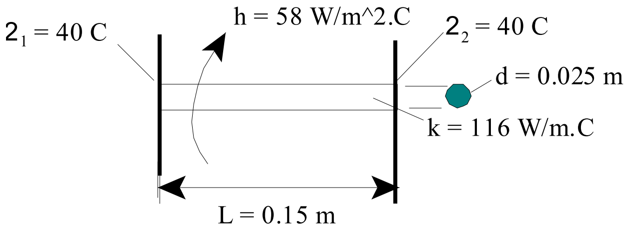


Fig.Prob.1E.13

EES Solution:

"Data:"

L = 0.15[m]
d = 0.0125[m]
k = 116 [W/m-C]
h = 58 [W/m^2-C]
theta_1 = 40 [C]
theta_2 = 40 [C]

"Calculations:"

P = pi * d "[m]…perimeter"

A_c = pi * d^2 / 4 "[m^2] … area of cross-section of rod "

m = sqrt((h * P)/(k * A_c)) "[1/m] … fin parameter"

"Now, for a fin with specified temps at ends end:

Excess temp at both ends = 40 deg.…by data.

i.e. both ends are at equal temperatures. Get the corresponding eqns from Table 1E.1:"

"Temp distribution at a distance x from LHS: is given by: (here, x = 0.075 m)"

theta_x = (theta_1 * sinh(m * (L – x)) + theta_1 * sinh(m * x)) / sinh(m * L) "[C] … excess temp at mid-point of rod"

x = 0.075 [m]

"Rate of heat transfer, Q:"

Q = k * A_c * m * (2 * theta_1) * (cosh(m * L) – 1) / sinh(m * L) "[W] .. heat transfer from rod surface to ambient"

Results:

Unit Settings: SI C kPa kJ mass deg

A_c = 0.0001227 [m^2]	d = 0.0125 [m]	h = 58 [W/m^2-C]	k = 116 [W/m-C]
L = 0.15 [m]	m = 12.65 [1/m]	P = 0.03927 [m]	Q = 10.65 [W]
θ_1 = 40 [C]	θ_2 = 40 [C]	θ_x = 26.94 [C]	x = 0.075 [m]

Thus:

theta_x = 26.94 C …. Excess temp over the ambient at the centre of rod …. Ans.

Q = 10.65 W ….. heat transfer from the rod to ambient …. Ans.

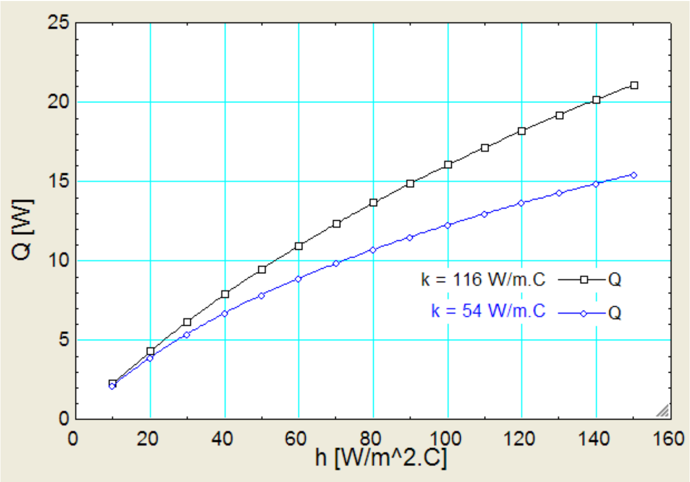
In addition:

Plot the variation of Q with h for rods with k = 116 W/m.K Al) and k = 54 W/m.K.(Carbon steel)

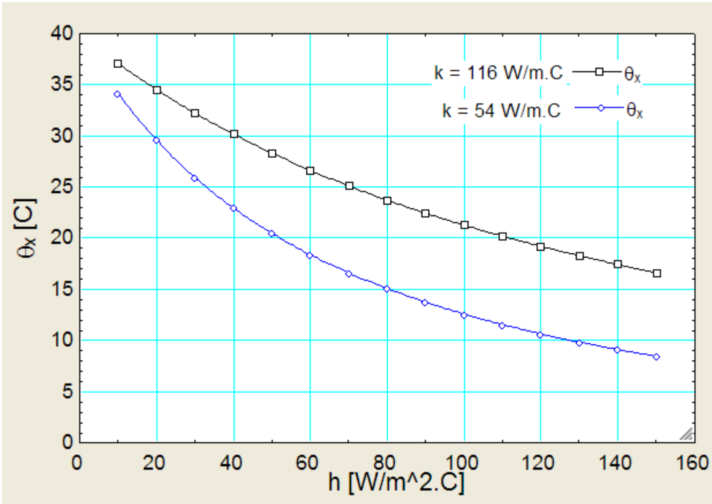
Let h vary from 10 to 150 W/m^2.K.

Prepare parametric tables and plot the graphs:

Q vs. h:



theta_x vs h:



Note that lower the thermal conductivity, lower is the amount of heat transferred to ambient. And, lower the k, lower is the value of theta_x, i.e. higher is the centre temp of the rod.

==

Prob.1E.14. The temp. of air in an air stream in a tube is measured by a thermometer placed in a protective well filled with oil. The thermo-well is made of steel tube 1.5 mm thick sheet of length 120 mm. The thermal cond. of steel = 58.8 W/m.K. and h = 23.3 W/m^2.K. If the air temp. recorded was 84 C, estimate the measurement error if the temp. at the base of the well was 40 C. [M.U. May 1997]

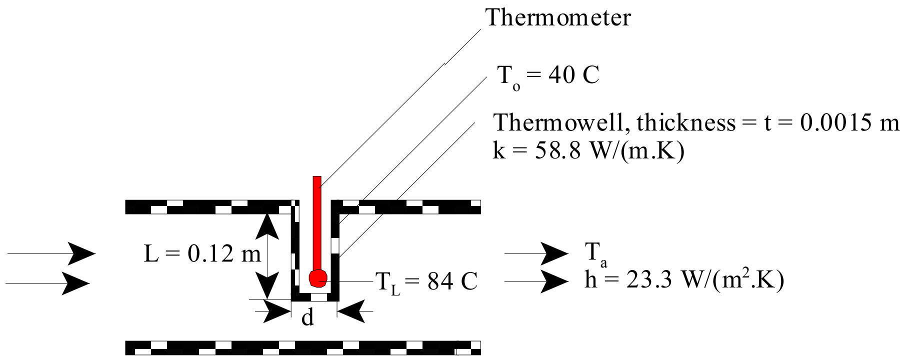


Fig.Prob.1E.14

Mathcad Solution:

Data:

$L := 0.12$ m $\quad t := 0.0015$ m $\quad h := 23.3$ W/m^2.K

$k := 58.8$ W/m.K $\quad T_0 := 40$ C $\quad T_L := 84$ C

Let Ta= temp. of air flow to be measured

Calculations:

$$m = \sqrt{\frac{h \cdot P}{k \cdot A_c}} = \sqrt{\frac{h \cdot \pi \cdot d}{k \cdot (\pi \cdot d \cdot t)}}$$ ....for a thin walled tube

i.e. $m := \sqrt{\frac{h}{k \cdot t}}$ i.e. $m = 16.253$ 1/m .... fin parameter

Treating the thermowell as a fin, insulated at its end:

Temp distribution is given by:

$$\frac{T_L - T_a}{T_0 - T_a} = \frac{1}{\cosh(m \cdot L)}$$ ...temp at the end of the fin (x = L) is given by this eqn.

Here, however, TL is known, and Ta is the unknown which can be determined:

Use the Solve Block of Mathcad:

Start with a trial value for Ta:

$T_a := 100$ ...Trial value

Given

$$\frac{T_L - T_a}{T_0 - T_a} = \frac{1}{\cosh(m \cdot L)}$$

$\text{Find}(T_a) = 101.009$

i.e. Actual temp of Air Ta is 101.009 C.....Ans.

$T_a := 101.009$ C

i.e. $T_a - T_L = 17.009$ **C ... error in thermometer reading ... Ans.**

And,

$$\frac{(T_a - T_L) \cdot 100}{T_a} = 16.839$$

So, Percentage error in measurement of temp= 16.84 % .... Ans.

==

"**Prob.1E.15.** A motor body is 25 cm in dia and 20 cm long. Its surface temp should be limited to 55 C when dissipating 150 W. Longitudinal fins of 1.2 cm thickness and 3 cm height are to be used. The convection heat transfer coeff is 40 W/m^2.K. The thermal cond. is 40 W/m.K. Determine the no. of fins required. The atmospheric temp is 40 C. [VTU – VI Se. B.E. July–Aug. 2003]"

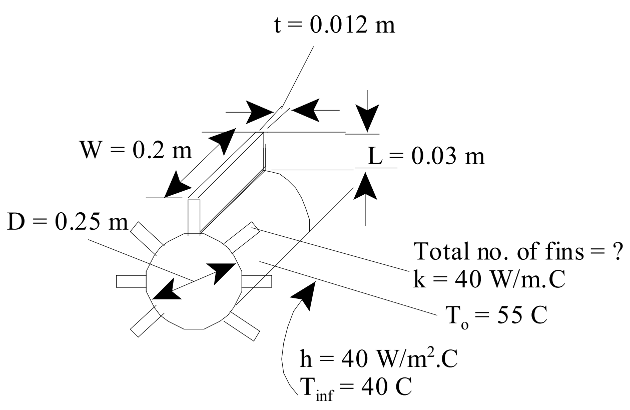


Fig.Prob.1E.15

EES Solution:

"Data:"

D = 0.25 [m]

W = 0.2[m]"…Width of fin = Length of cyl."

T_0 = 55[C]

Q_tot = 150[W]

L=0.03[m]"..length of fin"

t = 0.012[m]

h=40[W/m^2-C]

T_inf = 40[C]

k=40[W/m-C]

"Calculations:"

"This is the case of fin with convection off its end.

We will use the formula for Q for a fin with insulated end, but with the corrected length Lc:

Lc = (L + t/2) for a rect. fin and Lc = (L + r/2) for a pin fin."

Lc = L+t/2 "[m]… corrected length"

P = 2*(W+t)"[m]… perimeter"

A = W*t "[m^2]… cross-sectional area of fin"

m = sqrt((h*P)/(k*A)) "[1/m]… fin parameter"

Qperfin = k*A*m*(T_0-T_inf)*tanh(m*Lc) "[W]… heat transferred per fin"

Qfins = N_fins_calc*Qperfin "[W]… heat tr. for N fins"

"N_fins_calc is the exact value of calculated no. of fins, i.e. it may not be an exact integer"

A_unfin = (pi*D – N_fins_calc*t)*W "[m^2] … un-finned area or prime area on cyl surface"

Q_unfin = h*A_unfin*(T_0-T_inf)"[W] … heat tr. from un-finned area on cylinder surface"

Q_tot = Qfins+Q_unfin "[W] … total heat transferred"
N_actual = Ceil(N_fins_calc) " Actual no. of fins …. This is the no. of fins rounded off to higher integer"

"To get final values of heat transferred with corrected integer value of fins:"

Qfins_actual = N_actual*Qperfin
A_unfin_actual = (pi*D – N_actual*t)*W
Q_unfin_actual = h*A_unfin_actual*(T_0-T_inf)
Q_tot_actual = Qfins_actual+Q_unfin_actual

Results:

Unit Settings: SI C kPa J mass rad

$A = 0.0024$ [m^2]	$A_{unfin} = 0.1382$ [m^2]	$A_{unfin,actual} = 0.1379$ [m^2]
$D = 0.25$ [m]	$h = 40$ [W/m^2-C]	$k = 40$ [W/m-C]
$L = 0.03$ [m]	$Lc = 0.036$ [m]	$m = 13.29$ [1/m]
$N_{actual} = 8$	$N_{fins,calc} = 7.877$	$P = 0.424$ [m]
$Qfins = 67.09$ [W]	$Qfins_{actual} = 68.14$ [W]	$Qperfin = 8.518$ [W]
$Q_{tot} = 150$ [W]	$Q_{tot,actual} = 150.9$ [W]	$Q_{unfin} = 82.91$ [W]
$Q_{unfin,actual} = 82.73$ [W]	$t = 0.012$ [m]	$T_0 = 55$ [C]
$T_{inf} = 40$ [C]	$W = 0.2$ [m]	

Thus:

N_actual = 8 … actual (integer) no. of fins … Ans.,

whereas calculated value of fins = N_fins_calc = 7.877

Q_tot = 150 W .. by data, whereas, with actual no. of 8 fins Q_tot_actual = 150.9 W … Ans.

In addition:

1. **Total heat to be transferred remaining 150 W, find out the variation of N_actual and Q_tot_actual with h varying from h = 20 to 100 W/m^2.K:**

The parametric table is produced below:

Parametric Table — Table 1

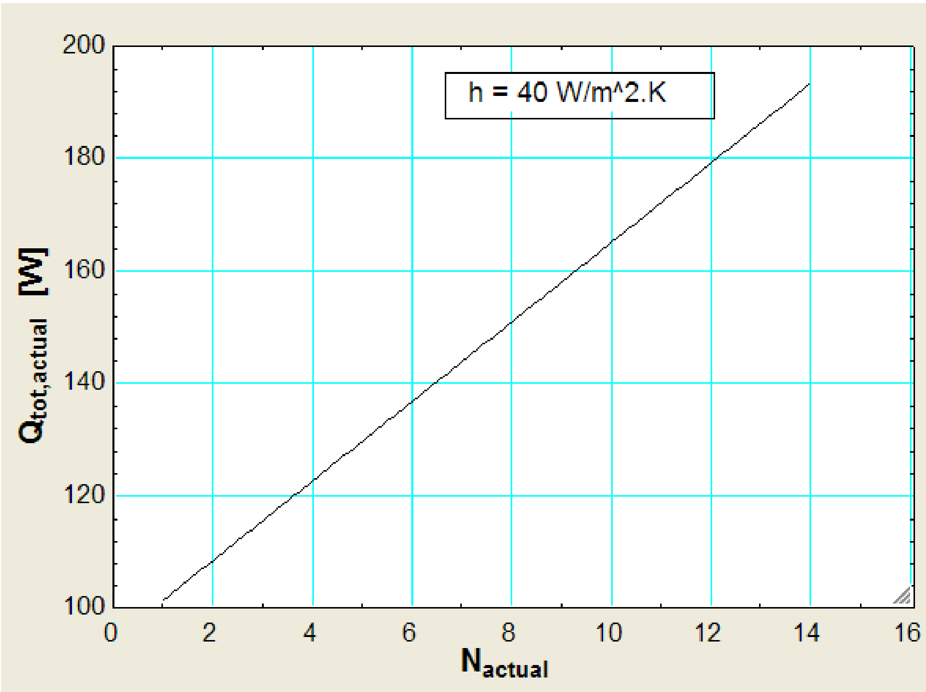
1..10	h [W/m2-C]	N_{actual}	$Q_{tot,actual}$ [W]
Run 1	20	28	150.5
Run 2	30	15	152
Run 3	40	8	150.9
Run 4	50	4	152.5
Run 5	60	1	151.6
Run 6	70	-1	153.3
Run 7	80	-2	162.3
Run 8	90	-4	154.3
Run 9	100	-5	156.9

It may be observed that at h = 20 W/m^2.K, no. of fins required is 28, and total Q = 150.5 W;

And, at h = 60 W/m^2.K, only one fin will be enough to give a total heat transfer of 151.6 W.

For values of h above 60 W/m^2.K, *no fins are necessary*.

2. **Plot the variation of Q_actual as actual no. of fins is varied from 1 to 14, with h remaining constant at 40 W/m^2.K:**



It may be noted that Q varies linearly with N_actual.

===

Prob. 1E.16. A short fin of 0.08 m length and 10 mm dia is exposed to air at 20 C. k is 15 W/m.K. The base temp is 250 C. Heat dissipated by the fin is 8 W. Determine the value of h and also the tip temp.

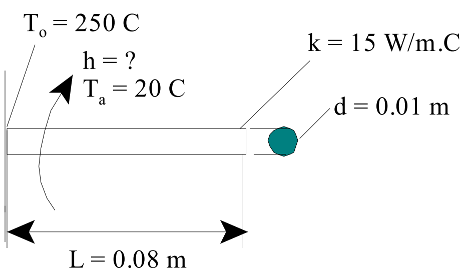


Fig.Prob.1E.16

Mathcad Solution:

Calculations:

$P := \pi \cdot d$ $P = 0.031$ m....perimeter

$A_c := \frac{\pi \cdot d^2}{4}$ $A_c = 7.854 \cdot 10^{-5}$ m^2... area of cross-section of fin

We need the fin parameter m; however, we do not know the value of h.

Since Qfin is given, we have, for a fin with insulated tip:

$Qfin = k \cdot A_c \cdot m \cdot \tanh(m \cdot L) \cdot (T_0 - T_a)$ ..where m is the fin parameter

and, $m = \sqrt{\frac{h \cdot P}{k \cdot A_c}}$

Therefore: $Qfin = k \cdot A_c \cdot \sqrt{\frac{h \cdot P}{k \cdot A_c}} \cdot \tanh\left(\sqrt{\frac{h \cdot P}{k \cdot A_c}} \cdot L\right) \cdot (T_0 - T_a)$

Let us write Qfin as a function of h:

$Q(h) := \sqrt{h \cdot P \cdot k \cdot A_c} \cdot \tanh\left(\sqrt{\frac{h \cdot P}{k \cdot A_c}} \cdot L\right) \cdot (T_0 - T_a)$ W....for fin with insulated end

Now, use the Solve Block of Mathcad to get h.

Start with a trial value of h:

$h := 10$ .....Trial value

Given

$Q(h) = 8$

$h := Find(h)$ ....finds h

i.e. $h = 33.78$ **W/m^2.K....Ans.**

To find the tip temp.:

We have, for temp. distribution:

$$\frac{T - T_a}{T_0 - T_a} = \frac{\cosh(m \cdot (L - x))}{\cosh(m \cdot L)}$$ ...T is the temp at any x

i.e. $$\frac{T_L - T_a}{T_0 - T_a} = \frac{1}{\cosh(m \cdot L)}$$ ..TL is the temp at x = L, i.e. at the tip

Now: $m := \sqrt{\frac{h \cdot P}{k \cdot A_c}}$

i.e. $m = 30.013$ 1/m...fin parameter

Therefore:

$$T_L := T_a + \frac{T_0 - T_a}{\cosh(m \cdot L)}$$

i.e $T_L = 61.346$ **C..Tip temp.....Ans.**

==

Prob. 1E.17. A square rod of side 10 mm and length 0.2 m has a heat generation rate, qg of 10^6 W/m^3. k of the material is 25 W/m.K and it is exposed to air at 25 C with h = 20 W/m^2.K. The ends are maintained at 60 C and 30 C. Determine the temp at the centre, location and value of max. temp and also the heat conducted at the ends and the heat convected.

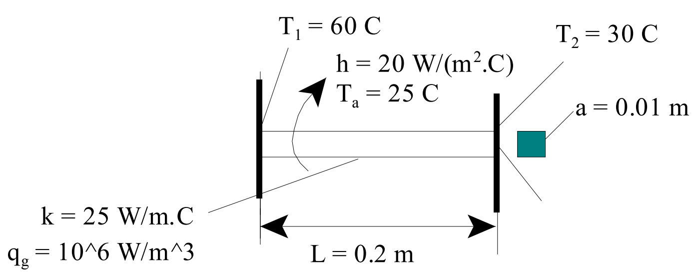


Fig.Prob.1E.17

Mathcad Solution:

Data:

$T1 := 60$ C $T2 := 30$ C $T_a := 25$ C $h := 20$ W/m^2.K

$a := 0.01$ m ... side of square rod $L := 0.2$ m $q_g := 10^6$ W/m^3

$k := 25$ W/m.K

Calculations:

$P := 4 \cdot a$ $P = 0.04$ m ... perimeter of fin

$A := a^2$ $A = 1 \cdot 10^{-4}$ m^2 ... area of cross-section of fin

$m := \sqrt{\frac{h \cdot P}{k \cdot A}}$ $m = 17.889$ 1/m ... fin parameter

Let:

$\theta_1 := T1 - T_a \qquad \theta_1 = 35$

$\theta_2 := T2 - T_a \qquad \theta_2 = 5$

$\theta'_1 := \theta_1 - \dfrac{q_g}{k \cdot m^2}$ and, $\theta'_1 = -90$

$\theta'_2 := \theta_2 - \dfrac{q_g}{k \cdot m^2}$ and, $\theta'_2 = -120$

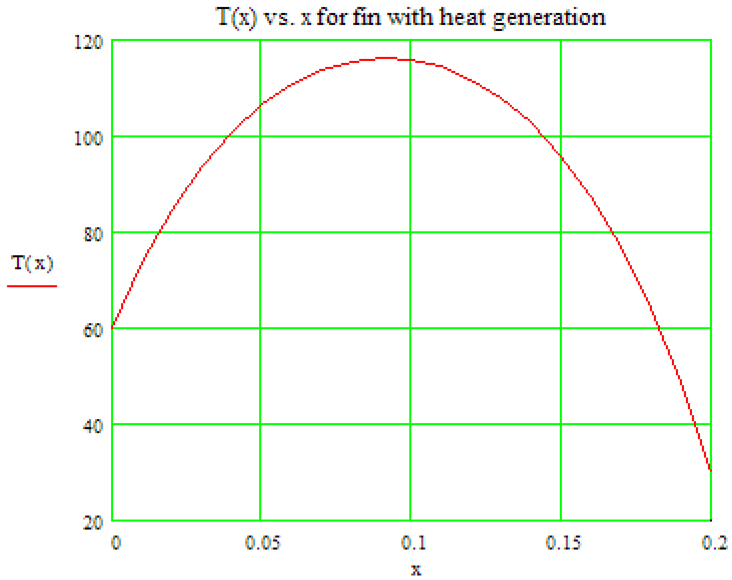
General solution for temp distribution is (Ref: [1]):

$$T(x) = T_a + \frac{q_g}{k \cdot m^2} + \frac{\theta'_1 \cdot \sinh(m \cdot (L - x)) + \theta'_2 \cdot \sinh(m \cdot x)}{\sinh(m \cdot L)}$$

Plot the temp distribution:

$x := 0, 0.01 .. 0.2$ ...define the range variable x from 0 to 0.2m

$$T(x) := \left[T_a + \frac{q_g}{k \cdot m^2} + \frac{\theta'_1 \cdot \sinh(m \cdot (L - x)) + \theta'_2 \cdot \sinh(m \cdot x)}{\sinh(m \cdot L)} \right]$$



T(x) in deg.C
x in metres

Temp at centre: $T(0.1) = 115.852$ **C .... Ans.**

What is the value of max. temp, and where does it occur?

Min. temp occurs where dT/dx = 0 We use the 'root function' of Mathcad to find where dT/dx = 0

Let: $T'(x) := \frac{d}{dx}T(x)$

Let Tmax occur at a distance xmax from LHS. Start with a trial value for xmax:

$x := 0.06$ ...trial value of x

$x_{max} := \text{root}(T'(x), x)$

$x_{max} = 0.091$ **m....i.e. Tmax occurs at x = 0.091 m from left... Ans.**

Therefore: $T_{max} := T(x_{max})$

i.e. $T_{max} = 116.244$ **C... value of max. temp.... Ans.**

Heat conducted at the ends:

We have:

$$T(x) := \left[T_a + \frac{q_g}{k \cdot m^2} + \frac{\theta'_1 \cdot \sinh(m \cdot (L - x)) + \theta'_2 \cdot \sinh(m \cdot x)}{\sinh(m \cdot L)}\right]$$

Let: $T'(x) := \frac{d}{dx}T(x)$

$Q_{left} := -k \cdot A \cdot T'(0)$ $Q_{left} = -3.731$ W.....heat flowing from left end to ambient

$Q_{right} := -k \cdot A \cdot T'(0.2)$ $Q_{right} = 5.15$ W.....heat flowing from right end to ambient

$|Q_{left}| + |Q_{right}| = 8.881$ **W....Total heat loss by conduction**

Heat loss by convection from surface:

$$Q_{conv} := h \cdot P \cdot \int_{0}^{0.2} \left(T(x) - T_a\right) dx$$

$Q_{conv} = 11.119$ **W.......heat loss by convection**

Check:

Total heat gen.: $Q_{gen} := q_g \cdot A \cdot L$

$Q_{gen} = 20$ W

Total heat loss by condn.+conv.:

$$Q_{tot} := \left|Q_{left}\right| + \left|Q_{right}\right| + Q_{conv}$$

i.e. $Q_{tot} = 20$ **W....checks.**

==

"**Prob.1E.18.** A hot surface at 100 C is to be cooled by attaching 3 cm long, 0.25 cm dia Aluminium fins (k = 237 W/m.K) to it, with a centre to centre distance of 0.6 cm. Temp. of surrounding air is 30 C and h = 35 W/m^2.K on the surface. Calculate the rate of heat transfer from the surface for a 1 m × 1 m section of the plate. Also, determine the overall effectiveness of the fins. [VTU – M.Tech. – Dec. 2010]"

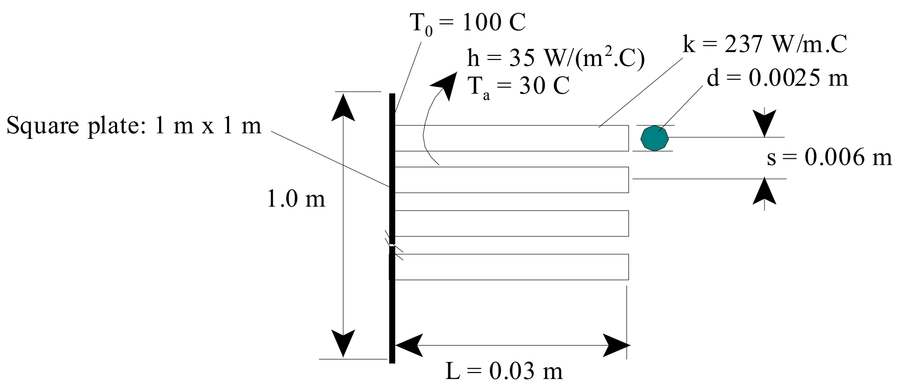


Fig.Prob.1E.18

EES Slution:

“Data:”

d = 0.0025 [m]
L = 0.03 [m]
T_0 = 100[C]
h = 35 [W/m^2-C]
T_a = 30[C]
k = 237 [W/m-C]
s = 0.006[m] “..centre to centre distance of fins”

“Calculations:”

“This is the case of fin array.

We will use the concept of fin efficiency.

Refer to Table 1E.2 for formulas for fin efficiencies.”

P = pi * d "[m]… perimeter"
A_c = pi * d^2/4 "[m^2]… cross-sectional area of fin"
m = sqrt((h*P)/(k*A_c)) "[1/m]… fin parameter"

eta_fin = tanh(m * L)/(m * L) "..fin effcy."
N_fins = 1/s^2 "…no. of fins in 1 m × 1 m area"

A_fins = ((pi * d * L) + (pi * d^2 / 4)) * N_fins "[m^2] … total surface area of fins"
A_unfin = 1 – N_fins * (pi * d^2 / 4) "[m^2] …. total un-finned area, or 'prime area'"

"Out of the fin area of A_fins, only the area (eta_fin * A_fins) is effective,

whereas all of A_unfin (or, prime area) is effective:

Therefore:"

Q_finned = h * (eta_fin * A_fins) * (T_0 – T_a) "[W] …. heat transfer from finned area"
Q_unfinned = h * A_unfin * (T_0 – T_a) "[W] …. heat transfer from un-finned area"
Q_total = Q_finned + Q_unfinned "[W] … total heat transfer fromthe array"

"When there are no fins:"

Q_nofin = h * 1 * (T_0 – T_a) "[W] …heat transfer when there are no fins; now area = 1 m^2"

"Therefore: Effectiveness of fin array:"

epsilon = Q_total / Q_nofin "…effectiveness"

Results:

Unit Settings: SI C kPa kJ mass deg

A_c = 0.000004909 [m^2]	A_{fins} = 6.681 [m^2]	A_{unfin} = 0.8636 [m^2]
d = 0.0025 [m]	ε = 7.108	η_{fin} = 0.9347
h = 35 [W/m^2-C]	k = 237 [W/m-C]	L = 0.03 [m]
m = 15.37 [1/m]	N_{fins} = 27778	P = 0.007854 [m]
Q_{finned} = 15300 [W]	Q_{nofin} = 2450 [W]	Q_{total} = 17416 [W]
$Q_{unfinned}$ = 2116 [W]	s = 0.006 [m]	T_0 = 100 [C]
T_a = 30 [C]		

Thus:

Q_total = 17416 W .. total heat transfer from the array … Ans.

ε = 7.108 …. Effectiveness of fin array …. Ans.

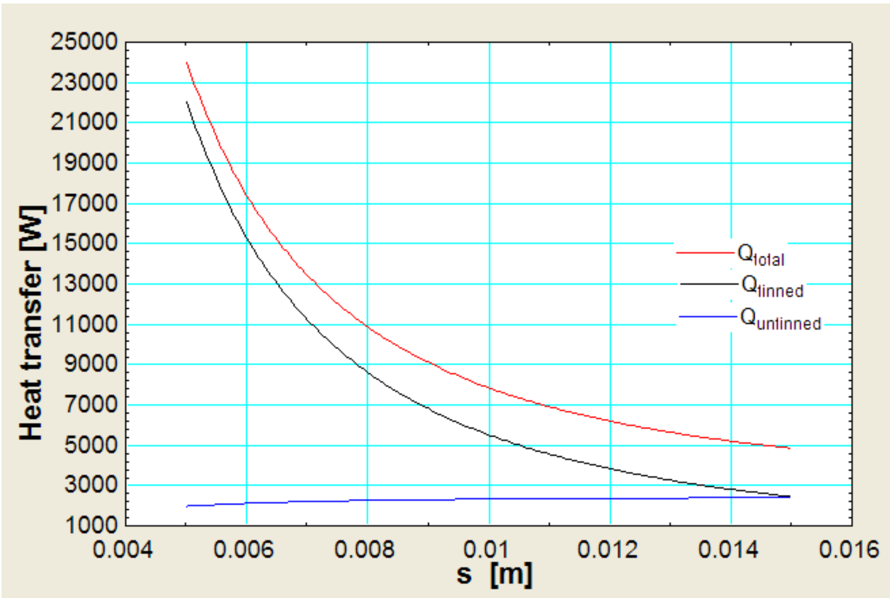
In addition:

Plot the variation of Q_finned, Q_unfinned and Q_total as centre to centre distance (s) is varied from 5 mm to 15 mm:

Parametric table:

1..11	s [m]	Q_{finned} [W]	$Q_{unfinned}$ [W]	Q_{total} [W]
Run 1	0.005	22032	1969	24001
Run 2	0.006	15300	2116	17416
Run 3	0.007	11241	2205	13445
Run 4	0.008	8606	2262	10868
Run 5	0.009	6800	2302	9101
Run 6	0.01	5508	2330	7838
Run 7	0.011	4552	2351	6903
Run 8	0.012	3825	2366	6191
Run 9	0.013	3259	2379	5638
Run 10	0.014	2810	2389	5199
Run 11	0.015	2448	2397	4845

Plot of Q_finned, Q_unfinned and Q_total against centre-to-centre distance, s:



Note that as the c/c distance increases, no. of fins decreases; so, fin area decreases, and the heat transfer from finned area decreases. But, un-finned area increases and heat transfer from un-finned area increases. Since the heat transfer from the fins decreases at a faster rate, net result is a decrease in total heat transfer from the array.

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Prob. 1E.19. A plane plate extended surface cooler for air consists of Al tubing of I.D. 17 mm and O.D. 20 mm with 1 mm thick plane Al fins fixed on the outside of tubes at a spacing of 10 mm. The pipes are located at a centre to centre dist of 60 mm. The inside and outside heat tr. coeff are 3600 and 32.5 W/m^2.K. The refrigerant and airside temp are -28 and 20 C respectively (avg. values). Find the outside tube surface area and tube length for a heat exchange of 2.72 kW. k of Al is 200 W/m.K. Assume fin efficiency for plane rect. fin as tanh(mL)/mL [M.U. – Dec. 1997]

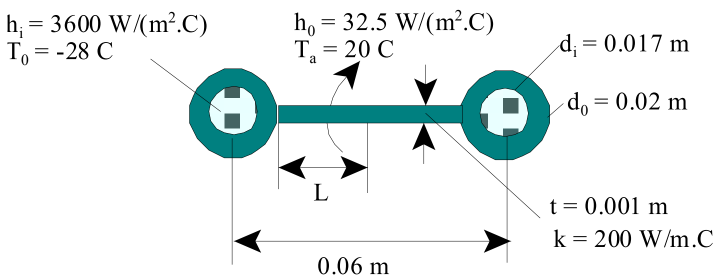


Fig.Prob.1E.19

Mathcad Solution:

Data:

$T_a := 20$ C $\quad T_o := -28$ C $\quad W := 1$ m ... width of fin = length of tube, assumed

$L := 0.02$ m ... length of fin. Since centre to centre distance between two tubes is 60 mm, fin length is half the space between them = (60 - 20) / 2 = 20 mm.

$d_i := 0.017$ m $\quad d_o := 0.02$ m $\quad t := 0.001$m

$h_i := 3600$ W/m^2.K $\quad h_o := 32.5$ W/m^2.K $\quad Qt := 2720$ W

$k := 200$ W/m.K $\quad s := 0.01$ m .. spacing between fins

$N := \frac{\pi \cdot d_o}{s}$ $\quad N = 6.283$ $\quad$ i.e. $N := 6$ ...no. of fins on the circumference

$P := 2 \cdot (W + t)$ $\quad P = 2.002$ m .... perimeter of fin

$A := W \cdot t$ $\quad A = 1 \cdot 10^{-3}$ m^2 ... area of cross-section of fin

$m := \sqrt{\frac{h_o \cdot P}{k \cdot A}}$ $\quad m = 18.037$ 1/m ...fin parameter

$\eta_f := \frac{\tanh(m \cdot L)}{m \cdot L}$ $\quad \eta_f = 0.959$ ...fin effcy.

$A_p := \pi \cdot d_o - N \cdot t$ $\quad A_p = 0.057$ ..m^2, prime surface area per metre length

$A_f := N \cdot W \cdot L \cdot 2$ $\quad A_f = 0.24$ ..m^2, fin surface area per metre length

$A_{to} := A_p + A_f$ $\quad A_{to} = 0.297$ ..m^2, total outer surface area per metre length

Note that of the total outer surface area, prime surface area is 100% effective, but not all of the fin area is effective. Effective fin area = fin effectiveness x total fin area = (η_f. A_f).

Therefore, effective outer surface area is:

$A_o := A_p + \eta_f \cdot A_f$

And,

$A_i := \pi \cdot d_i \cdot W$ $\quad$ i.e. $A_i = 0.053$ m^2...inside surface area of tube per metre length

To find overall heat transfer coeff.:

We have:

$$U_i \cdot A_i = U_o \cdot A_o = \frac{1}{R_{tot}} \quad \text{where:}$$

U_i = overall heat transfer coeff. based on inside area A_i,
U_o = overall heat transfer coeff. based on effective outer area A_o
R_{tot} = total thermal resistance
= convection resist on the inside, R_{conv1} + conduction resist of tube wall, R_w + convection resistance on outside, R_{conv2}

$$R_{conv1} := \frac{1}{h_i \cdot A_i} \qquad R_{conv1} = 5.201 \cdot 10^{-3}$$ C/W ... convection resist. on the inside

$$R_{conv2} := \frac{1}{h_o \cdot A_o} \qquad R_{conv2} = 0.107$$ C/W ... convection resist. on the outside

$$R_w := \frac{\ln\left(\frac{d_o}{d_i}\right)}{2 \cdot \pi \cdot k \cdot W} \qquad R_w = 1.293 \cdot 10^{-4}$$ C/WC/W ... conduction resist. of tube wall

$$R_{tot} := R_{conv1} + R_{conv2} + R_w \qquad R_{tot} = 0.113$$ C/W.... total resistance

Overall heat transfer coefficients:

$$U_o := \frac{1}{A_o \cdot R_{tot}} \qquad U_o = 30.961$$ W/m^2.K ..Overall U based on outside area

$$U_i := \frac{1}{A_i \cdot R_{tot}} \qquad U_i = 166.342$$ W/m^2.K ..Overall U based on inside area

Total outside area reqd: (A.tot)

$$A_{tot} := \frac{Qt}{U_o \cdot (T_a - T_o)}$$

i.e. $A_{tot} = 1.83$ **m^2...total outside area reqd....Ans.**

Length of tube reqd:

$$\text{Length} := \frac{A_{tot}}{A_o} \qquad \text{i.e. Length} = 6.379 \quad \textbf{m....Ans.}$$

==

Prob. 1E.20 : A cylinder 1 m long and 5 cm. dia is placed in an atmosphere at 45 C. It is provided with 10 longitudinal fins. k = 120 W/m.K. The height of 0.76 mm thk. fins is 1.27 cm from the cylinder surface. h between cylinder and atmosphere is 17 W/m^2.K. Calculate the rate of heat transfer and the temp. at the end of the fin, if the surface temp. of cylinder is 150 C. [VTU – VI Sem. B.E. – Aug. 2001]

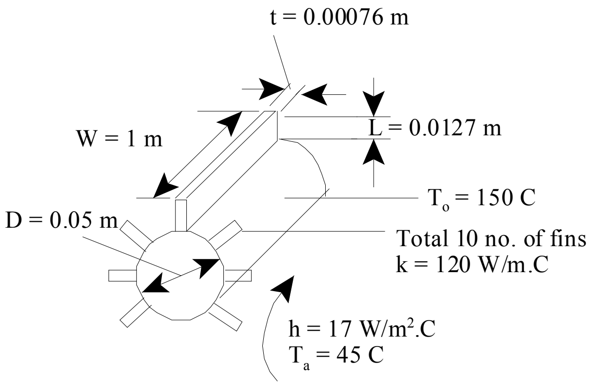


Fig.Prob.1E.20

Mathcad Solution:

Data:

Treat the fins as a case of fin with convection off its end.

$L := 0.0127$ m..length of fin $W := 1.0$ m; width of fin = length of cyl.

$D := 0.05$ m $t := 0.00076$ m

$T_0 := 150$ C $Ta := 45$ C $k := 120$ W/m.K

$h := 17$ W/m^2.K $N := 10$ no. of fins

Calculations:

$A := W \cdot t$ $A = 7.6 \cdot 10^{-4}$ m^2 ... area of cross-section of fin

$P := 2 \cdot W$ $P = 2$ m .. perimeter of fin

$m := \sqrt{\frac{h \cdot P}{k \cdot A}}$ $m = 19.308$ 1/m ... fin parameter

$\theta_0 := T_0 - Ta$ i.e. $\theta_0 = 105$ C

Heat transfer from a fin with convection off its tip:

$$Q_{perfin} := k \cdot A \cdot m \cdot \theta_0 \cdot \frac{\tanh(m \cdot L) + \frac{h}{m \cdot k}}{1 + \frac{h}{m \cdot k} \cdot \tanh(m \cdot L)}$$

i.e. $Q_{perfin} = 45.728$ ...Watts per fin

Then, for 10 fins, heat transfer:

$Q_{fins} := Q_{perfin} \cdot N$ i.e. $Q_{fins} = 457.275$ W .. heat transfer from N fins

For unfinned surface (base):

$A_b := (\pi \cdot D - N \cdot t) \cdot W$ i.e. $A_b = 0.149$ m^2 ... unfinned (prime or base) area

Therefore, total heat loss:

$$Q_{tot} := Q_{fins} + Q_{base}$$

i.e. $Q_{tot} = 724.096$ **Watts......Ans.**

Using simplified formula for fin with insulated end, with corrected length:

$Lc := L + \frac{t}{2}$ $\quad Lc = 0.013$ m .... corrected length

$Q_{onefin} := k \cdot A \cdot m \cdot \theta_0 \cdot \tanh(m \cdot Lc)$ $\quad Q_{onefin} = 45.728$ **W .. same as obtained earlier.**

To plot the variation of Qtot with h:

Let h vary from 10 to 200 W/m^2.K:

Write the involved parameters as functions of h:

$$m(h) := \sqrt{\frac{h \cdot P}{k \cdot A}}$$

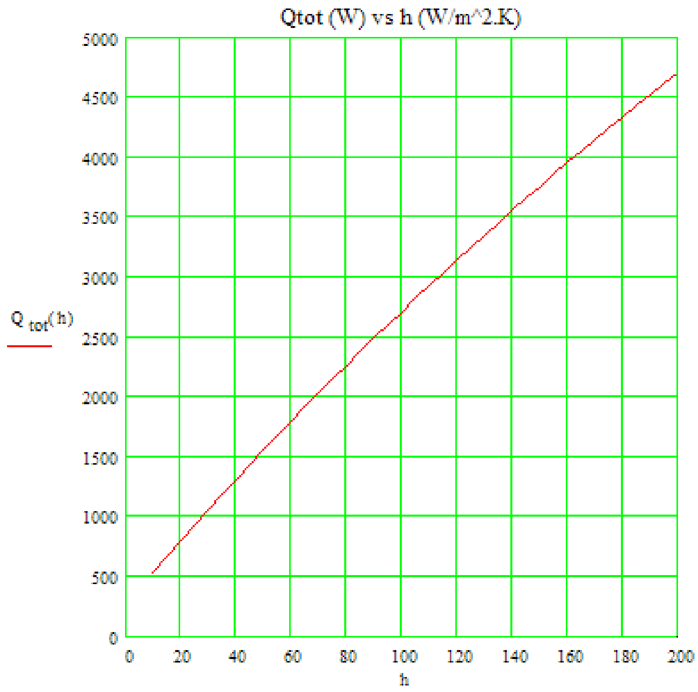
$$Q_{perfin}(h) := k \cdot A \cdot m(h) \cdot \theta_0 \cdot \frac{\tanh(m(h) \cdot L) + \frac{h}{m(h) \cdot k}}{1 + \frac{h}{m(h) \cdot k} \cdot \tanh(m(h) \cdot L)}$$

$$Q_{fins}(h) := Q_{perfin}(h) \cdot N$$

$$Q_{tot}(h) := Q_{fins}(h) + Q_{base}$$

Plot Qtot vs h:

$h := 10, 15 .. 200$ ...define a range variable h, from 10 to 200 W/m^2.K


Qtot (W) vs h (W/m^2.K)
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
Q tot(h)
0
20
40
60
80
100
120
140
160
180
200
h

To plot Qtot vs N for vatious values of h:

$Q_{fins}(h,N) := Q_{perfin}(h) \cdot N$ ...Qfins as a function of h and N

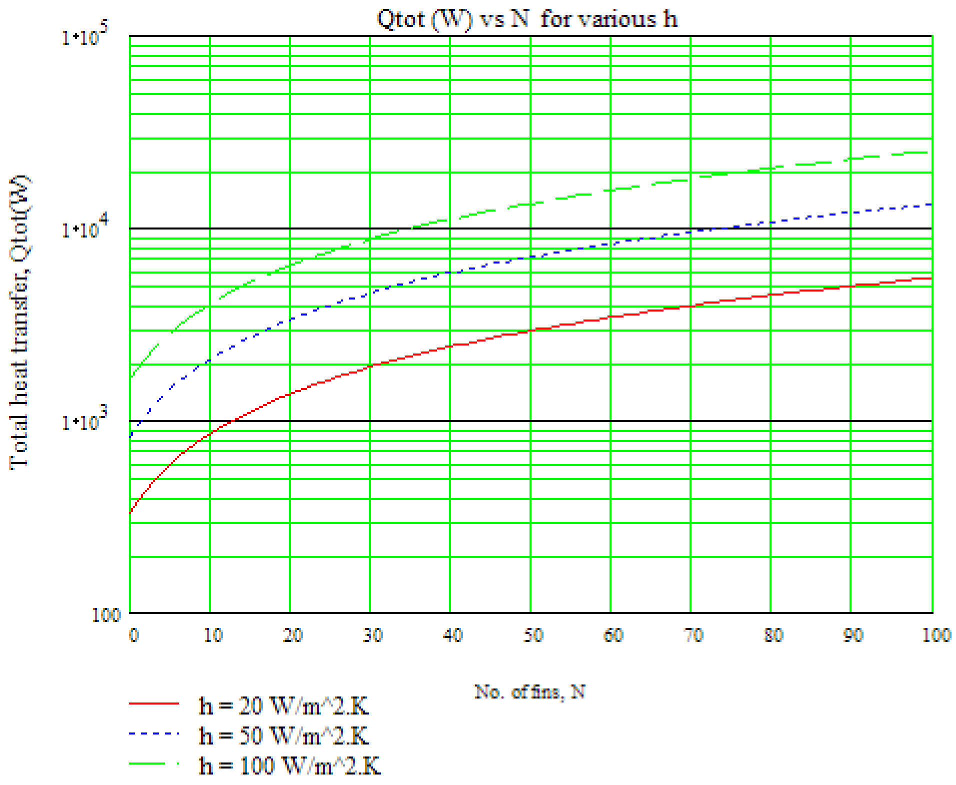
$A_b(N) := (\pi \cdot D - N \cdot t) \cdot W$ ... unfinned (prime or base) area as a function of N

$Q_{base}(h,N) := h \cdot A_b(N) \cdot \theta_0$ ...Qbase as a function of h and N

$Q_{tot}(h,N) := Q_{fins}(h,N) + Q_{base}(h,N)$ ...Qtot as a function of h and N

Let no. of fins vary from N = 0 to 100:

$N := 0, 1 .. 100$ ...define a range variable N, from 0 to 100



Note:

As number of fins increases, Qtot increases for a given h.

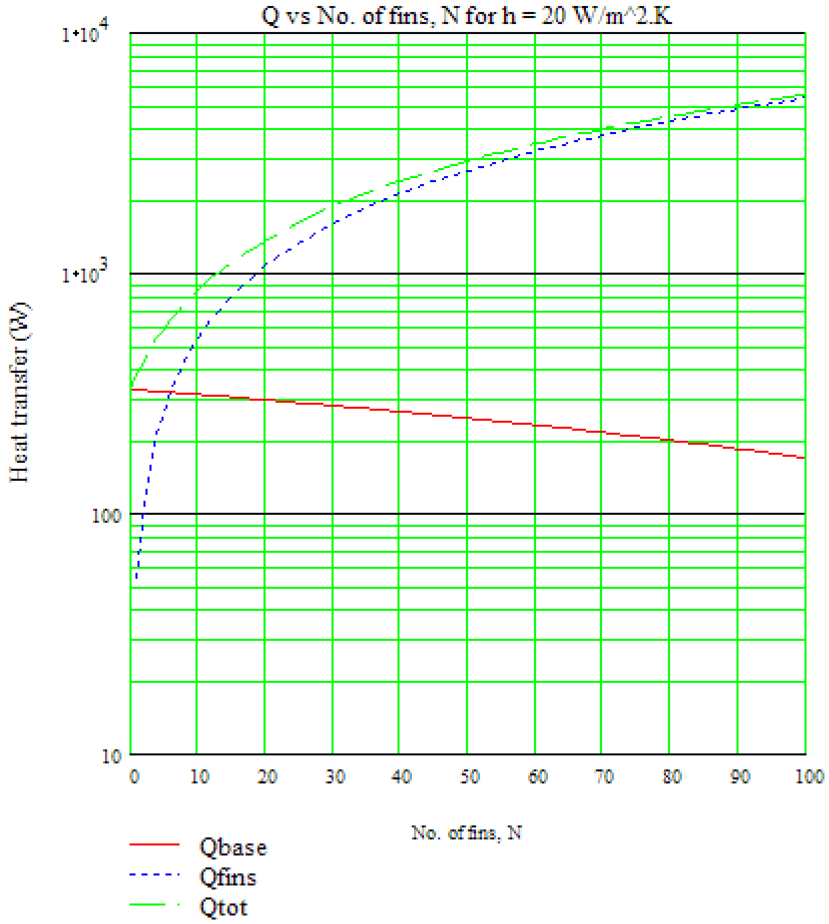
As h increases, Qtot increases for a given N.

N = 0 refers to the case of no fins, i.e. heat transfer is from the bare cylinder surface only. Then, for example, Qtot = 329.867 W for h = 20 W/m^2.K

Plot of Qbase, Qfins and Qtot against the no. of fins for a given h = 20 W/m^2.K:

h := 20 W/m^2.K

N := 0, 1.. 100 ...define the range variable N, varying from 0 to 100



Note the variation of Qbase, Qfins and Qtot with N at various values of h.

N = 0 refers to the case where no fins are used, i.e. heat transfer is from the bare cylindrical surface only.

==

"**Prob.1E.21**. A steel tube carries steam at a temp of 300 C. A thermometer pocket of iron (k = 52.3 W/m.K) of inside dia of 16 mm and thickness 1 mm is used to measure the temp. The error to be tolerated is 2% of maximum. Calculate the length of pocket required to measure temp within this error. How should the thermometer be located? Take the tube wall temp as 130 C and diameter as 90 mm. Assume the convective heat transfer coeff as 95 W/m^2.K. [VTU –VII Sem. B.E. – Dec. 2006–Jan. 2007]"

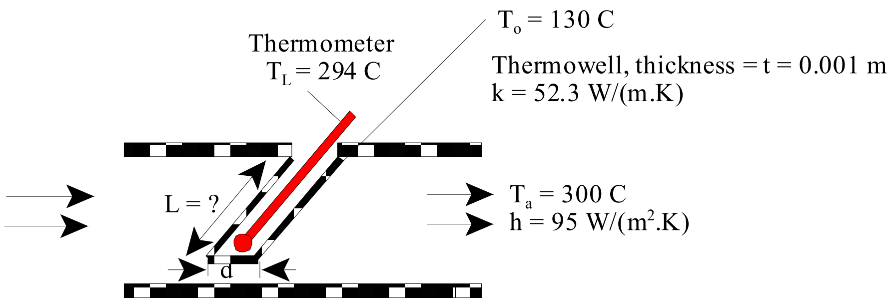


Fig.Prob.1E.21

EES Solution:

"Data:"

d = 0.0016 [m]

t = 0.001 [m]

T_0 = 130[C]

h = 95 [W/m^2-C]

T_a = 300[C]

k = 52.3 [W/m-C]

"Error to be tolerated = (T_a – T_L) = 0.02 * T_a …i.e. 2% of max.. Here, T_L is the thermometer temp."

"Calculations:"

Error = (T_a – T_L) "[C] ..error = (Actual temp. – measured temp.)"
Error = 0.02 * T_a "…error is 2% of max. … by data"

"Treating the pocket as a fin insulated at its end:"

d_o = d_i + 2 * t "[m] … OD of thermo-well"
P = pi * d_o "[m]… perimeter"
A_c = pi * d_o * t "[m^2]… cross-sectional area of fin"
m = sqrt((h * P) / (k * A_c)) "[1/m]… fin parameter"
(T_L – T_a) / (T_0 – T_a) = 1 / cosh(m * L) "…finds L "

Results:

Unit Settings: SI C kPa kJ mass deg

$A_c = 0.00005655$ [m^2]	$d_i = 0.016$ [m]	$d_o = 0.018$ [m]	Error = 6 [C]
h = 95 [W/m^2-C]	k = 52.3 [W/m-C]	**L = 0.09472 [m]**	**m = 42.62 [1/m]**
P = 0.05655 [m]	t = 0.001 [m]	$T_0 = 130$ [C]	$T_a = 300$ [C]
$T_L = 294$ [C]			

Thus:

L = 0.09472 m = 94.72 mm …. Length of pocket required. … Ans.

Note that dia of the steel tube in which the thermometer pocket is placed is 90 mm.
Therefore the pocket has be located obliquely in the tube. … Ans.

==

Prob. 1E.22. Annular Al fins (k = 240 W/m.K) of rectangular profile are attached to a circular tube having OD = 50 mm and an outer surface temp of 200 C. The fins are 4 mm thick and 15 mm long. The system is in ambient air at a temp of 20 C and h = 40 W/m^2.K.

a) What are the fin efficiency and effectiveness?
b) If there are 125 such fins per meter of tube length, what is the rate of heat transfer per unit length of tube? [Ref.3]

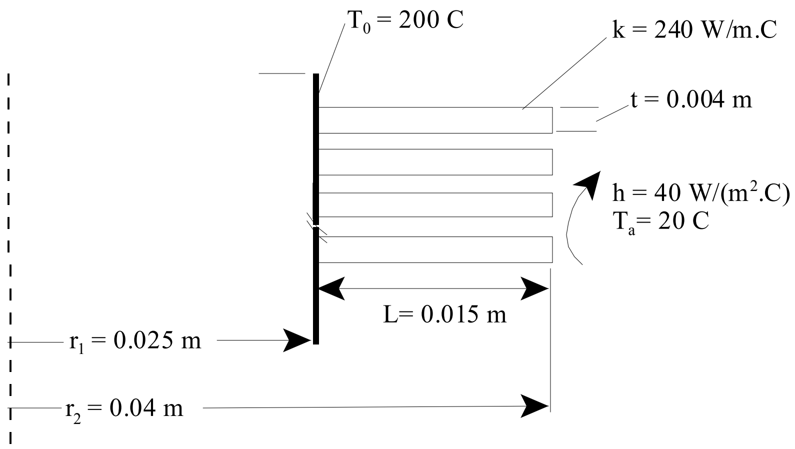


Fig.Prob.1E.22

Mathcad Solution:

Data:

$r_1 := 0.025$ m ... inner radius of circular fin = outer radius of tube

$r_2 := r_1 + 0.015$ m ...outer rad. of fin i.e. $r_2 = 0.04$ m

$t := 0.004$ m ... thickness of fin $k := 240$ W/m.K

$T_0 := 200$ C $T_a := 20$ C $h := 40$ W/m^2.K

$N_{fins} := 125$ ...no. of fins per metre length

Calculations:

$r_{2c} := r_2 + \frac{t}{2}$ i.e. $r_{2c} = 0.042$ m

$m := \sqrt{\frac{2 \cdot h}{k \cdot t}}$ i.e. $m = 9.129$ 1/m .... fin parameter

Fin efficiency: See Table 1E.2 for formulas.

$$C2 := \frac{\frac{2 \cdot r_1}{m}}{r_{2c}^2 - r_1^2}$$

$$\eta_f := C2 \cdot \frac{K1(m \cdot r_1) \cdot I1(m \cdot r_{2c}) - I1(m \cdot r_1) \cdot K1(m \cdot r_{2c})}{I0(m \cdot r_1) \cdot K1(m \cdot r_{2c}) + K0(m \cdot r_1) \cdot I1(m \cdot r_{2c})}$$

i.e. $\eta_f = 0.99$ ...**Fin efficiency ... Ans.**

Heat transfer from a single fin: [Ref. 3]

$$Q_f := 2 \cdot \pi \cdot k \cdot r_1 \cdot t \cdot (T_0 - T_a) \cdot m \cdot \frac{K1(m \cdot r_1) \cdot I1(m \cdot r_2) - I1(m \cdot r_1) \cdot K1(m \cdot r_2)}{I0(m \cdot r_1) \cdot K1(m \cdot r_2) + K0(m \cdot r_1) \cdot I1(m \cdot r_2)}$$

i.e. $Q_f = 43.761$ **W ... heat transfer from a single fin .... Ans.**

Now, heat transfer from the base area, if there were no fin:

$$Q_{nofin} := h \cdot (2 \cdot \pi \cdot r_1 \cdot t) \cdot (T_0 - T_a)$$

i.e. $Q_{nofin} = 4.524$ W

Therefore, fin effectiveness:

$$\varepsilon_f := \frac{Q_f}{Q_{nofin}}$$

i.e. $\varepsilon_f = 9.673$ ...**Fin effectiveness ... Ans.**

Temp profile in the fin:

$\theta = T\_T_a$ ...excess temp at radius r

$\theta_0 := T_0 - T_a$ ..excess temp at the base of fin, at r = r1

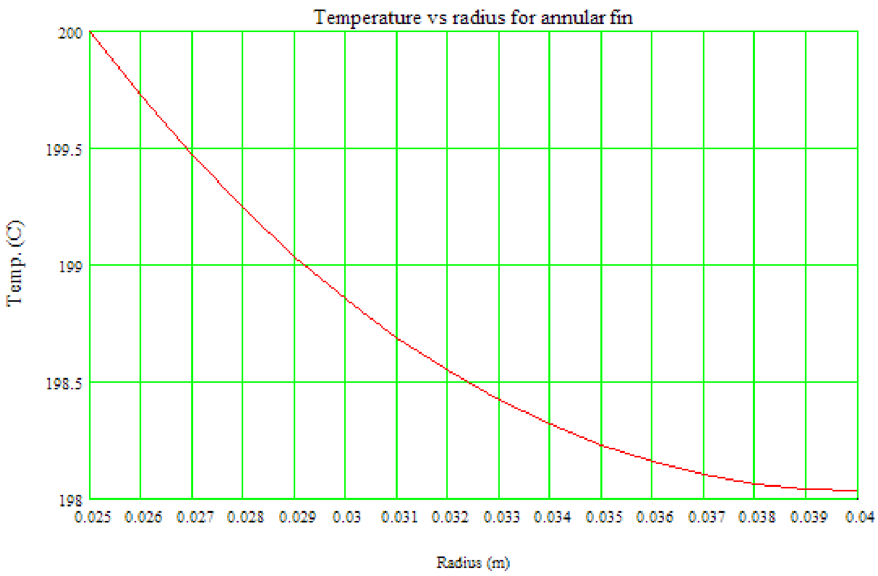
Then, we have, for temp distributon, from Ref.[3]:

$$\theta(r) = \theta_0 \cdot \frac{I0(m \cdot r) \cdot K1(m \cdot r_2) + K0(m \cdot r) \cdot I1(m \cdot r_2)}{I0(m \cdot r_1) \cdot K1(m \cdot r_2) + K0(m \cdot r_1) \cdot I1(m \cdot r_2)}$$

Draw the temp profile from r = r1 to r = r2:

$r := 0.025, 0.026 .. 0.04$ ....define the range variable r

$$\theta(r) := \theta_0 \cdot \frac{I0(m \cdot r) \cdot K1(m \cdot r_2) + K0(m \cdot r) \cdot I1(m \cdot r_2)}{I0(m \cdot r_1) \cdot K1(m \cdot r_2) + K0(m \cdot r_1) \cdot I1(m \cdot r_2)}$$



Total heat transfer from 1 m length of tube:

Consider surface area of (both surfaces) 125 fins and the un-finned area:

$$A_{fins} := N_{fins} \cdot \left[2 \cdot \pi \cdot \left(r_2^{\,2} - r_1^{\,2}\right)\right]$$

i.e. $A_{fins} = 0.766$ m^2 .... surface area of Nfins

$$A_{unfin} := \left(2 \cdot \pi \cdot r_1\right) \cdot \left(1 - N_{fins} \cdot t\right)$$

i.e. $A_{unfin} = 0.079$ m^2 .... unfinned surface area of cylinder

Now, note that of all the fin area, the effective area for heat transfer is: (Afins. hf),

whereas, all of the un-finned area is effective.

Therefore:

$$Q_{fins} := h \cdot \eta_f \cdot A_{fins} \cdot \left(T_0 - T_a\right)$$

i.e. $Q_{fins} = 5.457 \cdot 10^3$ **W .... heat transfer from fins**

$$Q_{unfin} := h \cdot A_{unfin} \cdot \left(T_0 - T_a\right)$$

i.e. $Q_{unfin} = 565.487$ **W ... heat transfer from unfinned area**

$$Q_{total} := Q_{fins} + Q_{unfin}$$

i.e. $Q_{total} = 6.022 \cdot 10^3$ **W ... total heat transfer from 1 metre length of tube...Ans.**

Draw the variation of Q_{total} with h and $N_{fins:}$

Express the relevant quantities as functions of h and N_{fins}:

$$A_{fins}(N_{fins}) := N_{fins}\left[2\cdot\pi\cdot\left(r_2^{\,2} - r_1^{\,2}\right)\right]$$

$$A_{unfin}(N_{fins}) := (2\cdot\pi\cdot r_1)\cdot(1 - N_{fins}\cdot t)$$

$$Q_{fins}(h, N_{fins}) := h\cdot\eta_f\cdot A_{fins}(N_{fins})\cdot(T_0 - T_a)$$

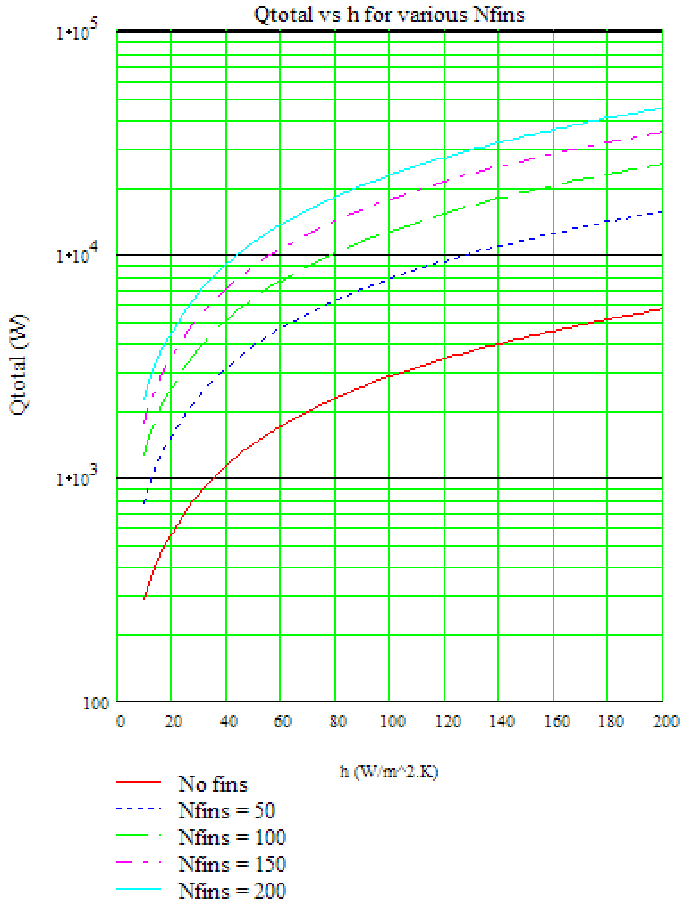
$$Q_{unfin}(h, N_{fins}) := h\cdot A_{unfin}(N_{fins})\cdot(T_0 - T_a)$$

$$Q_{total}(h, N_{fins}) := Q_{fins}(h, N_{fins}) + Q_{unfin}(h, N_{fins})$$

Now, draw the graphs:

Let h vary from 20 to 200 W/m^2.K, and Nfins vary from 0 (i.e. no fins) to 200 per metre length of tube:

$h := 10, 12 .. 200$ ...define a range variable h, from 10 to 200 W/m^2.K



Note that at a given h, Q_{total} increases as the no. of fins is increased.

Also, at a given N_{fins}, Q_{total} increases as h is increased.

===

"Prob.1E.23. Aluminium fins of triangular profile are attached to a plane wall whose surface temp is 250 C. The fin base thickness is 2 mm and its length is 6 mm. The system is in ambient air at a temp of 20 C and h = 40 W/m^2.K. (a) What are the fin efficiency and effectiveness? (b) What is the heat dissipated per unit width by a single fin? [Ref. 3]"

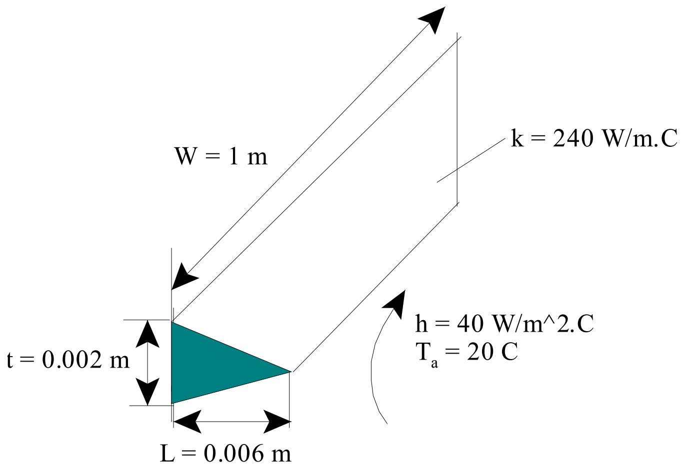


Fig.Prob.1E.23

EES Solution:

"Data:"

t = 0.002 [m]
L = 0.006 [m]
W = 1 [m] "…width of fin, assumed as 1 m"
T_0 = 250[C]
h = 40 [W/m^2-C]
T_a = 20[C]
k = 240 [W/m-C]

"Calculations:"

m = sqrt((2 * h) / (k * t)) "[1/m]… fin parameter"

A_f = 2 * W * (L^2 + (t/2)^2)^(1/2)

"Fin efficiency:"

eta_f = (1/(m * L)) * (BesselI(1, 2* m * L) / BesselI(0, 2 * m* L))

"Fin effectiveness:"

Q_fin = eta_f * A_f *h * (T_0 – T_a) "[W] … heat transfer from fin"

Q_base = h * (W * t) * (T_0 – T_a) "[W] … heat transfer without fin, from base area"

epsilon_ f= Q_fin / Q_base " … effectiveness"

“Temp. profile:

Note: In the following eqn for temp. profile, x is measured from the end of fin. i.e. x = 0 is the tip of fin and x = L is the base”

B = sqrt((2 * h * L) / (k * t))

x = 0 [m] “ …tip of fin”

(Tx – T_a) / (T_0 – T_a) = BesselI(0, 2* B * x^0.5) / BesselI(0, 2 * B* L^0.5) “…gives temp Tx at any x”

Results:

Unit Settings: SI C kPa kJ mass deg

A_f = 0.01217 [m^2]	B = 1	ε_f = 6.065 [-]	η_f = 0.997 [-]
h = 40 [W/m^2-C]	k = 240 [W/m-C]	L = 0.006 [m]	m = 12.91 [1/m]
Q_{base} = 18.4 [W]	Q_{fin} = 111.6 [W]	t = 0.002 [m]	Tx = 248.6 [C]
T_0 = 250 [C]	T_a = 20 [C]	W = 1 [m]	x = 0 [m]

Thus:

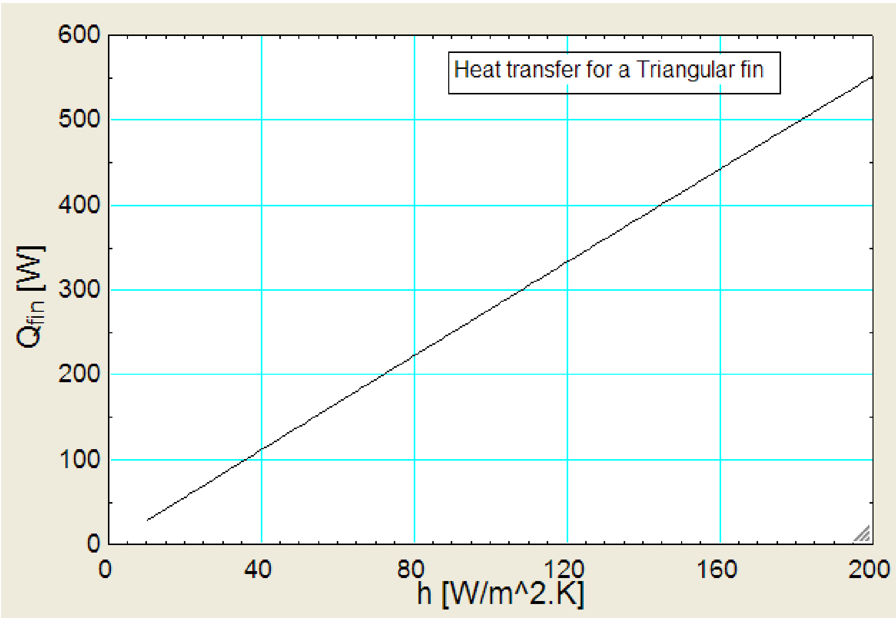
eta_f = 0.997 …. fin efficiency … Ans.

epsilon_f = 6.065 …. fin effectiveness …. Ans.

Qfin = 111.6 W …. Heat transfer from the fin … Ans.

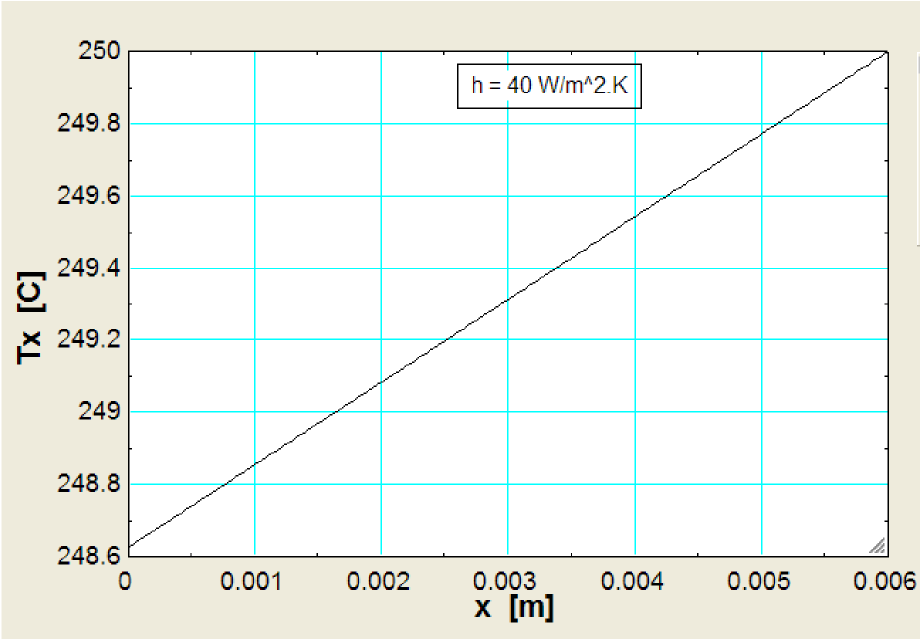
Tx = 248.6 C … temp at the tip of fin, i.e. at x = 0 … Ans.

Draw the plot of Q_fin vs h, wth h varying from 10 to 200 W/m^2.K:



Now, draw the temp. profile in the fin:

(Remember x = 0 is the tip and x = L is the base of fin)



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"Prob.1E.24. In order to decrease the thermal resistance from a surface of a vertical plane wall of size 500 mm × 500 mm, one hundred pin fins of 10 mm dia and 100 mm long are attached. If the fins are made of material of k = 350 W/m.K and the h = 17.5 W/m^2.K, calculate the decrease in thermal resistance. Also calculate the consequent increase in the heat transfer rate from the wall if it is maintained at 200 C and the surroundings are at 30 C. [VTU – VI Sem. B.E. – Feb. 2002]"

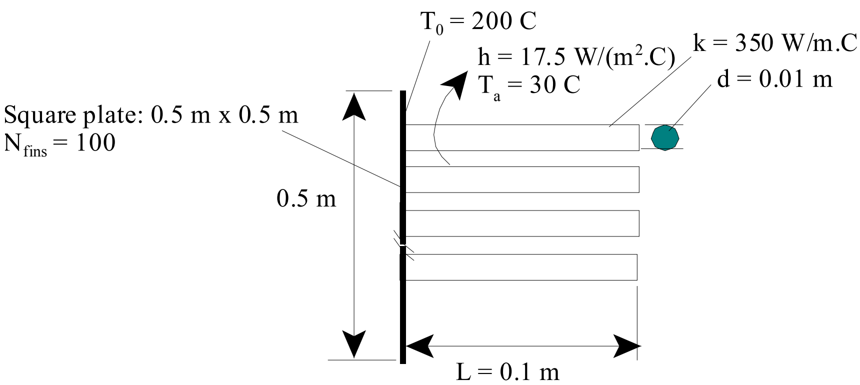


Fig.Prob.1E.24

EES Solution:

"Data:"

d = 0.010 [m]
L = 0.100 [m]
T_0 = 200[C]
h = 17.5 [W/m^2-C]
T_a = 30[C]
k = 350 [W/m-C]
N_fins = 100

"100 fins are fixed on a plate of size: 0.5 m × 0.5 m; i.e."

A_plate = 0.25 [m^2]

"Calculations:"

"This is the case of fin array.
We will use the concept of fin efficiency.

Refer to Table 1E.2 for formulas for fin efficiencies."

P = pi * d "[m]… perimeter"
A_c = pi * d^2/4 "[m^2]… cross-sectional area of fin"
m = sqrt((h * P) / (k * A_c)) "[1/m]… fin parameter"

eta_fin = tanh(m * L)/(m * L) "..fin effcy."

A_fins = ((pi * d * L) + (pi * d^2 / 4)) * N_fins "[m^2] … total fins area"
A_unfin = A_plate – N_fins * (pi * d^2 / 4) "[m^2] …. total un-finned area"

"Out of the fin area of A_fins, only the area (eta_fin * A_fins) is effective,
whereas all of A_unfin (or, prime area) is effective:

Therefore:"

Q_finned = h * (eta_fin * A_fins) * (T_0 – T_a) "[W] …. heat transfer from finned area"
Q_unfinned = h * A_unfin * (T_0 – T_a) "[W] …. heat transfer from un-finned area"
Q_total = Q_finned + Q_unfinned "[W] … total heat transfer fromthe array"

"When there are no fins:"

Q_nofin = h * A_plate * (T_0 – T_a) "[W] …heat transfer when there are no fins; now area = 0.25 m^2"

Q_increase = (Q_total – Q_nofin) "[C/W] … increase in heat transfer rate due to fixing the fins"

"Therefore: Effectiveness of fin array:"

epsilon = Q_total / Q_nofin "…effectiveness"

"Thermal resistances:"

R_no_fins = (T_0 – T_a) / Q_nofin "[C/W] … thermal resistance when there are no fins"

R_with_fins = (T_0 – T_a) / Q_total "[C/W] … thermal resistance when there are fins"

R_decrease = (R_no_fins – R_with_fins) "[C/W] … decrease in thermal resist. due to fixing the fins"

Results:

Unit Settings: SI C kPa kJ mass deg

A_c = 0.00007854 [m^2]	A_{fins} = 0.322 [m^2]	A_{plate} = 0.25 [m^2]
A_{unfin} = 0.2421 [m^2]	d = 0.01 [m]	ε = 2.177
η_{fin} = 0.9383	h = 17.5 [W/m^2-C]	k = 350 [W/m-C]
L = 0.1 [m]	m = 4.472 [1/m]	N_{fins} = 100
P = 0.03142 [m]	Q_{finned} = 898.9 [W]	$Q_{increase}$ = 875.5 [W]
Q_{nofin} = 743.8 [W]	Q_{total} = 1619 [W]	$Q_{unfinned}$ = 720.4 [W]
$R_{decrease}$ = 0.1236 [C/W]	$R_{no,fins}$ = 0.2286 [C/W]	$R_{with,fins}$ = 0.105 [C/W]
T_0 = 200 [C]	T_a = 30 [C]	

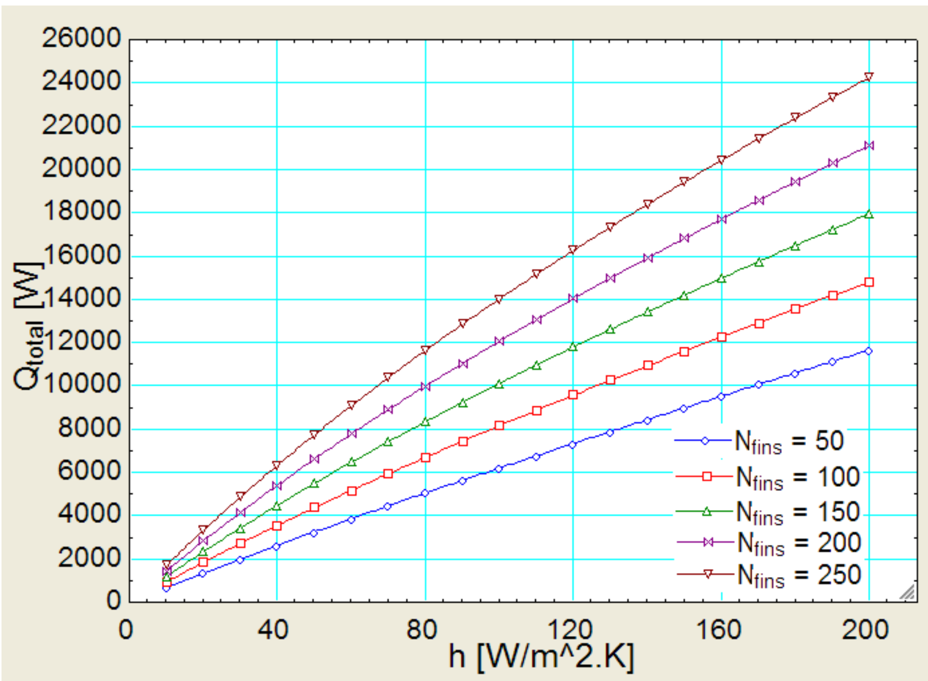
Thus:

R_decrease = 0.1236 C/W …. decrease in thermal resist. Due to putting fins …. Ans.

Q_increase = 875.5 W ….increase in heat transfer due to putting fins …. Ans.

Additionally:

Plot the variation of Q_total as h varies from 10 to 200 W/m^2.K and N_fins varies from 50 to 250:



Note that Q_total increases with h and also with No. of fins.

===

"Prob. 1E.25. One end of a copper rod (k = 380 W/m.C), 300 mm long is connected to a wall maintained at 300 C. Its other end is connected to a wall maintained at 100 C. Air is blown across the rod with h = 20 W/m^2.C. Diameter of the rod is 15 mm and the temp of air is 40 C. Determine: (i) the net heat transferred to air (ii) the heat conducted to the end at 100 C. [M.U.]"

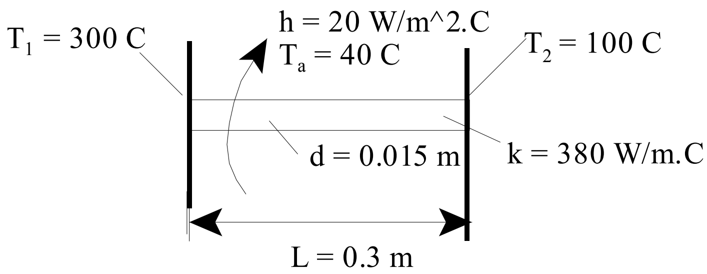


Fig.Prob.1E.25

EES Solution:

"Data:"

d = 0.015 [m]
L = 0.3 [m]
T_1 = 300 [C]
T_2 = 100 [C]
k = 380 [W/m-C]
h = 20 [W/m^2-C]
T_a = 40 [C]

"Calculations:"

"This is a fin with specified temps at the two ends.

Let us solve this problem from fundamentals:

General solution for temp distribution in a fin is:

theta(x) = A.cosh(m.x) + B. sinh (m.x) … eqn (1)

where A and B are constants, determined from Boundary conditions.

theta(x) = (Tx – Ta) and 'm' is the fin parameter."

A_c = pi * d^2 / 4 "[m^2] … area of cross-section of fin"
P = pi * d "[m] … perimeter"
m = sqrt ((h * P) / (k * A_c)) "[1/m] … fin parameter"

theta_1 = T_1 – T_a
theta_2 = T_2 – T_a

"BC's:

1. at x = 0, T = T_1

2. at x = L, T = T_2

Applying these BC's to eqn. (1), we get:"

theta_x = A * cosh(m * x) + B * sinh (m * x) "….eqn. (1)… general solution for theta at x."

theta_1 = A * cosh (0) + B * sinh (0) "…from BC – (1)"

theta_2 = A * cosh (m * L) + B * sinh (m * L) "…from BC – (2)"

x = 0.15 [m] "…mid-point …to draw temp. profile … will be commented out later"

theta_x = T_x – T_a "..finds Tx, temp at x"

"Heat transferred at the left and right ends:

Use Fourier eqn. Q = – k * A_c * dT/ dx

d(theta) / dx = dT / dx"

"At x = 0:"

dT\dx_0 = A * m * sinh(m * 0) + B * m * cosh (m * 0) "….eqn (a)"

"At x = L = 0.3:"

dT\dx_L = A * m * sinh(m * 0.3) + B * m * cosh (m * 0.3) "….eqn (b)"

"eqns. (a) and (b) determine values of constants A and B"

"Therefore:"

Q_left = – k * A_c * dT\dx_0 "[W] … heat transferred at left end"

Q_right = – k * A_c * dT\dx_L "[W] … heat transferred at left end"

"Therefore:"

Q_air = Abs(Q_left – Q_right) "[W]…. heat transferred to air"

Results:

Unit Settings: SI C kPa kJ mass deg

$A = 260$	$A_c = 0.0001767$ [m²]	$B = -277.8$
$d = 0.015$ [m]	$dT\backslash dx_0 = -1041$ [C/m]	$dT\backslash dx_L = -430.1$ [C/m]
$h = 20$ [W/m²-C]	$k = 380$ [W/m-C]	$L = 0.3$ [m]
$m = 3.746$ [1/m]	$P = 0.04712$ [m]	$Q_{air} = 41.01$ [W]
$Q_{left} = 69.89$ [W]	$Q_{right} = 28.88$ [W]	$\theta_1 = 260$ [C]
$\theta_2 = 60$ [C]	$\theta_x = 137.7$ [C]	$T_1 = 300$ [C]
$T_2 = 100$ [C]	$T_a = 40$ [C]	$T_x = 177.7$ [C]
$x = 0.15$ [m]		

Thus:

Q_left = 69.89 W … heat transferred to rod at the left end (from left to right, since +ve)

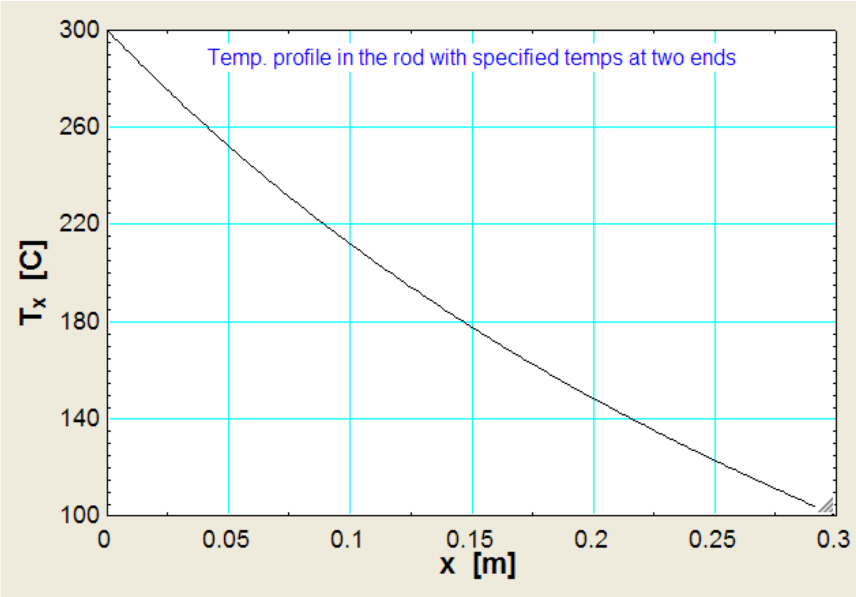
Q_right = 28.88 W … heat transferred from rod at the right end

(from left to right since +ve) … Ans.

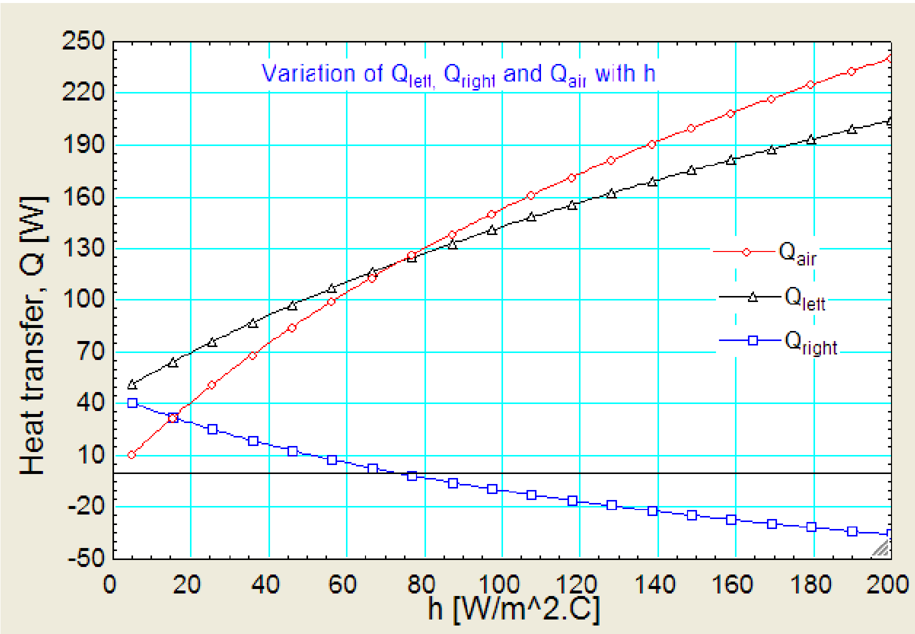
Q_qir = Abs(Q_left – Q_right) = 41.01 W …. Net heat transferred to air from rod (by convection) … Ans.

Tx = 177.7 C at x = 0.15 m, i.e. at mid-point of rod … Ans.

Plot the temp. profile in the rod:

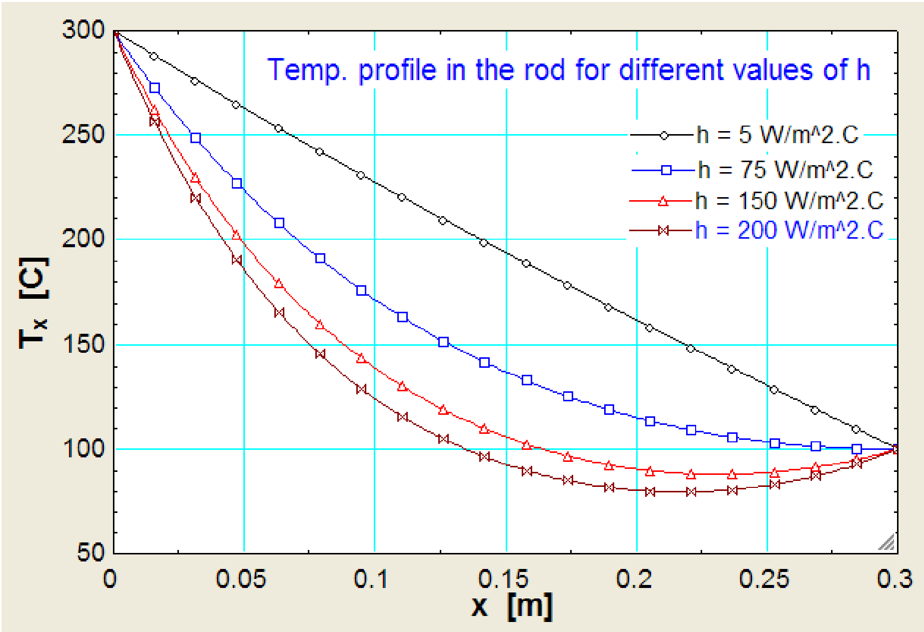


Further, plot the variation of Q_left, Q_right and Q_air as h varies from 5 to 200 W/m^2.C:



It is observed that Q_right becomes –ve, i.e. heat flow from the right end at 100 C becomes from right to left beyond a value of h = 75 W/m^2.C (approx.). Q_left i.e. from left end at 300 C is from left to right and of course, the Q_air is the sum of these two quantities and is dissipated to ambient by convection.

Let us also draw the temp. profile in the rod for values of h = 5, 75 and 150 W/m^2.C:



It can be seen that after a value of h = 75 W/m^2.C (approx.) is reached, the slope of the Tx vs x curve becomes – ve at the RHS, and a minimum in the curve occurs somewhere within x = 0 and x = L.

To determine the position and value of min. temp. in the rod, with h = 200 W/m^2.C:

"Let the minimum temp occur at x = x_min.

Then, at x = x_min, dT/dx is zero.

Put that condition and solve for x_min. Then, substitute this value of x_min in the eqn for temp profile (i.e. eqn (1)) and get T_min:"

dTdx_xmin = A * m * sinh(m * x_min) + B * m * cosh (m * x_min) "…..eqn. (c)"

dTdx_xmin = 0 "…..eqn. (d)"

"Solving eqns. (c) and (d) simultaneously, we get x_min.

Then, substitute this x_min in eqn. (1) to get Tx_min:"

theta_x_min = A * cosh(m * x_min) + B * sinh (m * x_min) "….eqn. (1)… solution for theta at x_min."

theta_x_min = Tx_min – T_a "..finds Tx_min, temp at x_min"

Result:

Unit Settings: SI C kPa kJ mass deg

A = 260	A_c = 0.0001767 [m^2]	B = -257
d = 0.015 [m]	$dTdx_{xmin}$ = 0	$dT\backslash dx_0$ = -3045 [C/m]
$dT\backslash dx_L$ = 535.6 [C/m]	h = 200 [W/m^2-C]	k = 380 [W/m-C]
L = 0.3 [m]	m = 11.85 [1/m]	P = 0.04712 [m]
Q_{air} = 240.4 [W]	Q_{left} = 204.4 [W]	Q_{right} = -35.97 [W]
θ_1 = 260 [C]	θ_2 = 60 [C]	θ_x = 52.62 [C]
$\theta_{x,min}$ = 39.45	Tx_{min} = 79.45 [C]	T_1 = 300 [C]
T_2 = 100 [C]	T_a = 40 [C]	T_x = 92.62 [C]
x = 0.15 [m]	x_{min} = 0.2172 [m]	

We observe that x_min = 0.2172 m …. position where min. temp. occurs,

and its value is: **Tx_min = 79.45 C.**

Now, see the plot above and observe that the values match.

===

"Prob.1E.26. One end of a long rod of 1 cm dia is maintained at a temp of 500 C by placing it in a furnace. The rod is exposed to air at 30 C with a heat transfer coeff of 35 W/m^2.K. The temp measured at a distance of 78.6 mm was 147 C. Determine the thermal conductivity of the material. [VTU – VI Sem. B.E. – May/June 2006]:"

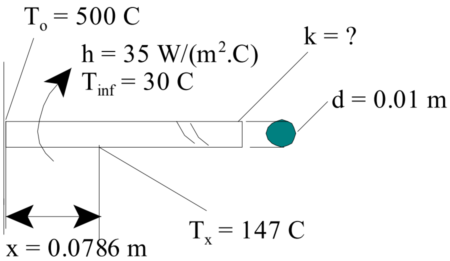


Fig.Prob.1E.26

This is the same Problem as Prob.1E.2.

But, now, we will solve it with EXCEL:

EXCEL Solution:

Following are the steps:

1. Set up the EXCEL worksheet, enter data, and name the cells:

T_x | fx 147

	A	B	C	D	E
1					
2		Data:			
3			d	0.01	m
4			T_0	500	C
5			T_inf	30	C
6			h	35	W/m^2.C
7			x	0.0786	m
8			T_x	147	C
9			k	10	...trial value

We have entered a trial value for k, since it has to be found out later by applying Goal seek in EXCEL.

2. Enter the preliminary calculations such as: cross-sectional area, A_c, Perimeter, P, and Fin parameter 'm':

m | =SQRT((h*P)/(k*A_c))

	A	B	C	D	E
4			T_0	500	C
5			T_inf	30	C
6			h	35	W/m^2.C
7			x	0.0786	m
8			T_x	147	C
9			k	10	...trial value
10		Calculations:			
11			A_c	7.85398E-05	m^2
12			P	0.031415927	m^2
13		Fin parameter	m	37.41657387	m^-1

3. Now, with the existing value of k (and 'm'), find out the value of temp T_x_1 at x = 0.0786 m. It should be equal to 147 C if the assumed value of k was correct. However, it is not equal to 147 C:

D14 | =T_inf+ EXP(-m*x)*(T_0-T_inf)

	A	B	C	D	E
1					
2		Data:			
3			d	0.01	m
4			T_0	500	C
5			T_inf	30	C
6			h	35	W/m^2.C
7			x	0.0786	m
8			T_x	147	C
9			k	10	...trial value
10		Calculations:			
11			A_c	7.85398E-05	m^2
12			P	0.031415927	m^2
13		Fin parameter	m	37.41657387	m^-1
14			T_x_1	54.82348023	C

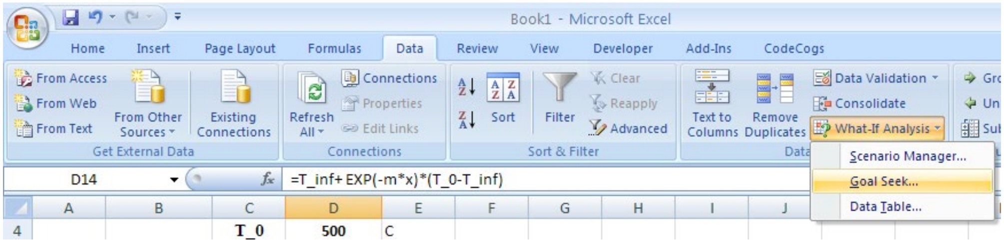
$$m = \sqrt{\frac{h \cdot P}{k \cdot A\_c}} \quad \text{....fin parameter}$$

$$\frac{T\_x - T\_inf}{T\_0 - T\_inf} = \exp(-m \cdot x) \quad \text{...for infinitely long fin}$$

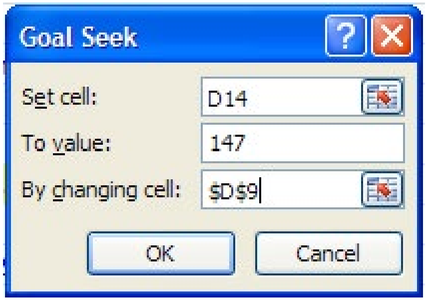
i.e. $T\_x = T\_inf + \exp(-m \cdot x) \cdot (T\_0 - T\_inf)$

Formula for Tx for an infinitely long fin can be seen in the Formula bar above.

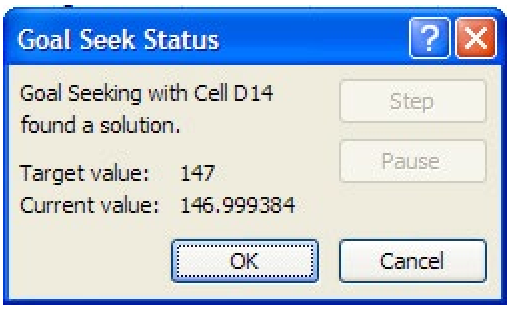
4. So, now, we apply Goal seek to make cell D14 equal to 147 by changing cell D10 (i.e. by changing k). Go to Data – What If Analysis – Goal seek:



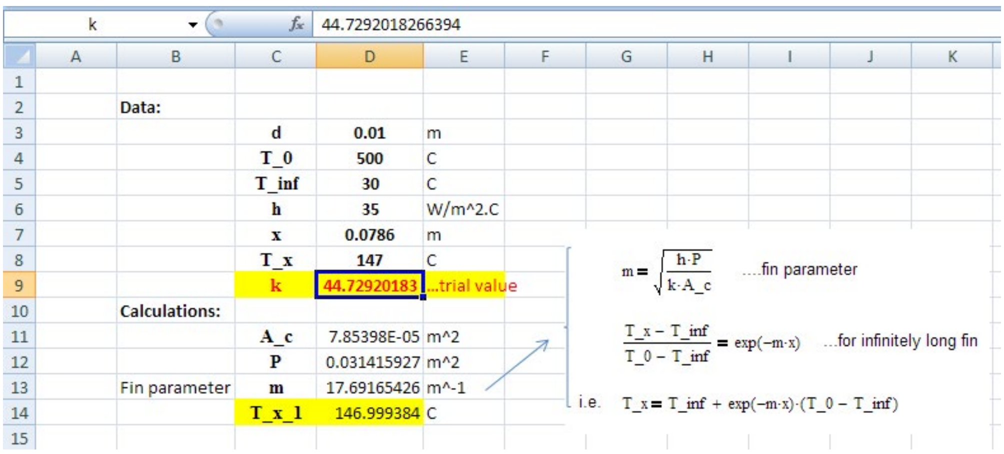
Click on Goal seek. We get:



In the above, suitable values have been filled up. Press OK. We get the message:



See that Target value has converged to 146.99938, which is almost equal to 147. Accept it by pressing OK. Note the value of k in cell D14:



	A	B	C	D	E
1					
2		Data:			
3			d	0.01	m
4			T_0	500	C
5			T_inf	30	C
6			h	35	W/m^2.C
7			x	0.0786	m
8			T_x	147	C
9			k	44.72920183	...trial value
10		Calculations:			
11			A_c	7.85398E-05	m^2
12			P	0.031415927	m^2
13		Fin parameter	m	17.69165426	m^-1
14			T_x_l	146.999384	C
15					

$$m = \sqrt{\frac{h \cdot P}{k \cdot A\_c}} \quad \text{....fin parameter}$$

$$\frac{T\_x - T\_inf}{T\_0 - T\_inf} = \exp(-m \cdot x) \quad \text{...for infinitely long fin}$$

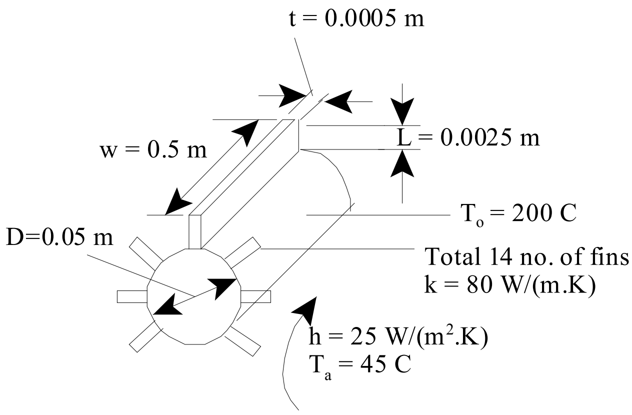
i.e. $T\_x = T\_inf + \exp(-m \cdot x) \cdot (T\_0 - T\_inf)$

Thus: Thermal conductivity, k = 44.729 W/m.C …. Ans.

===

Prob.1E.27. A cylinder 5 cm dia and 50 cm long, is provided with 14 longitudinal straight fins of 1mm thick and 2.5mm height. Calculate the heat loss from the cylinder per sec if the surface temperature of the cylinder is 200 C.

Take h = 25W/(m2 K), k= 80 W/(m.K), and Ta = 45 C.



This is the case of a *fin with convection from its end.*

For this case, for the fin, we have:

$$Q_{fin} := k \cdot A_c \cdot m \cdot \theta_o \cdot \frac{\left(\tanh(m \cdot L) + \frac{h}{m \cdot k}\right)}{\left(1 + \frac{h}{m \cdot k} \cdot \tanh(m \cdot L)\right)} \qquad \text{......(6.11)}$$

Total heat transfer is calculated as the sum of:

1) heat transferred from all the 14 fins, and
2) the convective heat transfer from the 'un-finned' base surface of the cylinder, which is at a temp. of 200 C.

EXCEL Solution:

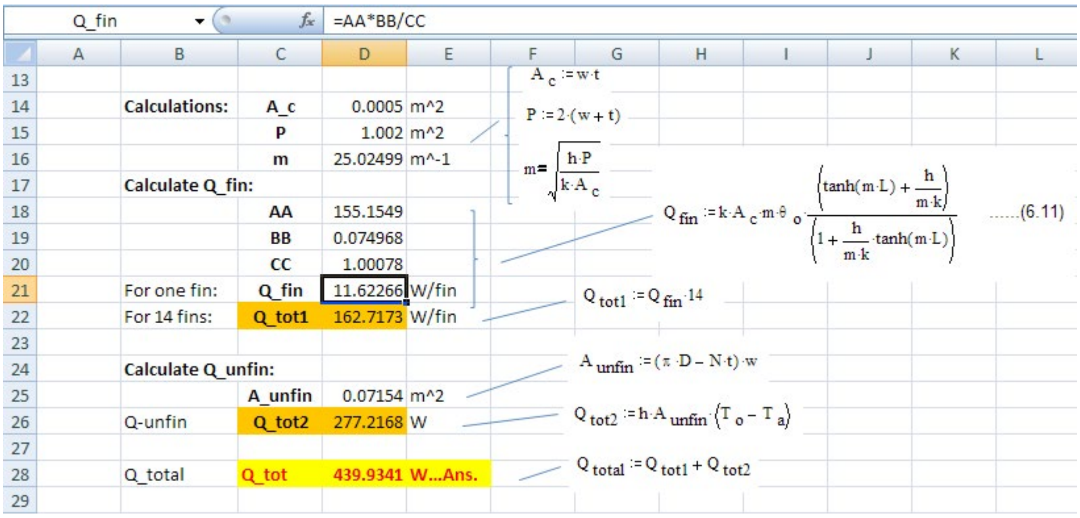
Following are the steps:

1. Set up the EXCEL worksheet, enter data and name the cells:

theta_0 | f_x =T_0-T_a

	A	B	C	D	E
1					
2		**Data:**			
3			**L**	**0.0025**	m
4			**w**	**0.5**	m
5			**t**	**0.001**	m
6			**N**	**14**	
7			**D**	**0.05**	m
8			**k**	**80**	W/m.C
9			**T_0**	**200**	C
10			**T_a**	**45**	C
11			**h**	**25**	W/m^2.C
12		Excess temp:	**theta_0**	**155**	

2. Make the calculations such as Area of cross-section A_c, Perimeter P, fin parameter, 'm', Q_fin… etc. as shown. The formulas used are also shown in the worksheet for ready reference.



Q_fin =AA*BB/CC

	A	B	C	D	E
13					
14		Calculations:	A_c	0.0005	m^2
15			P	1.002	m^2
16			m	25.02499	m^-1
17		Calculate Q_fin:			
18			AA	155.1549	
19			BB	0.074968	
20			CC	1.00078	
21		For one fin:	Q_fin	11.62266	W/fin
22		For 14 fins:	Q_tot1	162.7173	W/fin
23					
24		Calculate Q_unfin:			
25			A_unfin	0.07154	m^2
26		Q-unfin	Q_tot2	277.2168	W
27					
28		Q_total	Q_tot	439.9341	W...Ans.
29					

$A_c := w \cdot t$

$P := 2 \cdot (w + t)$

$m = \sqrt{\frac{h \cdot P}{k \cdot A_c}}$

$Q_{fin} := k \cdot A_c \cdot m \cdot \theta_o \cdot \frac{\left(\tanh(m \cdot L) + \frac{h}{m \cdot k}\right)}{\left(1 + \frac{h}{m \cdot k} \cdot \tanh(m \cdot L)\right)}$ ......(6.11)

$Q_{tot1} := Q_{fin} \cdot 14$

$A_{unfin} := (\pi \cdot D - N \cdot t) \cdot w$

$Q_{tot2} := h \cdot A_{unfin} \cdot (T_o - T_a)$

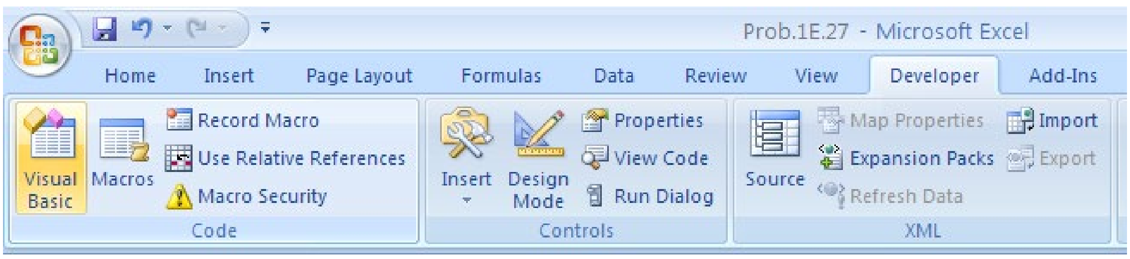
$Q_{total} := Q_{tot1} + Q_{tot2}$

Thus: Q_fin = 11.623 W for one fin, Q_tot1 = 162.717 W for 14 fins, and Q_tot2 = 277.217 W for the un-finned (or, base area) portion.

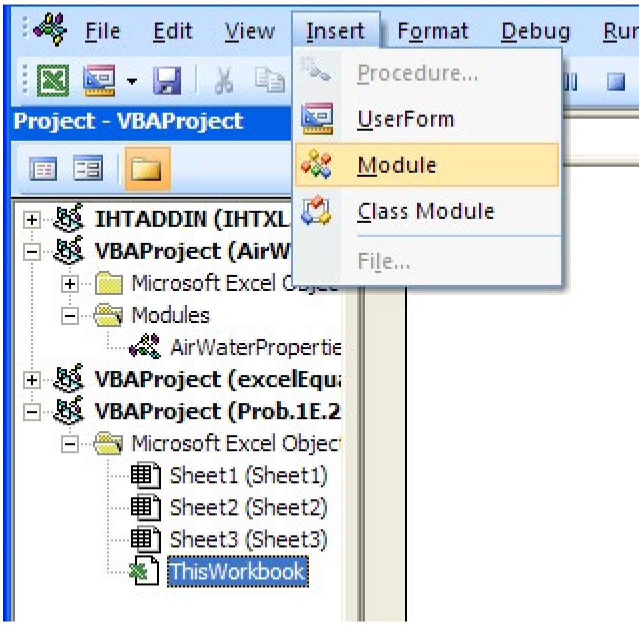
Therefore, Q_total = 439.934 W … Ans.

Note the way in which the calculation for Q_fin is done in parts. This way is convenient to work in EXCEL and avoids errors in entering long or complicated formulas.

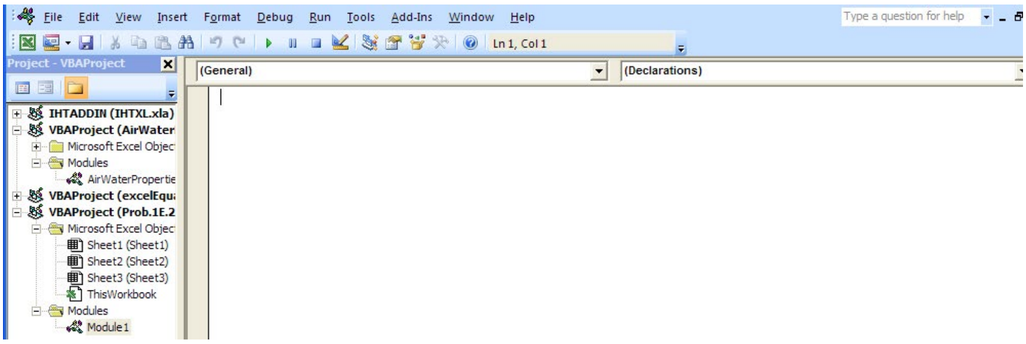
3. Now, if we have to make the calculations for different values of k, or h, it will be very convenient to have a VBA Function to calculate Q_fin. Q_unfin, of course, does not change with k.
4. Following is the procedure to write the VBA Function: Go to Developer-Visual Basic:



5. Press Visual Basic: We get the following window.



6. Click on Insert Module. We get:



7. Type the code in the blank window:

```
Option Explicit
Function Fin_convection_from_tip_Qfin(k As Double, Ac As Double, _
m As Double, _
T0 As Double, Tamb As Double, L As Double, h As Double) As Double
Dim A As Double
Dim B As Double
Dim C As Double
      A = k * Ac * m * (T0 - Tamb)
      B = Application.Tanh(m * L) + h / (m * k)
      C = 1 + (h / (m * k)) * Application.Tanh(m * L)
      Fin_convection_from_tip_Qfin = A * B / C
  End Function
```

Line 1: declares that all variables should be explicitly declared in the beginning of the code. This is recommended as a good programming practice.

Line 2, 3, 4: declares the Function name. The variables are also declared within the Function definition.

Note that code is continued to the next line with an underscore (i.e. _) at the end of the previous line.

Line 5, 6, 7: dimensions of dummy variables used inside the code only.

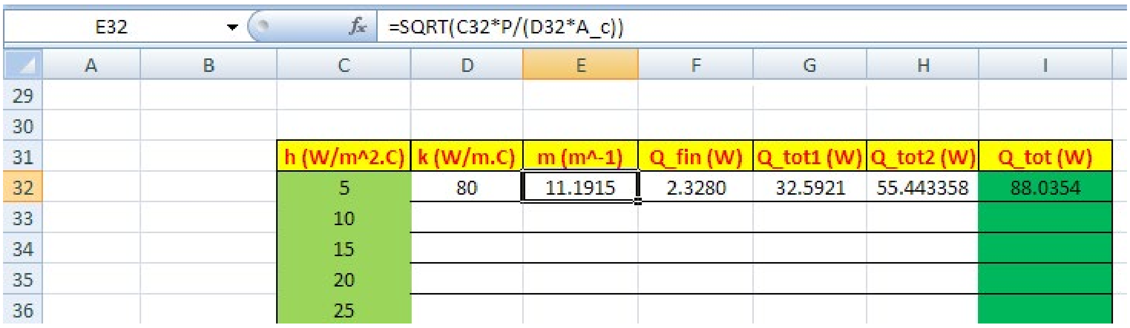
Lines 8, 9, 10: calculate the variables A, B and C

Line 11: Calculate Q_fin using A, B and C

Line 12: End statement of Function

While saving, save as 'Macro enabled worksheet'.

8. Now, this Function becomes available in the worksheet like any other built-in Function in EXCEL. And, let us find out Q_total for h varying from say, 5 to 100 W/m^2.C. Let us plan the worksheet as shown below. h and k are in separate columns and other variables are also in the same row:

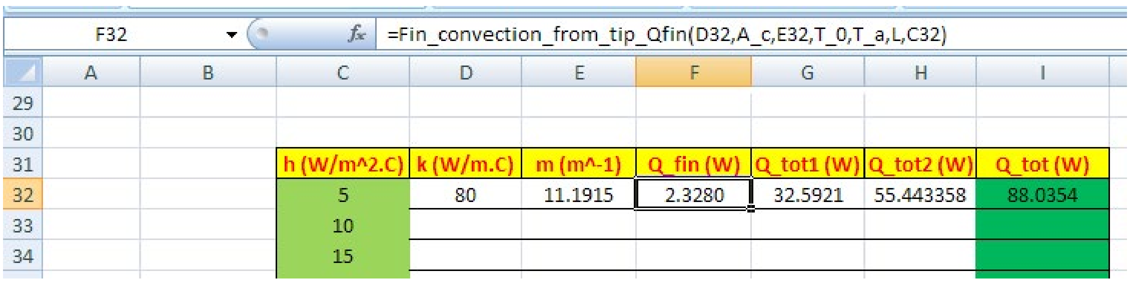

E32 =SQRT(C32*P/(D32*A_c))

	A	B	C	D	E	F	G	H	I
29									
30									
31			h (W/m^2.C)	k (W/m.C)	m (m^-1)	Q_fin (W)	Q_tot1 (W)	Q_tot2 (W)	Q_tot (W)
32			5	80	11.1915	2.3280	32.5921	55.443358	88.0354
33			10						
34			15						
35			20						
36			25						

In the above scheme, first, we change 'h', keeping 'k' constant at k = 80 W/m.C.

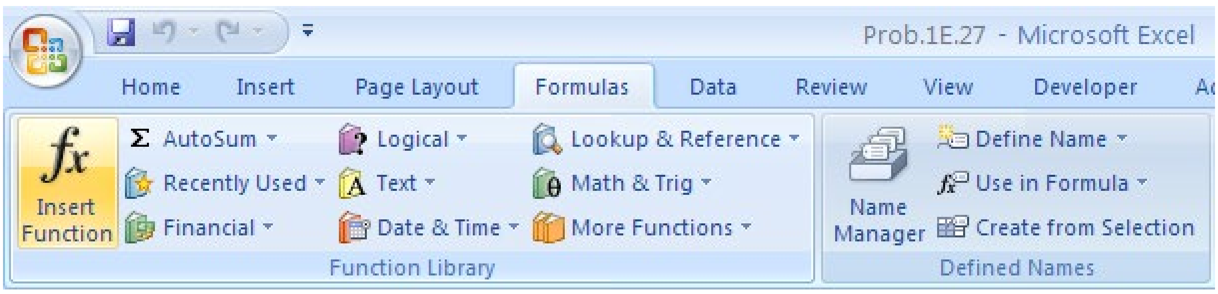
If we look at the formulas for fin parameter 'm' and Q_fin, we find that both contain h and k which are variables. See in the above screen shot how formula for 'm' is entered. 'h' and 'k' are written with relative reference as C32 and D32, so that when we copy by dragging, the cell references automatically up-date.

9. See how Function for Q_fin is entered in cell F32:

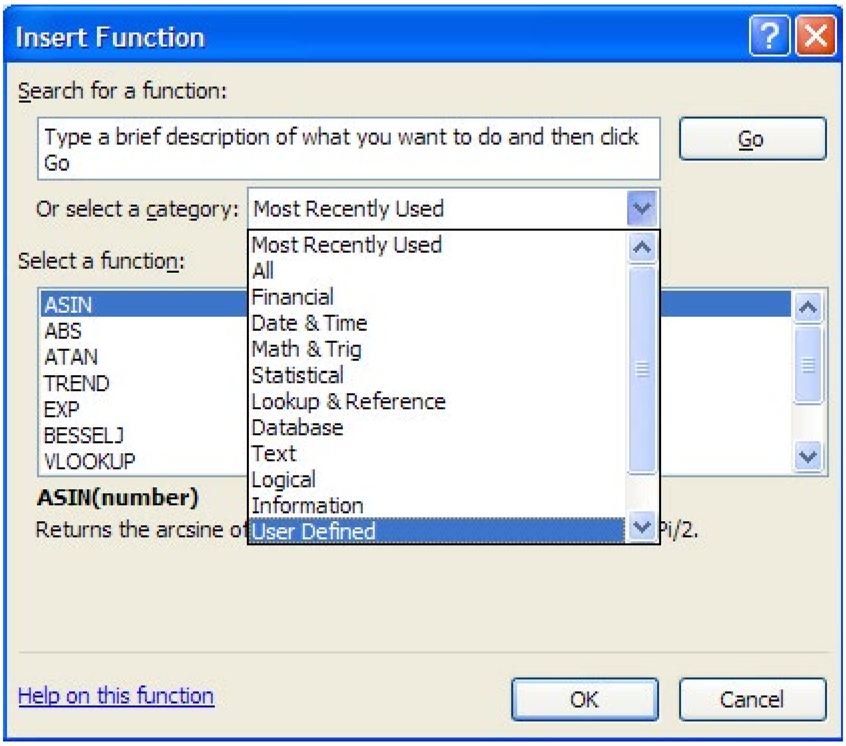

F32 =Fin_convection_from_tip_Qfin(D32,A_c,E32,T_0,T_a,L,C32)

	A	B	C	D	E	F	G	H	I
29									
30									
31			h (W/m^2.C)	k (W/m.C)	m (m^-1)	Q_fin (W)	Q_tot1 (W)	Q_tot2 (W)	Q_tot (W)
32			5	80	11.1915	2.3280	32.5921	55.443358	88.0354
33			10						
34			15						

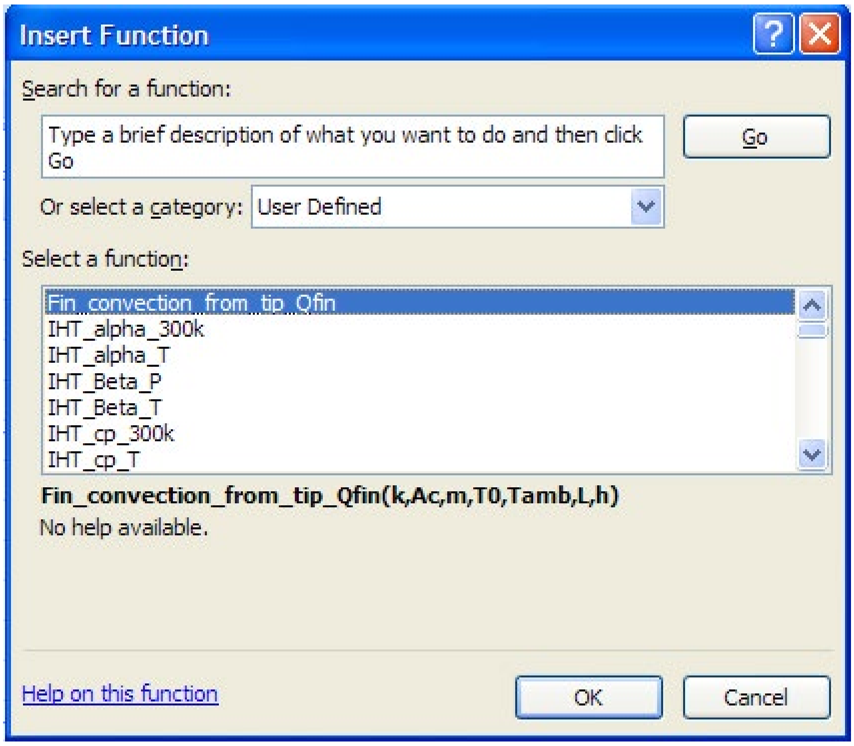
Note again that h and k are entered with relative reference and the other unchanging variables are entered as absolute references. Procedure to enter the Function is as follows: Go to Formulas – Insert Function (button on extreme left):



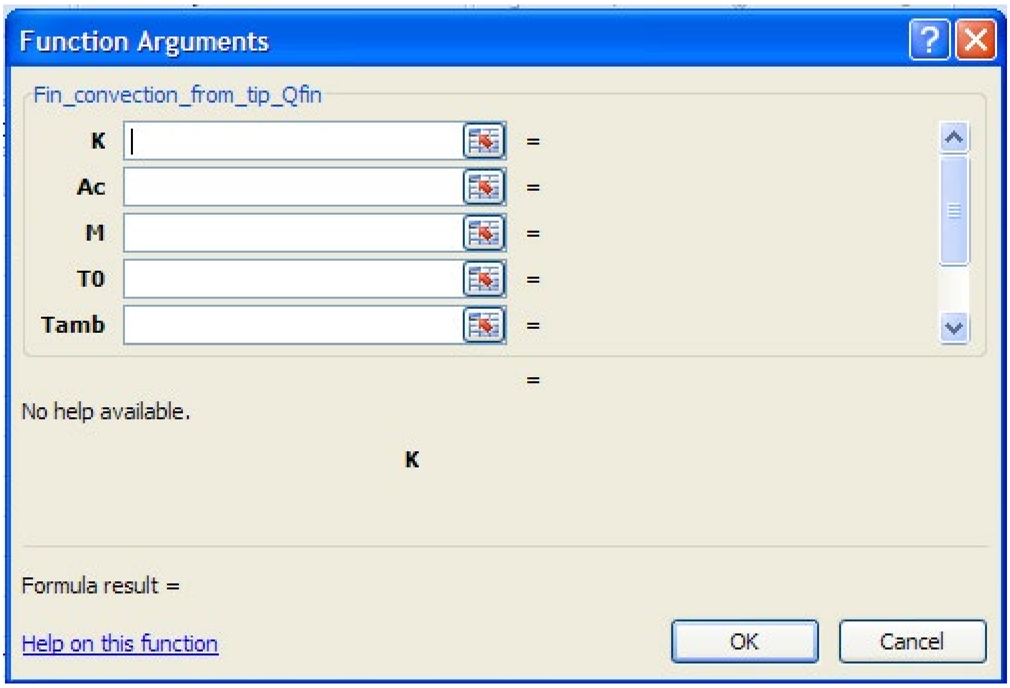
Click on *Insert Function.* We get:



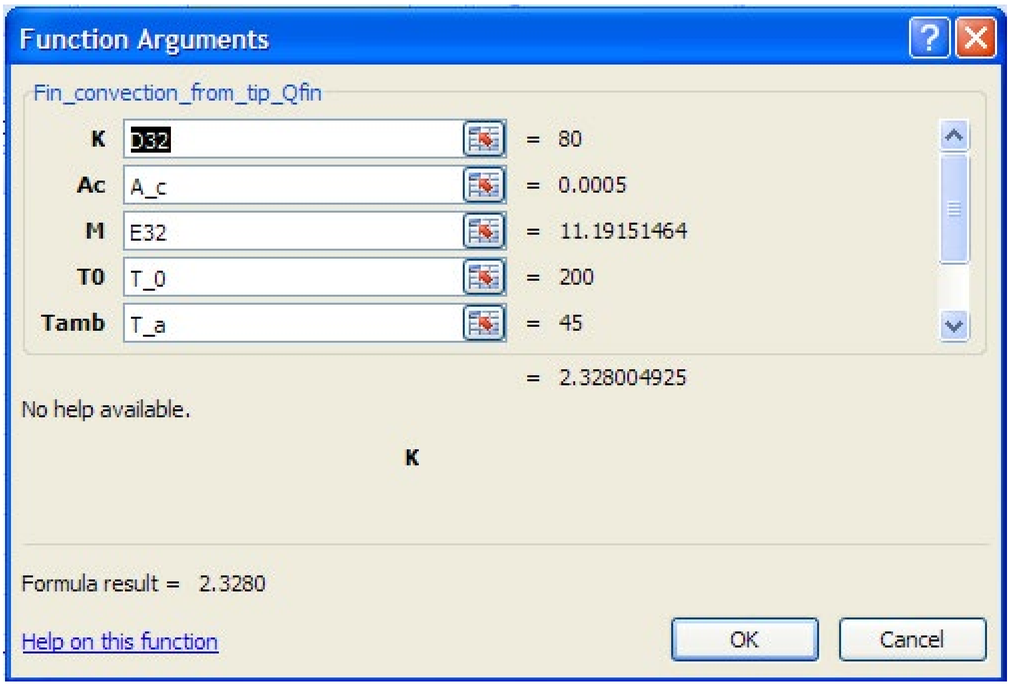
Select 'User Defined' category. We get:



Now, click OK and the Function entering window will appear:



In the above window, enter the variables (by pointing and clicking, to avoid errors), as shown below:



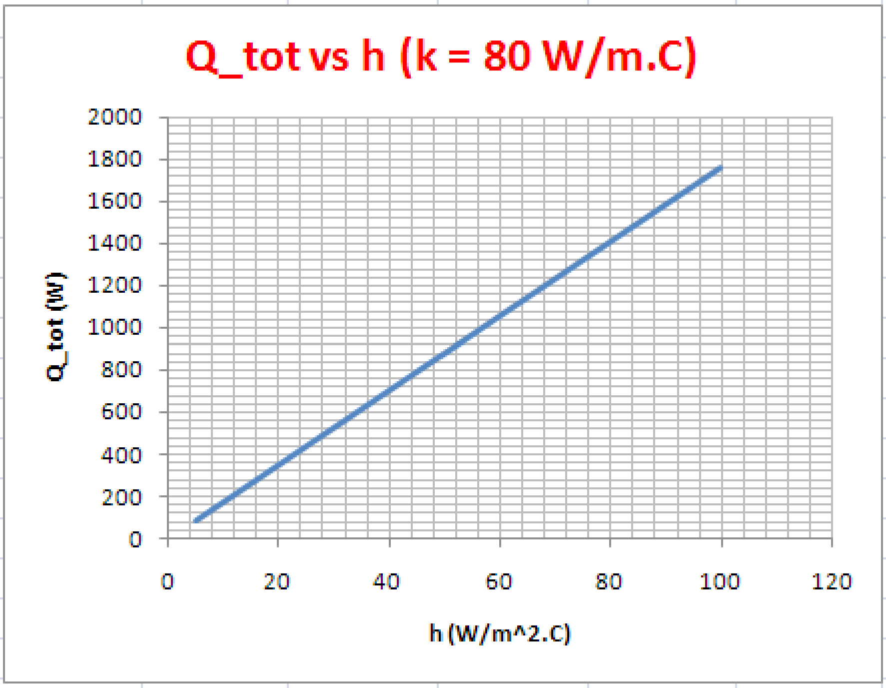
Note that final result also appears. Click OK.

10. Now, fill cells D33 to D51 with 80, and select cells E32 to I32 and drag and copy downwards up to cell I51. And, immediately, all calculations are made:

I51 =G51+H51

	B	C	D	E	F	G	H	I
31		h (W/m^2.C)	k (W/m.C)	m (m^-1)	Q_fin (W)	Q_tot1 (W)	Q_tot2 (W)	Q_tot (W)
32		5	80	11.1915	2.3280	32.5921	55.443358	88.0354
33		10	80	15.8272	4.6543	65.1598	110.8867	176.0465
34		15	80	19.3843	6.9788	97.7032	166.3301	264.0333
35		20	80	22.3830	9.3016	130.2224	221.7734	351.9958
36		25	80	25.0250	11.6227	162.7173	277.2168	439.9341
37		30	80	27.4135	13.9420	195.1880	332.6601	527.8481
38		35	80	29.6100	16.2596	227.6345	388.1035	615.7380
39		40	80	31.6544	18.5755	260.0568	443.5469	703.6037
40		45	80	33.5745	20.8897	292.4551	498.9902	791.4453
41		50	80	35.3907	23.2021	324.8293	554.4336	879.2629
42		55	80	37.1181	25.5128	357.1794	609.8769	967.0563
43		60	80	38.7685	27.8218	389.5055	665.3203	1054.8258
44		65	80	40.3516	30.1291	421.8076	720.7636	1142.5713
45		70	80	41.8748	32.4347	454.0858	776.2070	1230.2928
46		75	80	43.3445	34.7386	486.3401	831.6504	1317.9904
47		80	80	44.7661	37.0407	518.5704	887.0937	1405.6642
48		85	80	46.1438	39.3412	550.7769	942.5371	1493.3140
49		90	80	47.4816	41.6400	582.9596	997.9804	1580.9401
50		95	80	48.7827	43.9370	615.1185	1053.4238	1668.5423
51		100	80	50.0500	46.2324	647.2536	1108.8672	1756.1208

11. Now, draw the plot of Q_total vs h:

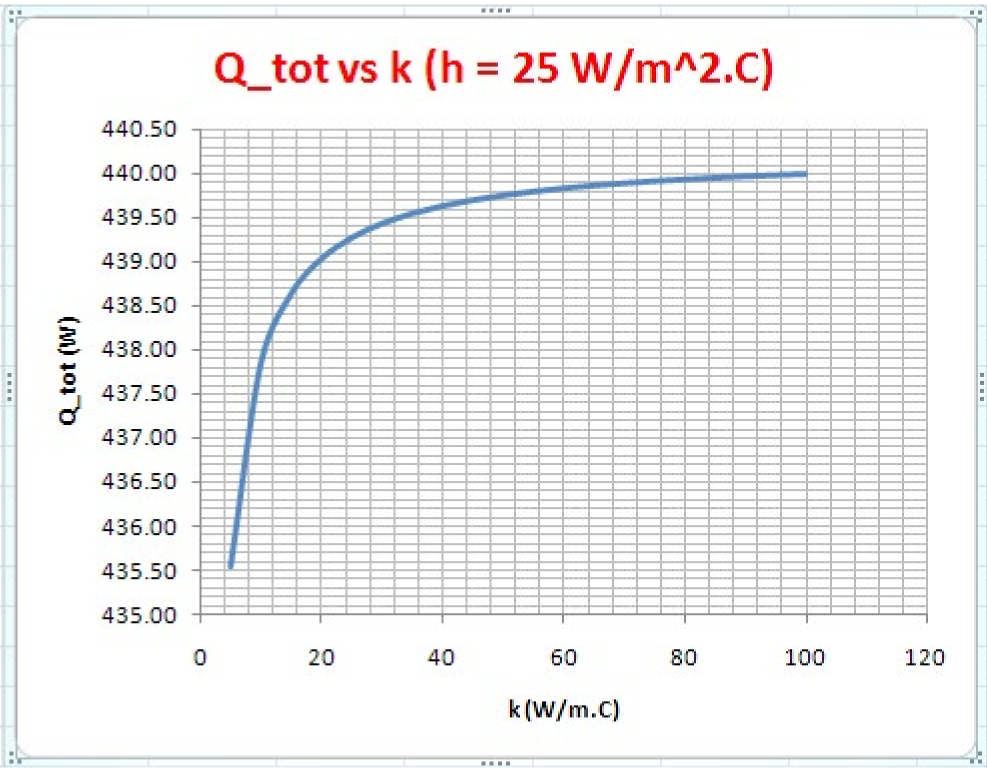


12. Now, if we have to calculate Q_total for say k = 5 to 100, keeping h = 25 W/m^2.C, simply change these values in columns C and D in the Table, and other values up-date themselves immediately:

I51 =G51+H51

	B	C	D	E	F	G	H	I
31		h (W/m^2.C)	k (W/m.C)	m (m^-1)	Q_fin (W)	Q_tot1 (W)	Q_tot2 (W)	Q_tot (W)
32		25	5	100.1000	11.3082	158.3151	277.21679	435.5319
33		25	10	70.7814	11.4733	160.6268	277.2168	437.8436
34		25	15	57.7927	11.5297	161.4155	277.2168	438.6323
35		25	20	50.0500	11.5581	161.8134	277.2168	439.0302
36		25	25	44.7661	11.5752	162.0533	277.2168	439.2700
37		25	30	40.8656	11.5867	162.2136	277.2168	439.4304
38		25	35	37.8342	11.5949	162.3284	277.2168	439.5452
39		25	40	35.3907	11.6010	162.4146	277.2168	439.6314
40		25	45	33.3667	11.6058	162.4818	277.2168	439.6986
41		25	50	31.6544	11.6097	162.5355	277.2168	439.7523
42		25	55	30.1813	11.6128	162.5795	277.2168	439.7963
43		25	60	28.8964	11.6154	162.6162	277.2168	439.8330
44		25	65	27.7627	11.6177	162.6473	277.2168	439.8641
45		25	70	26.7528	11.6196	162.6740	277.2168	439.8907
46		25	75	25.8457	11.6212	162.6971	277.2168	439.9138
47		25	80	25.0250	11.6227	162.7173	277.2168	439.9341
48		25	85	24.2778	11.6239	162.7351	277.2168	439.9519
49		25	90	23.5938	11.6251	162.7510	277.2168	439.9678
50		25	95	22.9645	11.6261	162.7652	277.2168	439.9820
51		25	100	22.3830	11.6270	162.7780	277.2168	439.9947

13. And, plot Q_tot against k:



14. For any other combination of h and k, simply change both h and k in a given row and look at the last column for the value of Q_tot. For example, if h = 80 and k = 60, we enter these values in cells C32 and D32 respectively, and see the result in cell I32. We get:

I32 f_x =G32+H32

	A	B	C	D	E	F	G	H	I
30									
31			h (W/m^2.C)	k (W/m.C)	m (m^-1)	Q_fin (W)	Q_tot1 (W)	Q_tot2 (W)	Q_tot (W)
32			80	60	51.6914	36.9677	517.5477	887.09372	1404.6414

We see that Q_tot = 1404.64 W.

Thus, EXCEL is very useful for parametric analysis and quick, graphical representation of results.

1F Conduction with heat generation:

Learning objectives:

1. When there is no internal heat generation in the system, the temperature distribution is determined purely by the boundary conditions. However, there are many practical cases where there is energy generation within the system and we would be interested to find out the temperature distribution within the body and the heat flux at any location, in such cases.
2. Examples of situations with internal heat generation are:
   a) Joule heating in an electrical conductor due to the flow of current in it
   b) Energy generation in a nuclear fuel rod due to absorption of neutrons
   c) Exothermic chemical reaction within a system (e.g. combustion), liberating heat at a given rate throughout the system
   d) Heat liberated in 'shielding' used in Nuclear reactors due to absorption of electromagnetic radiation such as gamma rays
   e) Curing of concrete
   f) Magnetization of iron
   g) Ripening of fruits and in biological decay processes

3. Temperature distribution and heat flux are of special interest in some cases where *safety of the system or personnel* is involved, e.g. 'burn – out' of nuclear fuel rods may occur due to excessive heat, causing a catastrophe, if suitable precautions for adequate cooling are not taken. Also, analysis of electrical machinery, transformers and electrical heaters would require that the generation of internal energy is taken into consideration.
4. We shall solve problems in heat transfer in simple geometries (i.e. plane slabs, cylinders and spheres), with uniform internal energy generation. Several possible boundary conditions will be considered. We will study the cases with constant thermal conductivity as well as temperature dependent thermal conductivity.
5. Solving the problems in this section would give a good foundation to solve many interesting, practical problems.

Formulas used:

Relations for steady state, one dimensional conduction with internal heat generation, and constant k

Relation	Plane wall (both sides at T_w)	Plane wall (sides at T1 and T2)	Plane wall (insulated on one side)
Governing diff. Eqn.	$\frac{d^2T}{dx^2} + \frac{q_g}{k} = 0$	$\frac{d^2T}{dx^2} + \frac{q_g}{k} = 0$	$\frac{d^2T}{dx^2} + \frac{q_g}{k} = 0$
Temp. distribution	$T(x) = T_w + \frac{q_g}{2\cdot k}\cdot(L^2 - x^2)$	$T(x) = T1 + \left[(L - x)\cdot\frac{q_g}{2\cdot k} + \frac{(T2 - T1)}{L}\right]\cdot x$	$T(x) = T_w + \frac{q_g}{2\cdot k}\cdot(L^2 - x^2)$
Heat transfer rate at the surface, Q, (W)	$Q = q_g\cdot A\cdot L$	$Q_{left} = -k\cdot A\cdot\frac{dT(x)}{dx}$ at x = 0 $Q_{right} = -k\cdot A\cdot\frac{dT(x)}{dx}$ at x = L	$q_g\cdot A\cdot L$
$T_{max} - T_w$, (C)	$\frac{q_g\cdot L^2}{2\cdot k}$	Equate dT(x)/dx to zero; Subst. Resulting x in T(x) to get T_{max}	$\frac{q_g\cdot L^2}{2\cdot k}$
Comments	L is **half thickness** of slab; Max. temp. occurs on the centre line	L is the thickness of slab	L is the thickness of slab; Max. temp. occurs on the insulated surface

Table 1F.1

Relations for steady state, one dimensional conduction with internal heat generation and k varying linearly with temperature as: $k(T) = k_o (1 + \beta T)$ $k_m = k_o (1 + \beta T_m)$; $T_m = (T1 + T2)/2$

Relation	Plane wall of thickness, L (sides at T1 and T2)
Governing diff. eqn.	$\frac{d}{dx}\left(k(T)\cdot\frac{dT}{dx}\right) + q_g = 0$
Temp. distribution	$T(x) = \frac{-1}{\beta} + \sqrt{\left(\frac{1}{\beta} + T1\right)^2 - \frac{2\cdot x}{\beta\cdot L}\cdot(T1 - T2)\cdot\left(1 + \beta\cdot T_m\right) + \frac{q_g\cdot x}{\beta\cdot k_o}\cdot(L - x)}$
Heat transfer rate at the surface, Q, (W)	$Q_{left} = -k\cdot A\cdot\frac{dT(x)}{dx}$ at x = 0 $Q_{right} = -k\cdot A\cdot\frac{dT(x)}{dx}$ at x = L
T_{max}, (C)	Equate dT(x)/dx to zero; Subst. resulting x in T(x) to get T_{max}

Table 1F.2

Relations for steady state, one dimensional conduction with internal heat generation, and constant k

Relation	Solid cylinder	Hollow cylinder (inside surface insulated)
Governing diff. Eqn	$\frac{d^2T}{dr^2} + \frac{1}{r}\cdot\frac{dT}{dr} + \frac{q_g}{k} = 0$	$\frac{d^2T}{dr^2} + \frac{1}{r}\cdot\frac{dT}{dr} + \frac{q_g}{k} = 0$
Temp. distribution	$T(r) = T_w + \frac{q_g}{4\cdot k}\cdot\left(R^2 - r^2\right)$	$T(r) = T_o + \frac{q_g\cdot r_i^2}{4\cdot k}\cdot\left[\left(\frac{r_o}{r_i}\right)^2 - 2\cdot\ln\left(\frac{r_o}{r}\right) - \left(\frac{r}{r_i}\right)^2\right]$
Heat transfer rate at the surface, Q, (W)	$q_g\cdot\pi\cdot R^2\cdot L$	$q_g\cdot\pi\cdot\left(r_o^2 - r_i^2\right)\cdot L$
$T_{max} - T_w$, (C)	$\frac{q_g\cdot R^2}{4\cdot k}$	$\frac{q_g\cdot r_i^2}{4\cdot k}\cdot\left[\left(\frac{r_o}{r_i}\right)^2 - 2\cdot\ln\left(\frac{r_o}{r_i}\right) - 1\right]$
Comments	L is length of cylinder; Max. temp. occurs at the centre	L is length of cylinder; Max. temp. occurs on the inside surface

Table 1F.3

Relations for steady state, one dimensional conduction with internal heat generation, and constant k

Relation	Hollow cylinder (outside surface insulated)	Hollow cylinder (surfaces at T1 and T2)
Governing diff. Eqn	$\frac{d^2T}{dr^2}+\frac{1}{r}\cdot\frac{dT}{dr}+\frac{q_g}{k}=0$	$\frac{d^2T}{dr^2}+\frac{1}{r}\cdot\frac{dT}{dr}+\frac{q_g}{k}=0$
Temp. distribution	$T(r)=T_i+\frac{q_g\cdot r_o^2}{4\cdot k}\cdot\left[2\cdot\ln\left(\frac{r}{r_i}\right)+\left(\frac{r_i}{r_o}\right)^2-\left(\frac{r}{r_o}\right)^2\right]$	$\frac{T(r)-T_i}{T_o-T_i}=\frac{\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)}+\frac{q_g}{4\cdot k}\cdot\frac{\left(r_o^2-r_i^2\right)}{\left(T_o-T_i\right)}\cdot\left[\frac{\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)}-\frac{\left(\frac{r}{r_i}\right)^2-1}{\left(\frac{r_o}{r_i}\right)^2-1}\right]$
Heat transfer rate at the surface, Q, (W)	$q_g\cdot\pi\cdot\left(r_o^2-r_i^2\right)\cdot L$	$Q_{inner}=-k\cdot A_i\cdot\frac{dT(r)}{dr}$ at $r = r_i$ $Q_{outer}=-k\cdot A_o\cdot\frac{dT(r)}{dr}$ at $r = r_o$
$T_{max} - T_{w,}$ (C)	$\frac{q_g\cdot r_o^2}{4\cdot k}\cdot\left[2\cdot\ln\left(\frac{r_o}{r_i}\right)+\left(\frac{r_i}{r_o}\right)^2-1\right]$	$\frac{q_g\cdot r_i^2}{4\cdot k}\cdot\left[\left(\frac{r_o}{r_i}\right)^2-2\cdot\ln\left(\frac{r_o}{r_i}\right)-1\right]$
Comments	L is length of cylinder ; Max. temp. occurs on the outside surface.	L is length of cylinder; Position of max. temp. is given by: $r_m=\sqrt{\frac{q_g\cdot\left(r_o^2-r_i^2\right)-4\cdot k\cdot\left(T_i-T_o\right)}{q_g\cdot 2\cdot\ln\left(\frac{r_o}{r_i}\right)}}$

Table 1F.4

Relations for steady state, one dimensional conduction with internal heat generation and k varying linearly with temperature as: $k(T) = k_o\,(1 + \beta\,T)$; $k_m = k_o\,(1 + \beta\,T_m)$; $T_m = (T1 + T2)/2$

Geometry	Temperature distribution, T(r)
Solid cylinder	$T(r) = \frac{-1}{\beta} + \sqrt{\left(\frac{1}{\beta} + T_w\right)^2 + \frac{q_g\cdot\left(R^2 - r^2\right)}{2\cdot\beta\cdot k_o}}$
Hollow cylinder with inside surface insulated	$T(r) = \frac{-1}{\beta} + \sqrt{\left(\frac{1}{\beta} + T_i\right)^2 - \frac{q_g\cdot r_i^2}{2\cdot\beta\cdot k_o}\cdot\left[\left(\frac{r}{r_i}\right)^2 - 2\cdot\ln\left(\frac{r}{r_i}\right) - 1\right]}$
Hollow cylinder with outside surface insulated	$T(r) = \frac{-1}{\beta} + \sqrt{\left(\frac{1}{\beta} + T_o\right)^2 - \frac{q_g\cdot r_o^2}{2\cdot\beta\cdot k_o}\cdot\left[2\cdot\ln\left(\frac{r_o}{r}\right) - \left(\frac{r_o}{r}\right)^2 - 1\right]}$

Table 1F.5

Relations for steady state, one dimensional conduction with internal heat generation, and constant k

Relation	Solid sphere
Governing diff. Eqn	$\frac{d^2T}{dr^2} + \frac{2}{r}\cdot\frac{dT}{dr} + \frac{q_g}{k} = 0$
Temp. distribution	$T(r) = T_w + \frac{q_g}{6\cdot k}\cdot\left(R^2 - r^2\right)$
Heat transfer rate at the surface, Q, (W)	$\frac{4}{3}\cdot\pi\cdot R^3\cdot q_g$
$T_{max} - T_{w,}$ (C)	$\frac{q_g\cdot R^2}{6\cdot k}$
Comments	Max. temp. occurs at the centre.

Table 1F.6

Relations for steady state, one dimensional conduction with internal heat generation and k varying linearly with temperature as: $k(T) = ko\,(1 + \beta\,T)$

$$k_m = k_o\,(1 + \beta\,T_m);\ Tm = (T1 + T2)/2$$

Relation	Solid sphere
Governing diff. Eqn	$\frac{d^2 T}{dr^2} + \frac{2}{r} \cdot \frac{dT}{dr} + \frac{q_g}{k} = 0$
Temp. distribution	$T(r) = \frac{-1}{\beta} + \sqrt{\left(\frac{1}{\beta} + T_w\right)^2 + \frac{q_g \cdot (R^2 - r^2)}{3 \cdot \beta \cdot k_o}}$
Heat transfer rate at the surface, Q, (W)	$\frac{4}{3} \cdot \pi \cdot R^3 \cdot q_g$
Comments	Max. temp. occurs at the centre.

Table 1F.7

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Prob. 1F.1. A plane wall 6 cm thick generates heat internally at the rate of 0.30 MW/m^3. One side of the wall is insulated, and the other is exposed to an environment at 93 C. The convection heat transfer coefficient between the wall and the environment is 570 $W/m^2.K$. Thermal conductivity of the wall is k=21 W/m.K. Calculate the maximum temperature in the wall. [M.U.]

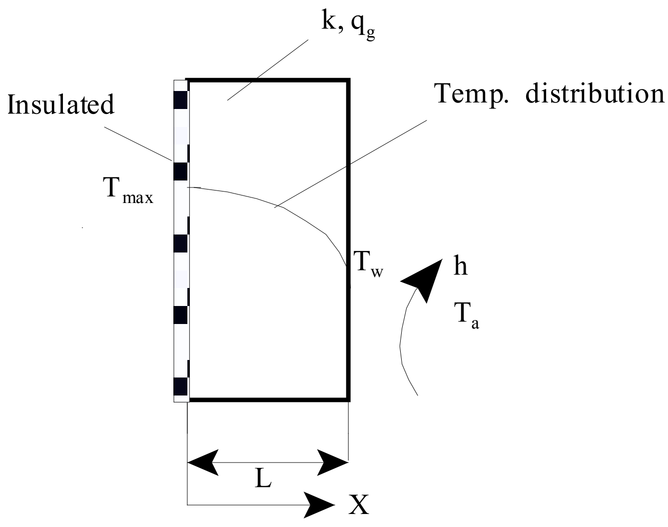


Fig.Prob.1F.1

Mathcad Solution:

Data:

$L := 0.06$ m ... thickness of insulated slab

$q_g := 0.3 \cdot 10^6$ W/m^3 .... heat gen. rate

$T_a := 93$ C ..ambient temp. $h := 570$ W/m^2.K ... convective heat transfer coeff

$k := 21$ W/m.K ... th. conductivity

Calculations:

$T_w := T_a + q_g \cdot \frac{L}{h}$ i.e. $T_w = 124.579$ **C ... wall temp. exposed to ambient**

Temp. distribution is given by:

$$T(x) := T_a + q_g \cdot \frac{L}{h} + \frac{q_g}{2 \cdot k} \cdot \left(L^2 - x^2\right)$$

Note: x is measured from insulated left side of slab

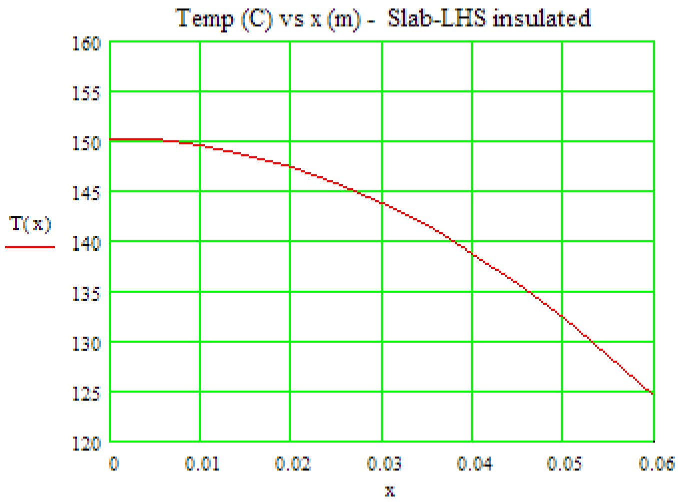
Now, max. temp occurs at the insulated surface, i.e. at x = 0.

Therefore:

$T(0) = 150.293$ **C...max temp. in the wall ... Ans.**

Draw the temp. profile in the slab:

$x := 0, 0.005 .. 0.06$ ...define a range variable x, varying from 0 to 0.06 m



Note that T_{max} occurs t the insulated face and its value is 150.293 C. On the right face, the temp is 124.579 C.

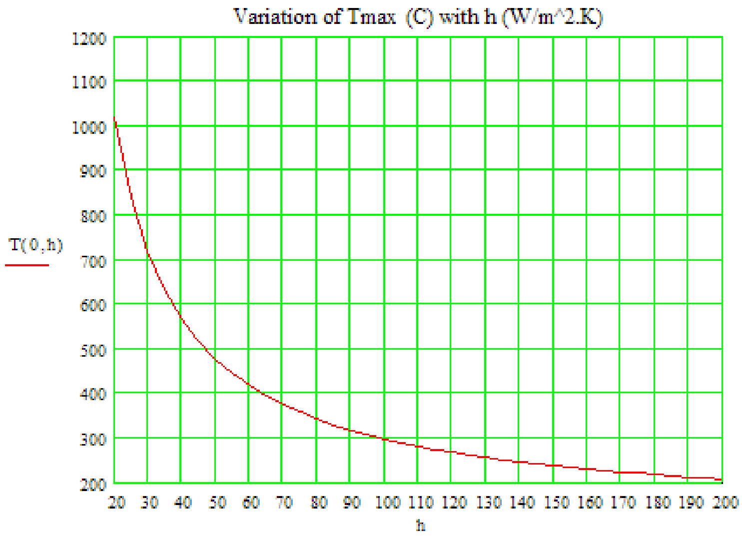
Also, plot the variation of max. temp in slab when h varies from 20 to 200 W/m^2.K:

Write the temp distribution as a function of x and h:

$$T(x,h) := T_a + q_g \cdot \frac{L}{h} + \frac{q_g}{2 \cdot k} \cdot \left(L^2 - x^2\right)$$ **...temp defined as a function of x and h**

x = 0 , since max. temp. occurs on the insulated surface

$h := 20, 25 .. 200$ ...define a range variable h, varying from 20 to 200 W/m^2.K

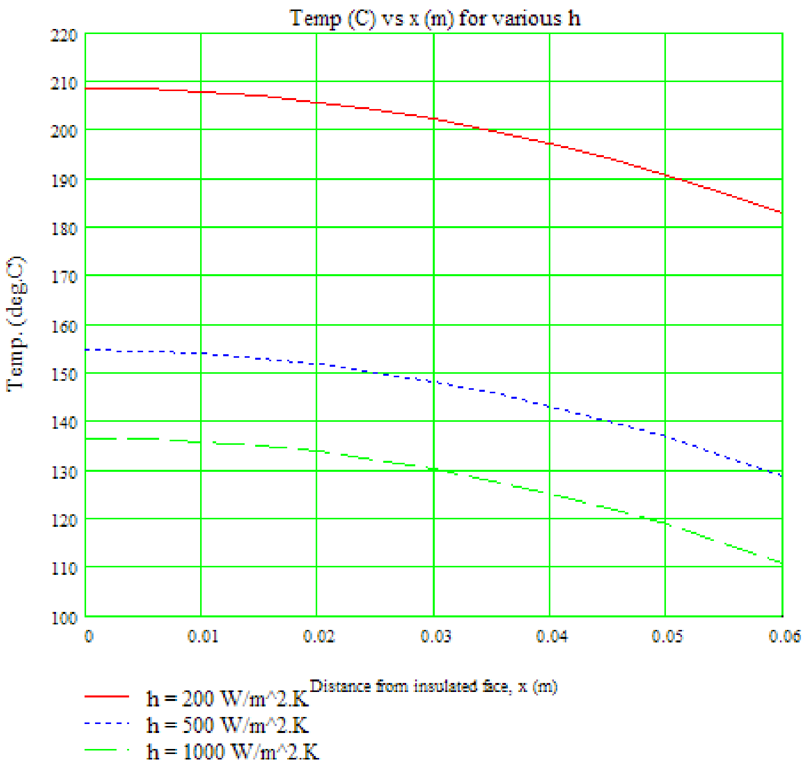


Note that the max. temp reached in the slab (on the insulated left face) goes on decreasing as h increases.

Temp. profile with h = 200, 500, and 1000 W/m^2.K:

$x := 0, 0.005 .. 0.06$ ...define a range variable x, varying from 0 to 0.06 m

$T(x,h) := T_a + q_g \cdot \frac{L}{h} + \frac{q_g}{2 \cdot k} \cdot (L^2 - x^2)$ **...temp defined as a function of x and h**



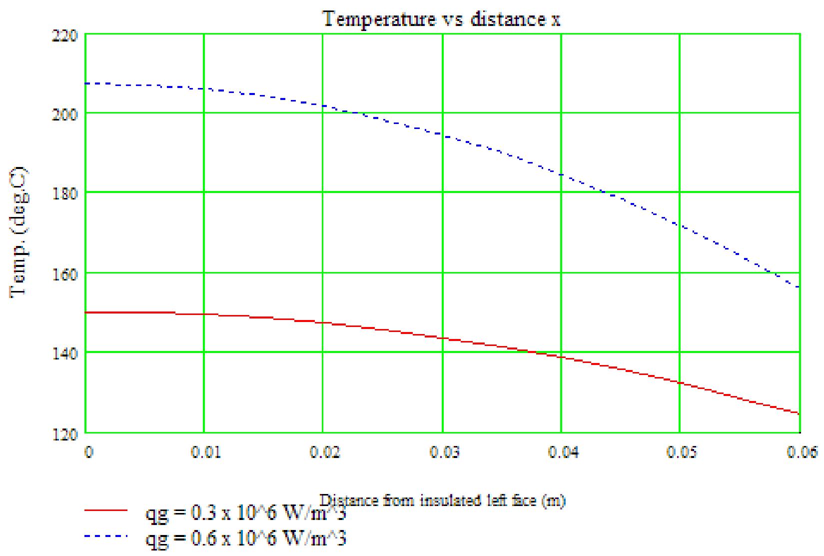
It may be noted that temp profiles are of the same shape, temperatures being uniformly higher for lower values of h.

Temp. profile when heat gen. rate is doubled: (with h = 570 W/m^2.K):

$h := 570$ W/m^2.K $q_g := 0.3 \cdot 10^6$ W/m^2.K

$T(x, q_g) := T_a + q_g \cdot \frac{L}{h} + \frac{q_g}{2 \cdot k} \cdot (L^2 - x^2)$ **....Temp as a function of x and qg**

$x := 0, 0.005 .. 0.06$ ....define a range variable x, varying from 0 to 0.06 m



Note that for higher value of q_g, temperatures in the slab are higher.

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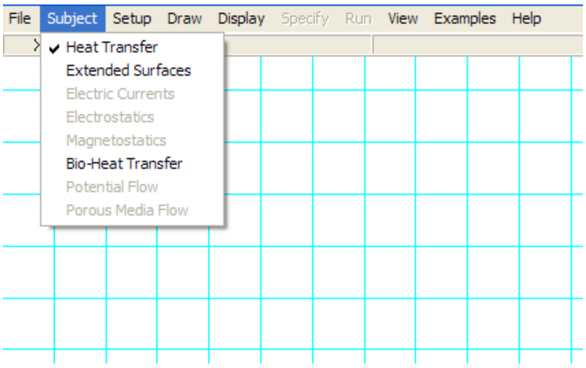
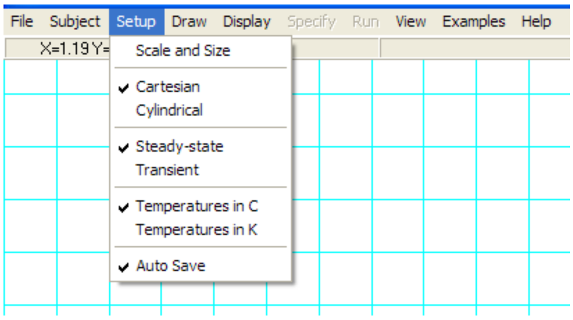
Solve the above problem with Finite Element Heat Transfer (FEHT) Software:

We shall show the steps involved:

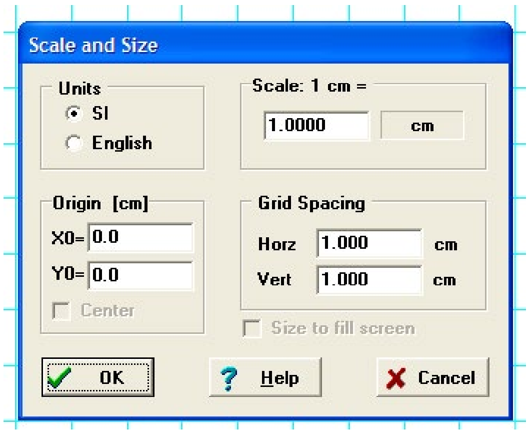
1. Select Subject – Heat Transfer after starting FEHT:



2. Then, from Setup –choose Cartesian cords – Steady state – Temp in deg. C:

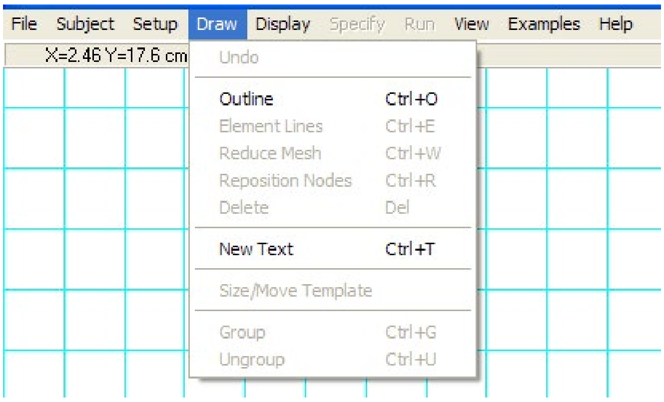


3. Again from Set up – choose the Scale and Size:

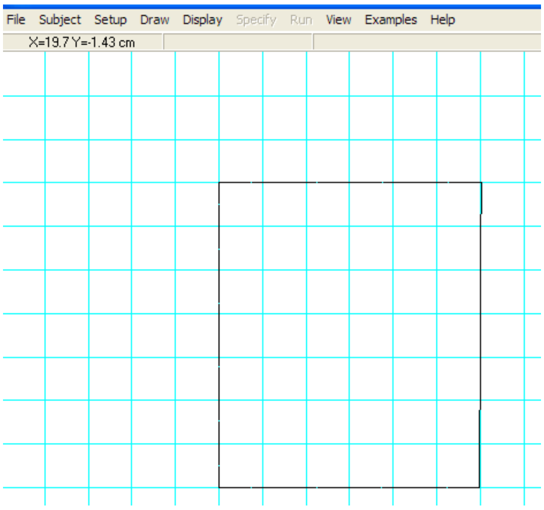


In the above, 1 cm on the screen represents 1 cm on the model and grid spacing is also chosen as 1 cm for both horizontal and vertical.

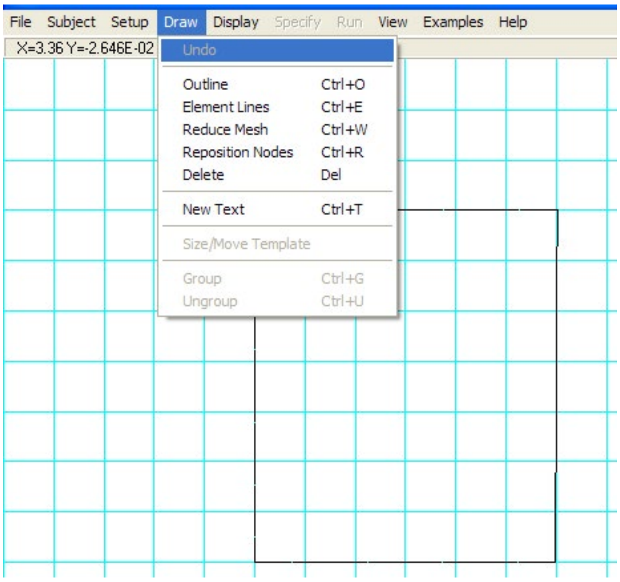
4. Next, from Draw menu, choose Outline to draw the model:



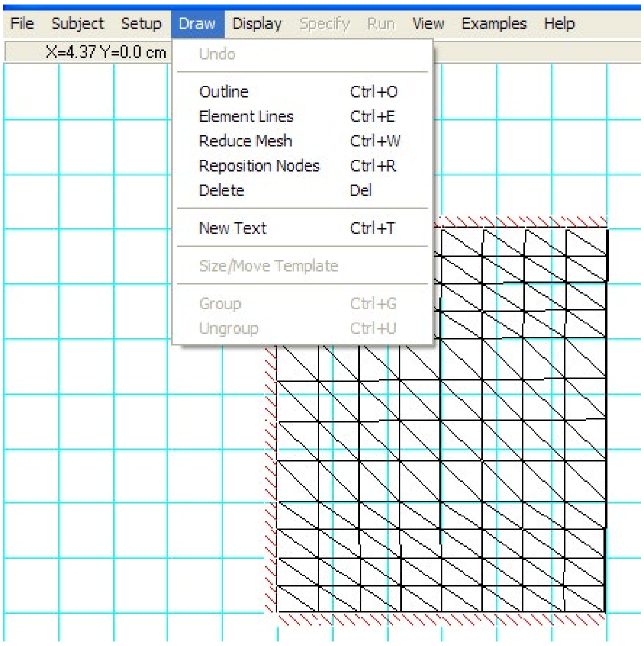
5. Draw the model:



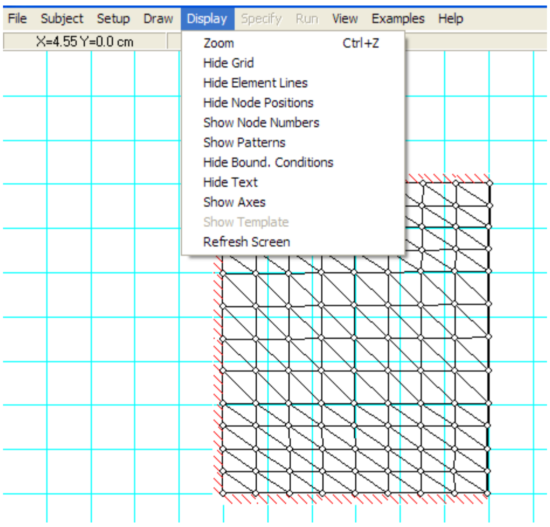
6. Now, choose Draw Element lines from the Draw menu:



7. And prepare a rough mesh of triangular elements:

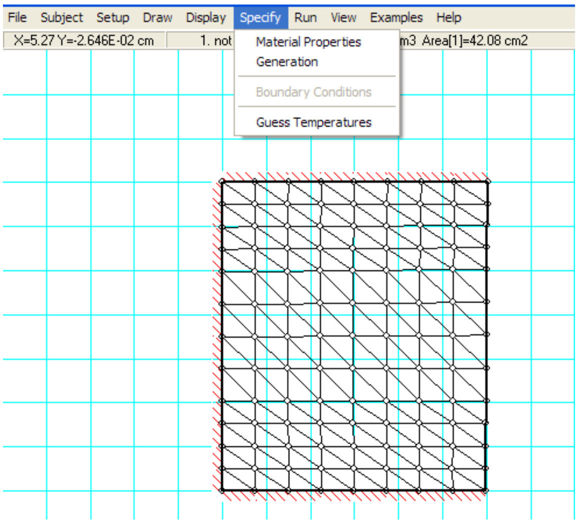


8. Then reduce the mesh by choosing Reduce Mesh from Draw menu:

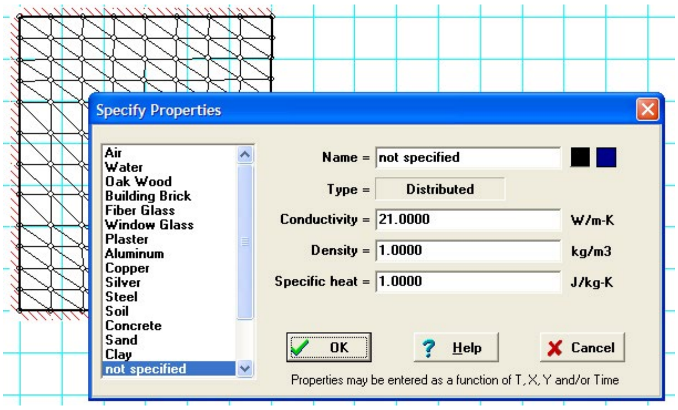


There are many choices to hide or show grid, Element Lines, Node positions, Node nos. etc.

9. After getting a fine mesh, go to Specify menu and enter Material properties, Heat generation:

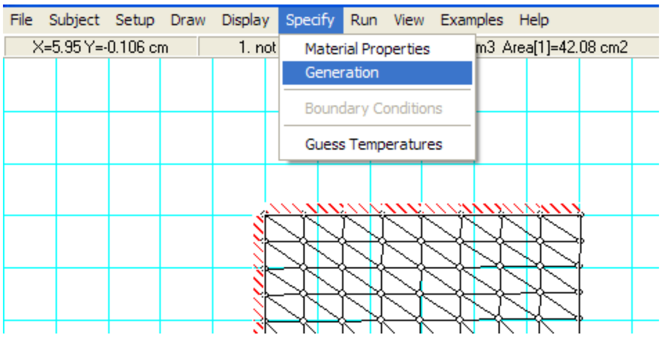


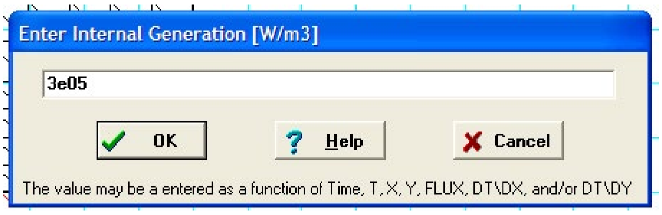
10. Material properties:



Note that k = 21 W/m.K is entered. For Steady state heat transfer, density and sp. Heat values are not important; leave them as they are.

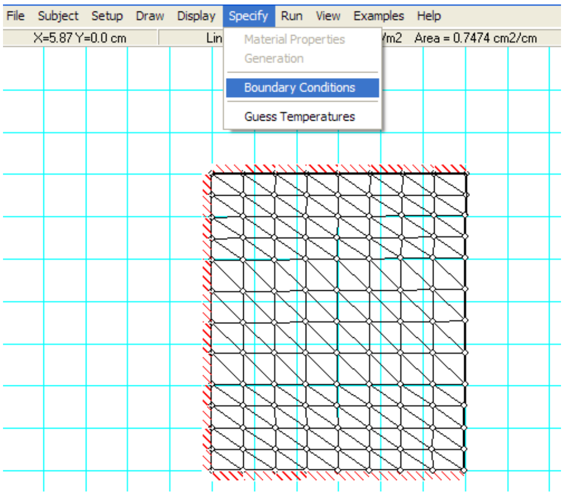
11. Now, enter heat gen. rate:

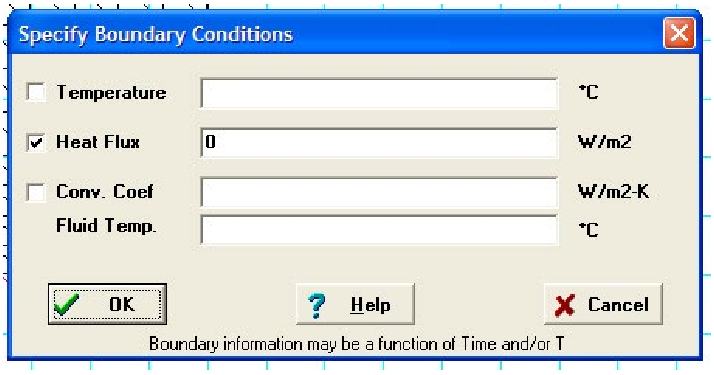




We have entered qg = 3E05 W/m^3, as given in data. Note that you can also enter qg as a function o T, X, Y etc.

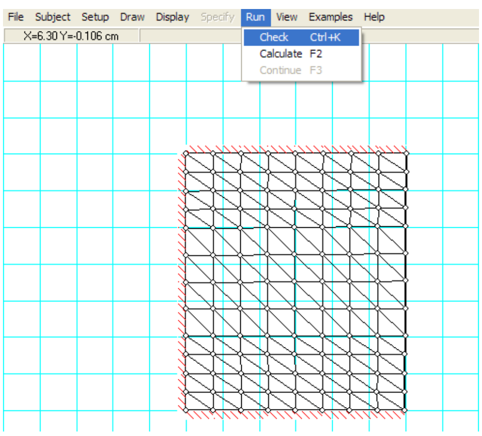
12. Next select the top, left and right sides to enter Boundary conditions: Left face is insulated, as given in data. Also show top and right sides as insulated, to make the heat transfer one-dimensional:



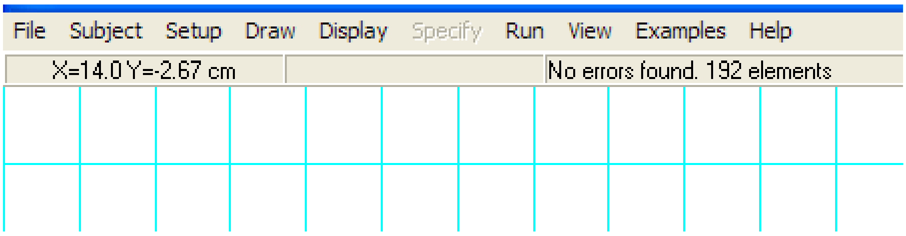


In the above, heat flux = 0 means that the face is insulated. Note that if the face is at a constant temp, tick the first choice and enter the value of temp. If there is convection condition on the face, tick that choice and enter values of h and T_inf.

13. After finishing entering all BC's, go to the Run menu. Press check:

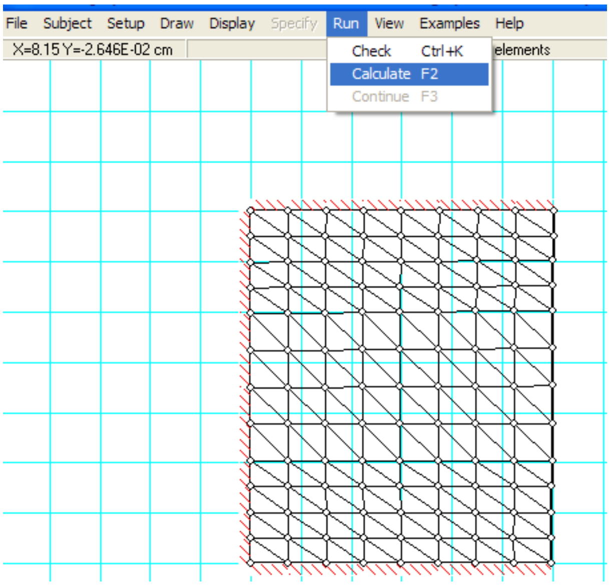


You get:

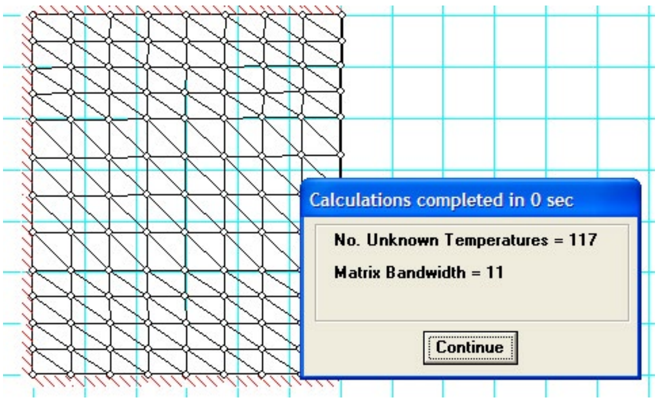


It shows that No errors are found, and the no. of elements in this case is: 192.

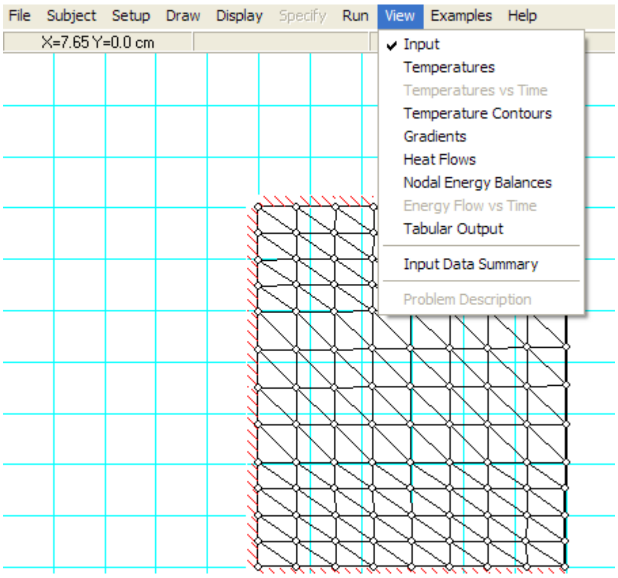
14. Next, from the Run menu click Calculate (or, press F2):



Following message appears: Click Continue:

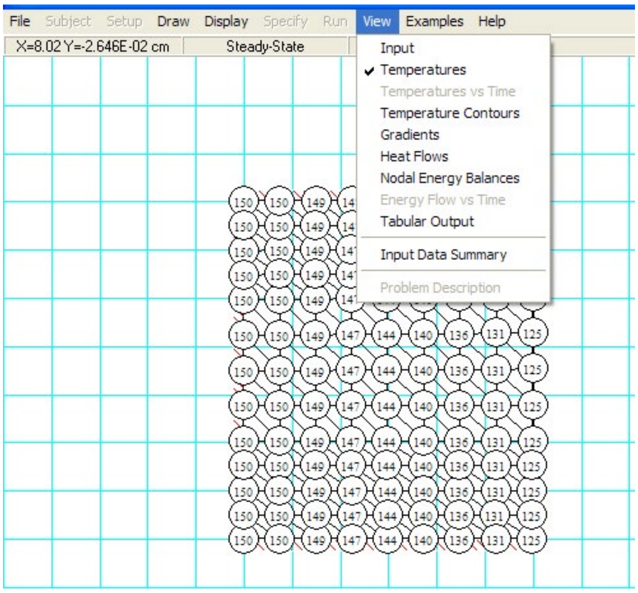


15. Now, view Results from the View menu: Many choices of output results are available:

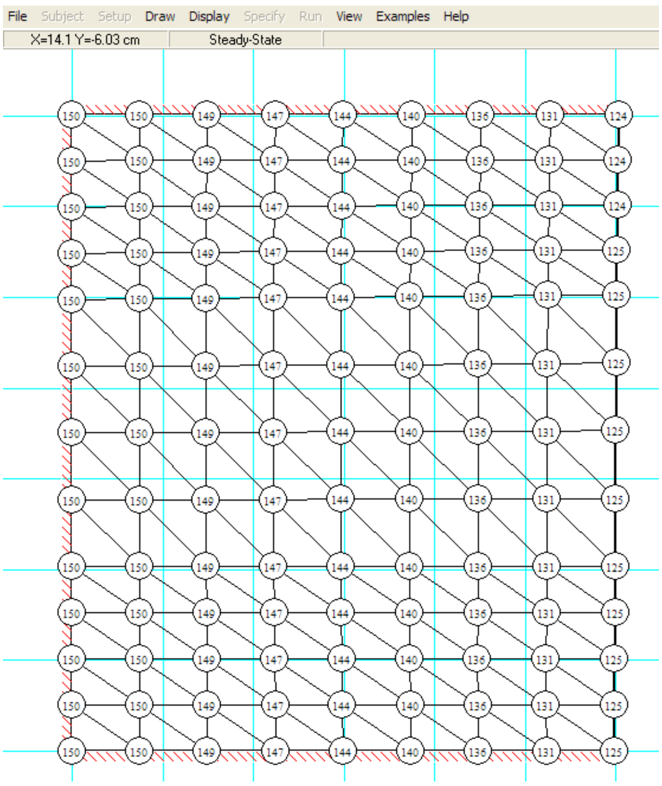


You can view Temperatures at different nodes, Temp. contours in many ways, Gradients, Heat flows, Node energy balances and you can also get the results in a Tabular form to copy and paste in some other application such as Excel for further processing or graphing etc.

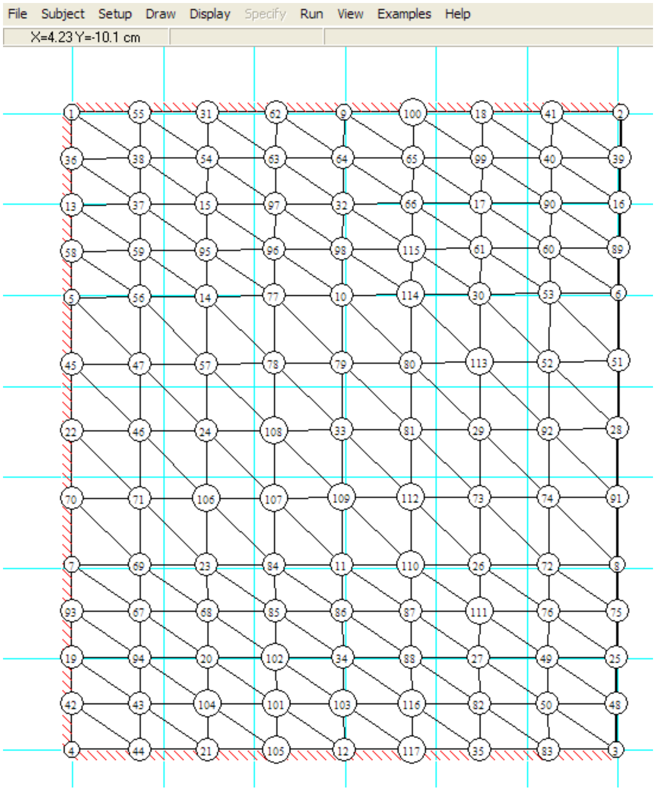
16. Now, view the temperatures at different Nodes:



You can Zoom the results, from the Display menu:



It is better to see these results with another window showing the Node Nos:



Now, to get the temp. profile: on the LHS, see the Node no. 22. Along that line we have the node nos. 22, 46, 24, 108, 33, 81, 29, 92 and 28. Corresponding temps are: (from the earlier picture): 150, 150, 149, 147, 144, 140, 136, 131 and 125 C approxly. To draw the temp profile accurately, view the Tabular Output in the View menu and copy and paste it to Excel, edit and draw the graph:

Tabular Results:

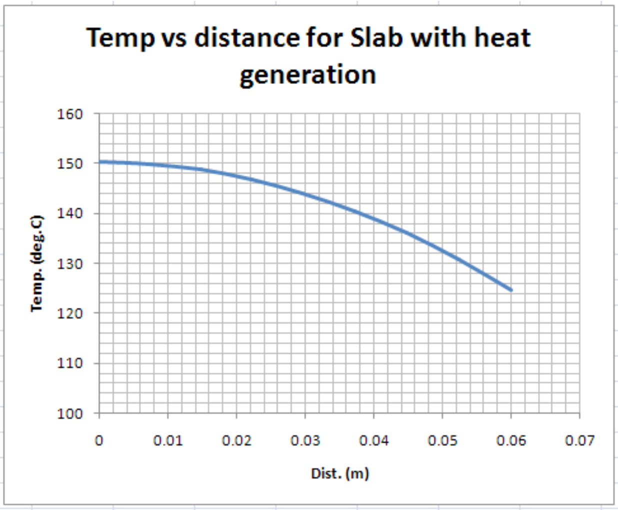
Tabular Nodal Results

Close | Print | Select All | Copy | Save as

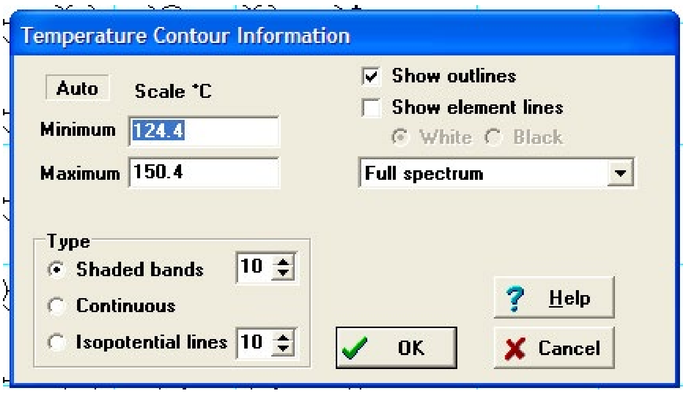
Node	X [m]	Y [m]	T [°C]	Node Balance [W/m]
1	0.05001	-0.0299	150.4	3.3307E-16
2	0.1103	-0.0299	124.4	-44.39
3	0.1098	-0.1	124.7	-46.06
4	0.05001	-0.1	150.2	0
5	0.05001	-0.05027	150.4	6.6613E-16
6	0.1101	-0.04974	124.6	-111.9
7	0.05001	-0.07964	150.3	-9.9920E-16
8	0.1099	-0.07964	124.6	-113.3
9	0.0799	-0.0299	144	1.1102E-16
10	0.07964	-0.05001	144.1	-8.8818E-16
11	0.07964	-0.07964	144	0
12	0.0799	-0.1	143.9	-4.4409E-16
13	0.05001	-0.04008	150.4	-4.4409E-16

We get:

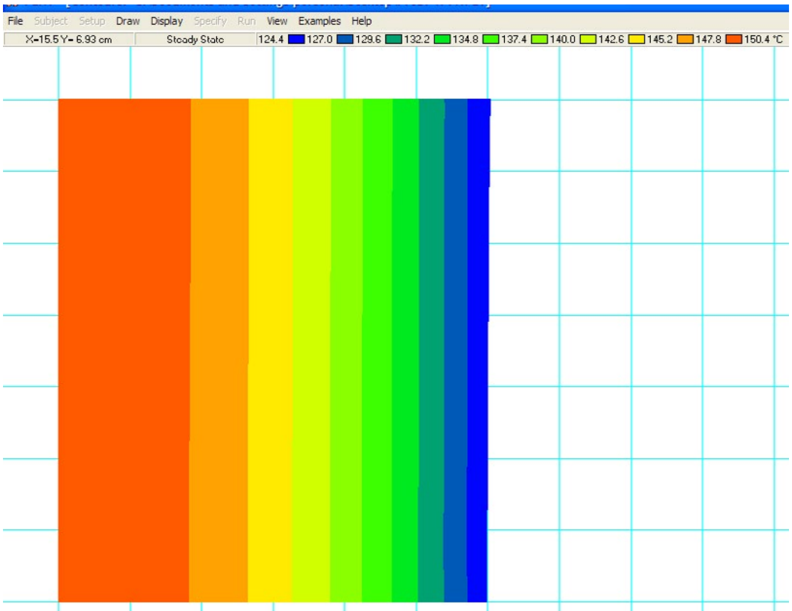
Node No.	X (m)	T(deg.C)
22	0	150.3
46	0.0074	149.9
24	0.01481	148.7
108	0.02222	146.8
33	0.02963	144
81	0.0372	140.4
29	0.04478	136
92	0.05239	130.7
28	0.05999	124.6



17. From the View menu, we can also get Temp contours in a variety of ways: Click on Temp contours. A screen appears:

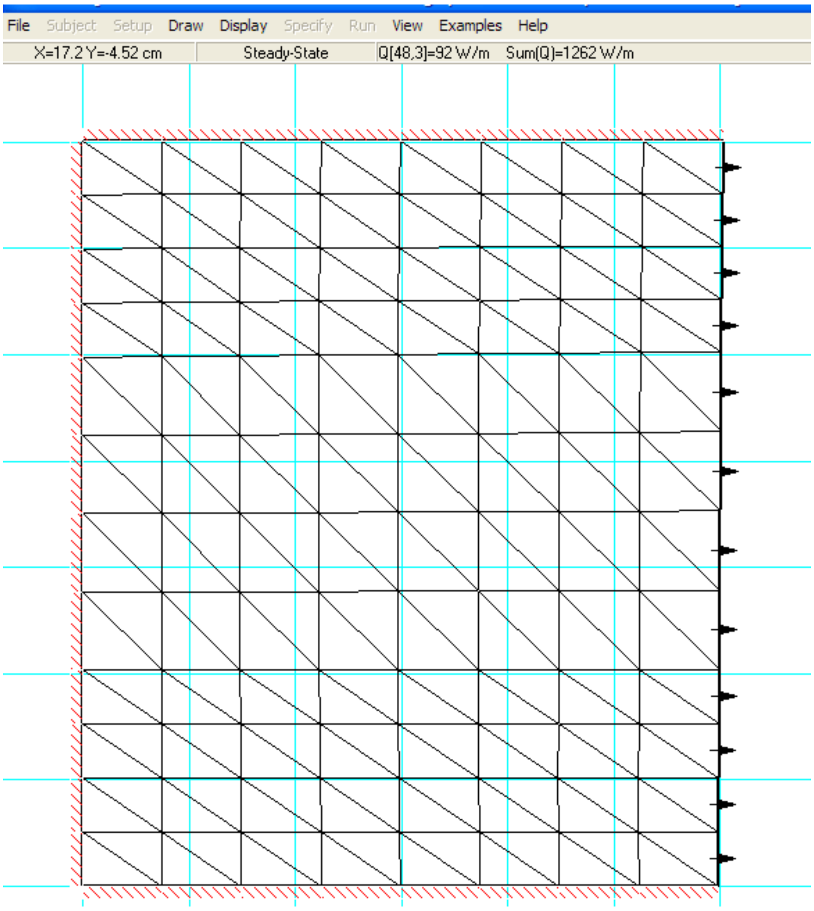


Press OK: Note that you can show outlines or element lines, Contours can be of Shaded bands, continuous or Iso-potential lines . For the choice shown above you get shaded bands as follows:



Note that on the top of the window, colour codes for temp range are shown.

18. If we click on Heat flow in the View menu and select the surface on the RHS through which heat will flow out, we get the following screen. Note on top of that window that total heat flow is 1262 W/m.:



As a check: Note that for this slab, L = 0.06 m (by data), H is arbitrarily taken as 0.07 m (see the fig.), and the depth perpendicular to the screen is 1 m.

Now, the total heat transferred from RHS must be equal to the heat generated in the slab. It is equal to: Volume × heat generation rate = (0.06 × 0.07 × 1 × 3 * 10^5) W = 1260 W. This matches well with the result Q = 1262 W/m.

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Prob. 1F.2. Heat is generated uniformly in a stainless steel plate having k = 20 W/m.K. The thickness of the plate is 1 cm and heat generation rate is 500 MW/m3. If the two sides of the plate are maintained at 100 and 200 C respectively, calculate the temperature at the centre of the plate. Also find the distance of the plate at which maximum temperature occurs from the 200 C surface.

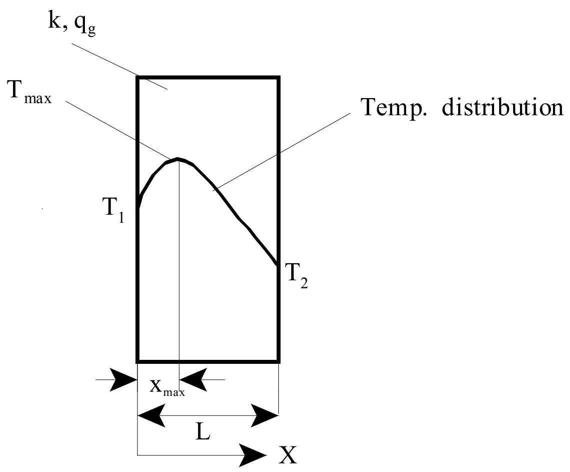


Fig.Prob.1F.2

Mathcad Solution:

Data:

$L := 0.01$ m ... thickness of plate

$q_g := 500 \cdot 10^6$ W/m^3 .... heat gen. rate

$T_1 := 200$ C ..temp. on left face $T_2 := 100$ C ..temp. on right face

$k := 20$ W/m.K ... th. conductivity

Calculations:

Temp distribution is given by:

$$T(x) := T_1 + \left[(L - x) \cdot \frac{q_g}{2 \cdot k} + \frac{T_2 - T_1}{L}\right] \cdot x$$

Note: x is measured from the left face. L is the thickness of plate.

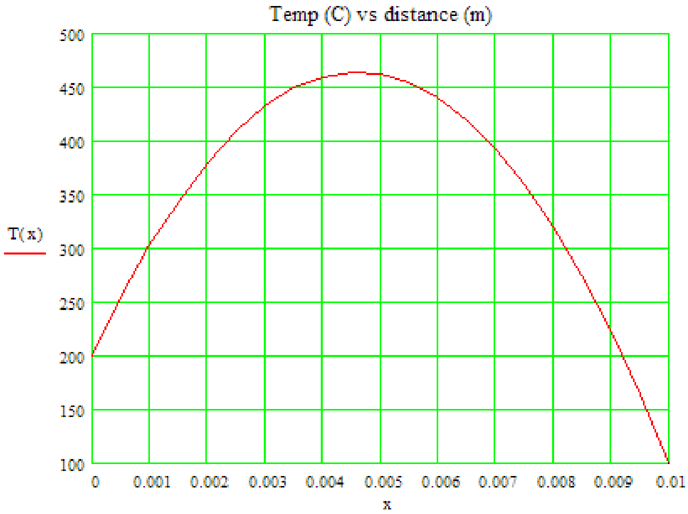
Check: $T(0) = 200$ C $T(0.01) = 100$ C

Therefore, temp. at the centre of the plate, i.e. at x = 0.005 m:

$T(0.005) = 462.5$ **C... Temp. at centre... Ans.**

Plot the temp distribution in the plate:

x := 0, 0.0005 .. 0.01 ...define a range variale x, from 0 to 0.01 m



Where does the max. temp. occur, and what is its value?

At the location where Tmax occurs, dT/dx = 0.

Apply this condition and use the root function of Mathcad to find the xmax.

Then substitute the value of xmax in T(x) and get the value of max. temp:

We have: $T(x) := T_1 + \left[(L - x)\cdot\frac{q_g}{2\cdot k} + \frac{T_2 - T_1}{L}\right]\cdot x$

Let $T'(x) := \frac{d}{dx}T(x)$ **...define the first derivative of T(x) w.r.t. x**

$x := 0.004$ ...trial value for xmax

$x_{max} := root(T'(x), x)$ ..using the root function

i.e. $x_{max} = 4.6\cdot 10^{-3}$ **m ... location where Tmax occurs, distance from left....Ans.**

$T(x_{max}) = 464.5$ **C ... max. temp.....Ans.**

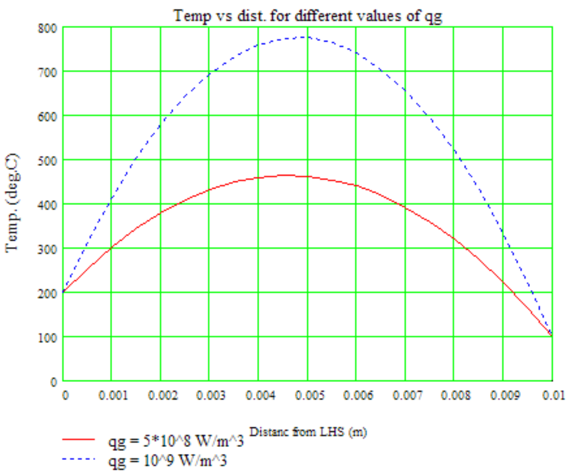
In addition, draw the temp profile if the heat gen. rate is doubled:

Define the temp as a function of x and qg:

$$T(x, q_g) := T_1 + \left[(L - x)\cdot\frac{q_g}{2\cdot k} + \frac{T_2 - T_1}{L}\right]\cdot x$$

$q_g = 5\cdot 10^8$ W/m^3

$x := 0, 0.0005 .. 0.01$ ...define a range variale x, from 0 to 0.01 m



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Prob. 1F.3. A 3.2 mm diameter stainless steel wire 30 cm long has a voltage of 10 V impressed on it. The outer surface temperature of the wire is maintained at 93 C. Calculate the centre temperature of the wire. Take the resistivity of the wire as 70 mW-cm and the thermal conductivity as 22.5 W/m.K.

(b). The heated wire in the above example is submerged in a fluid maintained at 93 C. The convection heat transfer coefficient is 5.7 kW/m.K. Calculate the centre temperature of the wire.

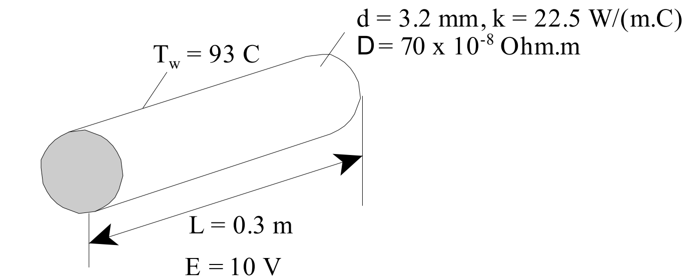


Fig.Prob.1F.3 (a) Wire with an impressed voltage, T_w is known

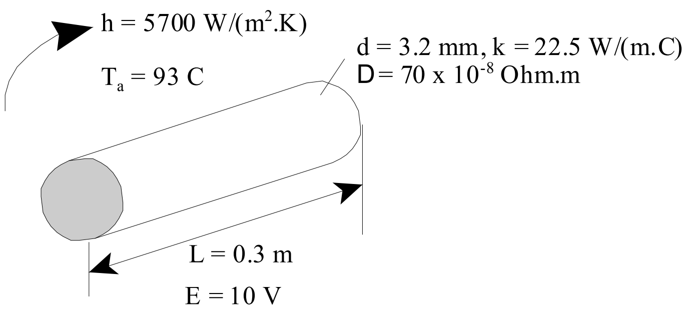


Fig.Prob.1F.3 (b) Wire with an impressed voltage, T_a is known

EES Solution:

"Data:"

d = 0.0032 [m] "..dia of wire"
L = 0.3 [m] "…length of wire"
E = 10 [V] "…voltage impressed"
T_w = 93 [C] "…outer surface temp of wire"
rho = 70e-08 [W-m] "…resistivity of wire"
k = 22.5 [W/m-C]

T_a = 93 [C]
h = 5.7e03 [W/m^2-C]

"Calculations:"

A_c = pi * d^2 /4 "[m^2] .. area of cross-section of wire"

Volume = A_c * L "[m^3] … total vol of wire"

Resist = rho * L / A_c "[ohms] …electrical resist of wire"

Q_gen = E^2 / Resist "[W] .. total heat generated"

q_g = Q_gen / Volume "[W/m^3] … heat gen. rate per unit volume, W/m3"

"Case 1:

For a cylindrical wire, we have the centre temp. (= max. temp), in the first case:"

T_centre1 = T_w + (q_g * (d / 2)^2) / (4 * k) "[C] .. centre temp, which is also the max. temp. in case 1"

"Case 2:

When there is convection at the surface:"

"By heat balance at the surface:"

Q_gen = h * (pi * d * L) * (T_wall – T_a) "…finds the wall temp. T_wall"

"Then, the centre temp in the second is given by:"

T_centre2 = T_wall + (q_g * (d / 2)^2) / (4 * k) "[C] .. centre temp, which is also the max. temp. in case 2"

Results:

Unit Settings: SI C kPa kJ mass deg

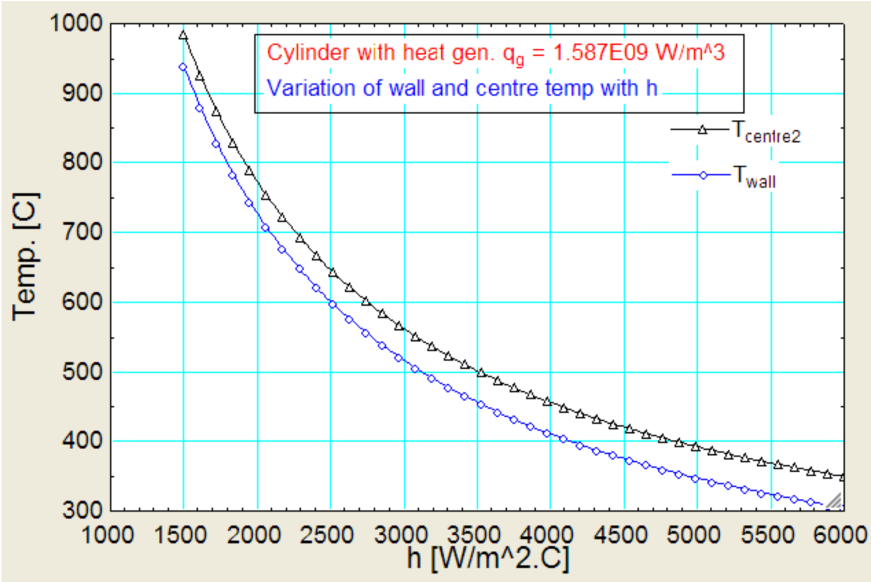
A_c = 0.000008042 [m^2]	d = 0.0032 [m]	E = 10 [V]
h = 5700 [W/m^2-C]	k = 22.5 [W/m-C]	L = 0.3 [m]
q_g = 1.587E+09 [W/m^3]	Q_{gen} = 3830 [W]	Resist = 0.02611 [Ω]
ρ = 7.000E-07 [W-m]	T_a = 93 [C]	$T_{centre1}$ = 138.1 [C]
$T_{centre2}$ = 360.9 [C]	T_w = 93 [C]	T_{wall} = 315.8 [C]
Volume = 0.000002413 [m^3]		

Thus:

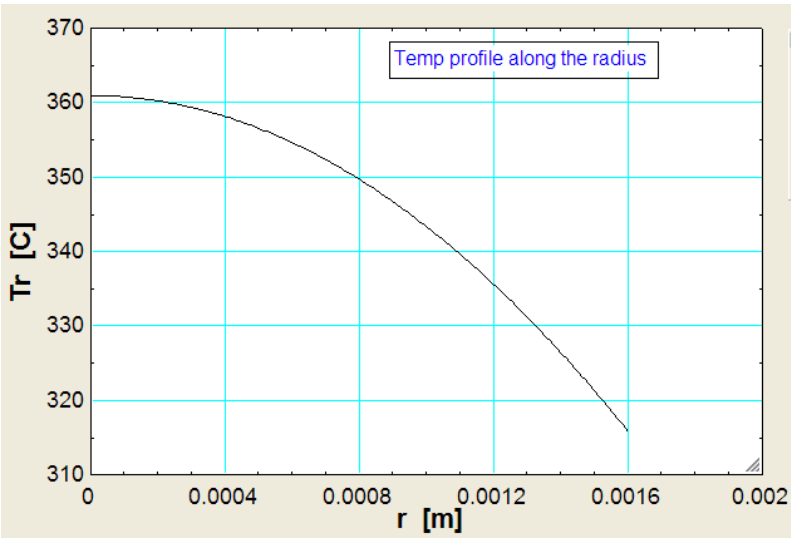
T_centre1 = 138.1 C …. Centre temp. when surface temp is 93 C… Ans.

T_centre2 = 360.9 C …. Centre temp. when there is convection at the surface to a fluid at 93 C… Ans.

Now, plot the variation of centre and wall temp. when h varies from 1000 to 6000 W/m^2.K:



Also draw the temp. profile along the radius when there is convection at the surface (i.e. case 2):



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"**Prob. 1F.4.** A long hollow cylinder has inner and outer radius as 5 cm and 15cm respectively. It generates heat at the rate of 1.0 kW/m^3. If the max temperature occurs at the radius of 10 cm and the temperature of the outer surface is 50 C, find

1) Temperature of the inner surface.
2) Maximum temperature in the cylinder.
   Assume k= 0.5 W/m.K. [M.U.]"

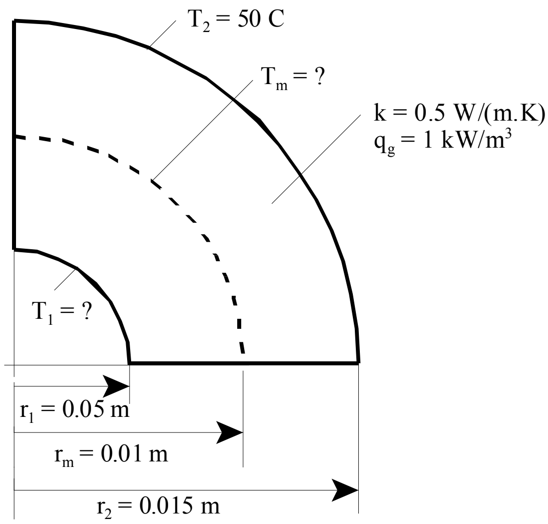


Fig.Prob.1F.4

EES Solution:

"Data:"

r_1 = 0.05 [m]
r_2 = 0.15 [m]
q_g = 1000 [W/m^3]
r_m = 0.1 [m]
T_2 = 50 [C]
k = 0.5 [W/m-C]

"Calculations:"

"It is a cylindrical system with heat generation.

So, the governing differential eqn is: d2T/dr2 + (1/r).(dT/dr) + (q_g / k) = 0

i.e. r.d2T/dr2 + dT/dr = (-q_g . r) / k

i.e. d/dr (r . dT/dr) = (-q_g . r) / k

Integrating once: r. dT/dr = -(q_g . r^2) / (2 .k) + C1

i.e. dT/dr = -(q_g . r) / (2 .k) + C1/r

And, integrating once again:

Tr = (-q_g . r^2) / (4 .k) + C1 . ln(r) + C2

Constants C1 and C2 are determined from BC's."

{r = 0.06 [m] "…trial value of r .. will be commented out later to draw the temp profile"}

Tr = -(q_g * r^2) / (4 * k) + C1 * ln(r) + C2 "eqn. (1) …. general solution for temp. profile"

"B.C.….(1): when r = r_2, Tr = T2:"

T_2 = -(q_g * r_2^2) / (4 * k) + C1 * ln(r_2) + C2 "…eqn.(2) … from BC-(1)"

"B.C.….(2): when r = 0.1, Tr = T_max: i.e. dT/dr = 0 at r_m = 0.1

Now, dT/dr = – (q_g . r) / (2 . k) + C1/r"

0 = (-q_g * r_m) / (2 * k) + C1 / r_m "….eqn (3) …. from BC-(2)"

"Eqns. (2) and (3) determine C1 and C2, and then eqn. (1) gives the temp profile:"

T_max = -(q_g * r_m^2) / (4 * k) + C1 * ln(r_m) + C2 "…gives the max. temp. at the radius r_m"

T_1 = -(q_g * r_1^2) / (4 * k) + C1 * ln(r_1) + C2 "…. gives the temp T_1 at the inner surface, i.e. at radius r_1"

Results:

Unit Settings: SI C kPa kJ mass deg

C1 = 10	C2 = 80.22	k = 0.5 [W/m-C]	q_g = 1000 [W/m^3]
r = 0.06 [m]	r_1 = 0.05 [m]	r_2 = 0.15 [m]	r_m = 0.1 [m]
Tr = 50.29 [C]	T_1 = 49.01 [C]	T_2 = 50 [C]	T_{max} = 52.2 [C]

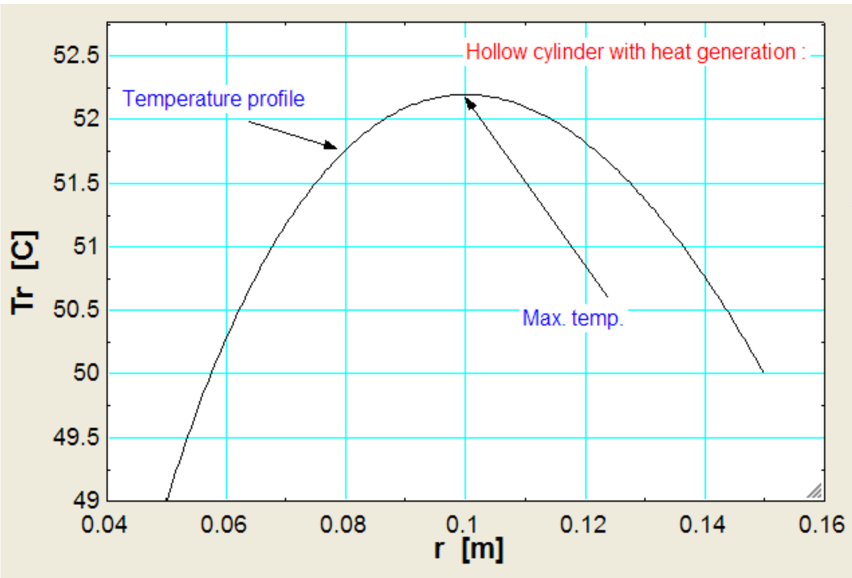
Thus:

T_1 = 49.01 C … temp at the inner surface … Ans.

T_max = 52.2 C ….max. temp …..Ans.

In addition:

Plot the temp. distribution along the radius:



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"Prob.1 F.5. A hollow conductor with r1 = 0.6 cm, r2 = 0.8 cm, and insulated on the outside, is made up of metal of k = 20 W/m.K and electrical resist. per metre of 0.03 ohms. Find the max. allowable current if the temp is not to exceed 50 C anywhere in the conductor. The cooling fluid at the inside is at 38 C. [M.U.]"

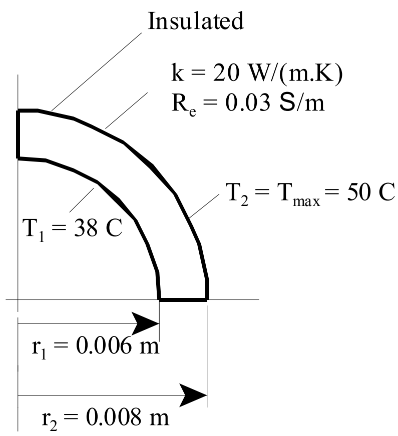


Fig.Prob.1F.5

EES Solution:

"Data:"

r_1 = 0.006 [m]
r_2 = 0.008 [m]
T_1 = 38 [C] "..temp. on inside surface"
T_2 = 50 [C] "….temp on outside surface"
k = 20 [W/m-C]
R_e = 0.03 [ohm/m]
L = 1 [m] "….length of conductor"

"Calculations:"

"Max. temp. occurs on the insulated surface, i.e. on the outer surface.:

"It is a cylindrical system with heat generation.

So, the governing differential eqn is: d2T/dr2 + (1/r).(dT/dr) + (q_g / k) = 0

i.e. r.d2T/dr2 + dT/dr = (-q_g . r) / k

i.e. d/dr (r . dT/dr) = (-q_g . r) / k

Integrating once: r. dT/dr = -(q_g . r^2) / (2 .k) + C1

i.e. dT/dr = -(q_g . r) / (2 .k) + C1/r

And, integrating once again:

Tr = (-q_g . r^2) / (4 .k) + C1 . ln(r) + C2

Constants C1 and C2 are determined from BC's."

r = 0.007 [m] "…trial value of r .. will be commented out later to draw the temp profile"

Tr = -(q_g * r^2) / (4 * k) + C1 * ln(r) + C2 "eqn. (1) …. general solution"

"B.C…..(1): when r = r_1, Tr = T_1:"

T_1 = -(q_g * r_1^2) / (4 * k) + C1 * ln(r_1) + C2 "…eqn.(2) … from BC-(1)"

"B.C…..(2): when r = r_2, Tr = T_2:"

T_2 = -(q_g * r_2^2) / (4 * k) + C1 * ln(r_2) + C2 "…eqn.(3) … from BC-(2)"

"BC .. (3): Also,T_2 is max. temp; i.e. dT/dr = 0 at r = r_2:"

0 = -(q_g * r_2) / (2 * k) + C1/r_2 " eqn. (4) ….i.e. temp is max at r = r_2"

"Eqns. (2), (3) and (4) determine C1, C2 and q_g ; then eqn. (1) gives the temp profile:"

Q_gen = I^2 * R_e "[W] … total heat generated by passage of electric current"

Vol = pi * (r_2^2 – r_1^2) * L "[m^3] .. vol. of conductor"

q_g = Q_gen / Vol "[W/m^3] … heat gen. per unit volume"

Results:

Unit Settings: SI C kPa kJ mass deg

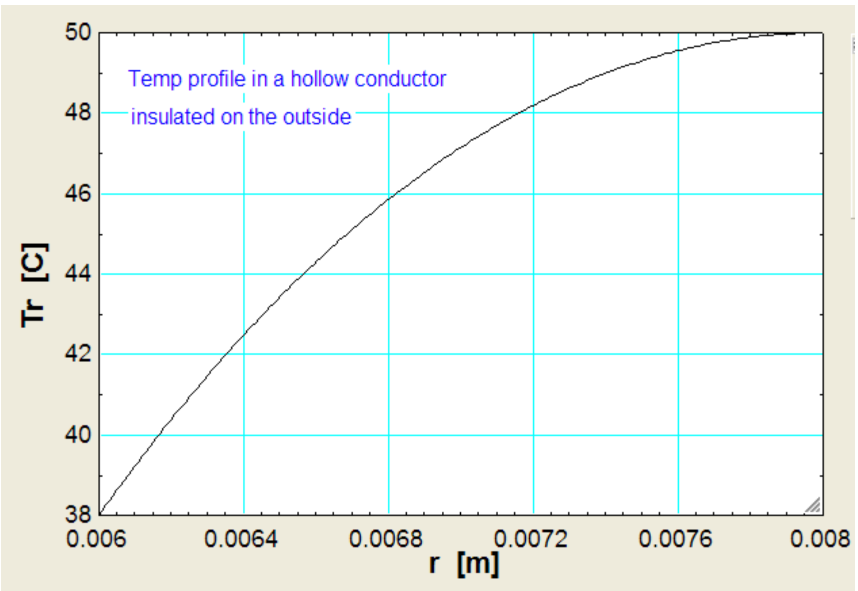
C1 = 174.1	C2 = 977.6	I = 564.8 [A]
k = 20 [W/m-C]	L = 1 [m]	q_g = 1.088E+08 [W/m^3]
Q_{gen} = 9571 [W]	r = 0.007 [m]	r_1 = 0.006 [m]
r_2 = 0.008 [m]	R_e = 0.03 [Ω/m]	Tr = 47.15 [C]
T_1 = 38 [C]	T_2 = 50 [C]	Vol = 0.00008796 [m^3]

Thus:

I = 564.8 A … max. allowable current … Ans.

q_g = 1.088E08 W/m^3 …. Volumetric heat gen. rate…. Ans.

Now, draw the temp. profile in the conductor:



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"**Prob. 1F.6.** A hollow cyl. 6 cm ID, 9 cm OD has a heat generation rate of 5*10^6 W/m^3. The inner surface is maintained at 400 C and outer surface at 320 C. k of material is 3 W/m. C. Determine: (i) the location and value of the max temp. (ii) the temp at mid radius, and (iii) fraction of heat generated going to the inner surface. [M.U.]"

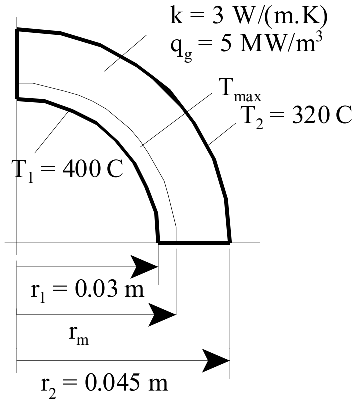


Fig.Prob.1F.6

EES Solution:

"Data:"

r_1 = 0.03 [m]
r_2 = 0.045 [m]
q_g = 5E06 [W/m^3]
T_1 = 400 [C] "..temp. on inside surface"
T_2 = 320 [C] "….temp on outside surface"
k = 3 [W/m-C]
L = 1 [m] "….length of conductor"

"Calculations:"

"It is a cylindrical system with heat generation.

So, the governing differential eqn is: d2T/dr2 + (1/r).(dT/dr) + (q_g / k) = 0

i.e. r.d2T/dr2 + dT/dr = (-q_g . r) / k

i.e. d/dr (r . dT/dr) = (-q_g . r) / k

Integrating once: r. dT/dr = -(q_g . r^2) / (2 .k) + C1

i.e. dT/dr = -(q_g . r) / (2 .k) + C1/r

And, integrating once again:

Tr = (-q_g . r^2) / (4 .k) + C1 . ln(r) + C2

Constants C1 and C2 are determined from BC's."

r = 0.0375 [m] "…mid-radius value of r .. will be commented out later to draw the temp profile"

Tr = -(q_g * r^2) / (4 * k) + C1 * ln(r) + C2 "eqn. (1) …. general solution"

"B.C…..(1): when r = r_1, Tr = T_1:"

T_1 = -(q_g * r_1^2) / (4 * k) + C1 * ln(r_1) + C2 "…eqn.(2) … from BC-(1)"

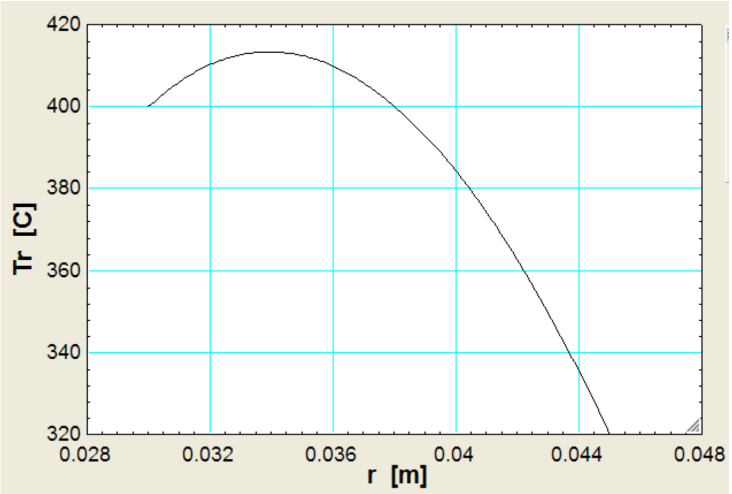
"B.C…..(2): when r = r_2, Tr = T_2: "

T_2 = -(q_g * r_2^2) / (4 * k) + C1 * ln(r_2) + C2 "…eqn.(3) … from BC-(2)"

"Eqns. (2), (3) determine C1, C2 ; then eqn. (1) gives the temp profile:"

"Location and value of max. temp.:

First, draw the temp profile:"



"It is seen that max. temp. occurs at about r = 0.034 m. and the approx. value is about 412 C.

We know that at the max temp, slope dT/dr = 0. Use this condition to get corresponding location, r_m.

Then, substitute r_m in the expression for temp distribution and get T_max:

Now, dT / dr = 0 gives:"

0 = (– q_g * 2 * r_m)/ (4 * k) + C1 / r_m "…gives value of r_m, where max temp occurs"

"Then, max. temp:"

T_max = -(q_g * r_m^2) / (4 * k) + C1 * ln(r_m) + C2 "[C] … value of max. temp."

"Fraction of heat generated going to the inner surface:"

Vol = pi * (r_2^2 – r_1^2) * L " [m^3] … vol. of conductor of 1 m length"

Q_gen = q_g * Vol "[W] ..heat gen. in the conductor"

Q_in = pi * (r_m^2 – r_1^2) * L * q_g "[W] … heat that is generated between r_m and r_1, which has to go to inside surface only, since r_m is the location of max temp"

"Therefore:"

Fraction = Q_in / Q_gen "… fraction of heat gen. that goes to the inner surface"

Results:

Unit Settings: SI C kPa kJ mass deg

C1 = 958.8	C2 = 4137	Fraction = 0.2227
k = 3 [W/m-C]	L = 1 [m]	q_g = 5.000E+06 [W/m^3]
Q_{gen} = 17671 [W]	Q_{in} = 3935 [W]	r = 0.0375 [m]
r_1 = 0.03 [m]	r_2 = 0.045 [m]	r_m = 0.03392 [m]
Tr = 403 [C]	T_1 = 400 [C]	T_2 = 320 [C]
T_{max} = 413.3 [C]	Vol = 0.003534 [m^3]	

Thus:

T_max = 413.3 C …. Value of max. temp … Ans.

r_m = 0.03392 m ….location of max. temp. … Ans.

Tr = 403 C … temp at mid-radius, i.e. at r = 0.0375 m … Ans.

Fraction of heat gen. going to inner surface = (Q_in / Q_gen) = 0.2227 = 22.27 % … Ans.

"Now, check these results with direct formulas for a hollow cyl. from Ref:[1]"

r_m_check = sqrt((q_g * (r_2^2 – r_1^2) – 4 * k * (T_1 – T_2)) / (q_g * 2 * ln (r_2 / r_1))) "…see the eqn for r_m"

"Temp. profile: is given by:"

(T_mid – T_1) / (T_2 – T_1) = A + B * (C – D) ".. T_mid is the temp at mid-radius.

Verify this with the value of Tr calculated above at mid-radius, r = 0.0375 m"

"where:"

A = ln (r/ r_1) /ln (r_2 / r_1)

B = q_g * (r_2^2 – r_1^2) / (4 * k * (T_2 – T_1))

C = ln(r / r_1) / ln (r_2 / r_1)

D =((r / r_1)^2 – 1) / ((r_2 / r_1)^2 – 1)

"---"

"Also, Check the max. temp."

(T_max_check – T_1) / (T_2 – T_1) = A11 + B11 * (C11 – D11)

A11 = ln (r_m_check/ r_1) /ln (r_2 / r_1)

B11 = q_g * (r_2^2 – r_1^2) / (4 * k * (T_2 – T_1))

C11 = ln(r_m_check / r_1) / ln (r_2 / r_1)

D11 =((r_m_check / r_1)^2 – 1) / ((r_2 / r_1)^2 – 1)

"We see that we get exactly the same values for r_m, T_mid and T_max:"

Unit Settings: SI C kPa kJ mass deg

A = 0.5503	A11 = 0.3028	B = -5.859
B11 = -5.859	C = 0.5503	C1 = 958.8
C11 = 0.3028	C2 = 4137	D = 0.45
D11 = 0.2227	Fraction = 0.2227	k = 3 [W/m-C]
L = 1 [m]	q_g = 5.000E+06 [W/m^3]	Q_{gen} = 17671 [W]
Q_{in} = 3935 [W]	r = 0.0375 [m]	r_1 = 0.03 [m]
r_2 = 0.045 [m]	r_m = 0.03392 [m]	$r_{m,check}$ = 0.03392
Tr = 403 [C]	T_1 = 400 [C]	T_2 = 320 [C]
T_{max} = 413.3 [C]	$T_{max,check}$ = 413.3	T_{mid} = 403
Vol = 0.003534 [m^3]		

Note that: Tr = T_mid. … mid-radius temperatures by both methods match.

r_m = r_m,check .. position where max, temp occurs calculated by both methods match.

Also, T_max = T_max_check = 413.3 C … value of max temp by both the methods are the same.

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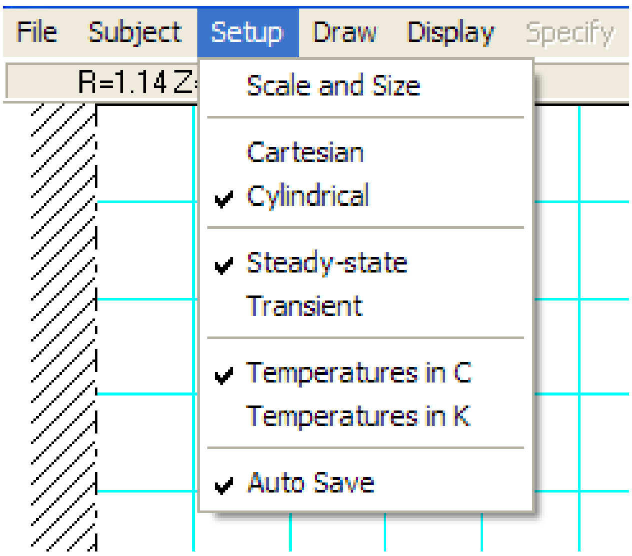
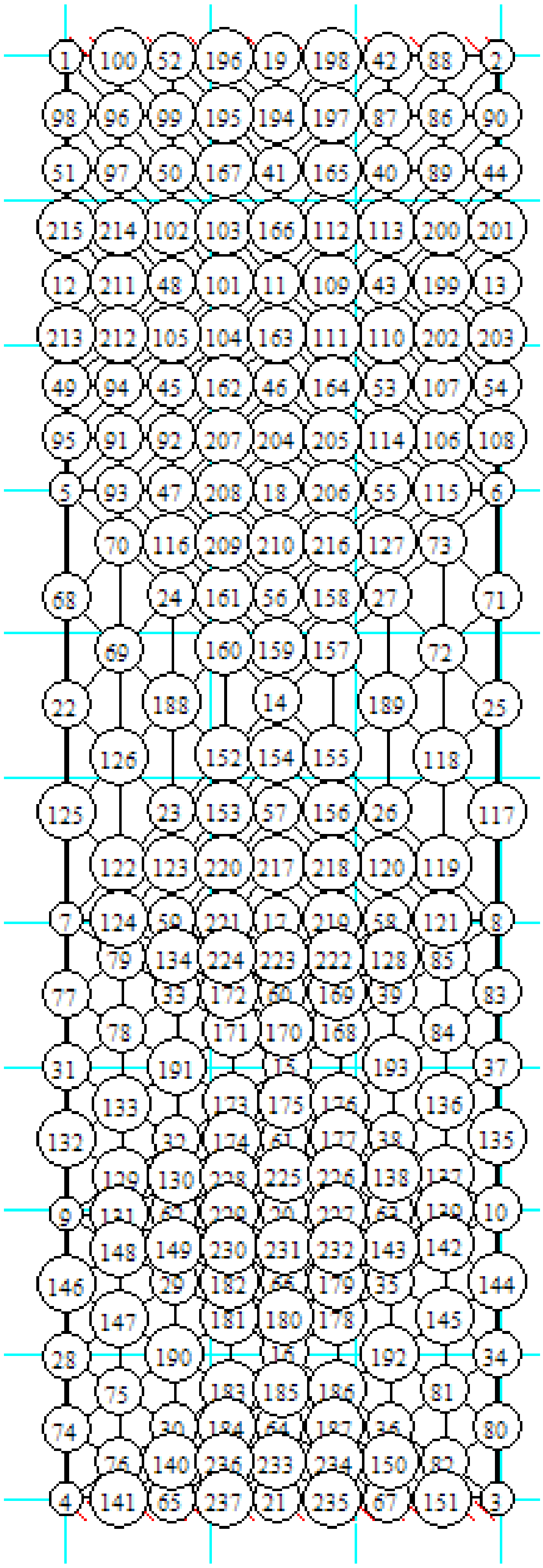
Solve the above problem by Finite Element Heat Transfer (FEHT) Software:

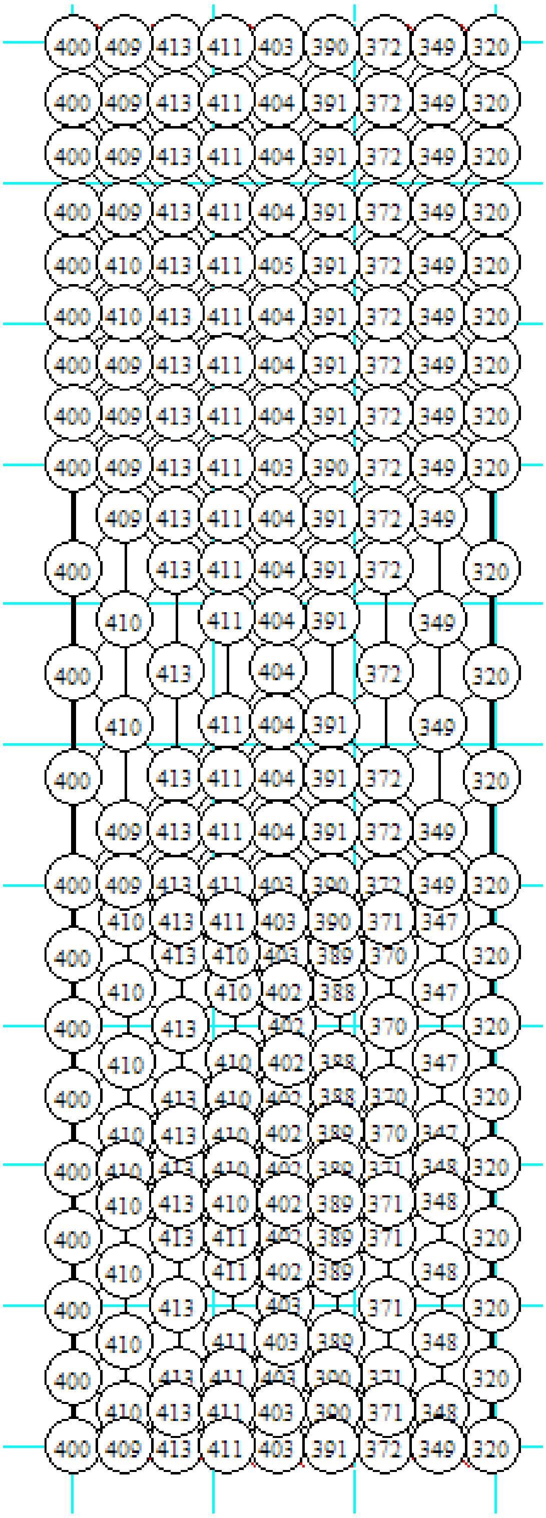
See prob. 1F.1. Steps are the same as for that problem. However, now choose Cylindrical from the Setup menu:



1. **Node positions and Node Numbers:**

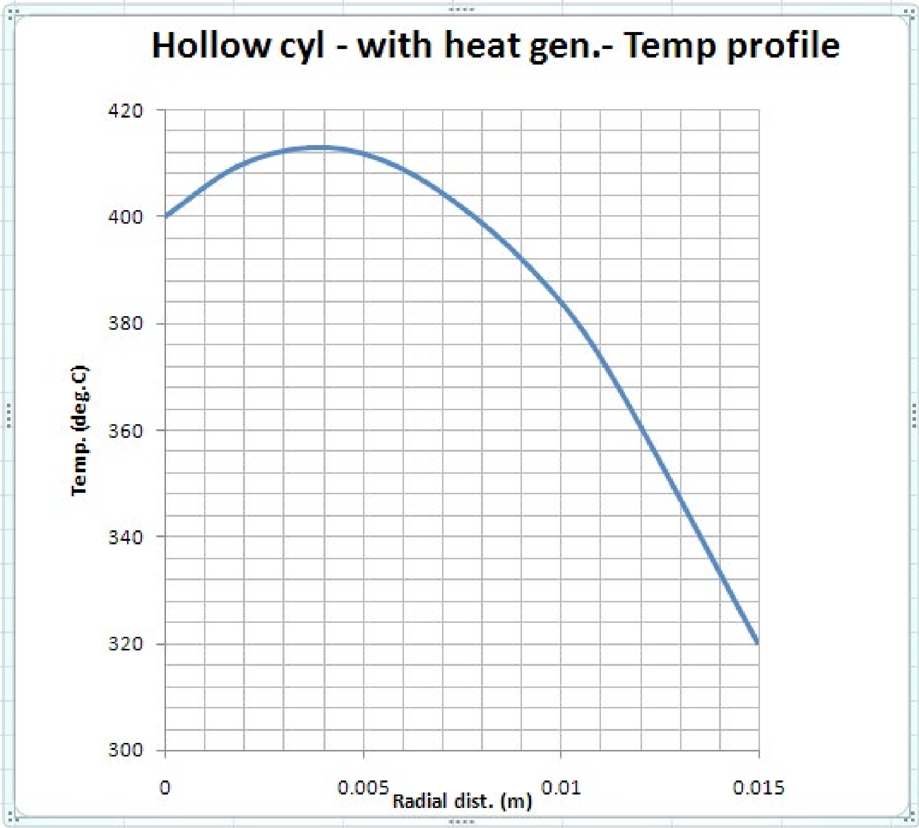


2. **Node Temperatures:**



3. **View Tabular output, copy to Excel, edit and draw the plot of T vs R:**

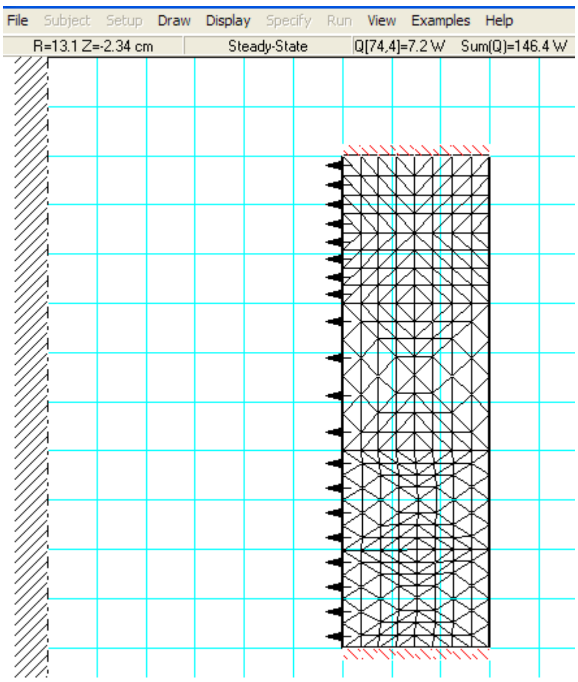
Node No.	R(m)	T(deg.C)
5	0	400
93	0.00182	409.4
47	0.00364	412.9
208	0.00546	410.8
18	0.00728	403.1
206	0.00919	390.4
55	0.01111	372.1
6	0.01495	320



Note that Tmax = about 412.9 C at R = 0.0036 m.

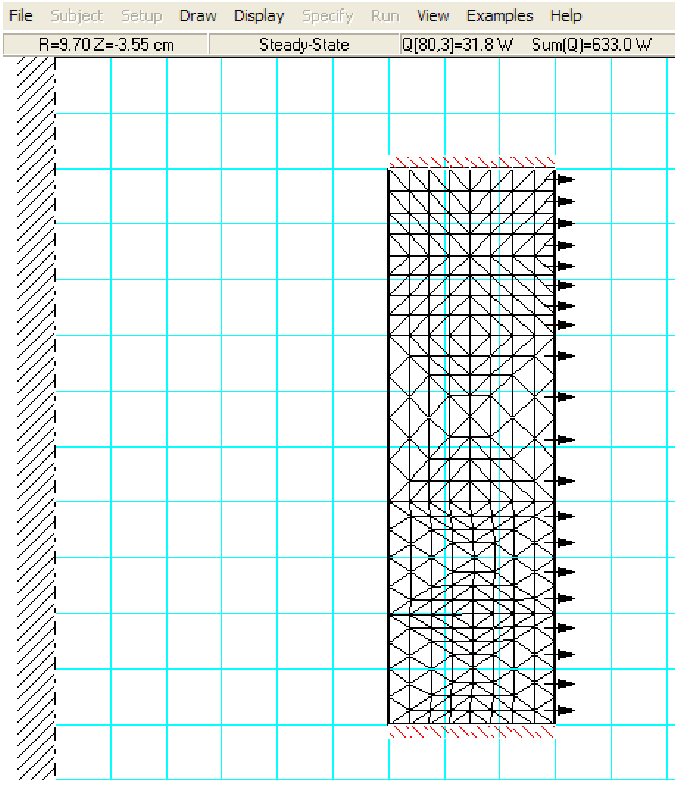
T = about 403 C at R = 0.0075, i.e. at mid-radius.

4. **Heat transfer on the left side, Q_L:**



Q_L = 146.4 W for a cyl length of 5 cm in the model.
Therefore, for 1 m length of cyl, Q_L = 146.4 × 20 = 2928 W.

Similarly for heat tr on the RHS, Q_R:



For 1 m length of cyl, Q_R = 633 × 20 = 12660 W.

Therefore, Total Q = Q_L + Q_R = 15588 W.

So, Q_L as a percentage of Q_Total = 18.8%

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Prob. 1F.7. A thin hollow tube with 4 mm inner diameter and 6 mm outer diameter carries a current of 1000 amperes. Water at 30 C is circulated inside the tube for cooling the tube. Taking heat transfer coefficient on the water side as 35,000 W/m2'c and k for the material as 18 W/m°c, estimate the surface temperature of the tube if its outer surface is insulated. Electrical resistance of the tube is 0.0065 ohms per meter length. [M.U.]

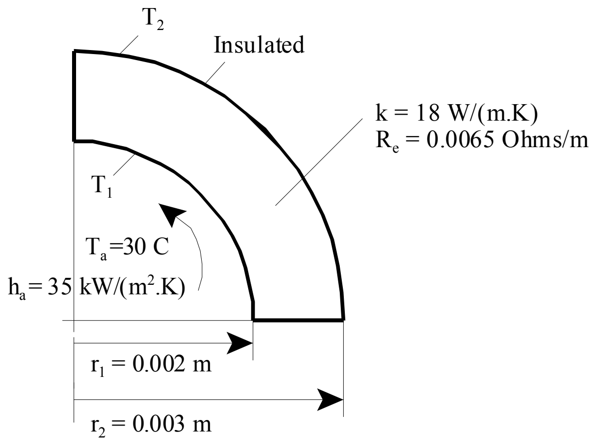


Fig.Prob.1F.7

Mathcad Solution:

This is the case of a cylinder, cooled on inside, insulated on outside:

We shall first solve this problem from fundamentals and then verify the results by applying the direct formulas given in the Table.

Data:

$r_1 := 0.002$ m... inner radius $r_2 := 0.003$ m... outer radius $I := 1000$ A

$k := 18$ W/m.K $R_e := 0.0065$ Ohms/m $L := 1$ m .. length of tube

$T_a := 30$ C $h_a := 35000$ W/m^2.C

Outer surface is insulated.

Calculations:

$Vol := \pi \cdot \left(r_2^{\,2} - r_1^{\,2}\right) \cdot L$ m^3 ... vol. of tube

$Q_{gen} := I^2 \cdot R_e$ i.e. $Q_{gen} = 6.5 \cdot 10^3$ W

$q_g := \frac{Q_{gen}}{Vol}$ i.e. $q_g = 4.138 \cdot 10^8$ W/m^3 .... volumetric heat gen. rate

Since the outer surface is insulated, all the heat generated has to travel only to inside surface and then to cooling water by convection.

Let T1, T2 be the temp on the inner and outer surfaces respectively.

Then, by heat balance at the inner surface:

$$h_a \cdot \left(2 \cdot \pi \cdot r_1 \cdot L\right) \cdot \left(T1 - T_a\right) = Q_{gen}$$

Therefore: $T1 := \frac{Q_{gen}}{\left[h_a \cdot \left(2 \cdot \pi \cdot r_1 \cdot L\right)\right]} + T_a$

i.e. $T1 = 44.779$ **C .... temp. on the inner surface**

Now, for a cylindrical system with heat gen. the controlling differential eqn is:

$$\frac{d^2}{dr^2}T + \frac{1}{r}\cdot\frac{d}{dr}T + \frac{q_g}{k} = 0$$

i.e. $$r \cdot \frac{d^2}{dr^2}T + \frac{d}{dr}T = \frac{-q_g \cdot r}{k}$$

i.e. $$\frac{d}{dr}\left(r \cdot \frac{d}{dr}T\right) = \frac{-q_g \cdot r}{k}$$

Integrating once:

$$\frac{d}{dr}T = \frac{-q_g \cdot r}{2 \cdot k} + \frac{C1}{r} \qquad \text{...eqn.(a)}$$

And, integrating again, we get its general solution as:

$$T(r) = \frac{-q_g \cdot r^2}{4 \cdot k} + C1 \cdot \ln(r) + C2 \qquad \text{...eq.(1)}$$

where C1 and C2 are obtained by applying the Boundary Conditions.

Now, for this problem:

BC..(1): at r = r1, T= T1

BC….(2): at r = r2, dT/dr = 0 since insulated.

Apply these conditions and get C1 and C2 and substitute them in eqn.(1) to get the solution for temp distribution:

Use the Solve Block of Mathcad. Start with guess values for C1 and C2:

$C1 := 10 \qquad C2 := 10 \qquad$ ...guess values

Given

$$T1 = \frac{-q_g \cdot r_1^{\,2}}{4 \cdot k} + C1 \cdot \ln(r_1) + C2$$

$$0 = \frac{-q_g \cdot r_2}{2 \cdot k} + \frac{C1}{r_2}$$

$$\text{Find}(C1, C2) = \begin{bmatrix} 103.451 \\ 710.673 \end{bmatrix}$$

i.e. $\quad C1 := 103.451 \quad C2 := 710.673$

Therefore, temp distribution is given by:

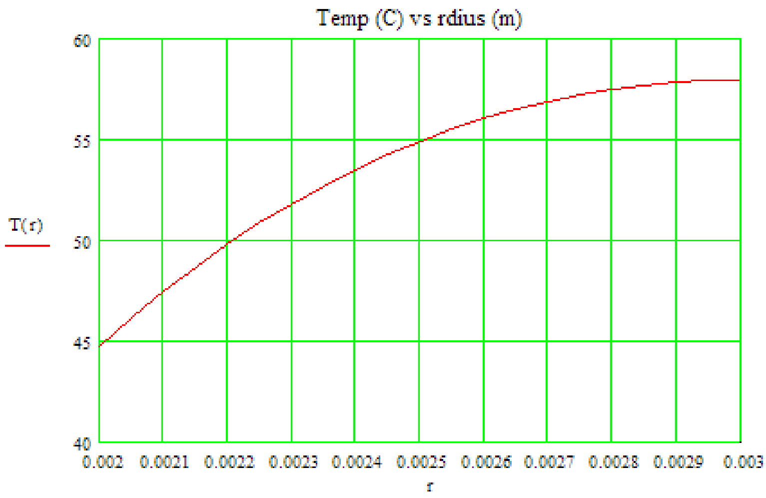
$$T(r) := \left(\frac{-q_g \cdot r^2}{4 \cdot k} + C1 \cdot \ln(r) + C2 \right) \quad ...eq.(1)$$

Then, temp of insulated, outer surface is:

$T(r_2) = 57.986$ **C .... max. temp. occurring at the insulated, outer surface ... Ans.**

Immediately, let us draw the temp. profile:

$r := 0.002, 0.00205 .. 0.003$ ...define the range variable r



Verify the result by applying the direct formula:

For an annular tube, insulated on the outside:

$$T(r) := T1 + \frac{q_g \cdot r_2^{\,2}}{4 \cdot k} \cdot \left[2 \cdot \ln\left(\frac{r}{r_1}\right) + \left(\frac{r_1}{r_2}\right)^2 - \left(\frac{r}{r_2}\right)^2 \right]$$

Therefore: temp on the outer surface, i.e. at r = r2:

$T(r_2) = 57.988$ **C .... max. temp. occurring at the insulated, outer surface ...**

same Ans. obtained earlier.

Now, plot the variation of T2 with ha. What happens to T2 if the coolant water stops? (i.e. h = 0)

First, write T2 as a function of r and h_a:

$$T1(h_a) := \frac{Q_{gen}}{\left[h_a \cdot (2 \cdot \pi \cdot r_1 \cdot L)\right]} + T_a$$

$$T(r, h_a) := T1(h_a) + \frac{q_g \cdot r_2^{\,2}}{4 \cdot k} \cdot \left[2 \cdot \ln\left(\frac{r}{r_1}\right) + \left(\frac{r_1}{r_2}\right)^2 - \left(\frac{r}{r_2}\right)^2\right]$$

$T(r_2, h_a) = 57.988$ C....temp. at r = r2 with ha = 35000 W/m^2.C .... verified.

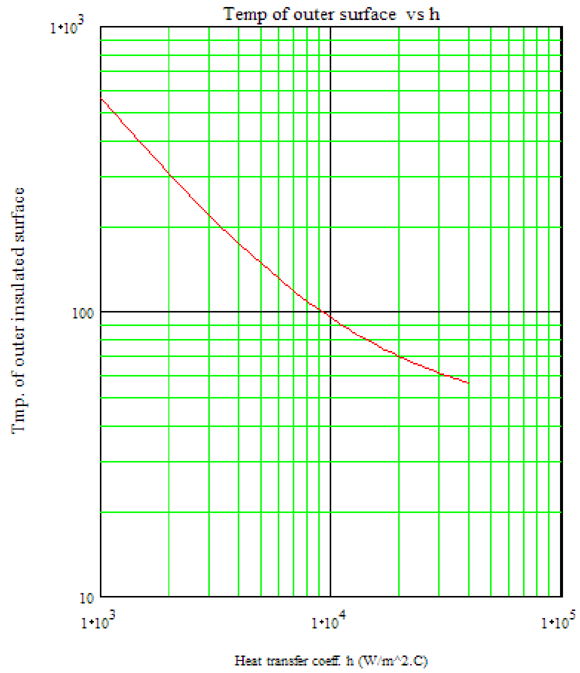
If the coolant stops, i.e. h_a = 0:

$T(r_2, 0.00001) = 5.173 \cdot 10^{10}$ ... C !!!!

i.e. T2 attains a very high value that the material will melt. Note that we have put h_a = 0.00001, a very small value, instead of 0, to avoid dividing by zero in the expression for T1.

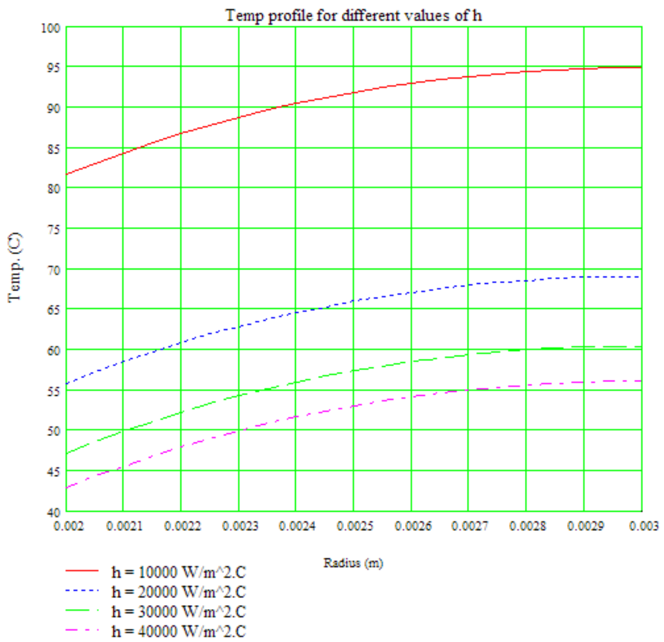
Plot of T2 vs ha:

$h_a := 1000, 2000 .. 40000$ ... define a range variable h_a.



Plot of Temp profle for h = 10000, 20000 and 40000 W/m^2.C:

$r := 0.002, 0.00205 .. 0.003$ ...define a range variable r



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"Prob. 1F.8. A nuclear fuel element is in the form of a hollow cylinder insulated at the inner surface. Its inner and outer radii are 50 mm and 100 mm respectively. Outer surface gives heat to a fluid at 50 C when the unit surface conductance is 100 W/m^2.C. Thermal cond. of the material is 50 W/m.C. Find the rate of heat generation so that max temp in the system will not exceed 200 C. [P.U.]"

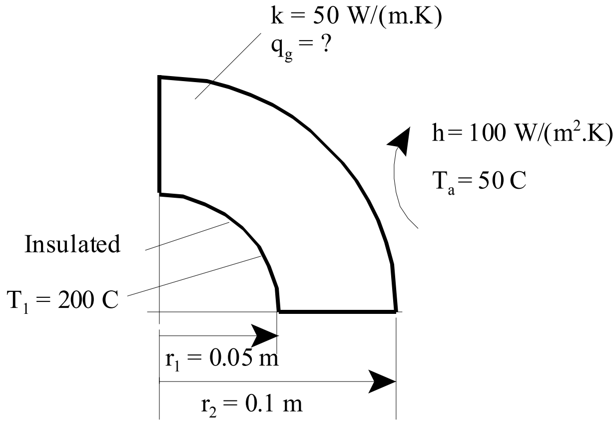


Fig.Prob.1F.8

"Data:"

r_1 = 0.05 [m]
r_2 = 0.1 [m]
T_1 = 200 [C] "..temp. on inside surface, max. since insulated"
k = 50 [W/m-C]
h = 100 [W/m^2.C]
T_a = 50 [C]
L = 1 [m] "….length of conductor"

"Calculations:"

"All the heat generated in the hollow tube is dissipated by convection at the outer surface.

Let q_g be the heat gen. rate per unit volume. And, let T_2 be the temp of outer surface. Then:"

Q_gen = q_g * (pi * (r_2^2 – r_1^2) * L) "[W] … total heat generated … eqn. (A)"

Q_gen = h * (2 * pi * r_2 * L) * (T_2 – T_a) ".. by heat balance …. eqn. (B)"

"It is a cylindrical system with heat generation.

So, the governing differential eqn is: d2T/dr2 + (1/r).(dT/dr) + (q_g / k) = 0

i.e. r.d2T/dr2 + dT/dr = (-q_g . r) / k

i.e. d/dr (r . dT/dr) = (-q_g . r) / k

Integrating once: r. dT/dr = -(q_g . r^2) / (2 .k) + C1

i.e. dT/dr = -(q_g . r) / (2 .k) + C1/r

And, integrating once again:

Tr = (-q_g . r^2) / (4 .k) + C1 . ln(r) + C2

Constants C1 and C2 are determined from BC's."

{r = 0.06 [m] "… any radius r .. will be commented out later to draw the temp profile"}

Tr = -(q_g * r^2) / (4 * k) + C1 * ln(r) + C2 "eqn. (1) …. general solution"

"B.C.....(1): when r = r_1, Tr = T_1:"

T_1 = -(q_g * r_1^2) / (4 * k) + C1 * ln(r_1) + C2 "…eqn.(2) … from BC-(1)"

"B.C.....(2): when r = r_2, Tr = T_2: "

T_2 = -(q_g * r_2^2) / (4 * k) + C1 * ln(r_2) + C2 "…eqn.(3) … from BC-(2)"

"Also, max. temp occurs at the insulated inner surface:, i.e. dT/dr = 0 at r = r_1:"

0 = -(q_g * r_1) / (2 *k) + C1/r_1 "….eqn.(4)"

"Eqns. (2), (3) and (4) determine C1, C2 ; then eqn. (1) gives the temp profile.

Eqn. (A) gives the heat gen. rate per unit volume, q_g, and eqn. (B) gives T_2"

"---"

"**Now, check these results** with direct formulas for a hollow cyl. insulated on the inner surface: from Ref:[1]"

(T_1 – T_2) = ((q_g_check * r_1^2) / (4 * k)) *((r_2 / r_1)^2 – 2 * ln(r_2 / r_1) – 1) "…gives qg_check"

"---"

"**Temp. profile: is given by eqn. (1).**

Using it plot the Tr vs r:"

Results:

Unit Settings: SI C kPa kJ mass deg

C1 = 9.49	C2 = 233.2	h = 100 [W/m2-C]
k = 50 [W/m-C]	L = 1 [m]	q_g = 379582 [W/m^3]
Q_{gen} = 8944 [W]	$q_{g,check}$ = 379582 [W/m^3]	r = 0.06 [m]
r_1 = 0.05 [m]	r_2 = 0.1 [m]	Tr = 199.6 [C]
T_1 = 200 [C]	T_2 = 192.3 [C]	T_a = 50 [C]

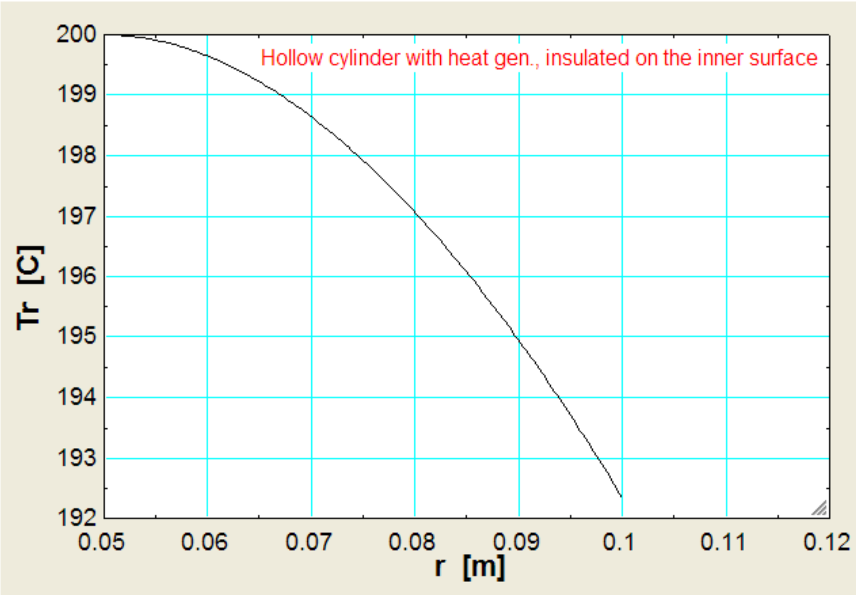
Thus:

q_g = 379582 W/m^3 …. Heat gen. rate per unit vol…..Ans.

q_g as per ready formula is: qg_check = 379582 W/m^3 …. Checks.

T_2 = 192.3 C …. Temp. of outer surface …. Ans.

Temp. profile along the radius:



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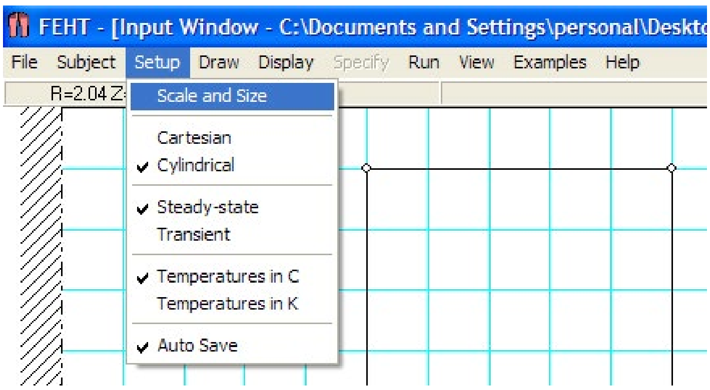
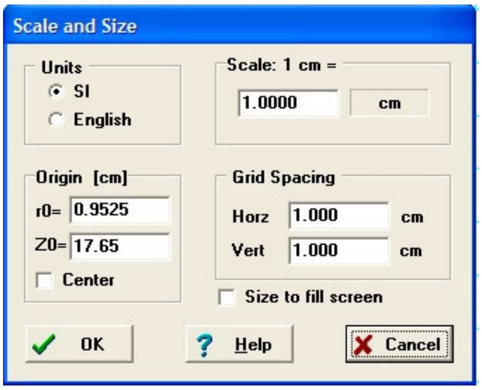
Solve Problem 1F.8 with Finite Element Heat Transfer (FEHT):

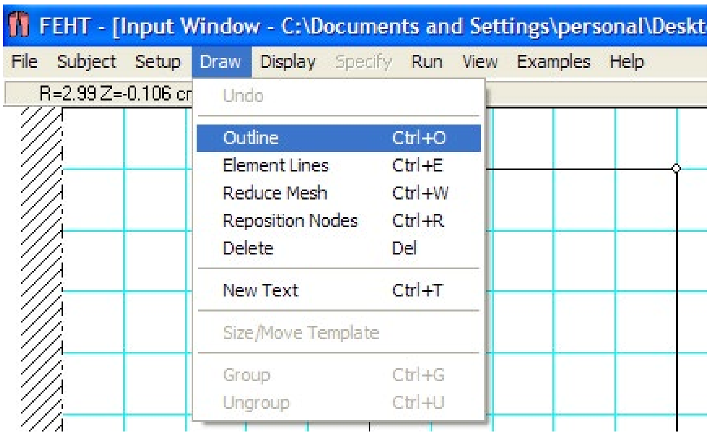
1. After starting FEHT, select Setup-cylindrical, and Steady State, and Scale and Size:



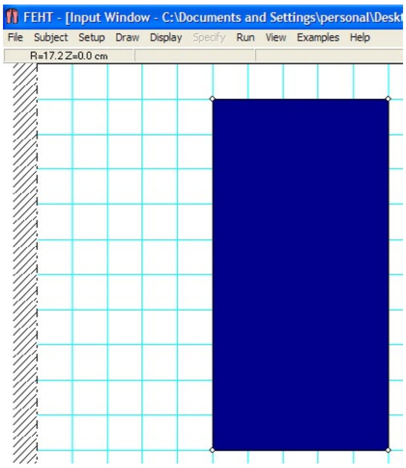
2. Choose Scale: 1 cm = 1 cm:



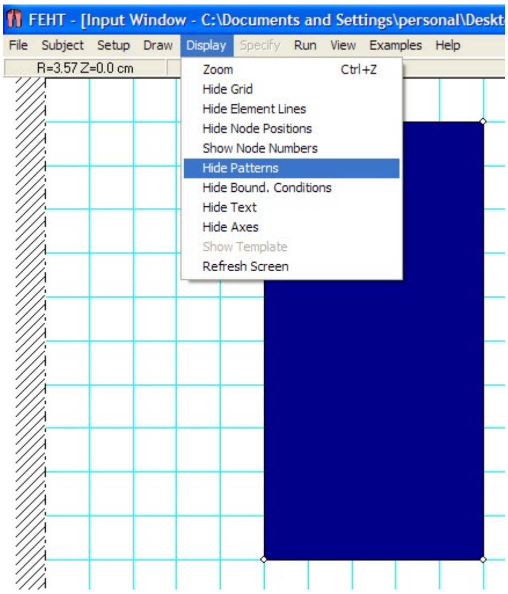
3. To draw the model: click on Draw-Outline:



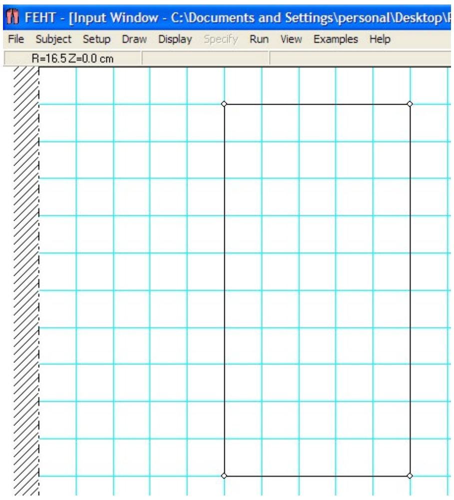
And, draw the outline:



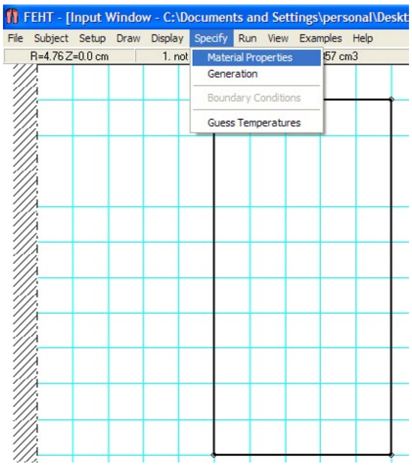
4. Select Draw-Hide Patterns to have a clear picture of model:



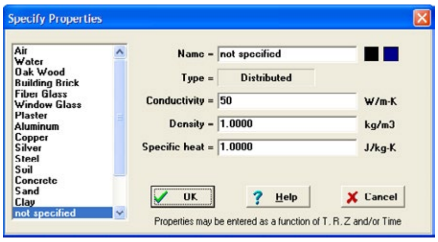
And, we get:



5. Select Specify-Material Properties:



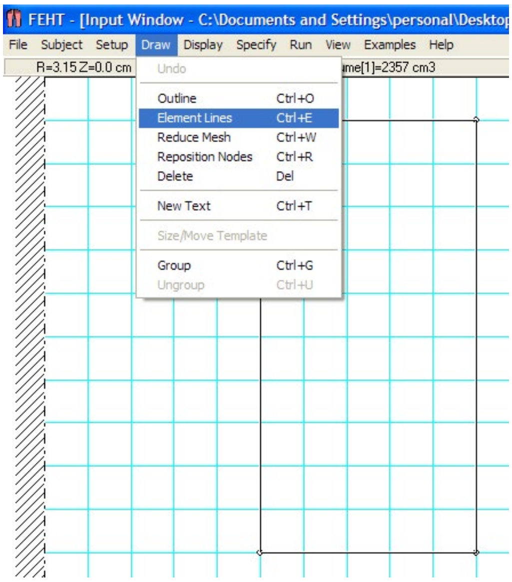
We get the following screen:



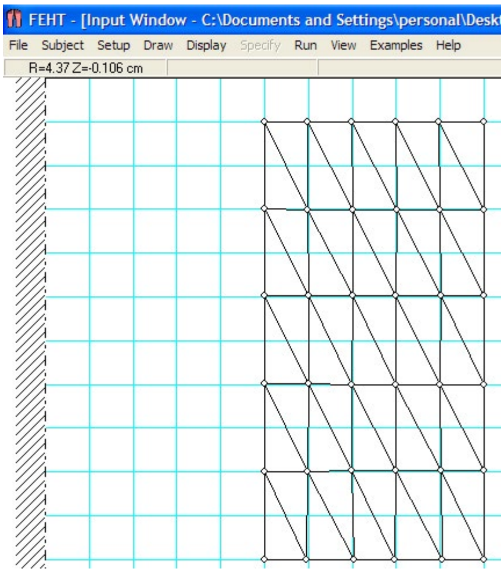
Enter thermal cond. value = 50 W/m.C.

For steady state problem, Density and Sp.heat are not required; so, leave the default values as they are.

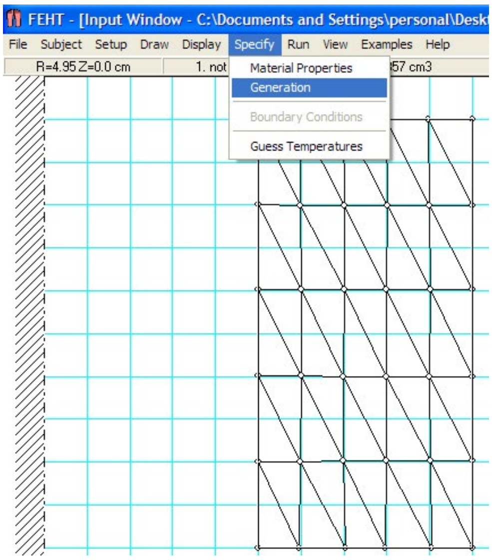
6. Now, draw triangular elements: Select Draw-Element lines:



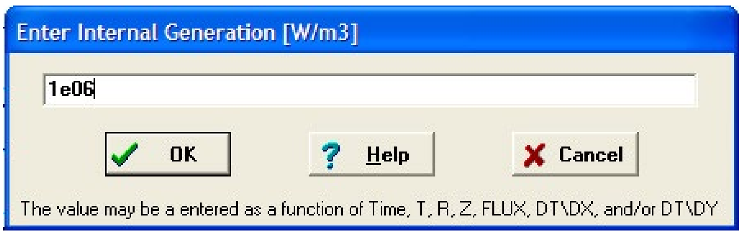
And, complete drawing element lines:



7. Now, select the model and click on Specify-Generation to enter the Heat gen. rate in the material:

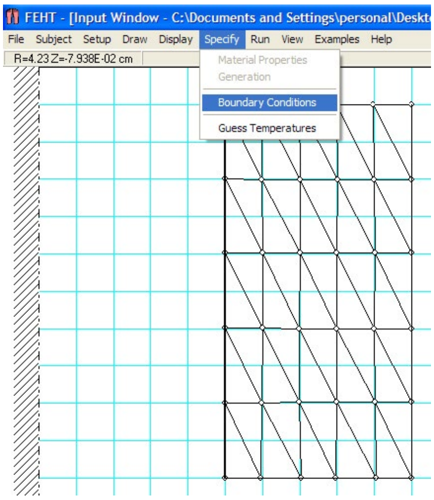


We get the following screen:

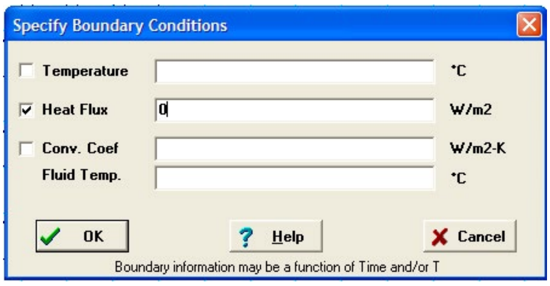


In the above, we entered qg = 1e06 W/m^3. The reason is: we don't know the heat gen. rate and we have to find it out such that for the given Boundary conditions(BC's), we obtain the inside surface temp T1 as 200 C. So, qg = 1e06 W/m^3 is the first trial value.

8. Now, to enter the BC's: Select the entire left side boundary, and select Specify-Boundary conditions:

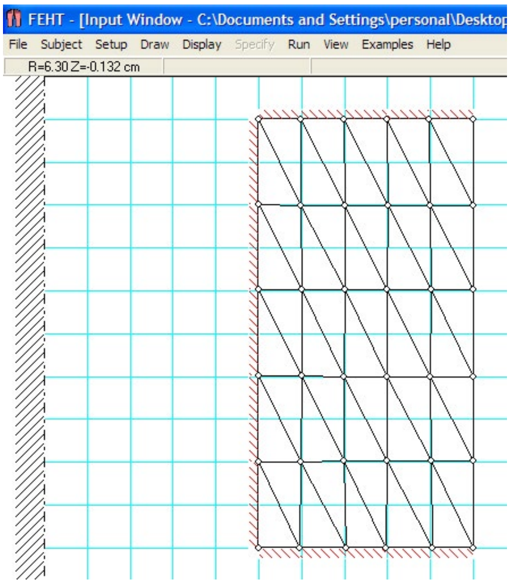


We get the following screen. Enter heat flux = 0 since, by data, the left surface is insulated:

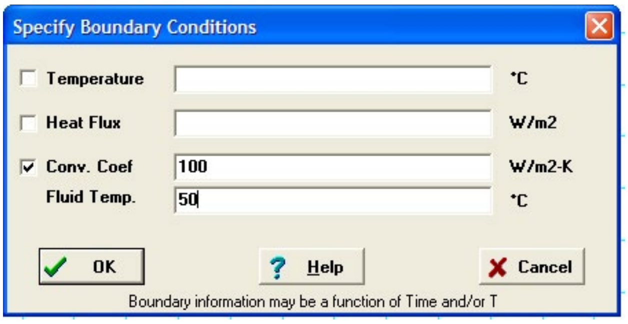


Hit OK.

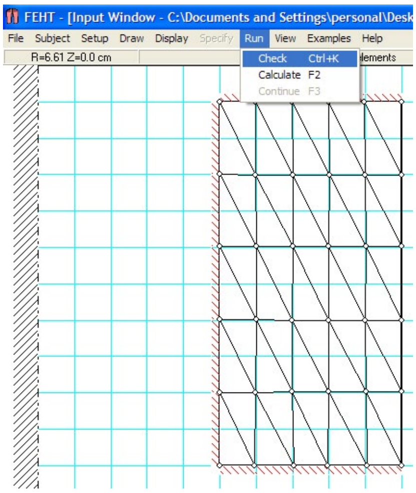
Now, repeat the same BC's for top and bottom boundaries also, so that the is 1D conduction in the R-direction only:



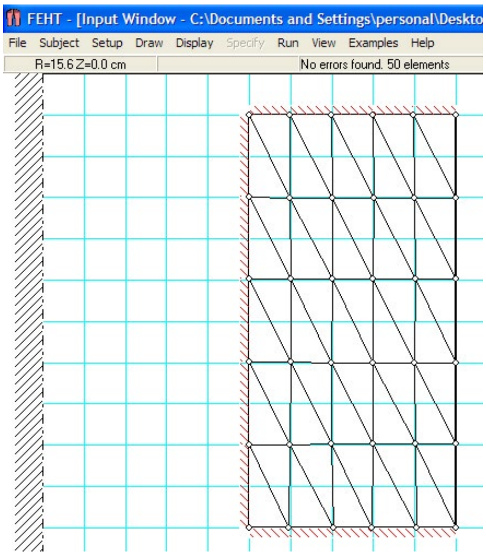
9. Now, select the entire boundary on RHS and Specify-BC with given h and T_inf values:



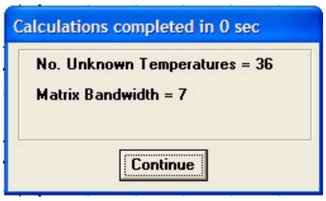
10. Now, click on Run-Check to check whatever is done so far:



We get following screen: No errors, and there are 50 elements:

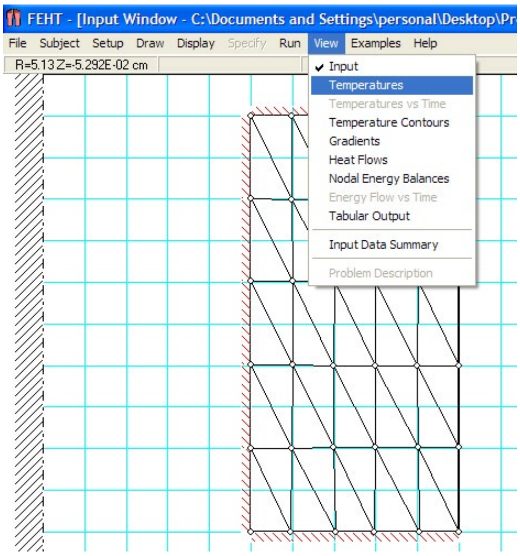


11. Now, click on Run-Calculate: And we get following screen:

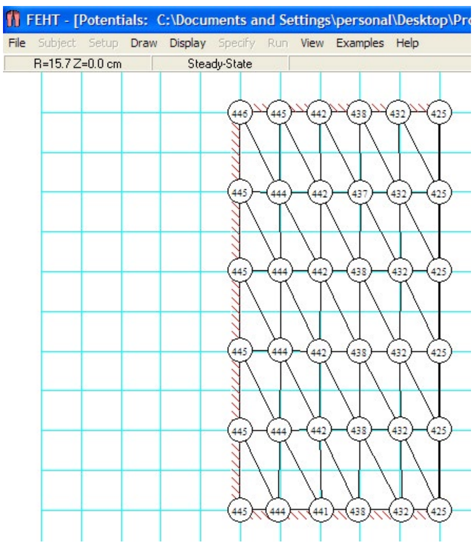


Click on Continue.

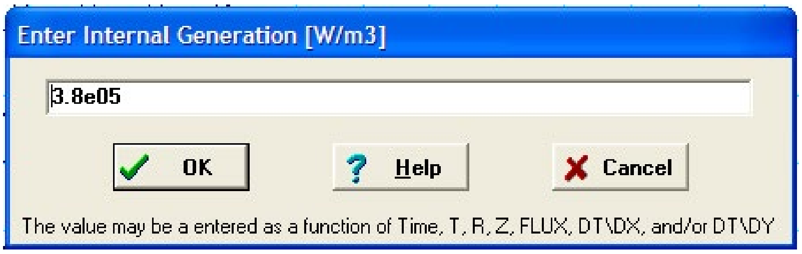
12. Now, we are ready to view the results. Click on View-Temperatures:



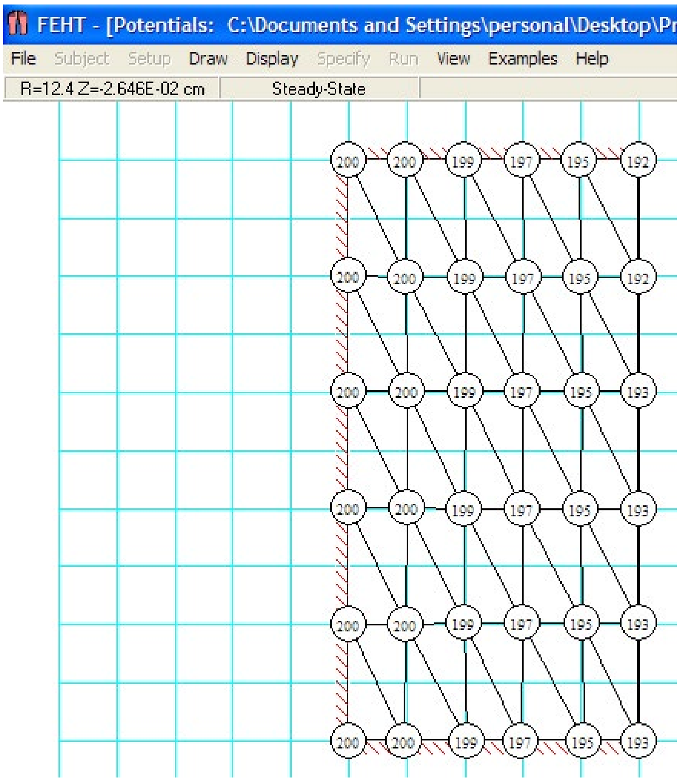
We get the Temperatures at various Nodes:



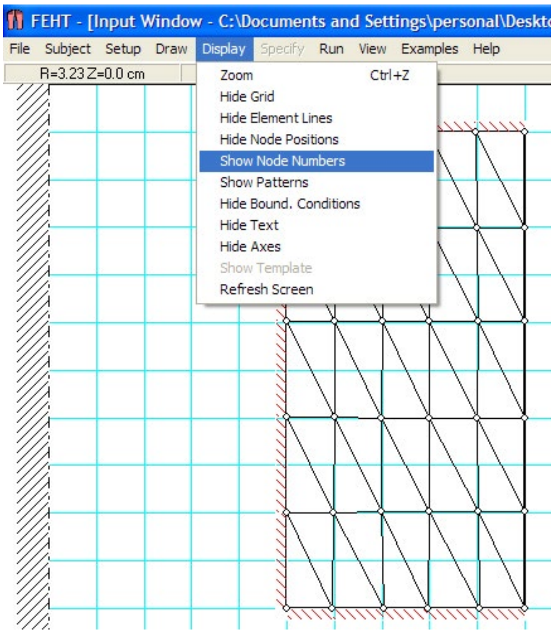
13. Now, observe that Temp on LHS is 445 deg.C. But, we have to get it as 200 C. So, reduce the heat gen. rate and repeat the calculations. This is an easy trial and error process.



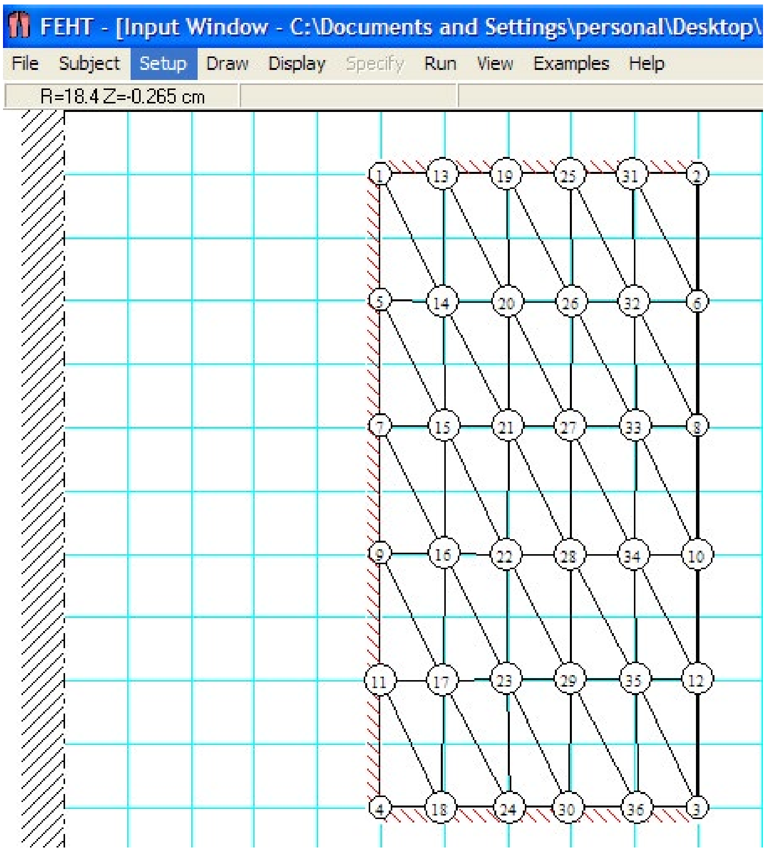
Finally, with qg = 3.8e06 W/m^3, we get the temp on LHS as 200 C:



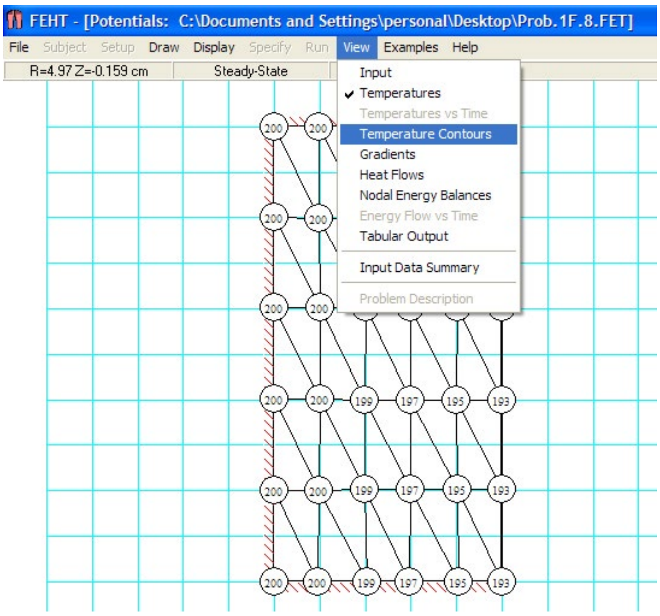
14. Next, we would like to relate temperatures with Node nos. so that we can draw temp. profile in the cyl. Shell. Choose Display-Show Node Numbers:



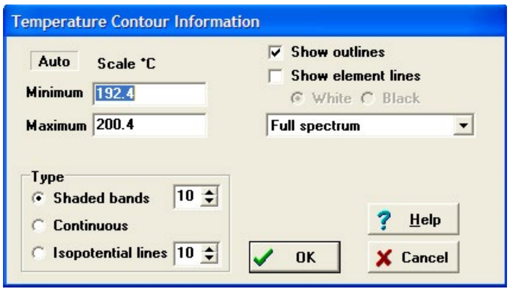
And the Node nos. are displayed:



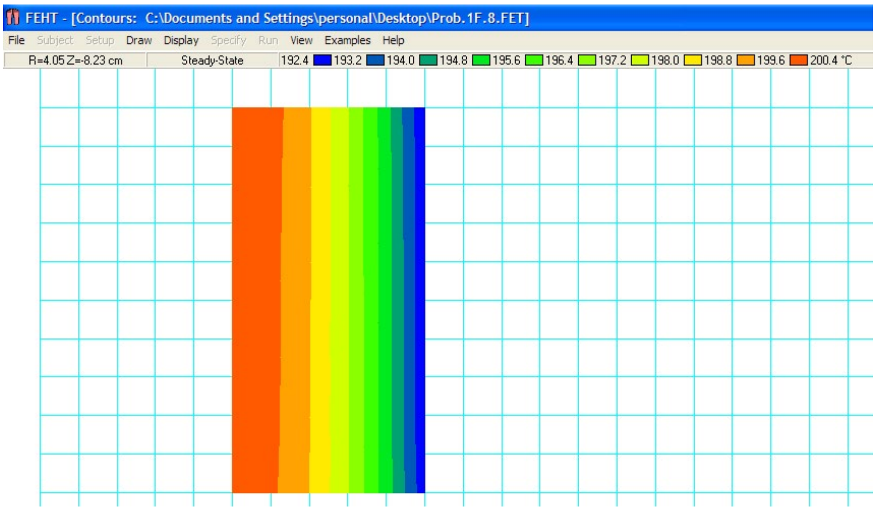
15. Next, let us view temp contours: Select View-Temp contours:



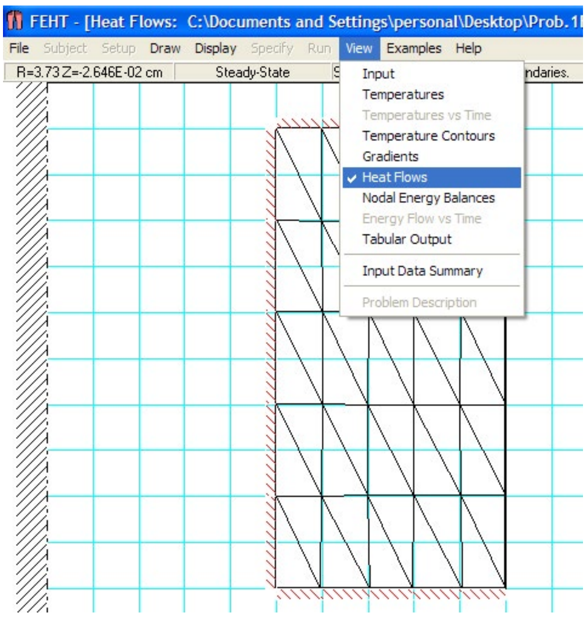
We get following screen:



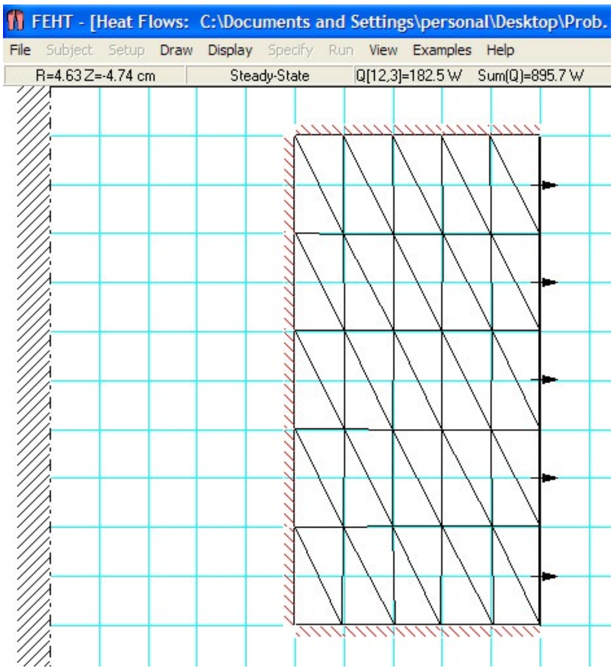
Click OK to accept default values. We get the Temp contours in color bands as shown below:



16. Next, we would like to find out heat transfer from the RHS: Select the entire boundary on the RHS and click on View-Heat flows:



We get:



Observe on the right hand corner of the above fig. that heat transfer from RHS is 895.7 W.

Now, remember that model length in the Z-direction was arbitrarily taken as 10 cm.

i.e. Q = 895.7 for 10 cm long section.

Therefore, for 1 m length: Q = 8957 W.

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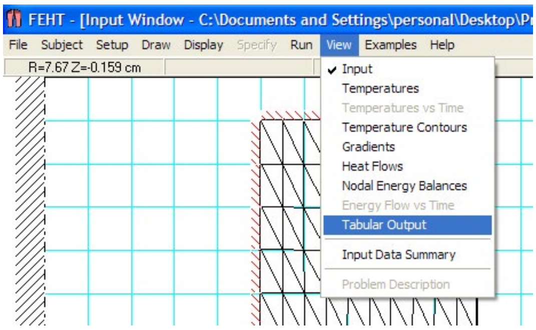
17. To draw the Temp. profile in the cyl. Shell: Click on View-Tabular output:

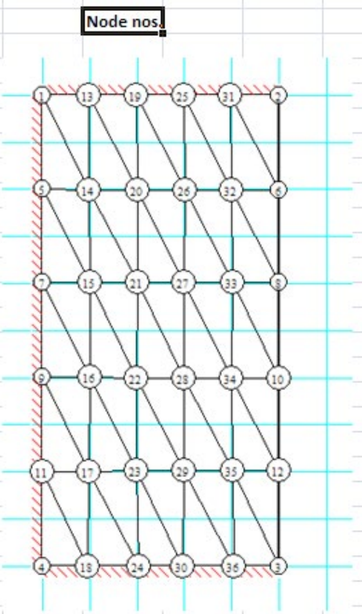


We get Tabular results…

18. Now, select and copy this Table to Excel to edit and draw the graph:

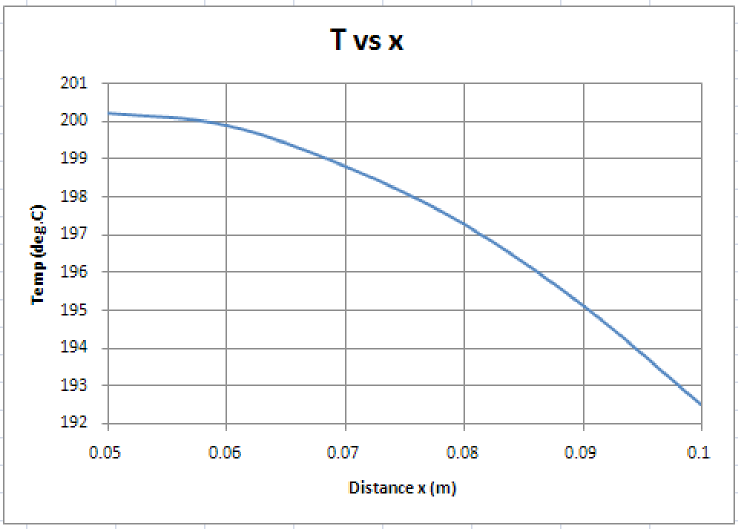
Again, part of the Table copied to Excel is shown below:

Node No.	R(m)	Z(m)	T(deg.C)	Node bal.(W)
1	0.05001	-0.01005	200.4	0
2	0.1	-0.01005	192.4	-89.99
3	0.1	-0.1101	192.6	-91.28
4	0.05001	-0.1101	200.1	3.49E-16
5	0.05001	-0.0299	200.3	0
6	0.1	-0.03016	192.5	-177.7
7	0.05001	-0.04974	200.2	0
8	0.1	-0.04974	192.5	-178.9
9	0.05001	-0.06985	200.2	-6.98E-16
10	0.1	-0.07011	192.5	-178.9
11	0.05001	-0.08996	200.2	0
12	0.1	-0.08969	192.5	-178.9
13	0.0598	-0.01005	200	0
14	0.0598	-0.03003	199.9	-2.09E-15
15	0.06006	-0.04974	199.9	-2.79E-15
16	0.06006	-0.06998	199.8	-6.98E-16
17	0.0598	-0.08983	199.8	0
18	0.05953	-0.1101	199.8	-3.49E-16
19	0.06985	-0.01005	198.9	3.49E-16
20	0.07011	-0.0301	198.8	-3.49E-15
21	0.07011	-0.04974	198.8	6.98E-16
22	0.06985	-0.07005	198.9	0
23	0.06985	-0.08976	198.8	-1.40E-15



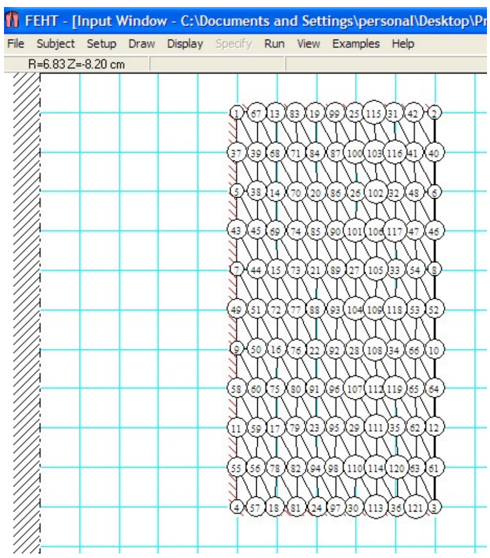
19. Let us choose Node Nos. 7, 15, 21, 27, 33 and 8 and plot the temps at these nodes against the radial distance, in Excel:

Node No.	Dist.(m)	T(deg.C)
7	0.05001	200.2
15	0.06006	199.9
21	0.07011	198.8
27	0.0799	197.3
33	0.09022	195.1
8	0.1	192.5

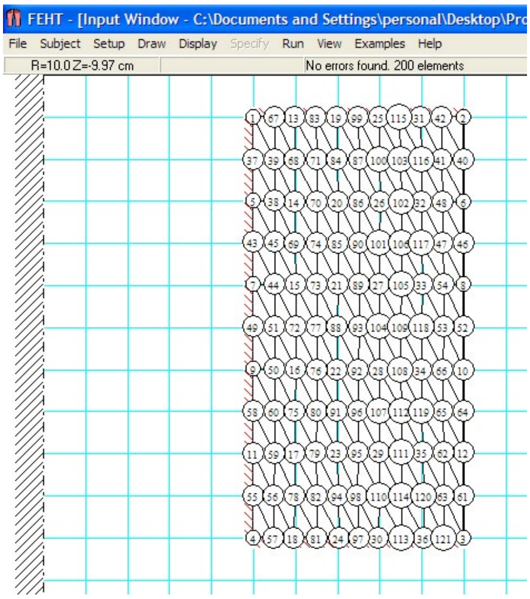


20. Now, let us see if there is any effect on temperatures if we use a finer mesh of elements:

Click on Draw-Reduce Mesh:

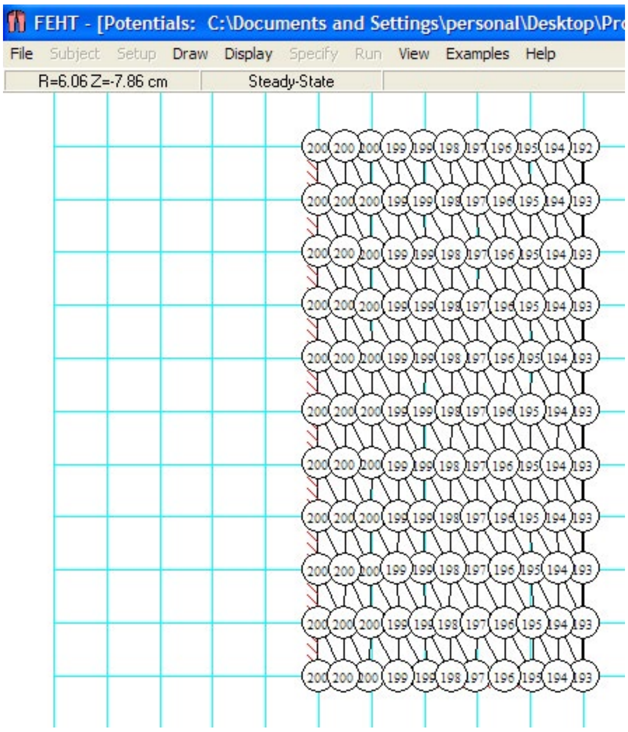


Run-Check: gives



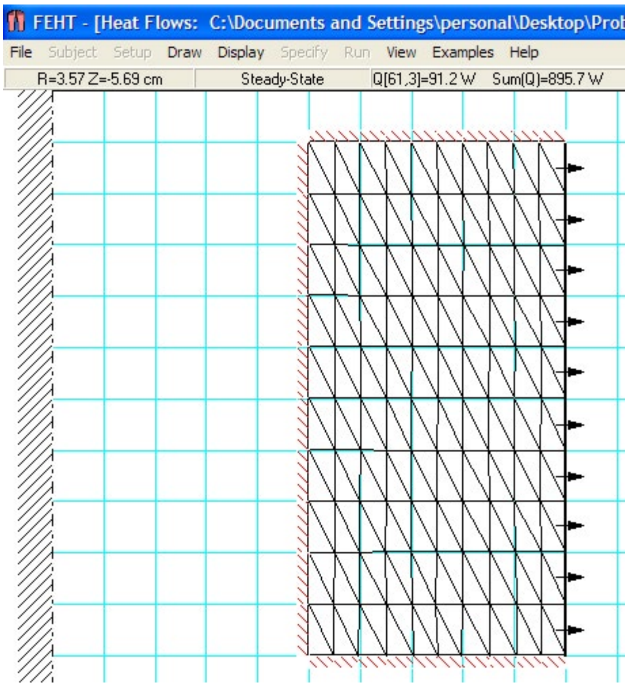
i.e now, there are 200 elements as shown on the right hand corner of above fig.

21. Click on View-Temps:



Observe that temperatures have remained almost the same as in the earlier case.

22. Also, clicking on View-Heat Flows gives:



Again, we get:

Q = 895.7 for 10 cm long section….. same as with coarse mesh, got earlier.

Therefore, for 1 m length: Q = 8957 W.

Prob. 1F.9. A solid sphere of radius R = 10 mm and k = 25 W/m.K is exposed to a fluid at 200 C on its surface with h = 1000 W/m^2.K. There is internal heat generation at the rate of 20 MW/m^3. Determine the surface temp, and the max. temp in the sphere. Draw the temp profile along the radius.

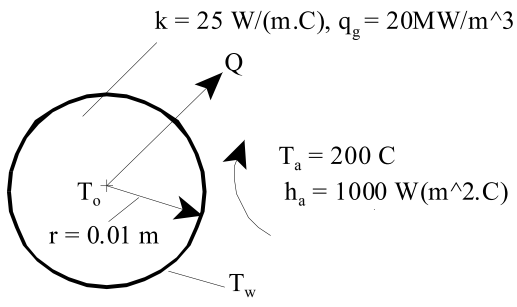


Fig.Prob.1F.9

Mathcad Solution:

This is the case of a sphere, cooled on outside:

We shall first solve this priblem from fundamentals and then verify the results by applying the direct formulas given in the Table.

Data:

$r := 0.01$ m... radius

$k := 25$ W/m.K $\quad q_g := 20\cdot10^6$ W/m^3

$T_a := 200$ C $\quad h_a := 1000$ W/m^2.C

Outer surface is subjected to convection.

Calculations:

$Q_{gen} := \left(\frac{4}{3}\cdot\pi\cdot r^3\right)\cdot q_g$ W .... total heat generated

Since the outer surface is subjected to convection, all the heat generated has to be removed at the outer surface;

Let T0, Tw be the temp at the centre and outer surface of the sphere respectively.

Then, by heat balance at the outer surface:

$$h_a \cdot \left(4 \cdot \pi \cdot R^2\right) \cdot \left(T_w - T_a\right) = Q_{gen}$$

Therefore: $$T_w := \frac{Q_{gen}}{\left[h_a \cdot \left(4 \cdot \pi \cdot r^2\right)\right]} + T_a$$

i.e. $T_w = 266.667$ C .... **temp. at the surface of sphere ... Ans.**

Now, for a Spherical system with heat gen. the controlling differential eqn is:

$$\frac{d^2}{dr^2} T + \frac{2}{r} \cdot \frac{d}{dr} T + \frac{q_g}{k} = 0$$

i.e. $$r^2 \cdot \frac{d^2}{dr^2} T + 2 \cdot r \cdot \frac{d}{dr} T = \frac{-q_g \cdot r^2}{k}$$

i.e. $$\frac{d}{dr}\left(r^2 \cdot \frac{d}{dr} T\right) = \frac{-q_g \cdot r^2}{k}$$

Integrating once:

$$r^2 \cdot \frac{d}{dr} T = \frac{-q_g \cdot r^3}{3 \cdot k} + C1$$

i.e. $$\frac{d}{dr} T = \frac{-q_g \cdot r}{3 \cdot k} + \frac{C1}{r^2} \qquad \text{...eqn.(a)}$$

And, integrating again, we get its general solution as:

$$T(r) = \frac{-q_g \cdot r^2}{6 \cdot k} - \frac{C1}{r} + C2 \qquad \text{...eq.(1)}$$

where C1 and C2 are obtained by applying the Boundary Conditions.

Now, for this problem:

BC..(1): at r = 0.01 m, T= Tw

BC....(2): at r = 0, dT/dr = 0 since temp is max at the centre.

Apply these conditions and get C1 and C2 and substitute them in eqn.(1) to get the solution for temp distribution:

Apply BC - (2) in eqn. (a):

$$\frac{d}{dr} T = \frac{-q_g \cdot r}{3 \cdot k} + \frac{C1}{r^2}$$

i.e. $C1 := 0$ .since T is finite at the centre

Then, apply BC (1) in eqn.(1):

$$T_w = \frac{-q_g \cdot r^2}{6 \cdot k} - \frac{C1}{r} + C2 \qquad \text{...from B.C. (1)}$$

i.e. $C2 := T_w + \frac{q_g \cdot r^2}{6 \cdot k}$

i.e. $C2 = 280$

Therefore, temp distribution is given by:

$$T(r) := \left(\frac{-q_g \cdot r^2}{6 \cdot k} - \frac{C1}{r} + C2 \right) \qquad \text{...eq.(1)}$$

Then, temp of centre, which is max temp. is:

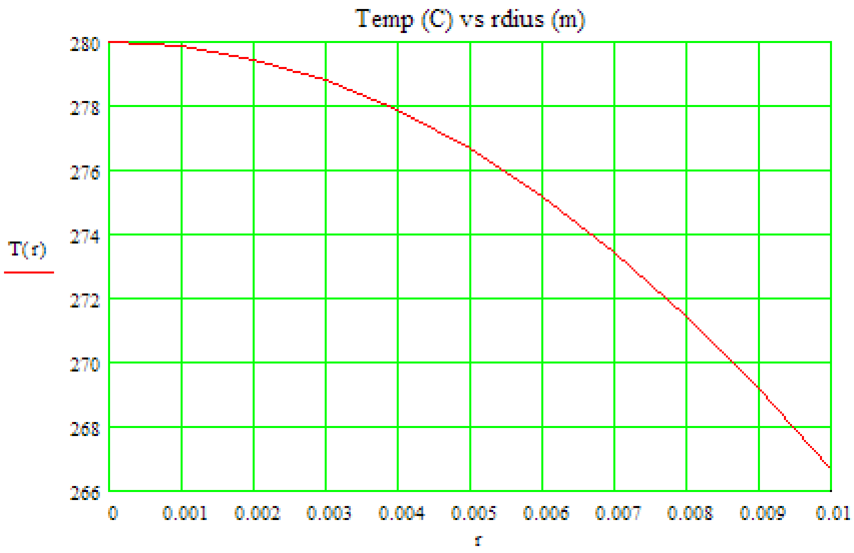
$T(0) = 280$ **C .... max. temp. occurring at the centre ... Ans.**

And,

$T(0.01) = 266.667$ **C .... temp. at the surface ... verified.**

Immediately, let us draw the temp. profile:

$r := 0, 0.001 .. 0.01$ ...define the range variable r



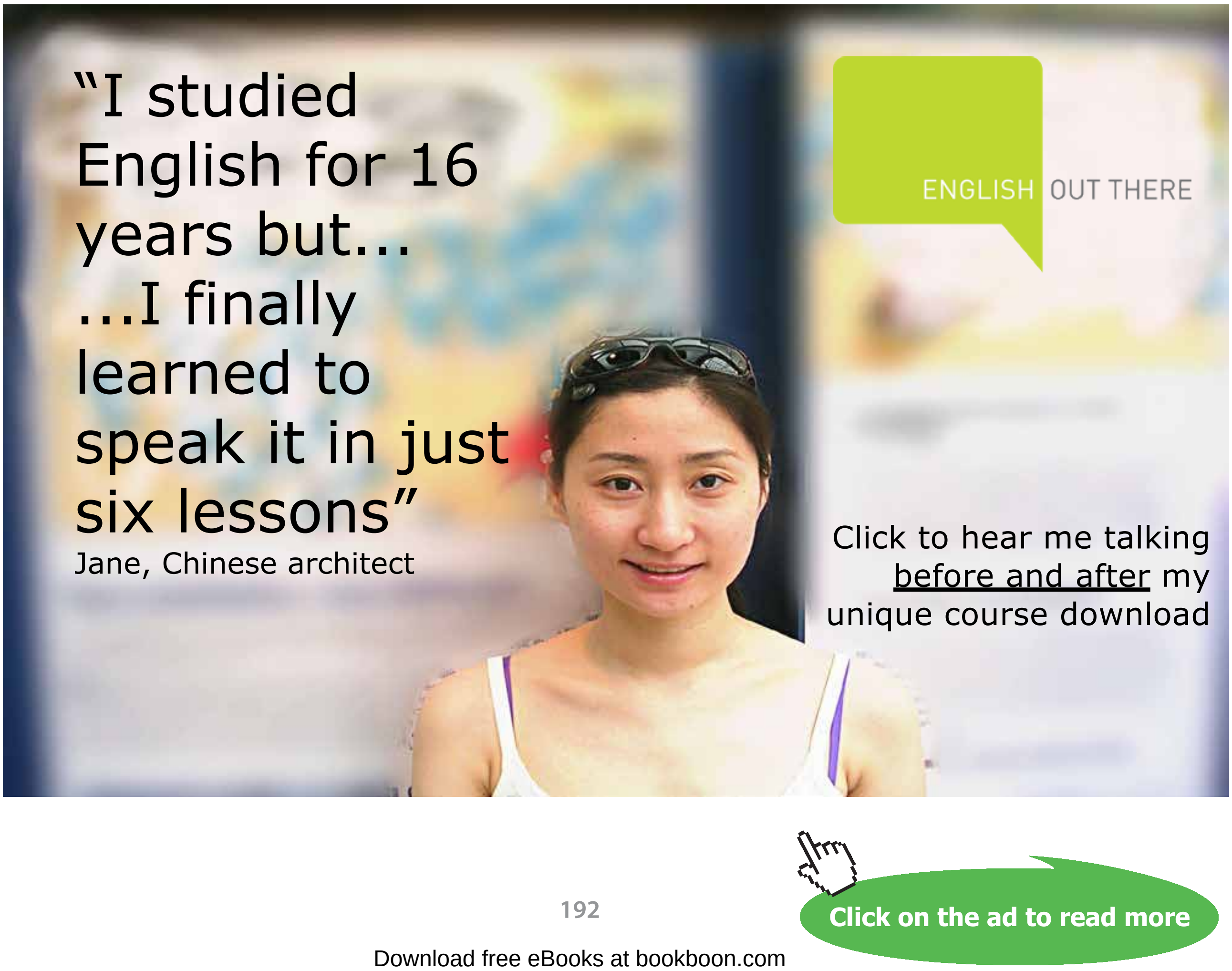
Verify the result by applying the direct formula:

For a sphere, with convection on the outside:

$r := 0.01$ m ... rad. of sphere

$$T_{max} := T_a + \frac{q_g \cdot r}{3 \cdot h_a} + \frac{q_g \cdot r^2}{6 \cdot k}$$ ....max. temp. i.e. at the centre of sphere

i.e. $T_{max} = 280$ C .... **max. temp. .... verified.**



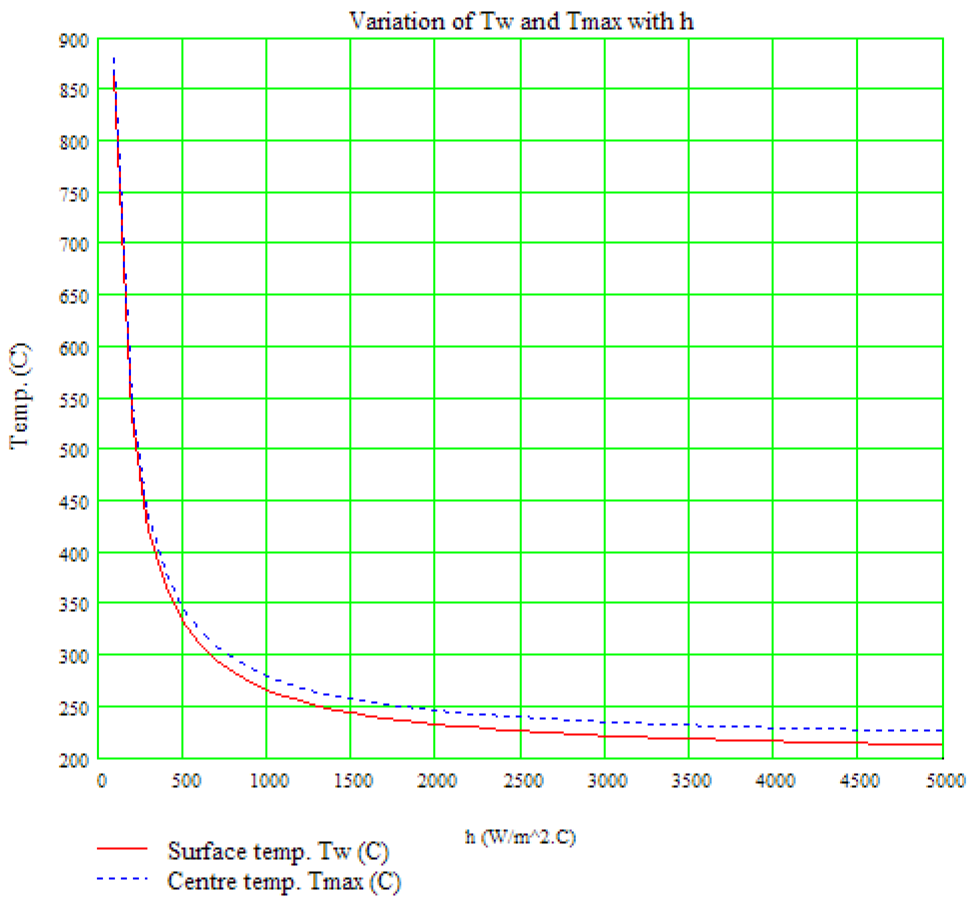
Plot the variation of Tmax and T0 with ha:

First express Tw and Tmax as function of r and ha and then draw the plot:

$$T_w(r, h_a) := \frac{q_g \cdot r}{3 \cdot h_a} + T_a$$ ...Tw as a function of r and h

$$T_{max}(r, h_a) := T_w(r, h_a) + \frac{q_g \cdot r^2}{6 \cdot k}$$ Tmax as a function of r and h

$h := 100, 200..5000$ ...define a range variable h



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"Prob. 1F.10. Radioactive wastes (k_rw = 20 W/m.K) are stored in a spherical, stainless steel (k_ss = 15 W/m.K) container of inner and outer radii equal to r_1 = 0.5 m and r_2 = 0.6 m. Heat is generated within the wastes at a rate of q_g = 10^5 W/m^3, and the outer surface of the container is exposed to a water flow with h = 1000 W/m^2.K and T_a = 25 C. (a) Evaluate the steady state outer surface temp. T_2, (b) Evaluate the steady state inner surface temp T_1 (c) Plot the temp distribution in the radioactive wastes and find the temp at the centre. [Ref: 3]"

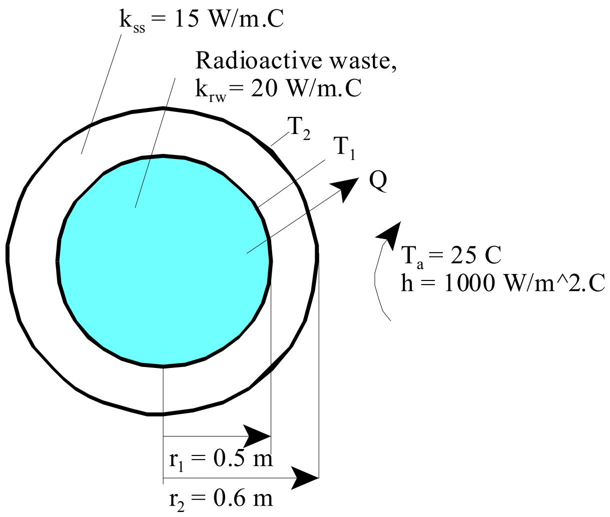


Fig.Prob.1F.10

EES Solution:

"Data:"

r_1= 0.5 [m]
r_2 = 0.6 [m]
k_rw = 20 [W/m-K]
k_ss = 15 [W/m-C]
h = 1000 [W/m^2-K]
T_a = 25 [C]
q_g = 1E05 [W/m^3]

"Calculations:"

Q_gen = q_g * (4/3) * pi * r_1^3 "[W] …. total heat gen. in the radioactive wastes"

Q_gen = h * (4 * pi * r_2^2) * (T_2 – T_a) "..heat balance .. finds T_2, the outer surface temp."

R_sph = (r_2 – r_1) / (4 * pi * k_ss * r_1 * r_2) "[K/W] … thermal resist of stainless steel spherical shell"

Q_gen = (T_1 – T_2) /R_sph "…. finds T_1, the inner surface temp"

"For the solid sphere of radioactive wastes:

It is a spherical system with heat generation.

So, the governing differential eqn is: d2T/dr2 + (2/r).(dT/dr) + (q_g / k) = 0

i.e. r^2.d2T/dr2 + 2.r.(dT/dr) = (-q_g . r^2) / k

i.e. d/dr (r^2 . dT/dr) = (-q_g . r^2) / k

Integrating once: r^2. dT/dr = -(q_g . r^3) / (3 .k) + C1

i.e. dT/dr = -(q_g . r) / (3 .k) + C1/r^2

And, integrating once again:

Tr = (-q_g . r^2) / (6 .k) – C1/ r + C2 ….. eqn for temp. distribution.

Constants C1 and C2 are determined from BC's."

"For the case of solid sphere:

C1 = 0 since dT/dr = 0 at r = 0 … for max. temp. occurs at the centre.

Other B.C. is that at r = R, i.e. at the surface: T = Tw

So, C2 = Tw + (q_g . R^2) / (6 . k)

And, T(r) = Tw + (q_g / 6 . k) . (R^2 – r^2) … temp distribn. in solid sphere with heat gen.

Also, (T_max – Tw) = (q_g . R^2) / (6 . k) "

"To plot the temp. distribution in sphere:"

Tr = T_1 + (q_g / (6 * k_rw)) * (r_1^2 – r^2) "… here, T_1 is wall temp. of radioactive waste sphere, r_1 is outer rad. of radioactive waste sphere, and r is any radius < r_1"

r = 0.0 [m] "…any radius < r_1 …will be commented out later, to plot graph"

Results:

Unit Settings: SI C kPa kJ mass deg

$h = 1000$ [W/m^2-K]	$k_{rw} = 20$ [W/m-K]	$k_{ss} = 15$ [W/m-C]
$q_g = 100000$ [W/m^3]	$Q_{gen} = 52360$ [W]	$r = 0$ [m]
$r_1 = 0.5$ [m]	$r_2 = 0.6$ [m]	$R_{sph} = 0.001768$ [C/W]
$Tr = 337.5$ [C]	$T_1 = 129.2$ [C]	$T_2 = 36.57$ [C]
$T_a = 25$ [C]		

Thus:

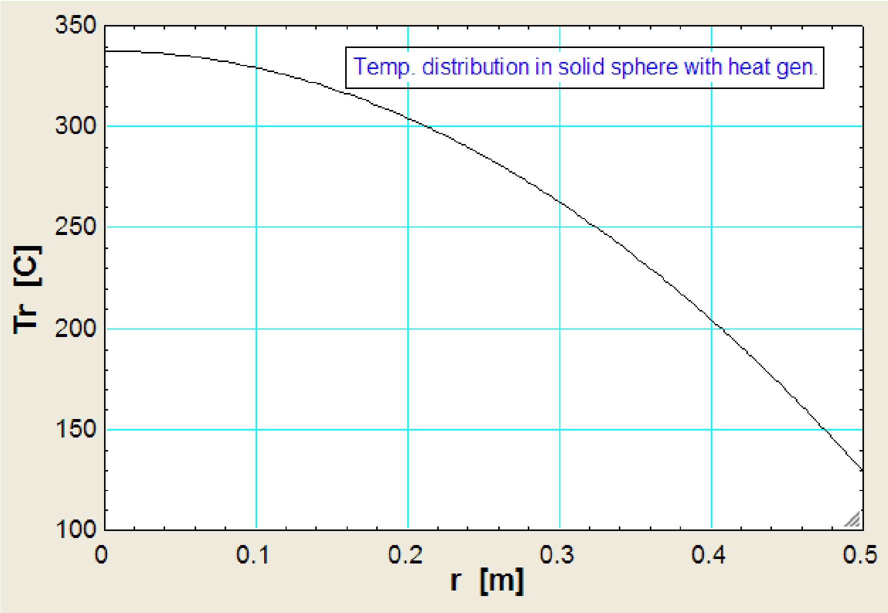
T_1 = 129.2 C …..temp of inner surface of spherical shell … Ans.

T_2 = 36.57 C …..temp of outer surface of spherical shell … Ans.

Tr = T_max = 337.5 C …..temp of centre of solid, radioactive sphere (i.e. at r = 0) … Ans.

In addition:

1. **Plot the temp distribution along the radius in the sphere:**



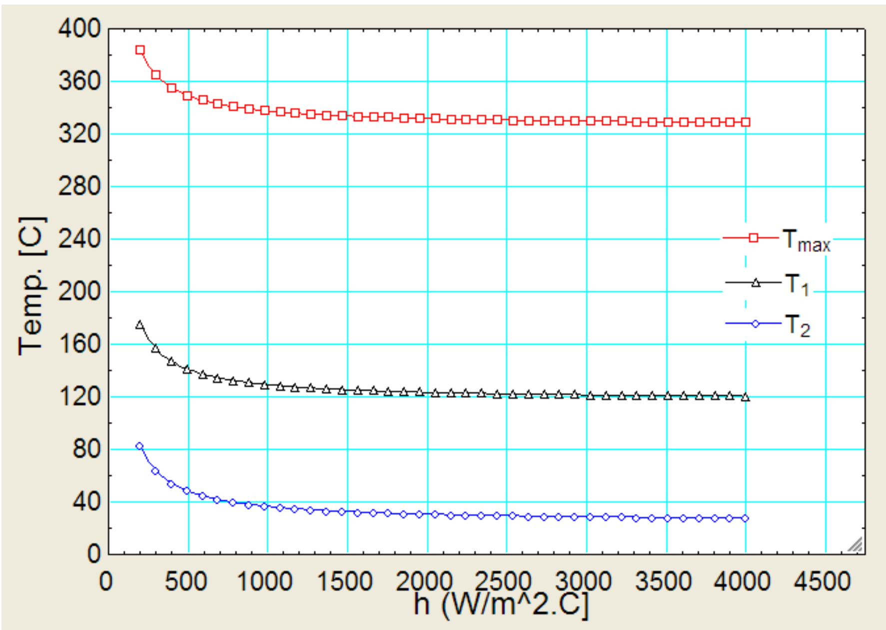
2. **What will be the value of Tmax if suddenly the coolant flow stops? (i.e. h = 0)**

Put h = 1E-06 i.e. a very small value (instead of putting h = 0), to avoid convergence problems during solution. Results are shown below. It is seen that Tmax = 1.157E10 i.e. a very large value; this means that the material will melt and the result will be catastrophic.

Unit Settings: SI C kPa kJ mass deg

$h = 0.000001$ [W/m^2-K]	$k_{rw} = 20$ [W/m-K]	$k_{ss} = 15$ [W/m-C]
$q_g = 100000$ [W/m^3]	$Q_{gen} = 52360$ [W]	$r = 0$ [m]
$r_1 = 0.5$ [m]	$r_2 = 0.6$ [m]	$R_{sph} = 0.001768$ [C/W]
$T_1 = 1.157E+10$ [C]	$T_2 = 1.157E+10$ [C]	$T_a = 25$ [C]
$T_{max} = 1.157E+10$ [C]		

3. **To plot the variation of T1, T2 and Tmax with h:**



We see that:

1) For values of h beyond about 2000 W/m^2.C all the three temperatures remain constant.
2) For values of h between 200 and 1000, all the three temps show a rather steep fall.
3) For h = 200, the max temp reached is about 380 C.

==

"**Prob. 1F.11.** Internal heat generation in a cyl. fuel rod of a nuclear reactor is given by the expression: q_g = q_0 . [1 – (r/r_0)^2] where q_g is local heat gen. rate (W/m^3), q_0 is heat gen. rate at the centre (W/m^3), and r_0 is the outer radius of fuel rod. Estimate the temp drop from centre line to the surface of a 3.5 cm OD rod if the heat removal rate at the outer surface is Q_per_sqm = 2.5 × 10^6 W/m^2. Thermal cond. k of fuel rod = 30 W/m.K

(b) if the temp of the fluid surrounding the rod is 100 C and the max wall surface temp is limited to 200 C, find the surface heat transfer coeff."

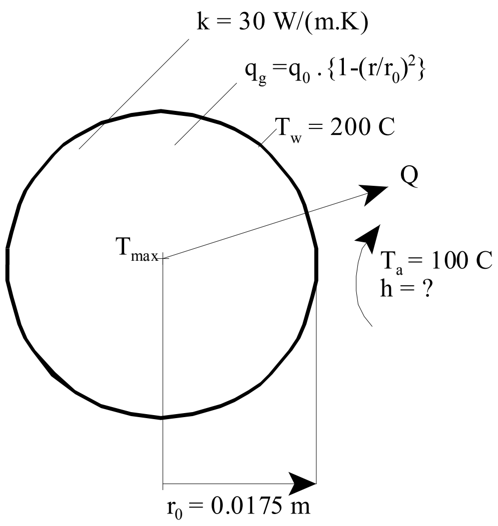


Fig.Prob.1F.11

EES Solution:

"Data:"

r_0 = 0.0175 [m] "…outer radius of fuel rod"
Q_per_sqm = 2.5E06 [W/m^2] "….heat removal rate from outside surface of rod, Watts per sq. meter area"
T_w = 200 [C]"…wall temp."
k = 30 [W/m-C] "….thermal cond. of rod"
q_g = q_0 * (1 – (r/r_0)^2) " where r is any radius .. radial variation of heat gen."
r = 0.005 [m] "…trial value of r .. will be commented out later to draw the temp profile"
T_a = 100 [C] "…surrounding fluid temp."
L = 1 [m] "..length of fuel rod"

"Calculations:"

"It is a cylindrical system with heat generation.

So, the governing differential eqn is: d2T/dr2 + (1/r).(dT/dr) + (q_g / k) = 0

i.e. r.d2T/dr2 + dT/dr + (q_g . r) / k = 0

i.e. d/dr (r . dT/dr) +((q_0 . r) / k) . [1 – (r / r_0)^2] = 0

Integrating once: r. dT/dr +(q_0 / k) . (r^2 /2 – r^4 / (4 . r_0^2)) = C1

BC – (1) : dT / dr= 0 at r = 0, i.e. at the centre, temp is a max. So, C1 = 0

i.e. dT/dr +(q_0 /k) . [r/2 – r^3 / (4 . r_0^2)] = 0

And, integrating once again:"

"Tr + (q_0 / k) * (r^2 / 4 – r^4 /(16 * r_0^2)) = C2 …**eqn(1) …. gives temp distribution"**

"Now, BC – (2) is: T = T_w at r = r_0. Then, we get:"

C2 = T_w + (q_0 / k) * (r_0^2 / 4 – r_0^2 /16) "…gives C2 …. eqn.(2)"

Tr + (q_0 / k) * (r^2 / 4 – r^4 /(16 * r_0^2)) = C2 "…eqn(1) …. gives temp distribution"

"Max. temp. at r = 0: Then, from eqn (1), we have : Tmax = C2, i.e."

T_max = C2 "[C] … temp at the centre"

"Heat transfer from 1 m length of rod:"

dT\dr_wall = (-q_0 / k) * (r_0 / 2 – r_0 /4) " … dT/dr at r = r_0"

Q = – k * (2 * pi * r_0 * L) * dT\dr_wall "..determines Q/ m length using Fourier's eqn."

"But, Q is also equal to heat removed by convection from the surface. So, we get:"

Q = h * (2 * pi * r_0 * L) * (T_w – T_a) "…. determines h"

Q = Q_per_sqm * (2 * pi * r_0 * L) "…heat removed from outside surface, by data"

"Also, plot the temp distribution in the fuel rod:"

Results:

Unit Settings: SI C kPa kJ mass deg

C2 = 1294	$dT\backslash dr_{wall}$ = -83333
h = 25000 [W/m²-C]	k = 30 [W/m-C]
L = 1 [m]	Q = 274889 [W]
q_0 = 5.714E+08 [W/m³]	q_g = 5.248E+08 [W/m³]
$Q_{per,sqm}$ = 2.500E+06 [W/m²]	r = 0.005 [m]
r_0 = 0.0175 [m]	Tr = 1177 [C]
T_a = 100 [C]	T_{max} = 1294 [C]
T_w = 200 [C]	

Thus:

Tmax = 1294 C … temp at centre

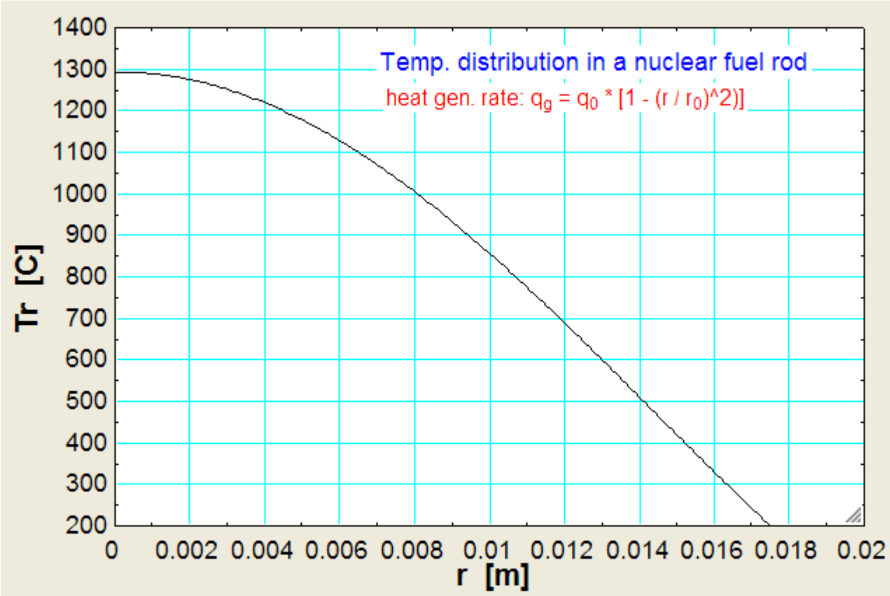
Tw = 200 C

Therefore, temp drop = (1294 – 200) = 1094 C …. Ans.

q_0 = 5.714 × 10^8 W/m^3 …. Heat gen. rate at the centre of rod.

h = 25000 W/m^2.C … heat transfer coeff. from cylinder wall to fluid … Ans.

Plot the temp. distribution:



===

Prob. 1F.12. Heat is generated uniformly in a stainless steel plate having k = 20 W/m.K. The thickness of the plate is 1 cm and heat generation rate is 500 MW/m3. If the two sides of the plate are maintained at 100 and 200 C respectively, calculate the temperature at the centre of the plate. Also find the distance of the plate at which maximum temperature occurs from the 200 C surface.

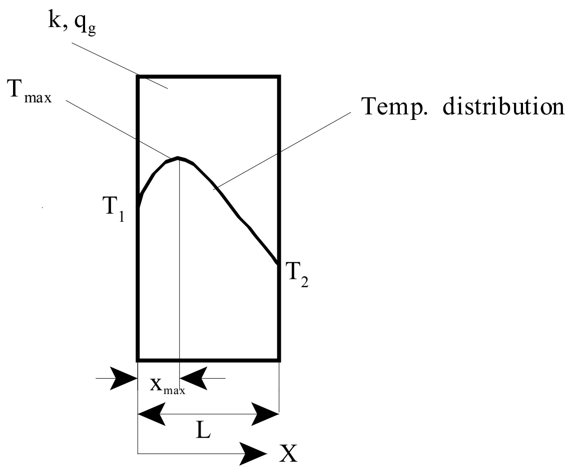


Fig.Prob.1F.12

This problem is the same as Prob.1F.2.

But, we will solve this problem with EXCEL:

EXCEL Solution:

Following are the steps:

1. Set up the EXCEL worksheet, enter data and name the cells:

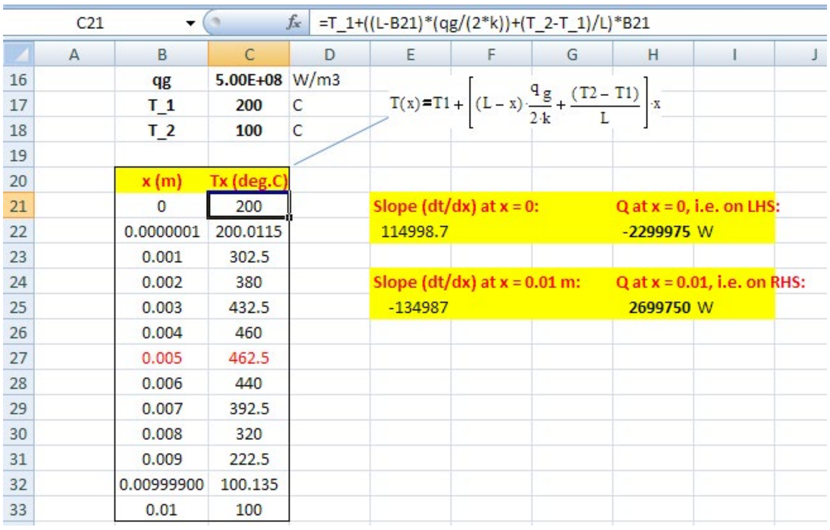
T_2 f_x 100

	A	B	C	D
13	Data:	L	0.01	m
14		A	1	m2
15		k	20	W/m.C
16		qg	5.00E+08	W/m3
17		T_1	200	C
18		T_2	100	C

2. Now, plan for calculating the temp. profile in the slab. For the Boundary Conditions of this problem, the temp distribution is given by:

$$T(x) = T1 + \left[(L - x)\cdot\frac{q_g}{2\cdot k} + \frac{(T2 - T1)}{L}\right]\cdot x$$

Since we have to find dT/dx at the LHS and RHS to determine the heat fluxes later, we prepare a Table of x and Tx as shown below. At x = 0 and at x = L, a very small increment in x is also given in the Table, so that dT/dx can be calculated at x = 0 and at x = L:



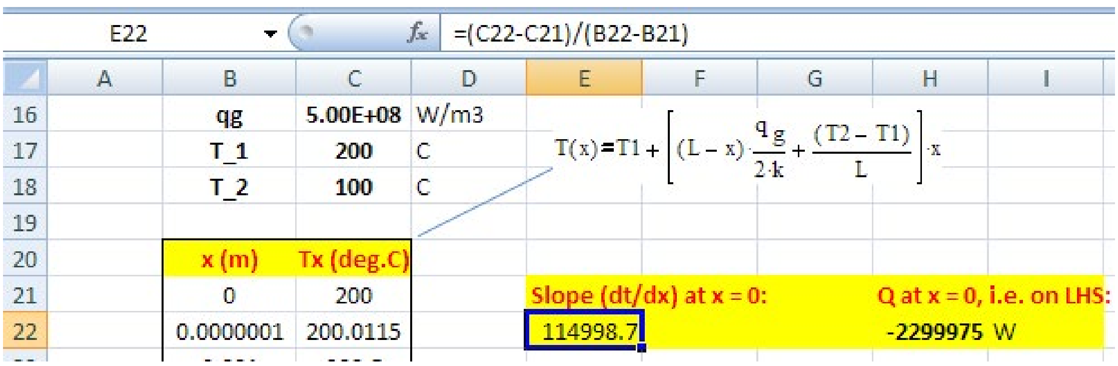
C21 =T_1+((L-B21)*(qg/(2*k))+(T_2-T_1)/L)*B21

	A	B	C	D	E	F	G	H
16		qg	5.00E+08	W/m3				
17		T_1	200	C				
18		T_2	100	C				
19								
20		x (m)	Tx (deg.C)					
21		0	200		Slope (dt/dx) at x = 0:		Q at x = 0, i.e. on LHS:	
22		0.0000001	200.0115		114998.7		-2299975 W	
23		0.001	302.5					
24		0.002	380		Slope (dt/dx) at x = 0.01 m:		Q at x = 0.01, i.e. on RHS:	
25		0.003	432.5		-134987		2699750 W	
26		0.004	460					
27		0.005	462.5					
28		0.006	440					
29		0.007	392.5					
30		0.008	320					
31		0.009	222.5					
32		0.00999900	100.135					
33		0.01	100					

$$T(x) = T1 + \left[(L - x)\cdot\frac{q_g}{2\cdot k} + \frac{(T2 - T1)}{L}\right]\cdot x$$

Referring to the above fig., note in the Formula bar the eqn entered in cell C21 for Tx. Eqn for Tx is also separately shown for clarity. After entering eqn for Tx in cell C21, value calculated is 200 C, as it should be, since it is the BC given on LHS. Now, select C21 and drag-copy till cell C33 and the Table is completed. As a check, temp on RHS, i.e. at x = 0.01 m is 100 C, which is the BC on RHS. **Temp at the centre of plate (i.e. at x = 0.005 m) is 462.5 deg.C … Ans.**

3. Next, to calculate the heat flow at LHS: Slope at x = 0 is calculated as: (C22–C21)/(B22–B21). This is entered in cell E22 and is calculated as 114998.7 C/m. Then immediately calculate the heat transferred, Q_left at LHS as Q_left = -k * A* (dT/dx). This is shown in cell H22 and is calculated as **Q_left = -2299975 W.** (-ve sign indicates that the heat flow is from right to left). See the following screen shot where eqn in cell E22 can be seen in the Formula bar:



E22 =(C22-C21)/(B22-B21)

	A	B	C	D	E	F	G	H
16		qg	5.00E+08	W/m3				
17		T_1	200	C				
18		T_2	100	C				
19								
20		x (m)	Tx (deg.C)					
21		0	200		Slope (dt/dx) at x = 0:			Q at x = 0, i.e. on LHS:
22		0.0000001	200.0115		114998.7			-2299975 W

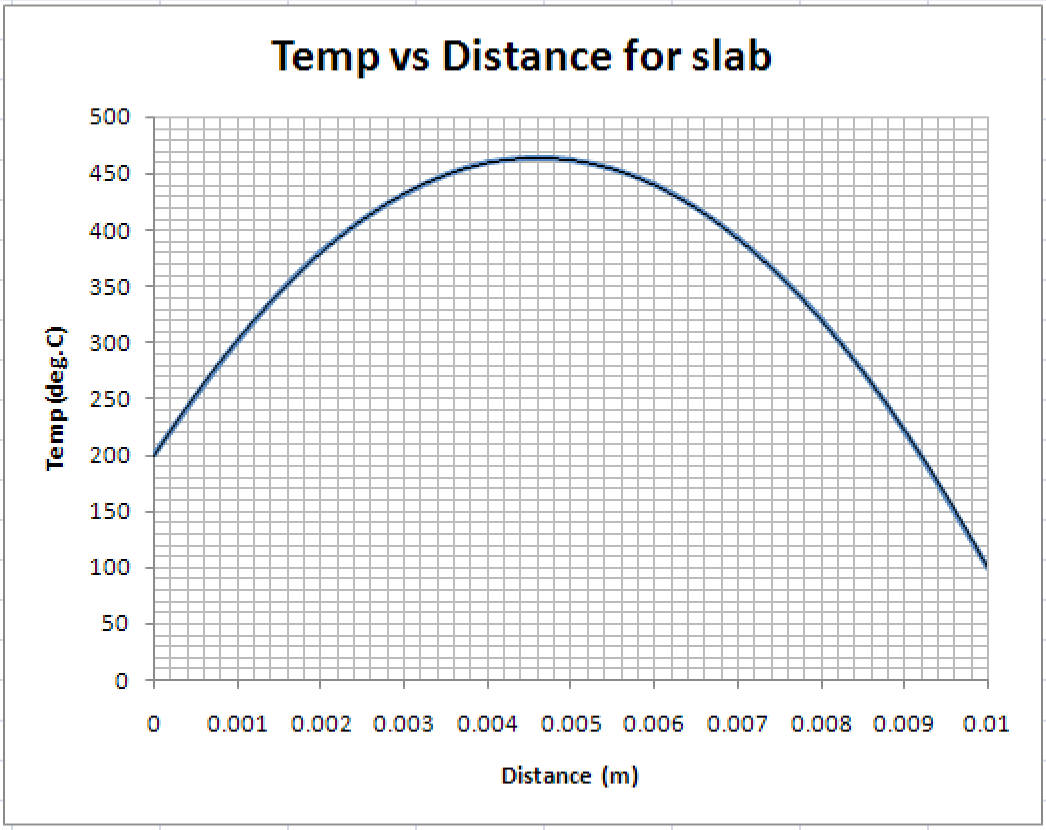
$$T(x) = T1 + \left[(L - x)\cdot\frac{q_g}{2\cdot k} + \frac{(T2 - T1)}{L}\right]\cdot x$$

4. Similarly, the heat flow at RHS is calculated. Slope, dT/d at x = 0.01 m is calculated in cell E25, and the heat flow Q_right in cell H25. We get: Q_right = 2699750 W.

E25 | fx =(C33-C32)/(B33-B32)

	A	B	C	D	E	F	G	H	I
23		0.001	302.5						
24		0.002	380		Slope (dt/dx) at x = 0.01 m:			Q at x = 0.01, i.e. on RHS:	
25		0.003	432.5		-134987			2699750 W	
26		0.004	460						
27		0.005	462.5						
28		0.006	440						
29		0.007	392.5						
30		0.008	320						
31		0.009	222.5						
32		0.00999900	100.135						
33		0.01	100						

5. Temp vs x is plotted in EXCEL:



6. **From the plot, we see that max. temp occurs at 0.046 m from LHS and is equal to about 465 deg.C… Ans.**

==

Prob.1F.13. Solve the above problem if the thermal conductivity varies linearly with x as:
K(T) = k0 * (1 + βT) W/m.C, where k0 = 14.695 W/m.C, and β = 10.208E-04 C-1 and T is in deg.C

EXCEL Solution:

For linearly varying k, w have for temp distribution:

$$T(x) = \frac{-1}{\beta} + \sqrt{\left(\frac{1}{\beta} + T1\right)^2 - \frac{2 \cdot x}{\beta \cdot L} \cdot (T1 - T2) \cdot \left(1 + \beta \cdot T_m\right) + \frac{q_g \cdot x}{\beta \cdot k_o} \cdot (L - x)}$$

where Tm = (T1 + T2)/2

Following are the steps in EXCEL Solution:

1. Set up the worksheet, enter data, name the cells:

T_2 | f_x 100

	A	B	C	D	E	F	G
4							
5	Data:	L	0.01	m			
6		A	1	m2			
7		k_0	14.695	W/m.C	k(T) = k_0 * (1+ beta *T)		
8		qg	5.00E+08	W/m3			
9		beta	1.02E-03	1/C	k(200) =	17.69513	W/m.C
10		T_1	200	C	k(100)=	16.19507	W/m.C
11		T_2	100	C			

2. Do preliminary calculations. Note that the eqn for Tx is solved in parts, to avoid errors in entering the eqn.i.e. C-!, C_2 and C_3 are calculated first, and then they are used to calculate Tx.

C_3 | f_x =qg/(beta*k_0)

	A	B	C	D	E	F	G
4							
5	Data:	L	0.01	m			
6		A	1	m2			
7		k_0	14.695	W/m.C	k(T) = k_0 * (1+ beta *T)		
8		qg	5.00E+08	W/m3			
9		beta	1.02E-03	1/C	k(200) =	17.69513	W/m.C
10		T_1	200	C	k(100)=	16.19507	W/m.C
11		T_2	100	C			
12		T_m	150	C			
13		C_1	1391512.367				
14		C_2	22592476.49				
15		C_3	33331875619				
16							

$$T_m = \frac{T1 + T2}{2}$$

$$C\_1 = \left(\frac{1}{\beta} + T1\right)^2$$

$$C\_2 = \frac{2\cdot(T1 - T2)\cdot\left(1 + \beta\cdot T_m\right)}{\beta\cdot L}$$

$$C\_3 = \frac{qg}{\beta\cdot k\_0}$$

7. Prepare the Table of x and Tx. Enter the eqn for Tx for x = 0 in cell C23. Note in the fig. below the eqn for Tx in the Formula bar. Then, drag copy it till the end of Table. Just as in the previous case, dT/dx is calculated at x = 0 and at x = 0.01 m. Then, heat transferred at LHS and RHS are also calculated. Q_left at LHS as Q_left = -k(200) * A* (dT/dx). Here, k(200) is the thermal cond. at the left face which is at 200 C. k(200) is calculated in cell F9, in the above Fig. Similarly, at the RHS, k(100) is calculated in cell F10. Formula for Q_left is entered in cell H24 and is calculated as **Q_left = -2330511 W.** (-ve sign indicates that the heat flow is from right to left). Similarly, the heat flow at RHS is calculated. Slope, dT/d at x = 0.01 m is calculated in cell H27, and the heat flow Q_right in cell H25. We get: **Q_right = 2699750 W.** Of course, sum of Q_left and Q_right should be equal to total heat generated in the slab (= A*L*qg). This is verified in cells H29 and H31:

$$T(x) = \frac{-1}{\beta} + \sqrt{\left(\frac{1}{\beta} + T1\right)^2 - \frac{2 \cdot x}{\beta \cdot L} \cdot (T1 - T2) \cdot \left(1 + \beta \cdot T_m\right) + \frac{q_g \cdot x}{\beta \cdot k_o} \cdot (L - x)}$$

i.e. $T(x) = \frac{-1}{\beta} + \sqrt{C\_1 - C\_2 \cdot x + C\_3 \cdot x \cdot (L - x)}$

x (m)	Tx (deg.C)
0	200
0.0000001	200.0131704
0.001	312.2379217
0.002	391.3748714
0.003	442.9457963
0.004	469.8959154
0.005	473.5956618
0.006	454.225025
0.007	410.8201405
0.008	341.0133571
0.009	240.2806634
0.00999900	100.1648031
0.01	100

Slope (dt/dx) at x = 0: 131703.5

Q at x = 0, i.e. on LHS: -2330511 W

Slope (dt/dx) at x = 0.01 m: -164803

Q at x = 0.01, i.e. on RHS: 2668997 W

(Q_left + Q_right)= 4999508 W

Total heat gen= 5000000 W

Verified.

Thus: Temp at mid-plane (i.e. at x = 0.005 m) is 473.596 deg.C …. Ans.

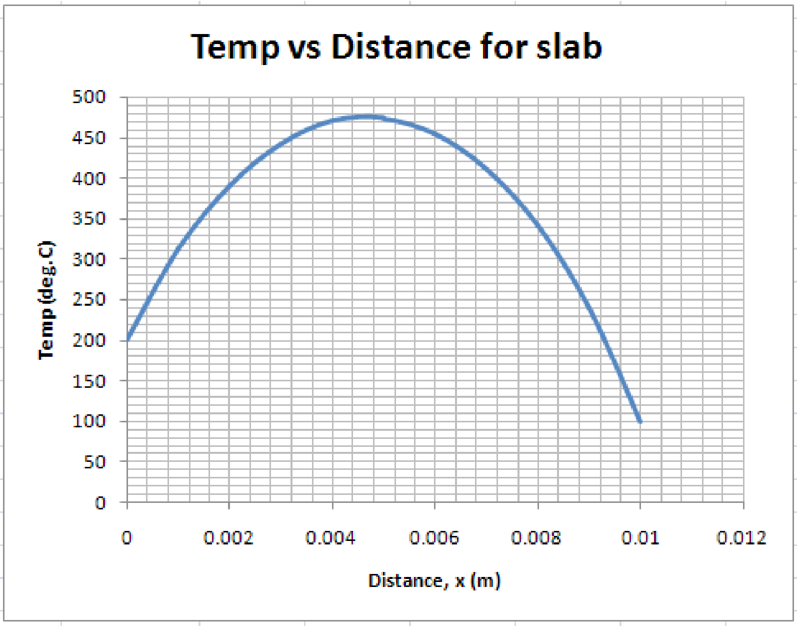
Q_left = 2.330511E06 W … Ans.

Q_right =2.668997E06 W … Ans.

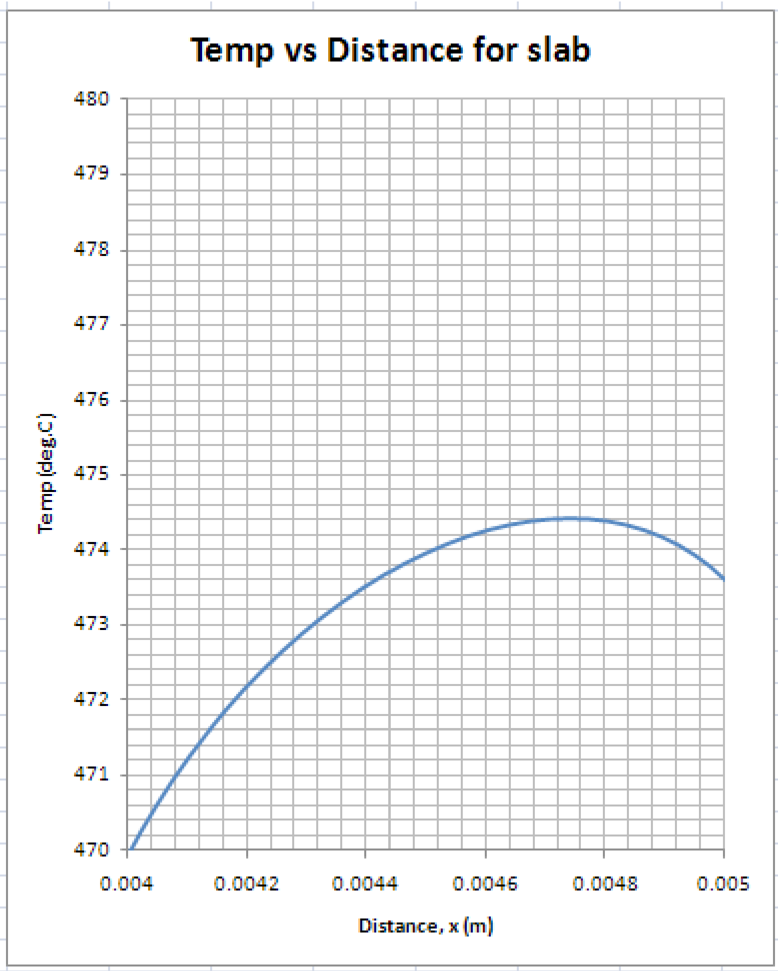
Check: Q_left + Q_right should be equal to the total heat generated in the slab (= **A*L*qg = 5E06 W);**

We get Q_left + Q_right = 4.999508E06 W … i.e. checks.

3. Plot Temp vs x:



4. **Max temp occurs at what distance? What is its value?** Draw the above graph to a magnified scale:



Now, we can read : **Max temp = T_{max} = 474.4 deg. C, x_{max} = 0.0047 6 m … Ans.**

==

Prob.1F.14. A large, 3 cm thick plate (k = 18 W/(m.K)) has an uniform heat generation rate of 5 MW/m3. Both the sides of the plate are exposed to an ambient at 25 C, with h = 150 W.m^2.C. Find out the max. temp. in the plate and where it occurs. Draw the temp. profile in the plate.

EXCEL Solution:

We note that temp distribution should be symmetrical around the centre-line of slab, since both sides are subjected to same boundary conditions. So, we consider only the right half of the slab. In effect, it is equivalent to a slab of thickness L subjected to convection on its RHS. See the following fig.

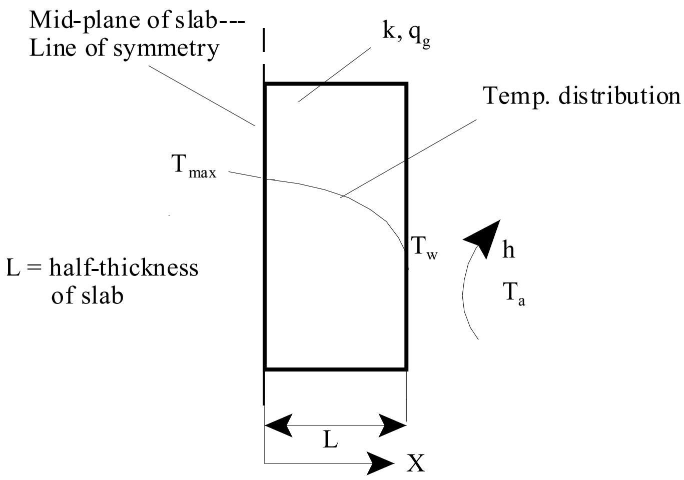


Fig.Prob.1F.14

For a slab, with internal heat generation, insulated on LHS and subjected to convection on RHS, **the max temp occurs on the insulated face** (since heat can flow out from that face towards the RHS only).

We have, **for temperature distribution:** (See Prob. 1F.1)

Temp. distribution is given by:

$$T(x) := T_a + q_g \cdot \frac{L}{h} + \frac{q_g}{2 \cdot k} \cdot \left(L^2 - x^2\right)$$

Note: x is measured from insulated left side of slab

Following are the steps in EXCEL solution:

1. Set up the EXCEL worksheet, enter data, name the cells:

h | fx 150

	A	B	C	D
1				
2	3. Prob.5.2...MT..p. 219...Slab with heat gen.... both sides convection:			
3				
4	Data:	L	0.015	m...half thickness
5		k	18	W/m.C
6		qg	5.00E+06	W/m^3
7		T_a	25	C
8		h	150	W/m^2.C
9				

Temp. distribution is given by:

$$T(x) := T_a + q_g \cdot \frac{L}{h} + \frac{q_g}{2 \cdot k} \cdot \left(L^2 - x^2\right)$$

Note: x is measured from insulated left side of slab

2. Set up the calculation table for Tx vs x: Enter the eqn for Tx in cell C11, as shown in Formula bar:

C11 | fx =T_a+qg*L/h+(qg/(2*k))*(L^2-B11^2)

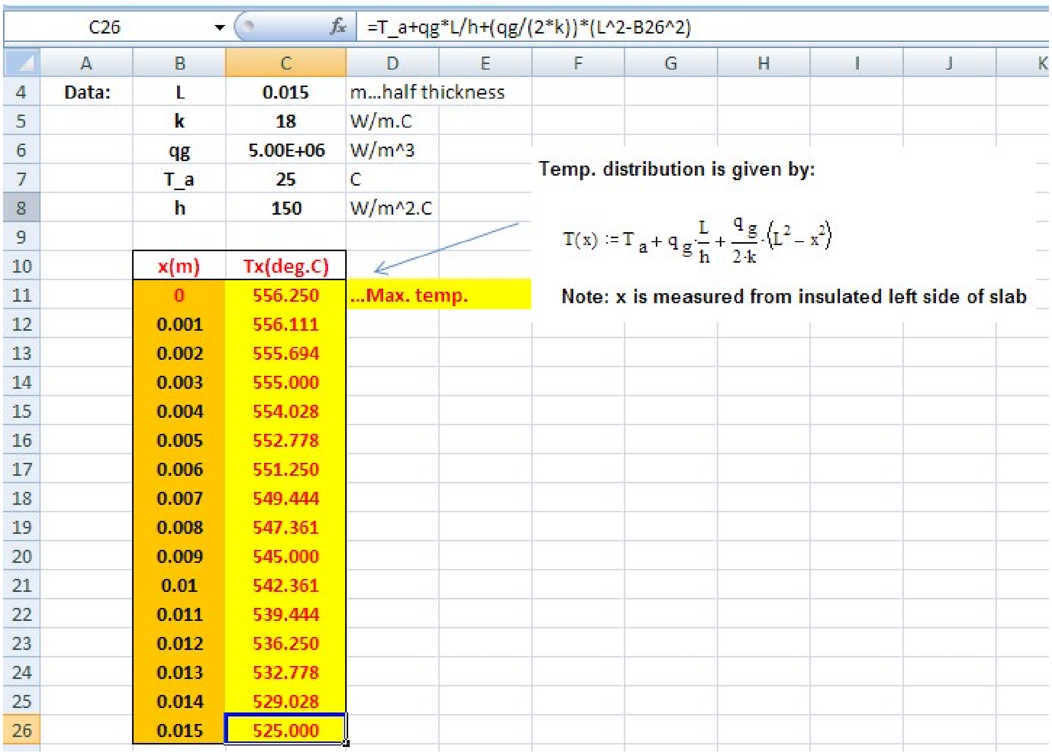
	A	B	C	D
4	Data:	L	0.015	m...half thickness
5		k	18	W/m.C
6		qg	5.00E+06	W/m^3
7		T_a	25	C
8		h	150	W/m^2.C
9				
10		x(m)	Tx(deg.C)	
11		0	556.250	...Max. temp.
12		0.001		
13		0.002		
14		0.003		
15		0.004		
16		0.005		
17		0.006		
18		0.007		
19		0.008		
20		0.009		
21		0.01		
22		0.011		
23		0.012		
24		0.013		
25		0.014		
26		0.015		

Temp. distribution is given by:

$$T(x) := T_a + q_g \cdot \frac{L}{h} + \frac{q_g}{2 \cdot k} \cdot \left(L^2 - x^2\right)$$

Note: x is measured from insulated left side of slab

3. Drag-copy the cell C11 downwards up to cell C26, to complete the calculations:



C26 | =T_a+qg*L/h+(qg/(2*k))*(L^2-B26^2)

	A	B	C	D
4	Data:	L	0.015	m...half thickness
5		k	18	W/m.C
6		qg	5.00E+06	W/m^3
7		T_a	25	C
8		h	150	W/m^2.C
9				
10		x(m)	Tx(deg.C)	
11		0	556.250	...Max. temp.
12		0.001	556.111	
13		0.002	555.694	
14		0.003	555.000	
15		0.004	554.028	
16		0.005	552.778	
17		0.006	551.250	
18		0.007	549.444	
19		0.008	547.361	
20		0.009	545.000	
21		0.01	542.361	
22		0.011	539.444	
23		0.012	536.250	
24		0.013	532.778	
25		0.014	529.028	
26		0.015	525.000	

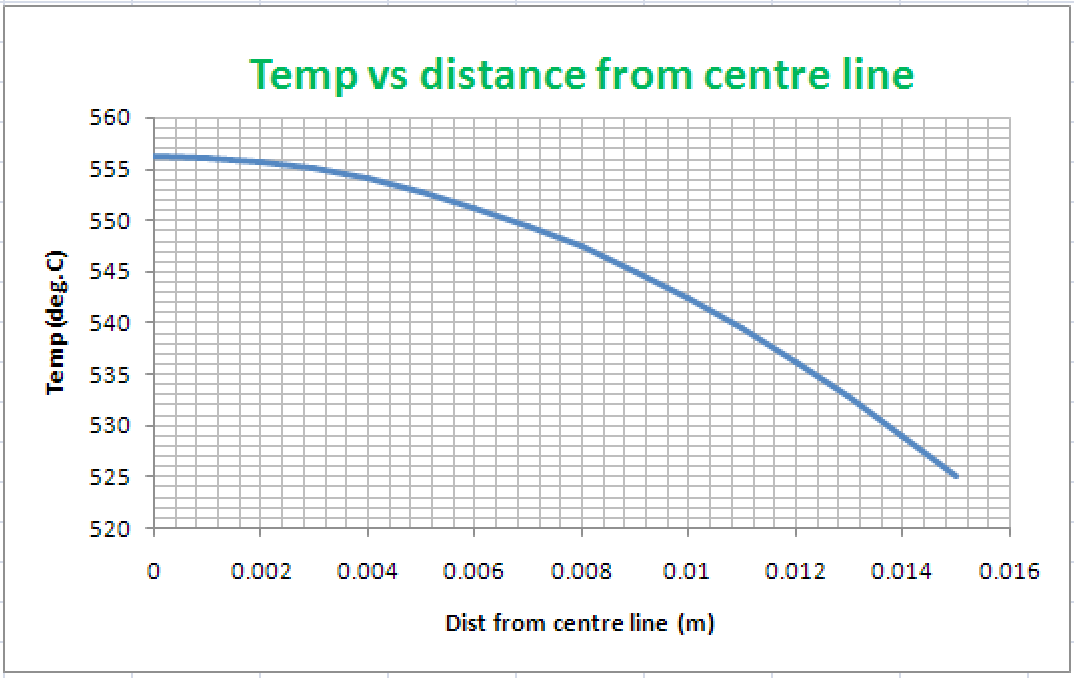
Temp. distribution is given by:

$$T(x) := T_a + q_g \cdot \frac{L}{h} + \frac{q_g}{2 \cdot k} \cdot \left(L^2 - x^2\right)$$

Note: x is measured from insulated left side of slab

We note that Max. temp is Tmax = 556.25 deg.C, at the centre of slab (i.e. at x = 0)

4. Plot Tx vs x:



Note that the above plot is for the right half of the slab. Temp profile is symmetrical in the left half.

5. **Plot the variation of centre-line temp with h:**

Let h vary from 100 to 300 W/m^2.C.
For temp. distribution, we have the eqn:

$$T(x) := T_a + q_g \cdot \frac{L}{h} + \frac{q_g}{2 \cdot k} \cdot \left(L^2 - x^2\right)$$

In this eqn. we put x = 0, since we need temps at the centre-line only, for different values of h.

Set up the EXCEL worksheet as shown:

D49 f_x =T_a+qg*L/C49+(qg/(2*k))*(L^2)

	A	B	C	D	E	F
46		Variation of centre-line temp with h:				
47						
48			h (W/m^2.C)	Tx (deg.C)		
49			100	806.250		
50			120			
51			140			
52			160			
53			180			
54			200			
55			220			
56			240			
57			260			
58			280			
59			300			

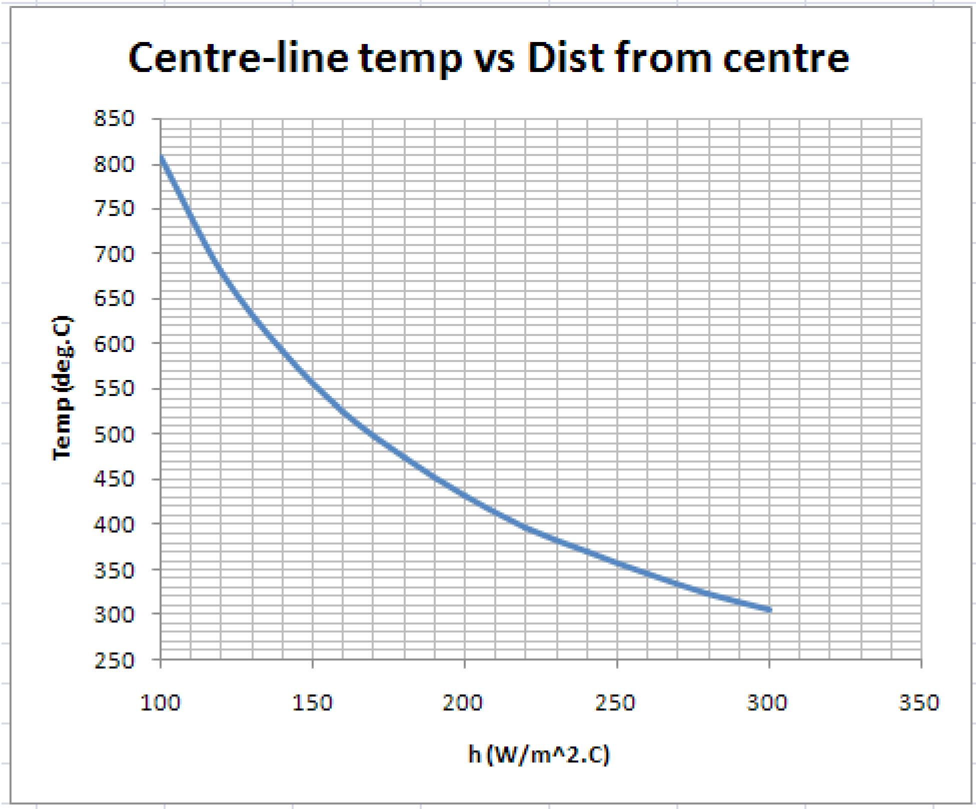
Note that the eqn entered in cell D49 is for h = 100 W.m^2.C, and can be seen in the Formula bar. In writing the eqn reference to 'h' is *relative reference* in EXCEL, so that when we drag-copy cell D49 to cell D48, the calculations automatically take care of varying h values:

Following is the result:

D59 f_x =T_a+qg*L/C59+(qg/(2*k))*(L^2)

	A	B	C	D	E	F
46		Variation of centre-line temp with h:				
47						
48			h (W/m^2.C)	Tx (deg.C)		
49			100	806.250		
50			120	681.250		
51			140	591.964		
52			160	525.000		
53			180	472.917		
54			200	431.250		
55			220	397.159		
56			240	368.750		
57			260	344.712		
58			280	324.107		
59			300	306.250		

6. Now, produce the plot:



==

Prob.1F.15. In a sphere of radius R, heat generation rate varies with the radius as: qg = qo [1-(r/R)2]. If the thermal conductivity k, is constant. If qo = 106 W/m3, R = 0.04 m, k = 12 W/(m.C), and if the centre temp. is 200 C, determine the surface temp. Also, find the heat flow rate at the surface. Draw the temp. profile.

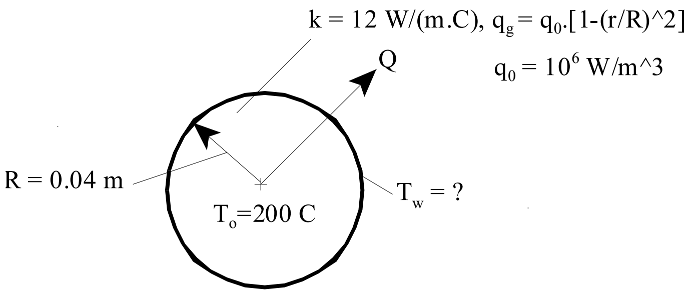


Fig.Prob.1F.15

EXCEL Solution:

This is a case solid sphere with variable rate of heat generation. From Ref [1], we have for temp distribution:

i.e.

$$T(r) = T_w + \frac{q_0}{6 \cdot k} \cdot (R^2 - r^2) - \frac{q_0}{20 \cdot k \cdot R^2} \cdot (R^4 - r^4) \qquad \text{.........(d)}$$

Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data and name the cells. Also, calculate wall temp (T_w) as shown:

`T_w` `=T_0-q_0*Radius^2/(6*k)+q_0*Radius^4/(20*k*Radius^2)`

	A	B	C	D	E
1					
2		**Data:**			
3			**Radius**	**0.04**	m
4			**q_0**	**1.00E+06**	W/m3
5		Centre temp	**T_0**	**200**	C
6			**k**	**12**	W/m.C
7			**r**	**0**	**...at centre**
8		Surface temp.	**T_w**	**184.4444**	**C...Ans.. Putting r = 0 in expression for Tr**
9					

$$q_g = q_0 \cdot \left[1 - \left(\frac{r}{R}\right)^2\right] \quad \text{....varable heat gen.}$$

$$T(r) = T_w + \frac{q_0}{6 \cdot k} \cdot (R^2 - r^2) - \frac{q_0}{20 \cdot k \cdot R^2} \cdot (R^4 - r^4)$$

It is seen that when centre temp T_0 = 200 deg.C, we get T_w = 184.444 deg.C … Ans.

2. Next, to plot temp profile, set up the worksheet and the Table of Tr vs r as shown below:

`D17` `=T_w+q_0*(Radius^2 -C17^2)/(6*k)-q_0*(Radius^4 - C17^4)/(20*k*Radius^2)`

To plot T as a function of r:

	r(m)	Tr(deg.C)
16	0	200.000
17	0.002	199.944
18	0.004	
19	0.006	
20	0.008	
21	0.01	
22	0.012	
23	0.014	
24	0.016	
25	0.018	
26	0.02	
27	0.022	
28	0.024	
29	0.026	
30	0.028	
31	0.03	
32	0.032	
33	0.034	
34	0.036	
35	0.038	
36	0.04	

$$T(r) = T_w + \frac{q_0}{6 \cdot k} \cdot (R^2 - r^2) - \frac{q_0}{20 \cdot k \cdot R^2} \cdot (R^4 - r^4)$$

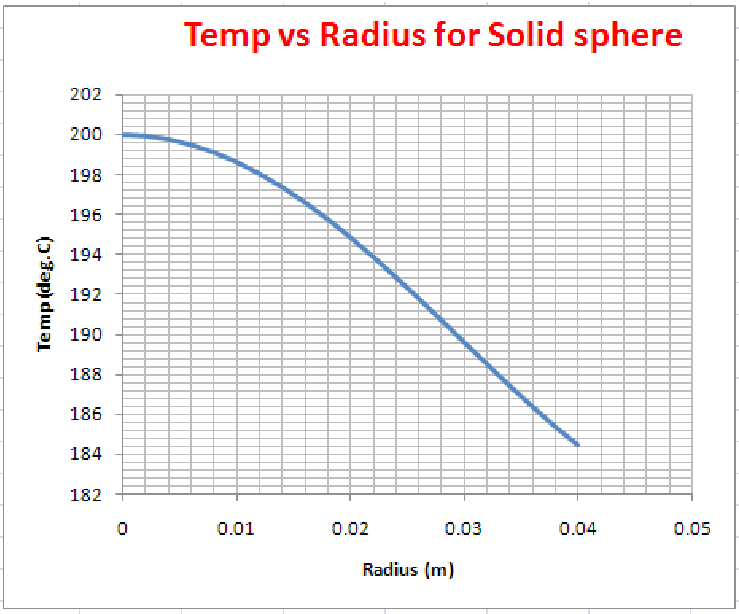
In the above fig., centre temp, Tr at r = 0, is entered directly as 200 C. In cell D17, we have entered the eqn for Tr for r = 0.002 m. See the Formula bar and verify the eqn.

Now, drag-copy the cell D117 up to cell D36. This fills up the Table:

	A	B	C	D
13		To plot T as a function of r:		
14				
15			r(m)	Tr(deg.C)
16			0	200.000
17			0.002	199.944
18			0.004	199.778
19			0.006	199.503
20			0.008	199.122
21			0.01	198.637
22			0.012	198.054
23			0.014	197.378
24			0.016	196.615
25			0.018	195.773
26			0.02	194.861
27			0.022	193.888
28			0.024	192.864
29			0.026	191.801
30			0.028	190.712
31			0.03	189.609
32			0.032	188.508
33			0.034	187.424
34			0.036	186.374
35			0.038	185.374
36			0.04	184.444

Note that at r = 0.04 m, i.e. at the surface the temp is 184.444 C as it should be. *It shows that the eqns are entered correctly.*

Now, plot Tr vs r:



3. To find the heat flow rate at the surface: Q = -k * A_s * (dT/dr) at the surface. A_s is the surface area of the sphere = 4 * π * R2. So, we have to find out dT/dr at the surface, i.e. at r = 0.04 m. We do this by considering the increase in Tr for a very small increase in r, say 0.000001 m. See the worksheet below:

F32 f_x =(D37-D36)/(C37-C36)

$$T(r) = T_w + \frac{q_0}{6 \cdot k} \cdot \left(R^2 - r^2\right) - \frac{q_0}{20 \cdot k \cdot R^2} \cdot \left(R^4 - r^4\right)$$

	C	D	F	G	H
14					
15	r(m)	Tr(deg.C)			
16	0	200.000			
17	0.002	199.944			
18	0.004	199.778			
19	0.006	199.503			
20	0.008	199.122			
21	0.01	198.637			
22	0.012	198.054			
23	0.014	197.378			
24	0.016	196.615			
25	0.018	195.773			
26	0.02	194.861			
27	0.022	193.888			
28	0.024	192.864			
29	0.026	191.801	A_s	0.0201062	m^2
30	0.028	190.712			
31	0.03	189.609	dT/dr (C/m)	Q(W)	
32	0.032	188.508	-444.45556	107.23571	
33	0.034	187.424			
34	0.036	186.374			
35	0.038	185.374			
36	0.039999	184.445			
37	0.04	184.444			

Note that we have introduced a very small increment in 'r' between cells C36 and C37, and calculated the corresponding increment in Tr in cells D36 to D37. And dT/dr at r = 0.04 is calculated in cell F32. See the simple formula used in the Formula bar.A_s is calculated in cell G28. Then, Q is calculated as: Q = -k * A_s * (dT/dr).

We get: Q = 107.236 W … Ans.

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1G Transient conduction:

Learning objectives:

1. 'Transient heat conduction' or, 'unsteady state conduction', means 'time dependent conduction'. Obviously, in transient conduction, temperature depends not only on position in the solid, but also on time.
2. Typical examples of transient conduction occur in:
   a) heat exchangers
   b) boiler tubes
   c) cooling of I.C.Engine cylinder heads
   d) heat treatment of engineering components and quenching of ingots
   e) heating of electric irons
   f) heating and cooling of buildings
   g) freezing of foods, etc.
3. Analysis where the internal resistance of the body for heat conduction is negligible and the whole body may be treated as a 'lump' as far as temperature increase or decrease is concerned, is known as **'lumped system analysis'**.
4. In this section, we shall first study problems on the lumped system analysis; then, we shall study analytical and chart solutions for some of the practically important transient conduction problems for the cases of a large slab, long cylinder, sphere and a semi-infinite medium.

Summary of Basic equations:

Basic relations for transient conduction

Relation	Comments
$\frac{d^2T}{dx^2} = \frac{1}{\alpha}\cdot\frac{dT}{d\tau}$	Governing differential eqn. in Cartesian cords. for one dimensional, transient cond. without heat generation.
$\frac{\theta}{\theta_i} = \frac{T(\tau)-T_a}{T_i-T_a} = \exp\left(\frac{-h\cdot A\cdot\tau}{\rho\cdot C_p\cdot V}\right)$ if Bi < 0.1.....(7.12)	Lumped system analysis, $Bi = \frac{h\cdot L_c}{k}$ and $L_c = \frac{V}{A}$
$\frac{\theta}{\theta_i} = \frac{T(\tau)-T_a}{T_i-T_a} = \exp(-Bi\cdot Fo)$ if Bi < 0.1.....(7.13)	$Fo = \frac{\alpha\cdot\tau}{L_c^2}$ = **Fourier number, or relative time**
$\frac{\rho\cdot C_p\cdot V}{h\cdot A} = t$	Time constant (seconds)
$Q(\tau) = m\cdot C_p\cdot\frac{dT(\tau)}{d\tau}$ W......(7.6,a) $Q(\tau) = h\cdot A\cdot(T(\tau)-T_a)$ W.....(7.6,b)	Instantaneous heat transfer rate
$Q_{tot} = m\cdot C_p\cdot(T(\tau)-T_i)$ J....(7.7,a) $Q_{tot} = \int_0^{\tau} Q(\tau)\,d\tau$ J.....(7.7,b)	Total heat transfer from time =0 to τ
$Q_{max} = m\cdot C_p\cdot(T_a-T_i)$ J.....(7.8)	Max. heat transfer
$\frac{T(\tau)-T_a}{T_i-T_a} = \exp(-a\cdot\tau) + \frac{\frac{b}{a}}{T_i-T_a}\cdot(1-\exp(-a\cdot\tau))$ ......(7.20) $a = \frac{h\cdot A}{\rho\cdot V\cdot C_p}$ $b = \frac{q\cdot A}{\rho\cdot V\cdot C_p}$	Temp. distribn. when transient condition is induced by mixed B.C. (eg. a slab with const. heat flux, q, at one surface and convection at the other surface)
$\tau = \frac{-1}{a}\cdot\ln\left[\frac{T(\tau)-T_a-\left(\frac{b}{a}\right)}{T_i-T_a-\left(\frac{b}{a}\right)}\right]$ ......(7.21)	Time reqd. to attain a given temp. in the above case

$T(\tau) = T_a + \frac{b}{a} = T_a + \frac{q}{h}$ .....(7.22)	Steady state temp. for the above case (obtained by putting τ = ∞, in eqn. (7.20)
$\theta(x,\tau) = \frac{T(x,\tau) - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \cdot \cos\left(\frac{\lambda_1 \cdot x}{L}\right)$ ...Fo > 0.2....(7.24, a)	One term approx. solution for plane wall
$\theta(x,\tau) = \frac{T(r,\tau) - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \cdot J_0\left(\frac{\lambda_1 \cdot r}{R}\right)$ ...Fo > 0.2....(7.24, b)	One term approx. solution for long cylinder
$\theta(x,\tau) = \frac{T(r,\tau) - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \cdot \frac{\sin\left(\frac{\lambda_1 \cdot r}{R}\right)}{\frac{\lambda_1 \cdot r}{R}}$ ...Fo > 0.2....(7.24, c)	One term approx. solution for a sphere
$\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo}$ ....(7.25, a)	One term approx.-centre temp. for plane wall
$\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo}$ ...(7.25, b)	One term approx.-centre temp. for long cyl.
$\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo}$ ...(7.25, c)	One term approx.-centre temp. for sphere
$\frac{Q}{Q_{max}} = 1 - \theta_0 \cdot \frac{\sin(\lambda_1)}{\lambda_1}$ ......(7.27, a)	Dimensionless heat transfer for large, plane wall
$\frac{Q}{Q_{max}} = 1 - 2 \cdot \theta_0 \cdot \frac{J_1(\lambda_1)}{\lambda_1}$ .....(7.27, b)	Dimensionless heat transfer for long cylinder
$\frac{Q}{Q_{max}} = 1 - 3 \cdot \theta_0 \cdot \left(\frac{\sin(\lambda_1) - \lambda_1 \cdot \cos(\lambda_1)}{\lambda_1^3}\right)$ .....(7.27, c)	Dimensionless heat transfer for a sphere
Semi-infinite slab: $\frac{T(x,\tau) - T_0}{T_i - T_0} = \mathrm{erf}\left(\frac{x}{2 \cdot \sqrt{\alpha \cdot \tau}}\right)$ ....(7.29)	Dimensionless temp. distribn. in a semi-infinite slab, surface temp. suddenly changed to T0

$T(x,\tau) = T_0 + (T_i - T_0)\cdot\frac{2}{\sqrt{\pi}}\cdot\int_0^{\frac{x}{\sqrt{4\cdot\alpha\cdot\tau}}} \exp(-u^2)\,du$ .....(7.31)	Temp. distribn. in a semi-infinite slab, surface temp. suddenly changed to T0
$Q_{surface} = k\cdot A\cdot\frac{(T_0 - T_i)}{\sqrt{\pi\cdot\alpha\cdot\tau}}$ W......(7.33)	Heat flow rate at the surface, for above case
$Q_{total} = 1.13\cdot k\cdot A\cdot(T_0 - T_i)\cdot\sqrt{\frac{\tau}{\alpha}}$ J......(7.34)	Total heat flow during time period τ for the above case

Semi-infinite slab:

Temp. distribn. in a semi-infinite slab, surface is subjected to const. heat flux, q_0:
$T(x,\tau) = T_i + \frac{2\cdot q_0\cdot\sqrt{\frac{\alpha\cdot\tau}{\pi}}}{k}\cdot\exp\left(\frac{-x^2}{4\cdot\alpha\cdot\tau}\right) - \frac{q_0\cdot x}{k}\cdot\left(1 - \mathrm{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}\right)\right)$ .....(7.35)

Semi-infinite slab:

Temp. distribn. in a semi-infinite slab, surface is subjected to convection at its surface:
$\frac{T(x,\tau)-T_i}{T_a-T_i}=1-\mathrm{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}\right)-\left(\exp\left(\frac{h\cdot x}{k}+\frac{h^2\cdot\alpha\cdot\tau}{k^2}\right)\right)\cdot\left(1-\mathrm{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}+\frac{h\cdot\sqrt{\alpha\cdot\tau}}{k}\right)\right)$ .....(7.36)

Multi-dimensional transient conduction:

Temp. distribn. for a body formed by intersection of three bodies:	
$\left(\frac{\theta}{\theta_i}\right)_{solid}=\left(\frac{\theta}{\theta_i}\right)_{system1}\cdot\left(\frac{\theta}{\theta_i}\right)_{system2}\cdot\left(\frac{\theta}{\theta_i}\right)_{system3}$	(7.38)
Temp. distribution in long, rectangular bar:	
$\left(\frac{T(x,y,\tau)-T_a}{T_i-T_a}\right)_{rect\_bar}=\theta_{wall}(x,\tau)\cdot\theta_{wall}(y,\tau)$	(7.40)
Temp. distribution in short cylinder:	
$\left(\frac{T(r,x,\tau)-T_a}{T_i-T_a}\right)_{short\_cyl}=\theta_{wall}(x,\tau)\cdot\theta_{cyl}(r,\tau)$	(7.41)
Heat transfer in two dimensional transient conduction:	
$\left(\frac{Q}{Q_{max}}\right)_{total}=\left(\frac{Q}{Q_{max}}\right)_1+\left(\frac{Q}{Q_{max}}\right)_2\cdot\left[1-\left(\frac{Q}{Q_{max}}\right)_1\right]$	(7.42)
Heat transfer in three dimensional transient conduction*:*	
$\left(\frac{Q}{Q_{max}}\right)_{total}=\left(\frac{Q}{Q_{max}}\right)_1+\left(\frac{Q}{Q_{max}}\right)_2\cdot\left[1-\left(\frac{Q}{Q_{max}}\right)_1\right]+\left(\frac{Q}{Q_{max}}\right)_3\cdot\left[1-\left(\frac{Q}{Q_{max}}\right)_1\right]\cdot\left[1-\left(\frac{Q}{Q_{max}}\right)_2\right]$	(7.42)

Table 1G.2 [Ref. 1]

Transient heat conduction in a plane wall, long cylinder and sphere-coefficients for one term approximation

---

Sphere **Plane wall** **Cylinder**

B_i	8_1	A_1	8_1	A_1	8_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

Table 1G.3

Zeroth and first order Bessel functions of the first kind

$x := 0, 0.1 .. 3.2$ ....define range variable x from 0 to 3.2, with an increment of 0.1

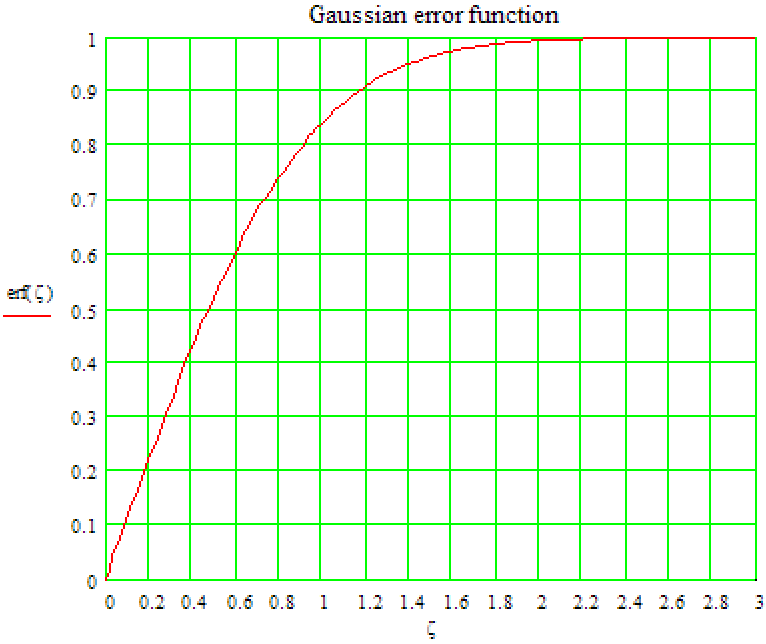
x	J0(x)	J1(x)
0	1	0
0.1	0.9975	0.04994
0.2	0.99002	0.0995
0.3	0.97763	0.14832
0.4	0.9604	0.19603
0.5	0.93847	0.24227
0.6	0.912	0.2867
0.7	0.8812	0.329
0.8	0.84629	0.36884
0.9	0.80752	0.40595
1	0.7652	0.44005
1.1	0.71962	0.4709
1.2	0.67113	0.49829
1.3	0.62009	0.52202
1.4	0.56686	0.54195
1.5	0.51183	0.55794
1.6	0.4554	0.5699
1.7	0.39798	0.57777
1.8	0.33999	0.58152
1.9	0.28182	0.58116
2	0.22389	0.57672
2.1	0.16661	0.56829
2.2	0.11036	0.55596
2.3	0.05554	0.53987
2.4	0.00251	0.52019
2.5	-0.04838	0.49709
2.6	-0.0968	0.47082
2.7	-0.14245	0.4416
2.8	-0.18504	0.40971
2.9	-0.22431	0.37543
3	-0.26005	0.33906
3.1	-0.29206	0.30092
3.2	-0.32019	0.26134

Table 1G.4. Values of 'error function'

ζ_1	$erf(\zeta_1)$	ζ_1	$erf(\zeta_1)$
0	0	1	0.8427
0.03	0.0338	1.05	0.8624
0.06	0.0676	1.1	0.8802
0.09	0.1013	1.15	0.8961
0.12	0.1348	1.2	0.9103
0.15	0.168	1.25	0.9229
0.18	0.2009	1.3	0.934
0.21	0.2335	1.35	0.9438
0.24	0.2657	1.4	0.9523
0.27	0.2974	1.45	0.9597
0.3	0.3286	1.5	0.9661
0.33	0.3593	1.55	0.9716
0.36	0.3893	1.6	0.9763
0.39	0.4187	1.65	0.9804
0.42	0.4475	1.7	0.9838
0.45	0.4755	1.75	0.9867
0.48	0.5027	1.8	0.9891
0.51	0.5292	1.85	0.9911
0.54	0.5549	1.9	0.9928
0.57	0.5798	1.95	0.9942
0.6	0.6039	2	0.9953
0.63	0.627	2.05	0.9963
0.66	0.6494	2.1	0.997
0.69	0.6708	2.15	0.9976
0.72	0.6914	2.2	0.9981
0.75	0.7112	2.25	0.9985
0.78	0.73	2.3	0.9989
0.81	0.748	2.35	0.9991
0.84	0.7651	2.4	0.9993
0.87	0.7814	2.45	0.9995
0.9	0.7969	2.5	0.9996
0.93	0.8116	2.55	0.9997
0.96	0.8254	2.6	0.9998
0.99	0.8385	2.65	0.9998
		2.7	0.9999
		2.75	0.9999
		2.8	0.9999

==

Graph of error function:



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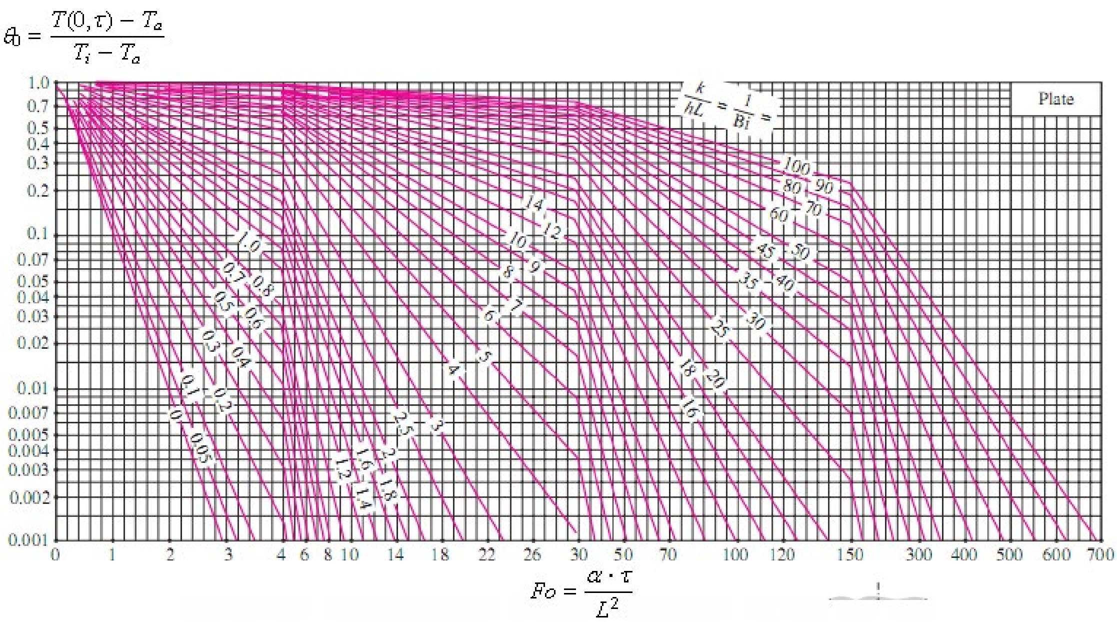
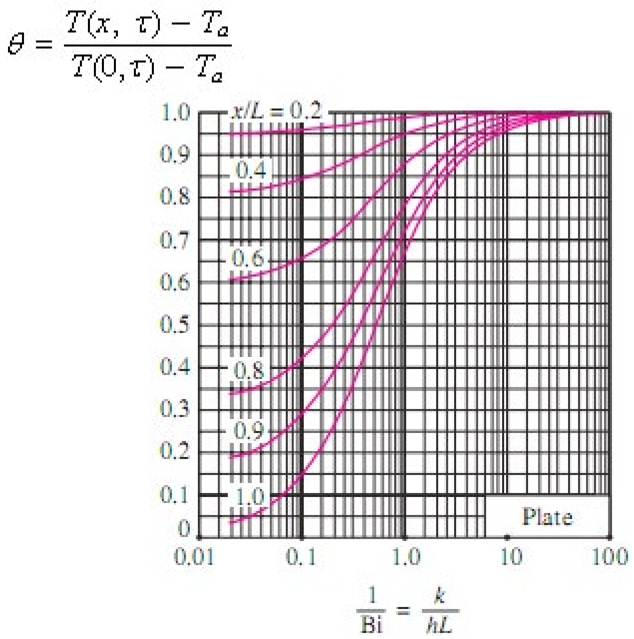
Transient conduction – Heisler charts and Grober charts: [Ref. 2]

1. **For Plane wall:**

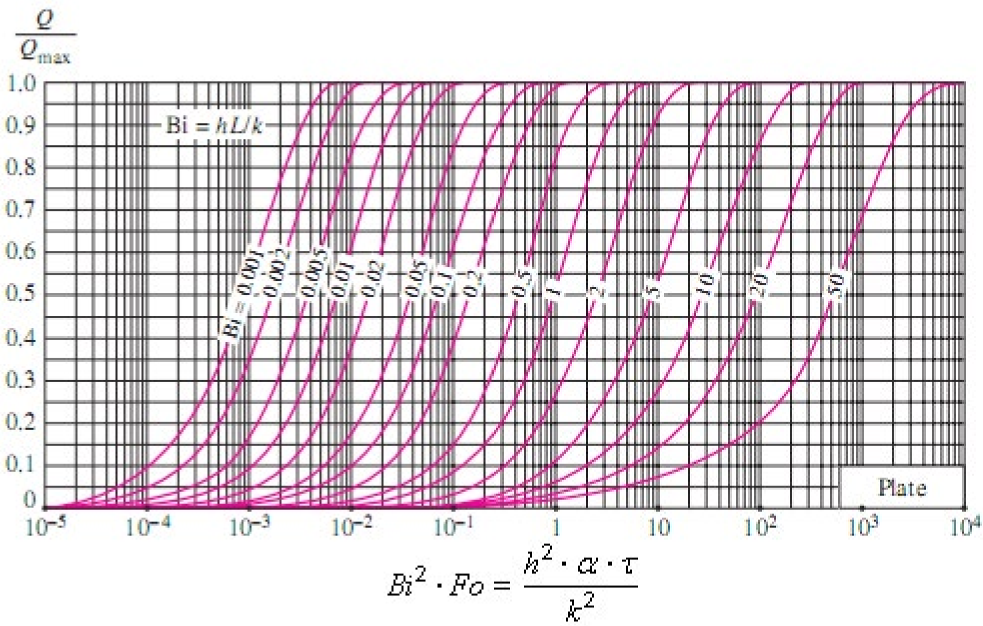
Mid-plane temp:



Position correction chart:

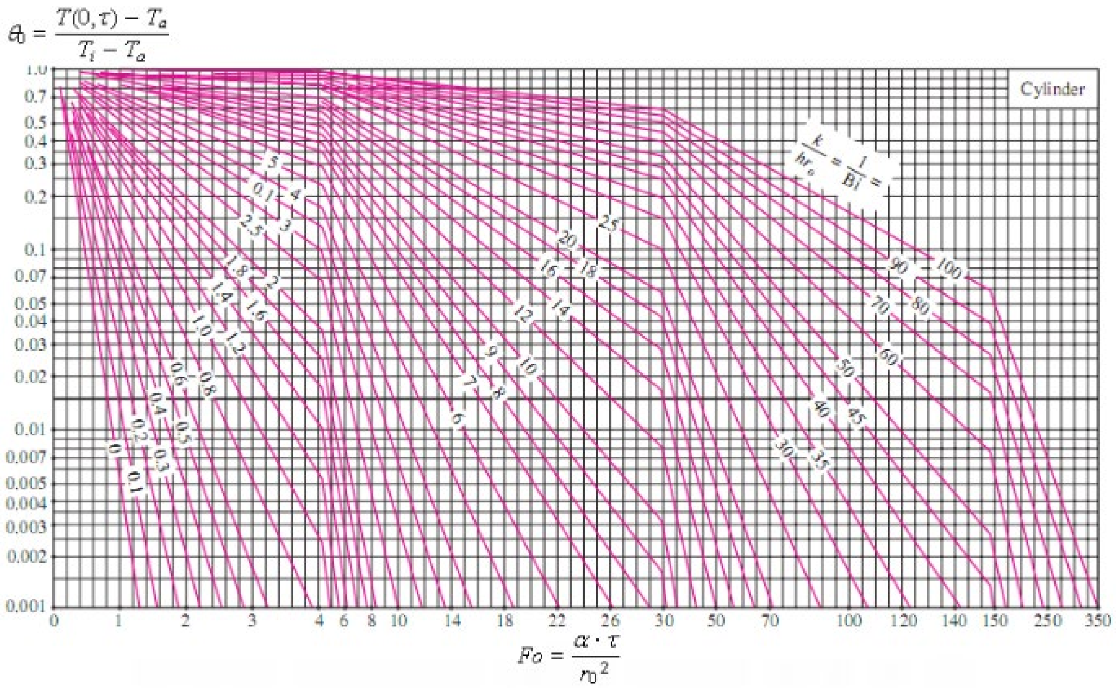


Grober chart for heat transfer:

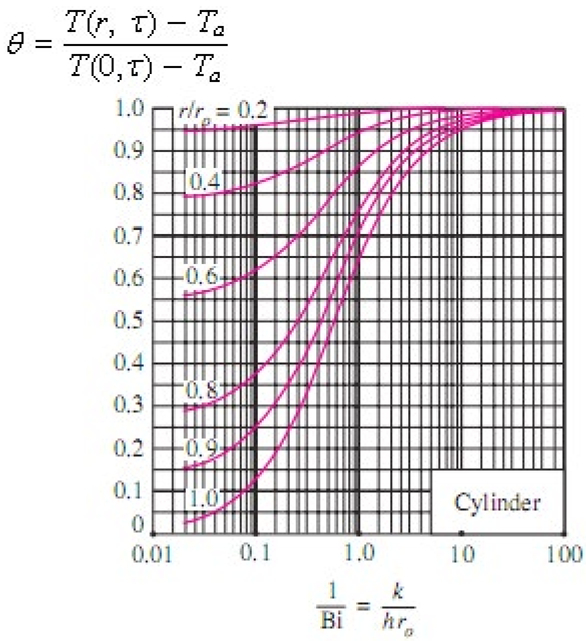


2. **For a long cylinder:X**

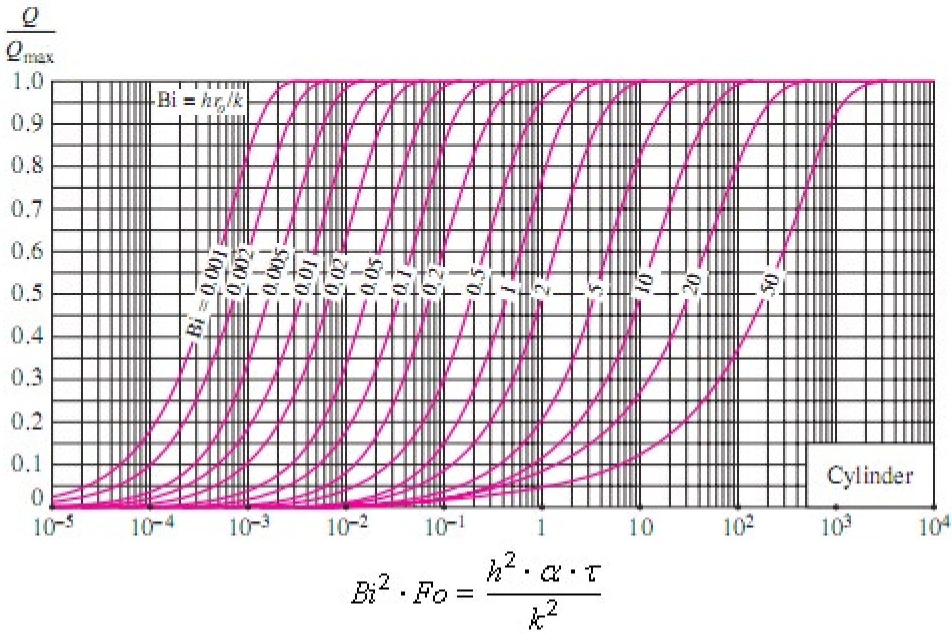
Centreline temp:



Position correction chart:

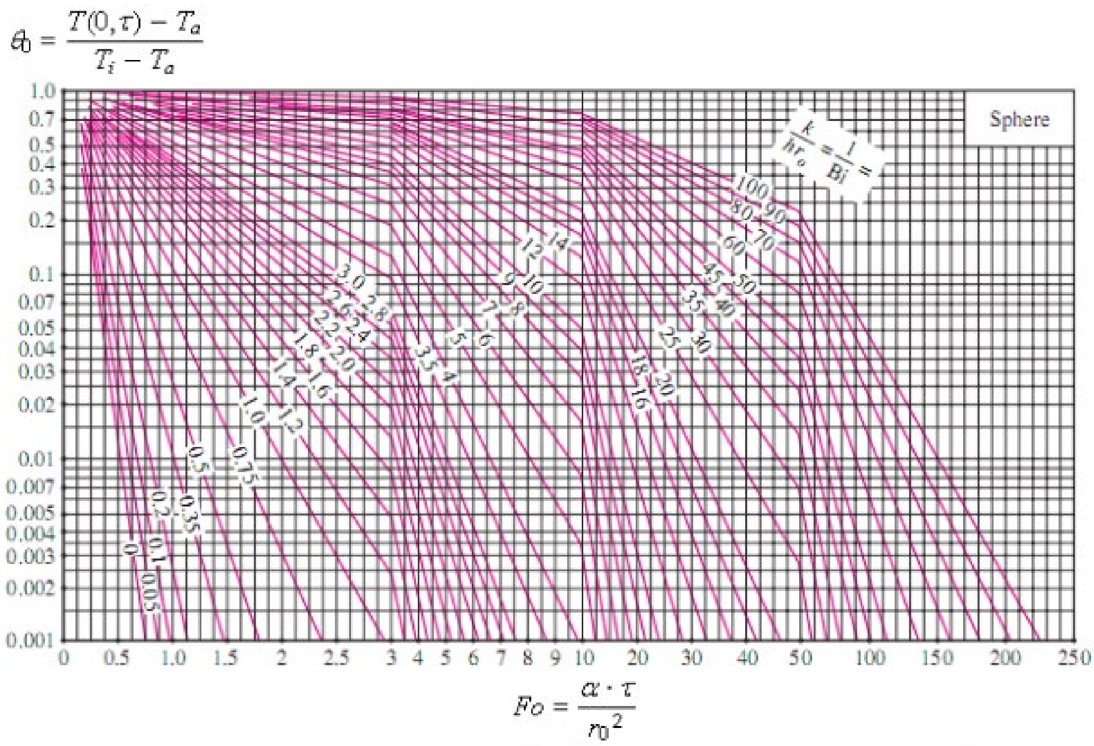


Grober chart for heat transfer:

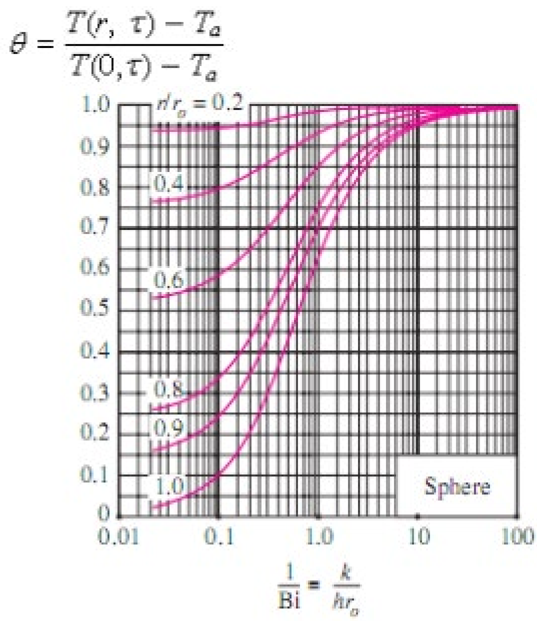


3. **For a Sphere:**

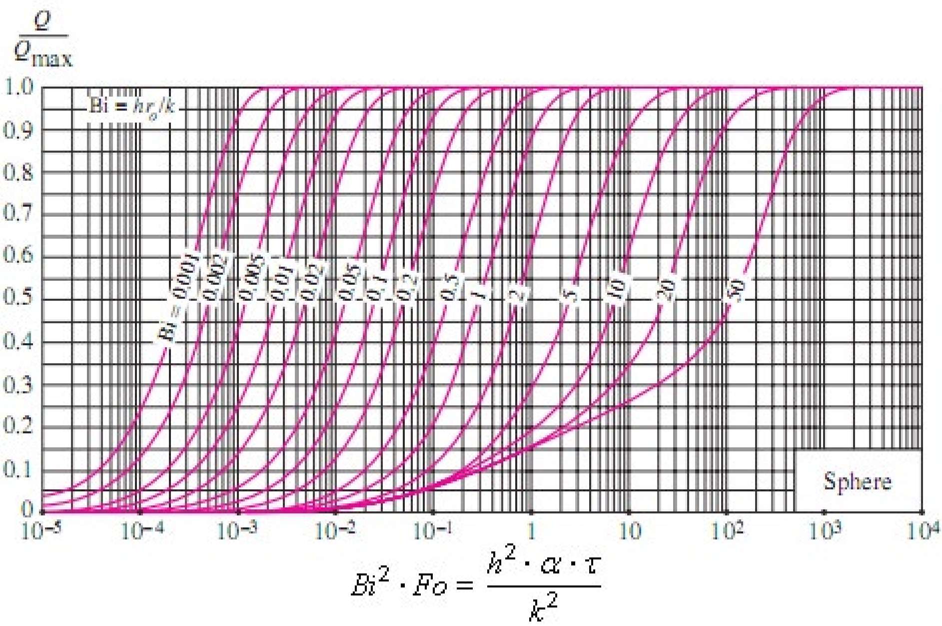
Centre temp:



Position correction chart:



Grober chart for heat transfer:



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"Prob. 1G.1. An Aluminium sphere weighing 5.5 kg and initially at a temp of 290 C is suddenly immersed in a fluid at 15 C. The convective heat transfer coeff is 58 W/m^2.K. Estimate the time required tio cool the aluminium to 95 C using lumped capacity method of analysis. For Aluminium: rho = 2700 kg/m^3, cp = 900 J/kg.K, k = 205 W/m.C. [VTU – VI Sem. B.E. – Dec. 2010]:"

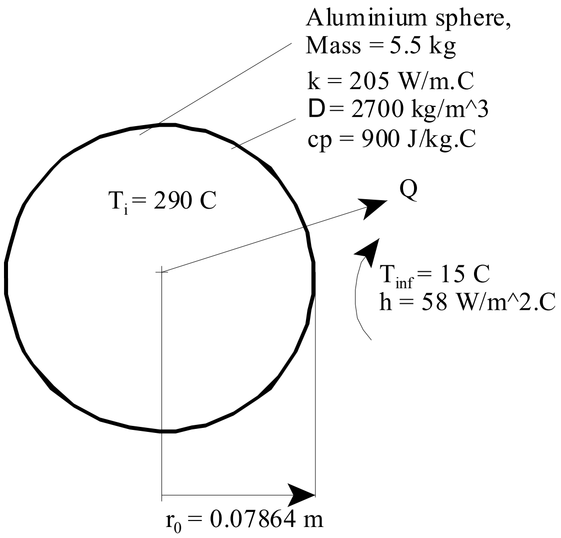


Fig.Prob.1G.1

EES Solution:

"Data:"

mass = 5.5[kg]
T_i = 290[C]
T_inf = 15[C]
h = 58[W/m^2-C]
T = 95[C]
k = 205[W/m-C]
rho = 2700 [kg/m^3]
cp = 900[J/kg-C]

"Calculations:"

V = mass / rho "[m^3] … finds Vol, V of sphere"
V = (4/3) * pi * r_o^3 "[m]… finds rad r_o"
alpha = k / (rho * cp) "[m^2/s] .. thermal diffusivity of Al"

Biot = (h * r_o / 3) / k "….Biot No."

"It is found that: Biot = 0.007416 < 0.1; **so, lumped system analysis is applicable:**"

"Then, we have:"

(T – T_inf) / (T_i – T_inf) = exp(-h * tau / (rho * cp * r_o / 3)) " Finds tau"

Results:

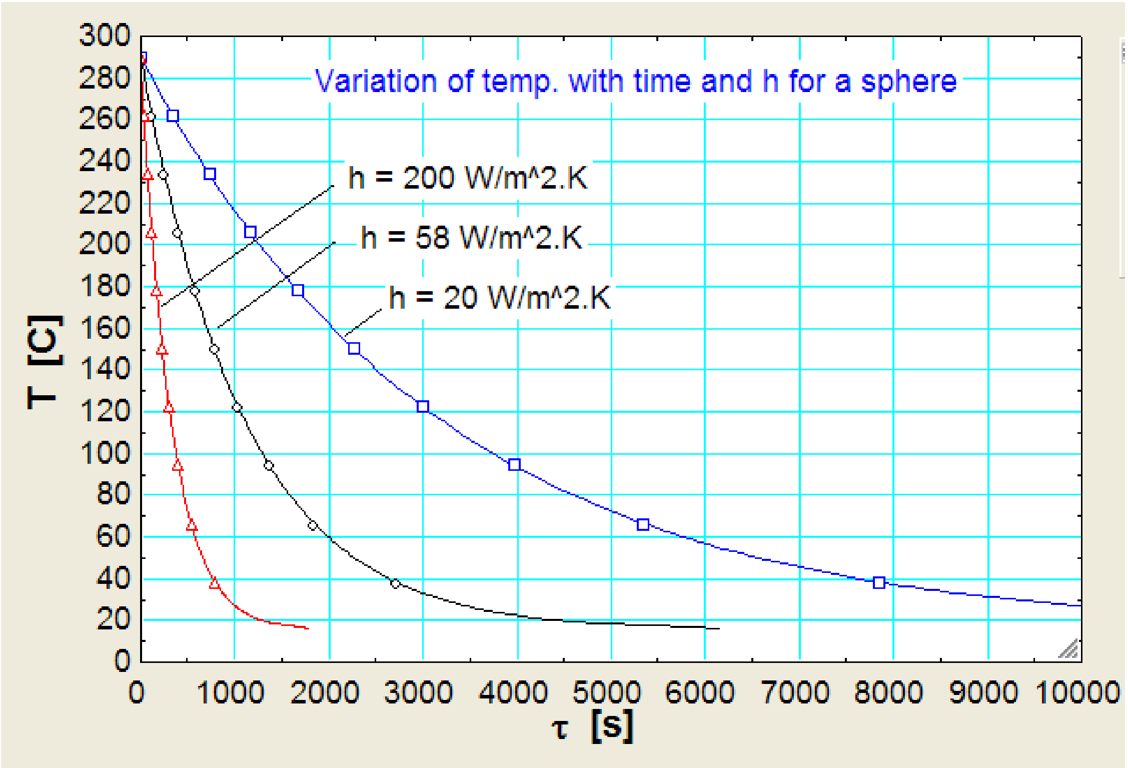
Unit Settings: SI C kPa kJ mass deg

α = 0.00008436 [m^2/s]	Biot = 0.007416 [-]
cp = 900 [J/kg-C]	h = 58 [W/m^2-C]
k = 205 [W/m-C]	mass = 5.5 [kg]
ρ = 2700 [kg/m^3]	r_o = 0.07864 [m]
T = 95 [C]	τ = 1356 [s]
T_i = 290 [C]	T_{inf} = 15 [C]
V = 0.002037 [m^3]	

Thus:

tau = 1356 s …. Time required to cool to 95 C … Ans.

Next, plot the variation of temp with time for h = 20, 50 and 200 W.m^2.K:



Observe, starting from T = 290 C, how much time elapses for steady state temp of 15 C is reached.

This time, of course, decreases as h increases.

It is noted that time required to reach a temp of 16 C is 17889, 6169 and 1789 s when h is 20, 58 and 200 W/m^2.K respectively.

==

Prob. 1G.2. A solid copper sphere of 10 cm dia (density = 8954 kg/m^3, cp = 383 J/kg.C, k = 386 W/m.C), initially at a uniform temp of 250 C is suddenly immersed in a well stirred fluid maintained at a uniform temp of 50 C. Heat transfer coeff between the sphere and the fluid is 200 W/m^2.C. Determine the temp of the copper block at 5 min after the immersion. [VTU – VI Sem. B.E. – June 2012]:

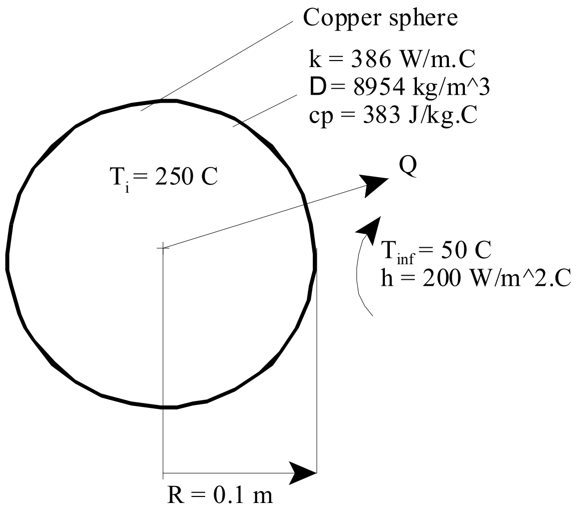


Fig.Prob.1G.2

Mathcad Solution:

Data:

$R := 0.1$ m $\rho := 8954$ kg/m^3 $cp := 383$ J/kg.C $k := 386$ W/m.C.

$h := 200$ W/m^2.K $T_i := 250$ C $T_{inf} := 50$ C

$tau := 300$ s

Calculations:

$Lc := \frac{R}{3}$ i.e. $Lc = 0.033$ m ... characteristic dimension

Biot Number:

$Bi := \frac{h \cdot Lc}{k}$ i.e. $Bi = 0.017$ **.....less than 0.1; Therefore Lumped analysis is applicable**

Now, we have:

$$\frac{T - T_{inf}}{T_i - T_{inf}} = \exp\left(-\frac{h \cdot A \cdot tau}{cp \cdot \rho \cdot V}\right)$$ ...where A is the surface area and V is the vol of the sphere

i.e. $$\frac{T - T_{inf}}{T_i - T_{inf}} = \exp\left(-\frac{h \cdot tau}{cp \cdot \rho \cdot Lc}\right)$$ ...since Lc = V/A = R/3 for a sphere

Therefore:

$$T := T_{inf} + \left(T_i - T_{inf}\right) \cdot \exp\left(-\frac{h \cdot tau}{cp \cdot \rho \cdot Lc}\right)$$

i.e. $T = 168.326$ **C .... temp after 5 min.....Ans.**

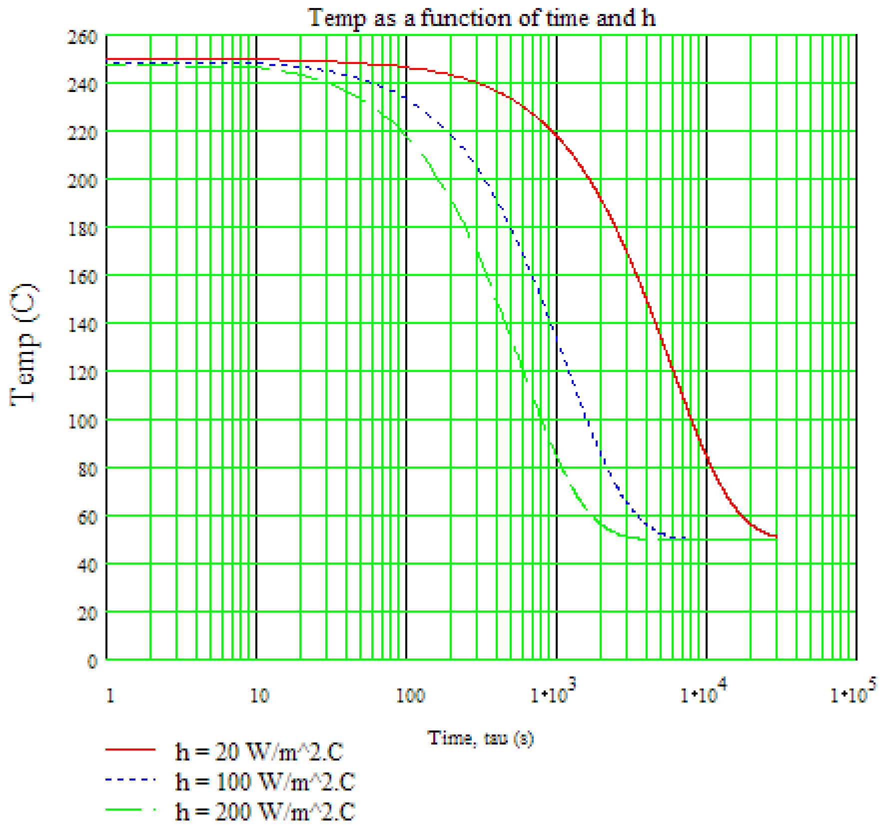
Plot the Temp vs time curve for different values of h:

Express T as a function of tau and h:

$Bi(h) := \frac{h \cdot Lc}{k}$ $Bi(200) = 0.017$

$$T(h, tau) := T_{inf} + \left(T_i - T_{inf}\right) \cdot \exp\left(-\frac{h \cdot tau}{cp \cdot \rho \cdot Lc}\right)$$

tau := 0.001, 10.. 30000....define a range variable tau



Note:

T(20, 30000) = 51.051   **C...temp. reached when h = 20 W/m^2.C and tau = 30000 s**

T(100, 6000) = 51.051   **C...temp. reached when h = 100 W/m^2.C and tau = 6000 s**

T(200, 5000) = 50.032   **C...temp. reached when h = 200 W/m^2.C and tau = 5000 s**

==

Prob. 1G.3. Heat transfer coeff for air flowing over a sphere is to be determined by observing the temp – time history of a sphere fabricated from pure copper. The sphere, which is 12.7 mm in dia, is at 66 C before it is inserted into an airstream having a temp of 27 C. A thermocouple on the outer surface of the sphere indicates 55 C after 69 s. Assume, and then justify, that the sphere behaves as a spacewise isothermal object and calculate the heat transfer coeff. [Ref: 3]

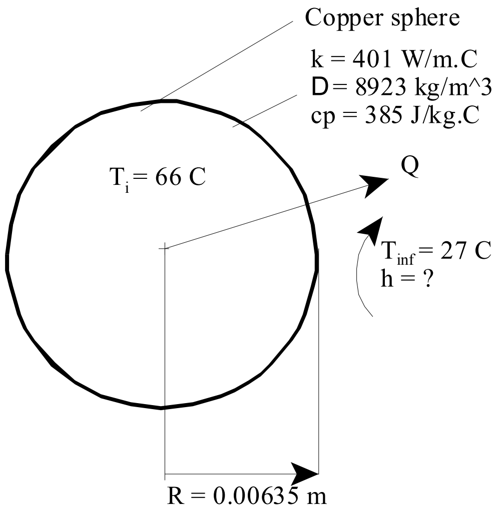


Fig.Prob.1G.3

Mathcad Solution:

Data:

$R := 0.00635$ m $\rho := 8923$ kg/m^3 $cp := 385$ J/kg.C $k := 401$ W/m.C.

$T_i := 66$ C. $T := 55$ C...temp after τ = 69 s

$\tau := 69$ s $T_{inf} := 27$ C

Calculations:

Now, assuming that lumped system analysis is applicable for this sphere (i.e. Bi < 0.1):

$Lc := \frac{R}{3}$ i.e. $Lc = 2.117 \cdot 10^{-3}$ m ... characteristic dimension

$$\frac{T - T_{inf}}{T_i - T_{inf}} = \exp\left(\frac{-h \cdot \tau}{cp \cdot \rho \cdot L_c}\right)$$

i.e. $$h := \frac{cp \cdot \rho \cdot Lc \cdot \ln\left(\frac{T - T_{inf}}{T_i - T_{inf}}\right)}{-\tau}$$

i.e. $h = 34.92$ **W/m^2.C ... heat transfer coeff. from sphere to air ... Ans.**

To justify that surface temp of sphere, is in fact, the temp throughout the body of the sphere, i.e. Biot No. should be less than 0.1.

Verify that Bi < 0.1:

Biot Number:

$Bi := \frac{h \cdot Lc}{k}$ ..Biot number

i.e. $Bi = 1.843 \cdot 10^{-4}$

Note that Bi < 0.1 ; Therefore, temp anywhere within the body does not differ by more than 5 % and Lumped parameter analysis is applicable for heat transfer calculations.

==

Prob. 1G.4. A Thermocouple (TC) junction is in the form of 4 mm sphere. Properties of the material are: cp = 420 J/kg. K, r = 8000 kg/m^3, k = 40 W/m.K, h = 45 W/m^2.K. Find, if the junction is initially at a temp of 40 C and inserted in a stream of hot air at 300 C:

1) the time const. of the TC
2) the TC is taken out from the hot air after 10 s and kept in still air at 30 C. Assuming heat transfer coeff. in air as 10 W/m^2.K, find the temp. attained by the junction 20 s after removing from hot air stream. [M.U. 1997]

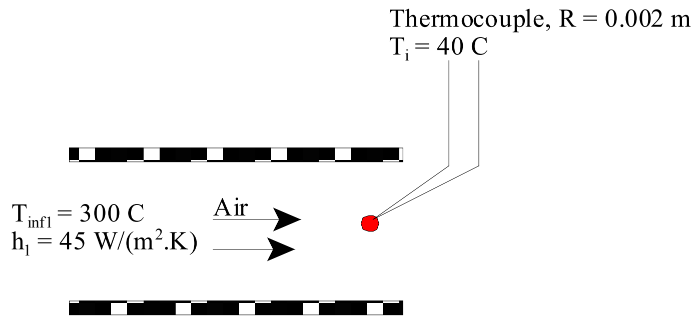


Fig.Prob.1G.4 (a) Temperature measurement, with thermocouple placed in the air stream

Mathcad Solution:

Data:

$R := 0.002$ m $\rho := 8000$ kg/m^3 $cp := 420$ J/kg.C $k := 40$ W/m.C.

$T_i := 40$ C. $T_{inf1} := 300$ C... temp of hot air $h_1 := 45$ W/m^2.K ... in hot air

$\tau_1 := 10$ s ... duration of stay of TC in hot air

$T_{inf2} := 30$ C...temp of still air $h_2 := 10$ W/m^2.K ... in still air

$\tau_2 := 20$ s ... duration of stay of TC in still air

Calculations:

$Lc := \frac{R}{3}$ i.e. $Lc = 6.667 \cdot 10^{-4}$ m ... characteristic dimension

Time constant, tstar:

We have: $\frac{h \cdot A \cdot \tau}{\rho \cdot cp \cdot V} = \frac{\tau}{tstar}$

i.e. $tstar = \frac{\rho \cdot cp \cdot V}{h \cdot A}$

i.e. $tstar := \frac{\rho \cdot cp \cdot Lc}{h_1}$ ..since Lc = V/A = R/3 for a sphere, h1 is the heat transfer coeff in first case.

i.e. $tstar = 49.778$ **s ... time constant for TC .... Ans.**

Case 1: When the TC is just taken out of hot air, its temp. T1 is given by::

$\frac{T1 - T_{infl}}{T_i - T_{infl}} = \exp\left(\frac{-h_1 \cdot \tau_1}{cp \cdot \rho \cdot L_c}\right)$ ..T1 is the temp of TC when it is just taken out of hot air

i.e. $T1 := T_{infl} + \left(T_i - T_{infl}\right) \cdot \exp\left(\frac{-h_1 \cdot \tau_1}{cp \cdot \rho \cdot Lc}\right)$

i.e. $T1 = 87.32$ **C .. temp. of TC when it is just taken out of hot air**

Now, TC is held in still air.

And, T1 becomes the initial temp for this case:

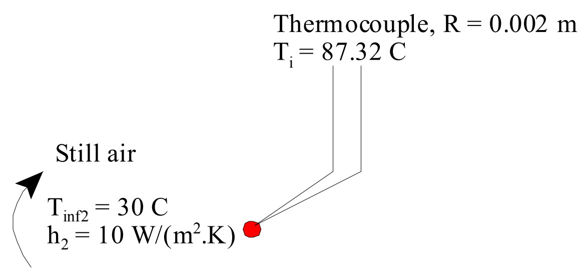


Fig.Prob.1G.4(b) Temperature measurement, with thermocouple placed in still air

Case 2: When the TC is taken out of hot air, and kept in still air for 10 s, its temp. T2 is given by::

$$\frac{T2 - T_{inf2}}{T1 - T_{inf2}} = \exp\left(\frac{-h_2 \cdot \tau_2}{cp \cdot \rho \cdot L_c}\right)$$ ..T2 is the temp of TC when it is just taken out of still air after 10 s

i.e. $$T2 := T_{inf2} + (T1 - T_{inf2}) \cdot \exp\left(\frac{-h_2 \cdot \tau_2}{cp \cdot \rho \cdot Lc}\right)$$

i.e. $T2 = 82.424$ **C .. temp. of TC when it is just taken out of still air ... Ans.**

==

"**Prob. 1G.5.** A 12 cm dia long bar initially at a uniform temp of 40 C is placed in a medium at 650 C with a convective coeff of 22 W/m^2.K. Calculate the time required for the bar to reach 255 C. Take k = 20 W/m.K, rho = 580 kg/m^3, and cp = 1050 J/kg.K. [VTU – VI Sem. B.E. – Dec. 2009–Jan. 2010]:"

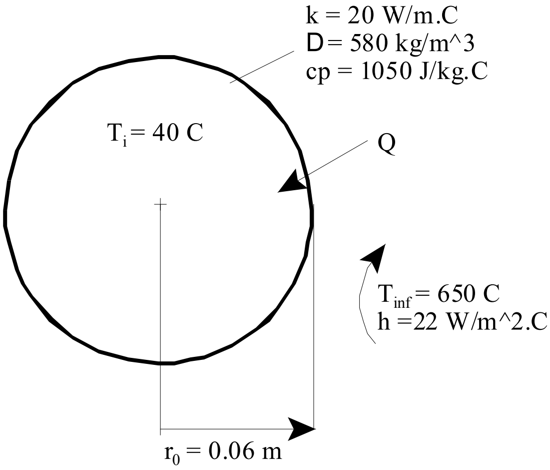


Fig.Prob.1G.5

EES Solution:

"Data:"

r_o = 0.06[m] "....radius of cyl."

T_i = 40[C] "...initial temp."

T_inf = 650[C] "..temp. of medium"

h = 22[W/m^2-C] "...heat tr. coeff."

k = 20[W/m-C] "..thermal cond."

rho = 580 [kg/m^3] "...density"

cp = 1050[J/kg-C] "...sp.heat"

T = 255 [C] "..final temp. reached"

"Calculations:"

"First check if lumped system analysis is applicable by calculating Biot No."

Biot = (h*r_o/2)/k "...since, for a cylinder Lc = V/A = r_0/2"

"We find that: Biot = 0.033 < 0.1; so, lumped system analysis is applicable:"

"Then, we have:"

(T – T_inf) / (T_i – T_inf) = exp(-h * tau / (rho * cp *r_o / 2)) "Finds tau"

Results:

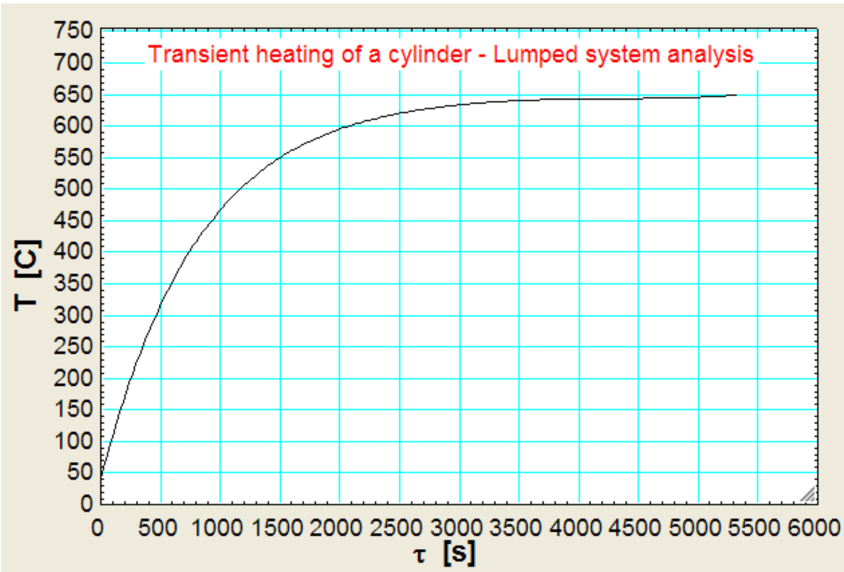
Unit Settings: SI C kPa J mass deg

Biot = 0.033 [-]	cp = 1050 [J/kg-C]	h = 22 [W/m^2-C]	k = 20 [W/m-C]
ρ = 580 [kg/m^3]	r_o = 0.06 [m]	T = 255 [C]	τ = 360.9 [s]
T_i = 40 [C]	T_{inf} = 650 [C]		

Thus:

tau = 360.9 s … time required to reach 255 C …. Ans.

Plot the temp vs time curve:



It is seen that a temp of 649 C is reached after 5326 s have elapsed.

==

“**Prob. 1G.6.** A Thermocouple (TC) junction, which may be approximated as a sphere, is to be used for temp measurement in a gas stream. The convection coeff between junction surface and the gas is 400 W/m^2.K and the junction thermo-physical properties are: k = 20 W/m.K, cp = 400 J/kg.K, rho = 8500 kg/m^3. Determine the junction dia needed for the TC to have a time constant of 1 s. If the junction is at 25 C and is placed in a gas stream at 200 C, how long will it take for the junction to reach 199 C? [VTU – VI Sem. B.E. – May–June 2010]:”

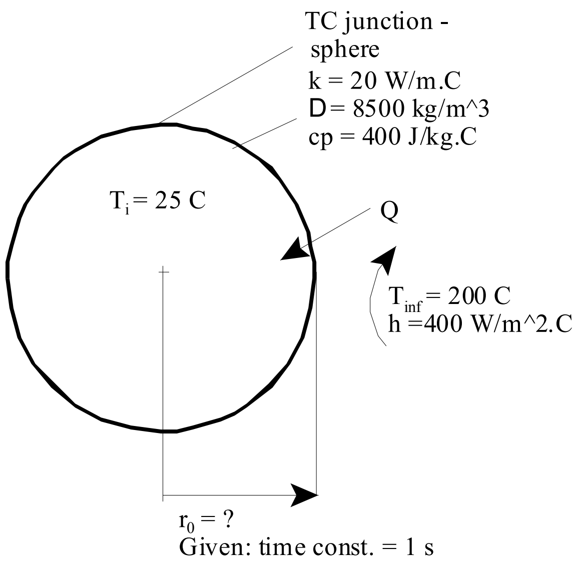


Fig.Prob.1G.6

EES Solution:

"Data:"

T_i = 25[C]

T_inf = 200[C]

h = 400[W/m^2-C]

k = 20[W/m-C]

rho = 8500 [kg/m^3]

cp = 400[J/kg-C]

T = 199[C]

"Calculations:"

"Time constant, by definition, is:

t_star = (rho * V * cp) / (h * A) = (rho * cp / h) * (r_o /3) since V/A = Lc = r_0 / 3 for a sphere

Therefore:"

(rho * cp / h) * (r_o /3) = 1 "finds r_o, since time const = 1 s, given"

d_o = 2 * r_o "[m]…finds junction dia"

Biot = (h * r_o / 3) / k "..Biot No."

"Biot = 0.002353 < 0.1; so, lumped system analysis is applicable:"

(T – T_inf)/(T_i – T_inf) = exp(-h*tau/(rho*cp*r_o/3)) " Finds tau"

Results:

Unit Settings: SI C kPa kJ mass deg

Biot = 0.002353 [-]	cp = 400 [J/kg-C]	d_o = 0.0007059 [m]	h = 400 [W/m^2-C]
k = 20 [W/m-C]	ρ = 8500 [kg/m^3]	r_o = 0.0003529 [m]	T = 199 [C]
τ = 5.165 [s]	T_i = 25 [C]	T_{inf} = 200 [C]	

Thus:

d_o = 0.0007059 m = 0.7059 mm … TC junction dia required …. Ans.

tau = 5.165 s …. Time taken for the TC junction to reach 199 C …. Ans.

===

"**Prob. 1G.7.** An egg with a mean dia of 40 mm and initially at 20 C is placed in boiling water for 4 min. and found to be boiled to the consumer's taste. For how long should a similar egg for same consumer be boiled when taken from a refrigerator at 5 C ? Take the following properties for the egg: k = 10 W/m.K, rho = 1200 kg/m^3, cp = 2 kJ/kg.K, and h = 100 W/m.K. [M.U. – May – 2000]:"

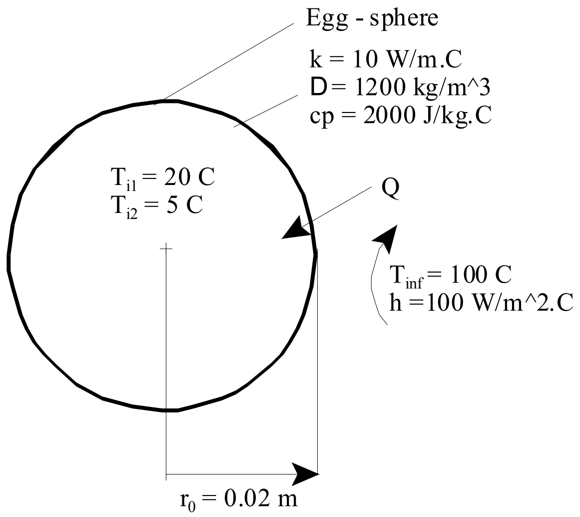


Fig.Prob.1G.7

EES Solution:

"Data:"

r_0 = 0.02 [m]

T_i_1 = 20[C] "…starting temp in first case"
T_i_2 = 5 [C] "… starting temp in case 2"
T_inf = 100[C] "..boiling water"
h = 100[W/m^2-C]
k = 10[W/m-C]
rho = 1200 [kg/m^3]
cp = 2000[J/kg-C]
tau = 240 [s] "… time duration of boiling"

"Calculations:"

"First case: Find the temp reached by the egg, starting with initial temp of 20 C, after 4 min. in boiling water.

Then, in second case: Find the time required for the egg to reach the same temp, starting with initial temp of 5 C."

"First, check Biot No."

Biot = (h * r_0 / 3) / k "..Biot No. .. = (h * Lc /k) where Lc = V / A = r_0 / 3 for a sphere"

"We get: Biot = 0.06667 < 0.1; **so, lumped system analysis is applicable**:"

(T1 – T_inf) / (T_i_1 – T_inf) = exp(-h * tau / (rho * cp * r_0/3)) "Finds T1 .. temp reached after

tau = 4 min. in first case"

"In the second case: starting temp is T_i_2 = 5 C, and final temp to be reached is T1, find the time required tau_2:"

(T1 – T_inf) / (T_i_2 – T_inf) = exp(-h * tau_2 / (rho * cp * r_0/3)) "Finds tau_2 .. time required to reach a temp of T1, starting with Ti = 5 C"

Results:

Unit Settings: SI C kPa kJ mass deg

Biot = 0.06667 [-]	cp = 2000 [J/kg-C]	h = 100 [W/m^2-C]
k = 10 [W/m-C]	ρ = 1200 [kg/m^3]	r_0 = 0.02 [m]
T1 = 82.15 [C]	τ = 240 [s]	τ_2 = 267.5 [s]
T_{inf} = 100 [C]	$T_{i,1}$ = 20 [C]	$T_{i,2}$ = 5 [C]

Thus:

T1 = 82.15 C …. Temp. of egg after 4 min. in boiling water, in first case.

tau_2 = 267.5 s …. Time required for the egg to reach a temp of T1 = 82.15 C, when the initial temp, before being dropped in boiling water, is 5 C …. Ans.

==

"**Prob.1G.8.** A steel ball of 50 mm dia and at 900 C is placed in still air at a temp of 30 C. Calculate the initial rate of cooling of the ball in C/min. Take rho = 7800 kg/m^3, cp = 2 kJ/kg.K and h = 30 W/m^2.K. [VTU – VI Sem. B.E. – July–Aug. 2003]:"

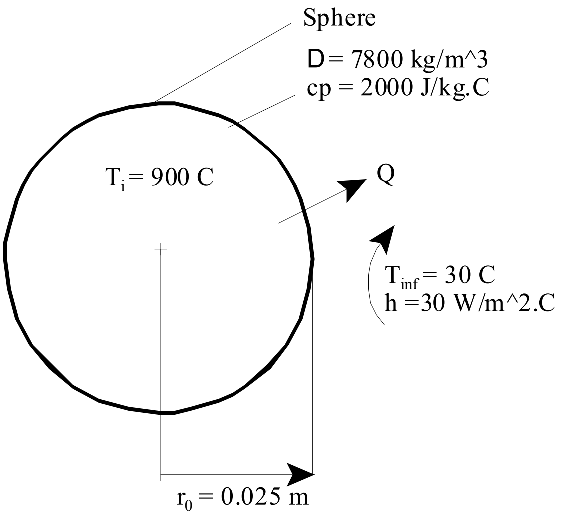


Fig.Prob.1G.8

EES Solution:

"Data:"

T_i = 900[C]
T_inf =30[C]
h = 30[W/m^2-C]
r_o = 0.025[m]
rho = 7800 [kg/m^3]
cp = 2000[J/kg-C]

"Calculations:"

"By an energy balance at the surface of the ball:

Rate of energy lost from the surface by convection = Rate of decrease of internal energy of the ball"

"So, we write:"

h * (4 * pi * r_o^2) * (T_i – T_inf) = rho * ((4 / 3) * pi * r_o^3) * cp * dTdtau "….finds dtdtau in C/s"

"In the above eqn. dTdtau is the rate of cooling in C/s.

Then, the rate of cooling in C/min is given by:"

dTdtau_per_min = dTdtau * 60 [s/min]

Results:

Unit Settings: SI C kPa kJ mass deg

cp = 2000 [J/kg-C]	dTdtau = 0.2008 [C/s]
$dTdtau_{per,min}$ = 12.05 [C/min]	h = 30 [W/m^2-C]
ρ = 7800 [kg/m^3]	r_o = 0.025 [m]
T_i = 900 [C]	T_{inf} = 30 [C]

Thus:

dT / dtau = 0.2008 C/s = 12.05 C/min. …. Ans.

==

Prob. 1G.9. The average heat transfer coeff for flow of 100 C air over a flat plate is measured by observing the temp – time history of 30 mm thick copper slab exposed to 100 C air. In one test run, the initial temp of the plate was 210 C, and in 5 minutes the temp decreased by 40 C. Calculate the heat transfer coeff for this case. Take the properties of copper as: ρ = 9000 kg/m^3, cp = 0.38 kJ/kg.K, k = 370 W/m.K. [VTU – VI Sem. B.E. – Feb. 2002]

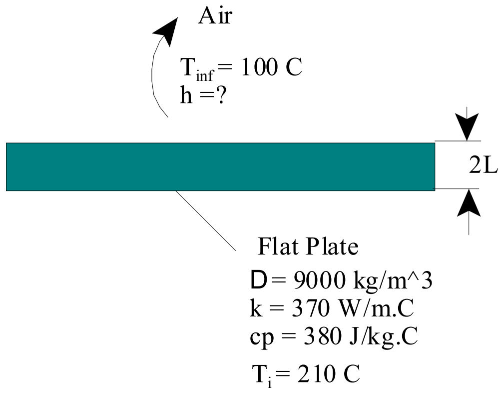


Fig.Prob.1G.9

Mathcad Solution:

Data:

$L := 0.015$ m... half thickness of slab $cp := 380$ J/kg.C

$\rho := 9000$ kg/m^3 $k := 370$ W/m.C.

$T_i := 210$ C. $T := 210 - 40$ C...temp after τ = 300 s

$\tau := 300$ s $T_{inf} := 100$ C

Calculations:

Now, assuming that lumped system analysis is applicable for this plate (i.e. Bi < 0.1):

When (2.L) is the thickness of the slab:

Surface area (on both sides) = 2.A

Volume, V = A.(2.L)

Therefore, Lc = Volume / Area = L, half- thickness

$Lc = \frac{V}{A} = L$ ..for a slab, half-thickness of the slab, L is the characteristic dimension to calculate the Biot No.

i.e. $Lc := L$

$$\frac{T - T_{inf}}{T_i - T_{inf}} = \exp\left(\frac{-h\cdot\tau}{cp\cdot\rho\cdot L_c}\right)$$

i.e. $$h := \frac{cp\cdot\rho\cdot Lc\cdot\ln\left(\frac{T - T_{inf}}{T_i - T_{inf}}\right)}{-\tau}$$

i.e. $h = 77.289$ **W/m^2.C ... heat transfer coeff. from sphere to air ... Ans.**

To justify that Lumped system analysis isvalid for this case, find out Biot No.,

i.e. Biot No. should be less than 0.1.

Verify that Bi < 0.1:

Biot Number:

$Bi := \frac{h\cdot Lc}{k}$ ..Biot number

i.e. $Bi = 3.133\cdot 10^{-3}$

Note that Bi < < 0.1 ; Therefore, temp anywhere within the body does not differ by more than 5 % and Lumped system analysis is applicable for heat transfer calculations.

==

"**Prob.1G.10.** A 5 cm thick iron plate with k = 60 W/m.K, cp = 460 J/kg.C, rho = 7850 kg/m^3, alpha = 1.6 × 10^-5 m^2/s, is initially at 225 C. Suddenly both the surfaces are exposed to an environmental temp of 25 C with a convective heat transfer coeff of 500 W/m^2.K. Calculate: (i) the centre temp at t = 2 min. after start of cooling, (ii) the temp at a depth of 1 cm from the surface at t = 2 min. after start of cooling, and (iii) the energy removed from the plate per m^2 during this time. [VTU – VI Sem. B.E. – June–July 2009]: "

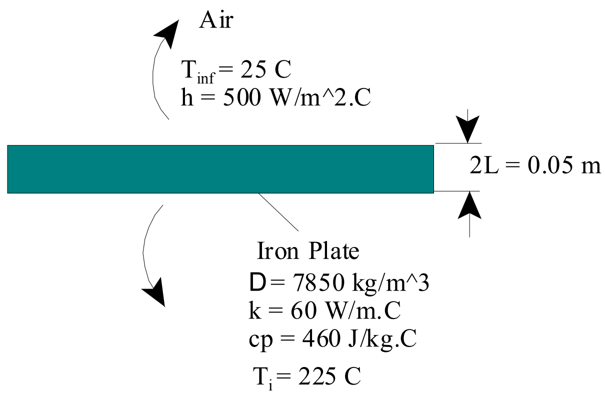


Fig.Prob.1G.10

EES Solution:

"Data:"

L = 0.025[m] "..L is half-thickness of plate"

T_i = 225[C]

T_inf = 25[C]

h = 500[W/m^2-C]

tau = 120[s]

{alpha = 1.6e-05[m^2/s]}

alpha = k / (rho * cp) "[m^2/s] … thermal diffusivity"

k = 60[W/m-C]

cp = 460 [J/kg-C]

rho=7850 [kg/m^3]

"Calculations:"

"First, calculate Biot No. to see if it is less than or more than 0.1:"

Biot = (h * L) / k "…Biot No."

"Biot = 0.2083 > 0.1; **So, use Heisler charts or one – term solution**:"

"For a Slab, Biot No. to be used for Charts (or One term solution) is the same as already calculated."

"Find Lambda_1 and A_1:"

$$\lambda_1 \cdot \tan(\lambda_1) = \text{Biot} \quad \text{Finds Lambda}_1$$

$$A_1 = 4 \cdot \left[\frac{\sin(\lambda_1)}{2 \cdot \lambda_1 + \sin(2 \cdot \lambda_1)} \right]$$

"Above eqns. to find lambda_1 and A_1 are entered in EES as:"

Lambda_1 * tan(Lambda_1) = Biot "Finds Lambda_1"

A_1 = 4 * sin(Lambda_1) / ((2 * Lambda_1 + sin(2 * Lambda_1)))

"Centre temp at t = 2 min:"

Theta_o =(T_o – T_inf) / (T_i – T_inf) "For Heisler chart verification"

$$\frac{T_o - T_{inf}}{T_i - T_{inf}} = A_1 \cdot \exp(-\lambda_1^2 \cdot Fo)$$

"Above eqn. to find T_0 is entered in EES as follows:"

(T_o – T_inf) / (T_i – T_inf) = A_1 * exp(-Lambda_1^2 * Fo) "Finds T_0"

Fo = alpha * tau / L^2 "Finds Fo"

"What is the temp at 1 cm from surface (i.e. x = 1.5 cm) after 2 min?"

x = 0.015 [m] "…this is the distance from the centre line"

$$\frac{T_x - T_{inf}}{T_i - T_{inf}} = A_1 \cdot \exp\left(-\lambda_1^2 \cdot Fo\right) \cdot \cos\left[\lambda_1 \cdot \frac{x}{L}\right]$$

"Above eqn. to find Tx is entered in EES as:"

(T_x – T_inf) / (T_i – T_inf) = A_1 * exp(-Lambda_1^2 * Fo) * cos(Lambda_1* x / L) "Finds T_x"

"For Heisler chart verification:"

xbyL = x / L

Theta = (T_x – T_inf) / (T_o – T_inf)

"Energy removed after 2 min:"

A = 1[m^2] "….area"

V = A * 2 * L "[m^3] … volume"

Qmax = V * rho * cp * (T_i – T_inf) "J]…. max. heat removed"

QbyQmax = 1 – (T_o – T_inf) / (T_i – T_inf) *sin(Lambda_1) / Lambda_1

Q = Qmax * QbyQmax "[J] …actual heat removed"

Results:

Unit Settings: SI C kPa kJ mass rad

$A = 1$ [m^2]	$\alpha = 0.00001662$ [m^2/s]	$A_1 = 1.032$
$Biot = 0.2083$ [-]	$cp = 460$ [J/kg-C]	$Fo = 3.19$
$h = 500$ [W/m^2-C]	$k = 60$ [W/m-C]	$L = 0.025$ [m]
$\lambda_1 = 0.4412$	$Q = 1.672E+07$ [J]	$QbyQmax = 0.463$
$Qmax = 3.611E+07$ [J]	$\rho = 7850$ [kg/m^3]	$\tau = 120$ [s]
$\theta = 0.9652$	$\theta_o = 0.5548$	$T_i = 225$ [C]
$T_{inf} = 25$ [C]	$T_o = 136$ [C]	$T_x = 132.1$ [C]
$V = 0.05$ [m^3]	$x = 0.015$ [m]	$xbyL = 0.6$

Thus:

T_0 = 136 C …. Centre temp after 2 min. .. Ans.
T_x = 132.1 C ….Temp at a depth of 1 cm from surface (i.e. 1.5 cm from centre) after 2 min.… Ans.

Q = 1.672E07 J …. Energy removed during this time …. Ans.

Alternatively: One can use the chart solution:

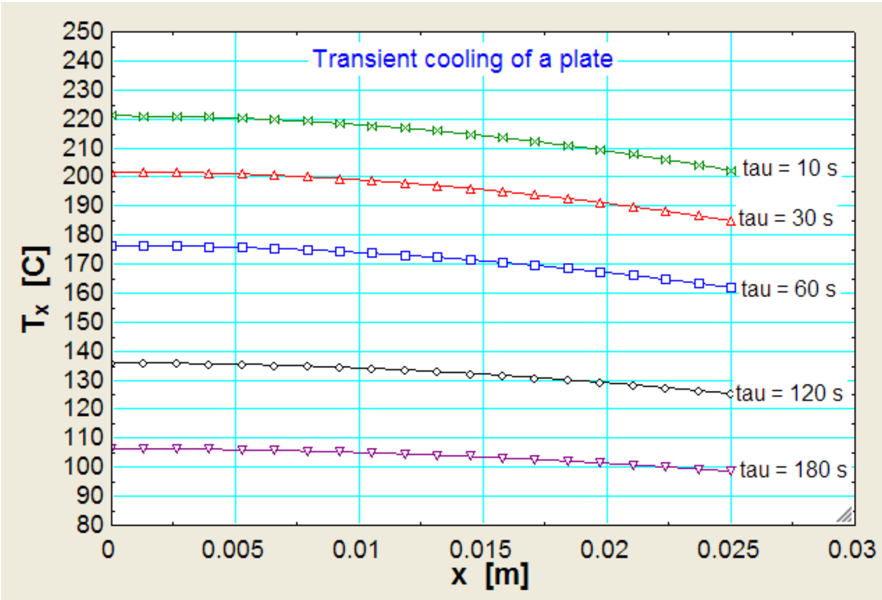
See the charts given above for a plane wall:

For Fo = 3.19, and Bi = 0.2083, check that theta_0 = 0.5548.

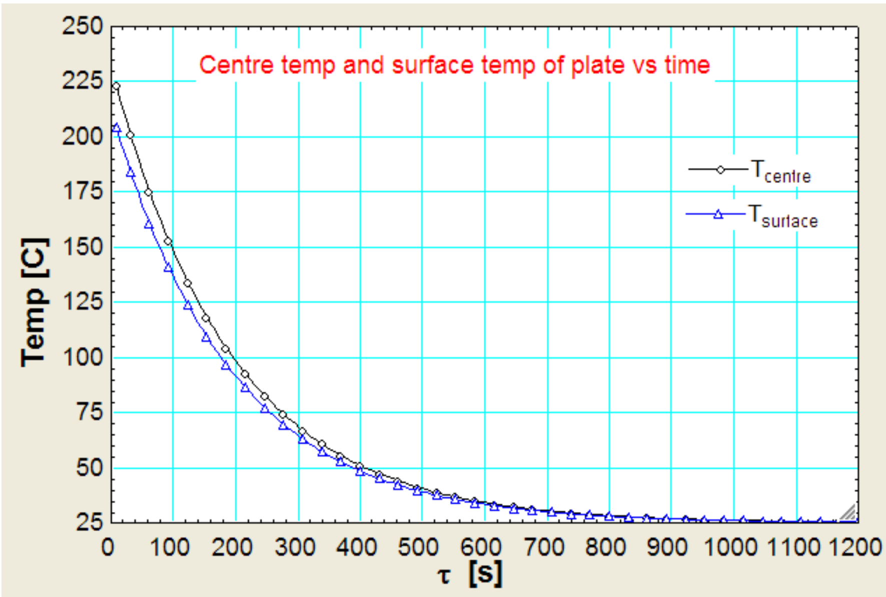
Also, x/L = 0.6 and Bi = 0.2083, check that theta = 0.9652.

Further, from Grober's chart, check that for x-axis value of Bi^2. Fo and for Bi = 0.2083, Q/Qmax = 0.463

Plot the temp profile in the plate (Tx vs x) for different times:



Also plot centre temp and surface temp of plate against time:



Note: In the above plot min value of tau is taken as 8 s since for one term solution to be valid, we should have Fo > 0.2, i.e. tau > 7.52 s.

With advancing time, the difference between the centre temp and surface temp goes on decreasing and after about 900 s, practically both the temperatures coincide.

==

Prob. 1G.11. A slab of Aluminium 10 cm thick is originally at a temp of 500 C. It is suddenly immersed in a liquid at 100 C resulting in a heat transfer coeff of 1200 W/m^2.K. Determine the temp at the centre line and the surface 1 minute after the immersion. Also calculate the total thermal energy removed per unit area of the slab during this period. Properties of Al for the given conditions are: α = 8.4 * 10^-5 m^2/s, ρ = 2700 kg/m^3, k = 215 W/m^2.K, cp = 0.9 kJ/kg.K. [VTU – VI SEm. B.E. – Dec. 2006–Jan. 07]

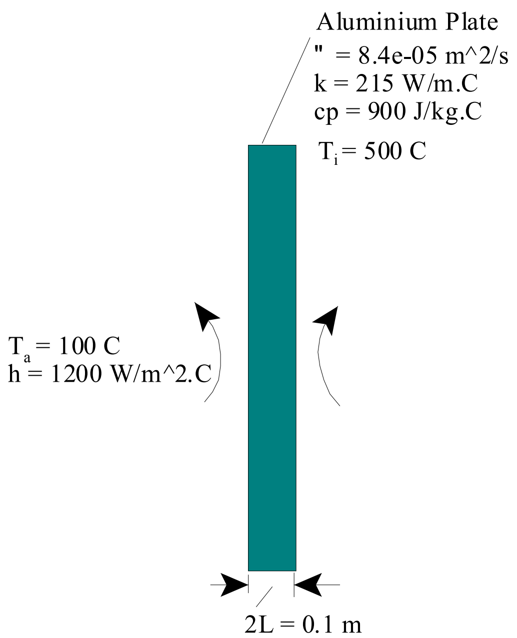


Fig.Prob.1G.11

Mathcad Solution:

Data:

$L := 0.05$ m...half thickness of plate

$\alpha := 8.4 \cdot 10^{-5}$ m²/s....thermal diffusivity of Al

$k := 215$ W/(m.C)....thermal cond. of Al

$cp := 900$ J/kg.C .... sp. heat

$T_i := 500$ C...initial temp. of plate

$T_a := 100$ C....temp. of medium

$h := 1200$ W/(m².C).....heat transfer coeff. between the surface and the medium

$\tau := 60$ s ... time after immersion

Let T0 be the centre temp. after τ = 60 s

To calculate the time τ, surface temp. and fraction of heat transferred Q/Qmax.

First check if lumped system analysis is applicable:

$Bi := \frac{h \cdot L}{k}$ ...define Biot number

i.e. $Bi = 0.279$ ...Biot number.

It is noted that Biot number is > 0.1; so, **lumped system analysis is not applicable.**

We will adopt Heisler chart solution and then check the results from one term approximation solution:

To find the time reqd. for the centre to reach 100 C:

For using the charts, Bi = hL/k, which is already calculated.

Therefore: $\frac{1}{Bi} = 3.583$ ..for use in the graph

Fourier number: $Fo := \frac{\alpha \cdot \tau}{L^2}$

i.e. $Fo = 2.016$ ...Fourier no. Note that this is greater than 0.2. So, one term approx. is applicable.

Now, refer to the graph for Plate. For 1/Bi = 3.583 and Fo = 2.016, read from the graph the value of q_0 as 0.6

i.e. $\theta_0 := 0.6$

But, $\theta_0 = \frac{T_0 - T_a}{T_i - T_a}$ ...definition of θ_0

i.e. $T_0 := \theta_0 \cdot (T_i - T_a) + T_a$ ..value of T_0

i.e. $T_0 = 340$ **C .... centre line temp of slab after 1 min ... Ans.**

Surface temperature:

At the surface, x/L =1. Enter position correction chart for a plate: on the x-axis with a value of 1/Bi = 3.583, move up to intersect the curve of x/L = 1, then move to left to read on y-axis the value of $\theta_0 = 0.85$

i.e. $\theta = \frac{T - T_a}{T_0 - T_a} = 0.85$

Therefore, $T := 0.85 \cdot (T_0 - T_a) + T_a$ C...temp. on the surface

i.e. $T = 304$ **C...temp. on the surface....Ans.**

Fraction of max. heat transferred, Q/Q_{max}:

We will use Grober's chart, for a Plate:

We need Bi^2Fo to enter the x-axis:

We get: $Bi^2 \cdot Fo = 0.157$

With this value of 0.157, enter the x-axis of Grober's chart, move vertically up to intersect the curve of Bi = 0.279, then move horizontally to read Q/Qmax = 0.45

i.e. from Fig. (7.7, c), we get: $\frac{Q}{Q_{max}} = 0.45$

i.e. 45% of the energy is removed by the time the centre temp. has reached 380 C.....Ans.

Now:

$Q_{max} := 0.1 \cdot \rho \cdot cp \cdot (T_i - T_a)$ J .... max heat transferred

i.e. $Q_{max} = 9.72 \cdot 10^7$ J

Therefore, actual heat transferred:

$Q := Q_{max} \cdot 0.45$

i.e. $Q = 4.374 \cdot 10^7$ **J ... actual heat transferred during τ = 1 min. ... Ans.**

We have, for the centre line temp of a Plate:

Centre of plane wall: (x = 0) $\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo}$

A_1 and λ_1 have to be found from Table 7.1, against Bi = 0.279

OR: use the equations for A1 and λ1:

i.e. λ1.tan(λ1) = Bi … to determine λ1, and,

A1 = [4. sin(λ1)] / [2.λ1 + sin(2.λ1)] … to determine A1

Use the Solve Block of Mathcad. Start with guess values for λ1:

$\lambda 1 := 1$ ...guess value

Given

$\lambda 1 \cdot \tan(\lambda 1) = Bi$

$\lambda 1 := Find(\lambda 1)$

i.e. $\lambda 1 = 0.505$ **....Also check this value of λ1 from the Table for λ1 against Bi.**

and, $$A1 := \frac{4 \cdot \sin(\lambda 1)}{2 \cdot \lambda 1 + \sin(2 \cdot \lambda 1)}$$

i.e. $A1 = 1.042$ **....Also check this value of A1 from the Table for A1 against Bi.**

Now,

$$\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A1 \cdot e^{-\lambda 1^2 \cdot Fo}$$

Therefore:

$$T_0 := T_a + (T_i - T_a) \cdot \left(A1 \cdot e^{-\lambda 1^2 \cdot Fo}\right)$$

i.e. $T_0 = 349.347$ **C ... centre line temp of slab after 60 s ... Ans.**

Compare this with the value of 340 C obtained earlier; the error is in reading the charts.

Surface temperature:

For surface temp, we have:

Plane wall: $$\theta(x, \tau) = \frac{T(x, \tau) - T_a}{T_i - T_a} = A1 \cdot e^{-\lambda 1^2 \cdot Fo} \cdot \cos\left(\frac{\lambda 1 \cdot x}{L}\right)$$ ...Fo > 0.2..

Here, x/L = 1, at the surface of the plate. So, we get:

$$T := (T_i - T_a) \cdot \left(A1 \cdot e^{-\lambda 1^2 \cdot Fo} \cdot \cos(\lambda 1)\right) + T_a$$

i.e. $T = 318.232$ **C...temp. at the surface...Ans.**

Compare this with the value of 304 C obtained earlier; the error is in reading the charts.

Fraction of max. heat transferred, Q/Qmax:

We have:

Plane wall: $\frac{Q}{Q_{max}} = 1 - \theta_0 \cdot \frac{\sin(\lambda 1)}{\lambda 1}$ ...where Qmax is the max. heat transferred

i.e. $Fraction := 1 - \frac{T_0 - T_a}{T_i - T_a} \cdot \frac{\sin(\lambda 1)}{\lambda 1}$ ....define Fraction, Q/Q_{max}

i.e. $Fraction = 0.403$

i.e. 40.3 % of the energy is removed by the time 1 min. is reached...Ans.

Compare this with the value of 45% obtained earlier; again, the error is in reading the charts.

Therefore, actual heat transferred:

$Q := Fraction \cdot Q_{max}$

i.e. $Q = 3.915 \cdot 10^7$ **J ... actual heat transferred during τ = 1 min. ... Ans.**

Note: It is apparent from this example that the error involved in reading the graphs can be substantial; this is because logarithmic scales are involved and also the lines are rather crowded in the graph. So, one term approximation with table of values of A1 and λ1 against Bi should be preferred.

To draw temp. profile in the plate at different times:

We have, for temp. distribution at any location:

Plane wall: $\theta(x,\tau) = \frac{T(x,\tau) - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda 1^2 \cdot Fo} \cdot \cos\left(\frac{\lambda 1 \cdot x}{L}\right)$ ...Fo > 0.2...

And, Centre of plane wall: (x = 0) $\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda 1^2 \cdot Fo}$ ....

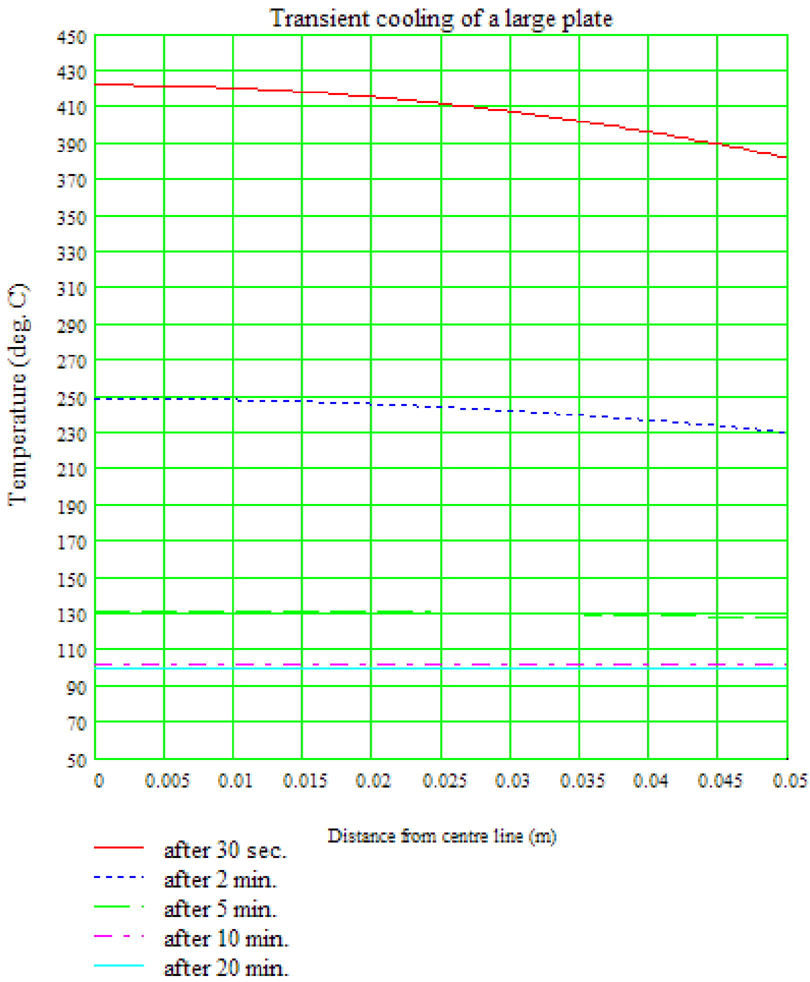
Fourier number as a function of τ: $Fo(\tau) := \frac{\alpha \cdot \tau}{L^2}$ ...for slab

By writing Fourier no. as a function of τ, and including it in eqn. (A) below, it is ensured that for each new τ, the corresponding new Fo is calculated.

Then, $T(x,\tau) := \begin{cases} T_a + (T_i - T_a) \cdot \left(A1 \cdot e^{-\lambda 1^2 \cdot Fo(\tau)}\right) & \text{if } x = 0 \\ T_a + (T_i - T_a) \cdot \left(A1 \cdot e^{-\lambda 1^2 \cdot Fo(\tau)} \cdot \cos\left(\frac{\lambda 1 \cdot x}{L}\right)\right) & \text{otherwise} \end{cases}$ .....eqn. (A) ....eqn. (B)

For a given τ, we will plot eqn.(A) against x; then, we will repeat for different times, τ:

$x := 0, 0.0005 .. 0.05$ ....define a range variable x varying from zero to 0.05 m, with an increment of 0.0005 m

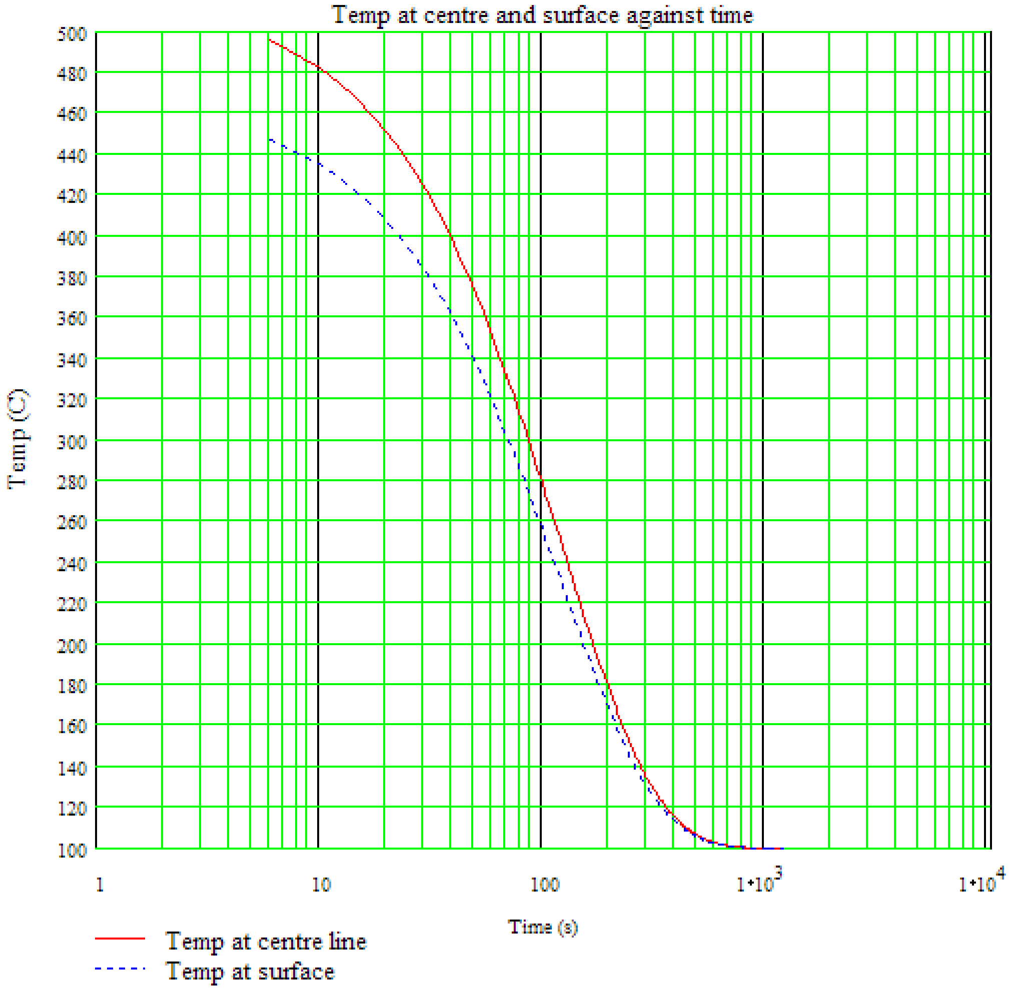


Note:

1) Note that the above graph shows temp. distribution for one half of the plate; for the other half, the temp. distribution will be identical.
2) See from the above fig. how cooling progresses with time. After a time period of about 20 min. the temperatures in the plate are almost uniform at 100 C.
3) eqn. (A and B) illustrates a small piece of Mathcad programming. It uses the "if…otherwise" condition, i.e. if x = 0, the temp. at the centre is given by eqn.(A); otherwise, temp. distribution is given by eqn. (B).

To plot temp. at the centre line and the surface against time:

$\tau := 6, 10..1200$ …define a range variable τ, from 0 to 1200 s



Note: τ is taken from 6 s since for the one term solution to be valid, Fourier No. must be > 0.2, i.e. τ should be more than 5.95 s.
It is noted from the plot that with increasing time, temperatures at the centre and surface approach each other, and after about 900 s, they are practically same. i.e. steady state is reached.

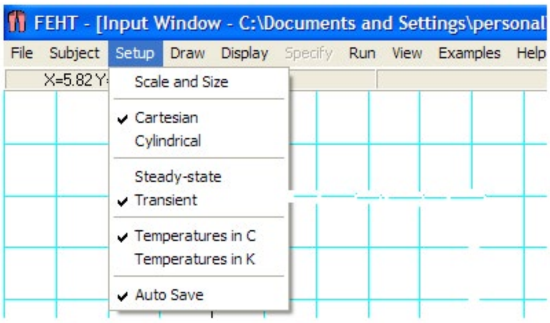
$T(0, 900) = 100.187$ **C ... temp at centre after 500 s**

$T(0.05, 900) = 100.164$ **C ... temp at surface after 500 s**

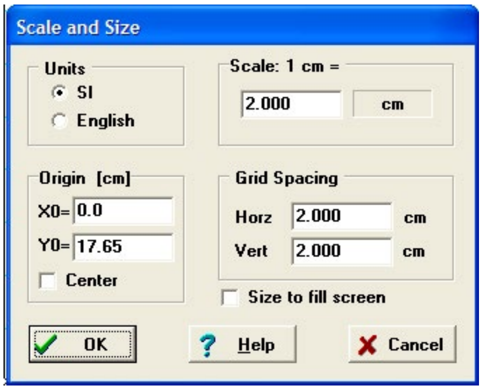
==

Solve the above Problem with FEHT:

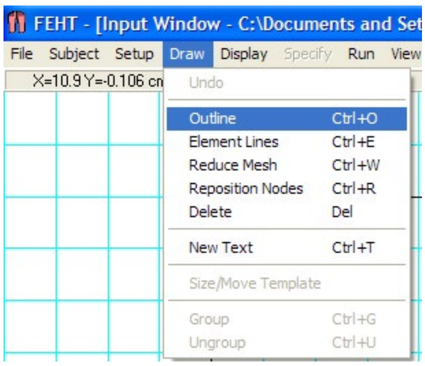
1. Start FEHT, choose Cartesian cords. and Transient condn.



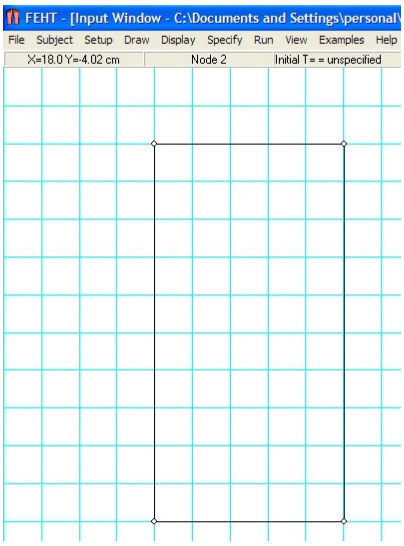
2. And click on Scale and Size. We get:



3. Select Draw-Outline:

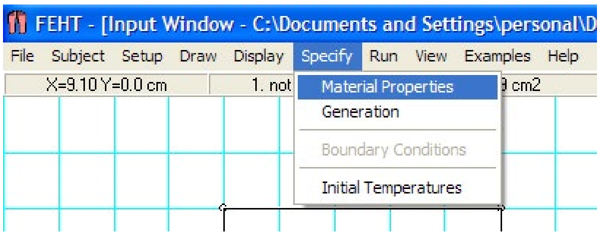


4. And complete the outline:

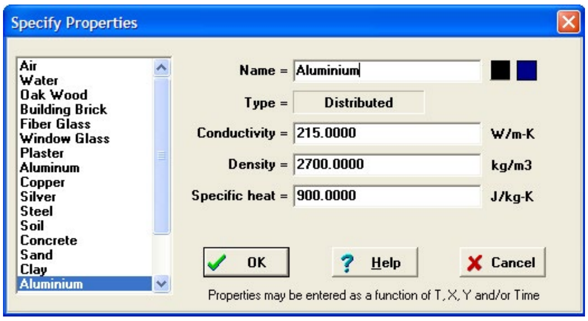


Note that width is 10 cm as given in data, and height of the section is arbitrarily taken as 20 Cm. Depth perpendicular to paper is 1 m, by default. So, 1D – heat transfer area on LHS is 1 × 0.2 = 0.2 m^2. Similarly, heat transfer area on RHS is also 0.2 m^2.

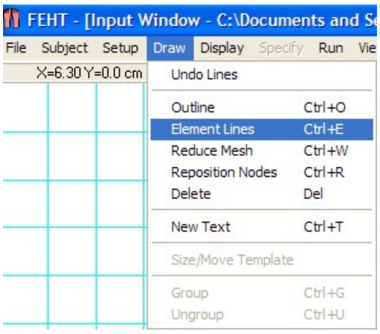
5. Choose Specify-Material Properties:



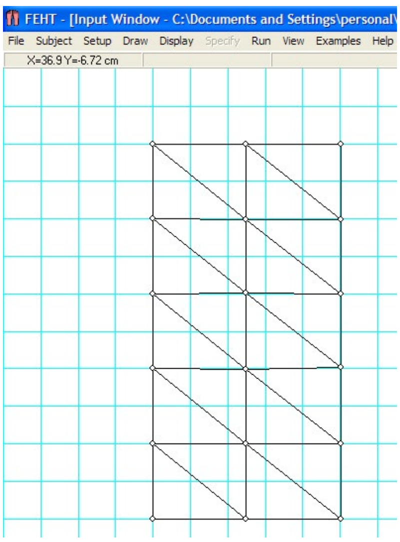
6. Choose Aluminium and fill in the k, rho and sp.heat values:



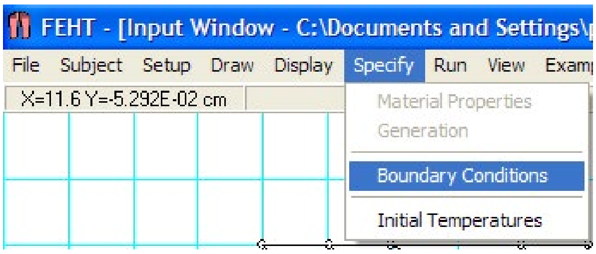
7. Now, choose Draw-Element lines:



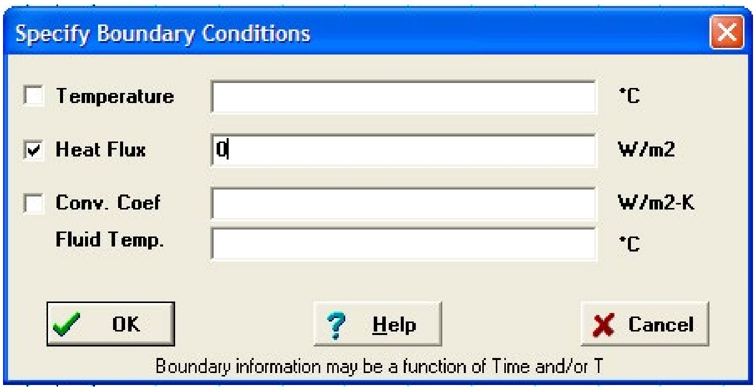
8. Complete the element lines:



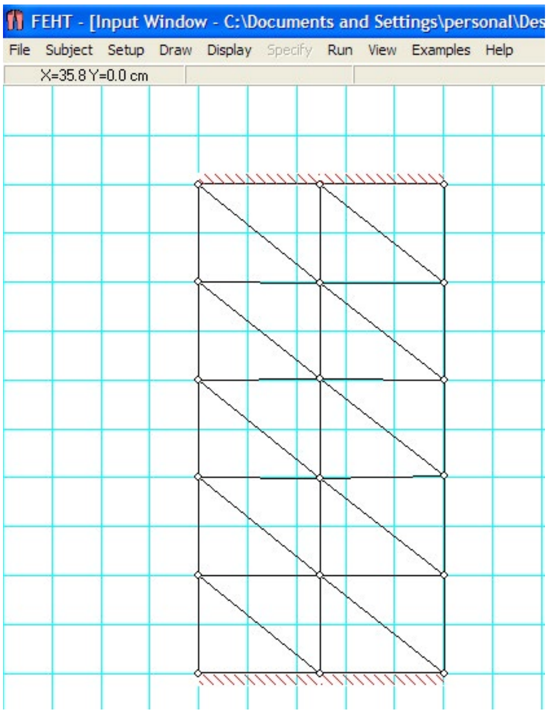
9. Select top and bottom surface boundaries and Specify-Boundary conditions:



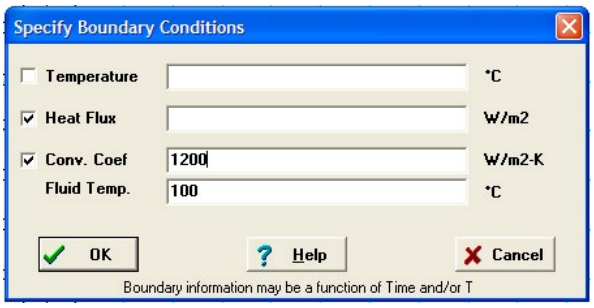
We get the following screen. Fill in heat flux = 0.



This makes the top and bottom surfaces insulated, and the heat transfer is forced to be one-dimensional. See the following screen:

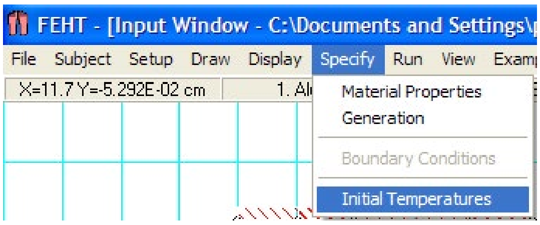


10. Similarly, the BC's for LHS and RHS:

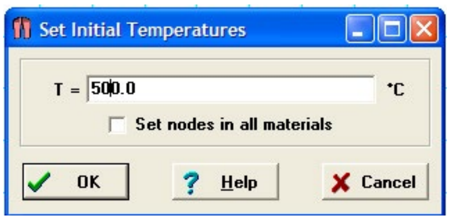


i.e. Fluid temp and conv. heat tr. coeff. values are entered.

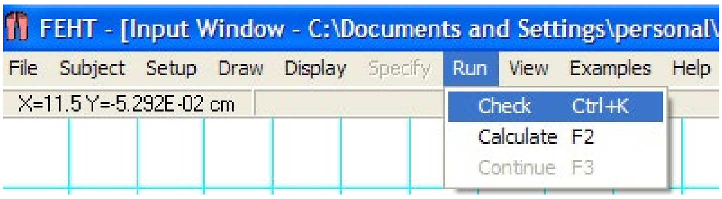
Next, Specify Initial temp by going to Specify-Initial temp:



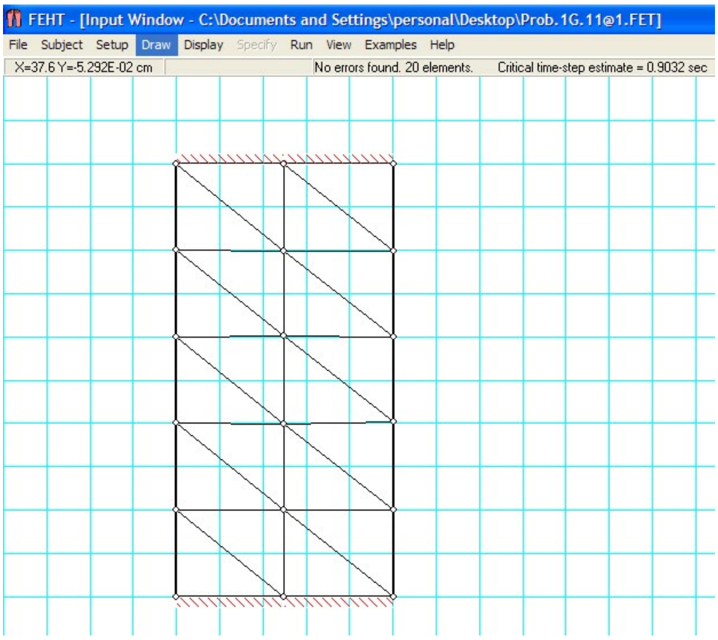
Fill in Initial temp = 500 C:



11. Now, check whatever is done so far by going to Run-Check:

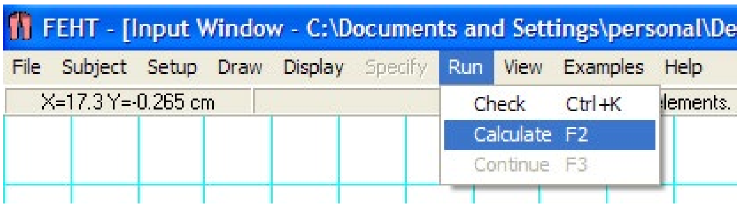


We get following screen:

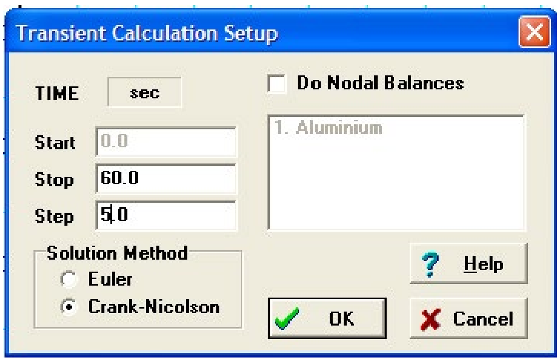


There are 20 elements, and no errors.

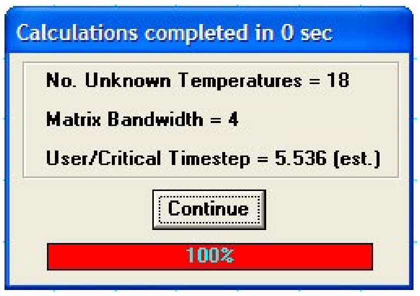
12. Now, click on Run-Calculate:



Following screen appears. Fill it up as shown:

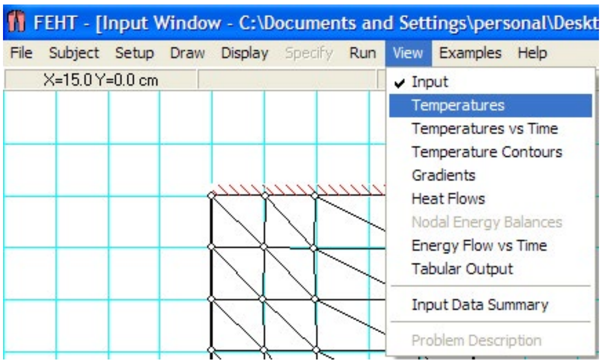


Click OK and we get:

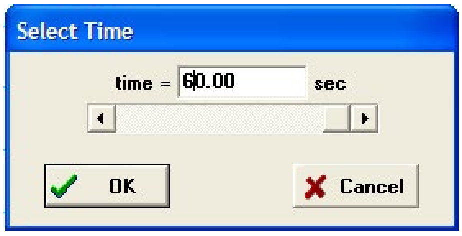


Click on Continue.

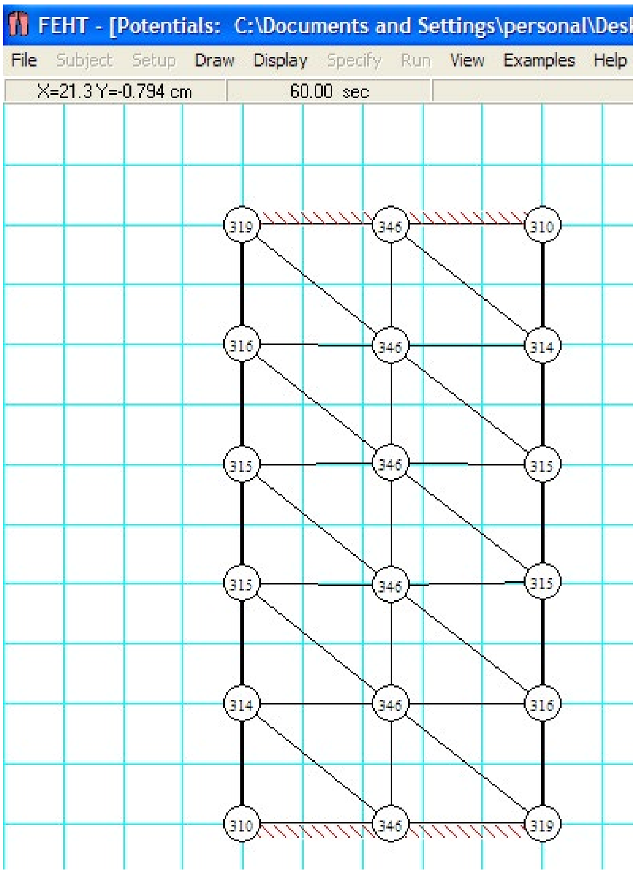
13. Now, we are ready to view results: Choose View-Temps:



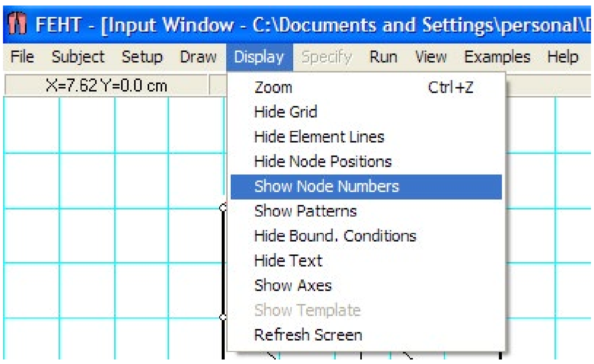
Following screen appears: Fill the time as 60 s:



Click OK, and Temps are displayed as shown below:

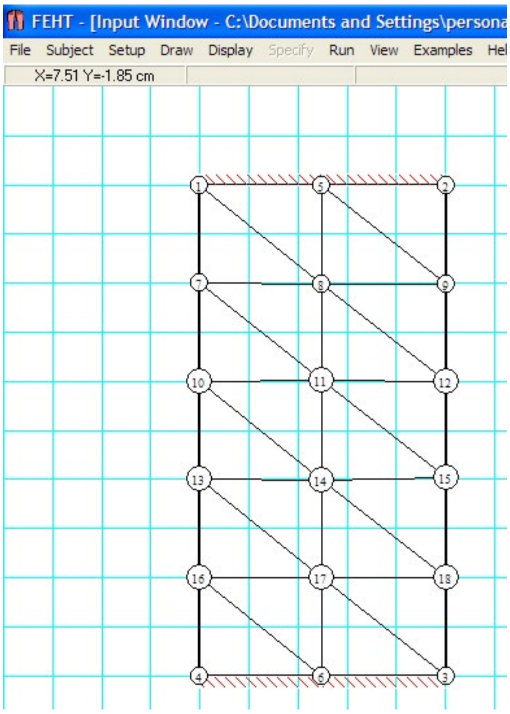


14. To see the Nodes, click on Display-Show Node Numbers.

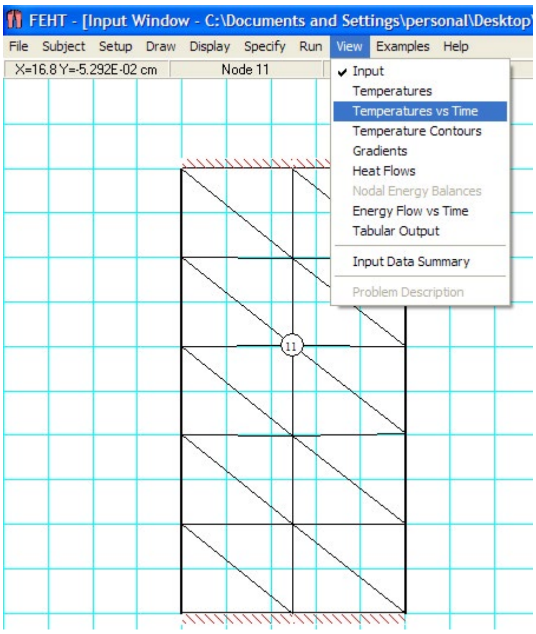


15. Node Nos. are displayed. Now, we can correspond the Node Temps with Node Nos.

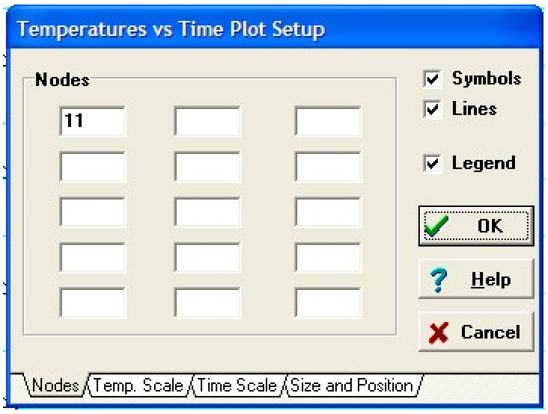
i.e. Centre Node no. 11 is at a temp of 316 C and Surface Node 12 is at 315 C after 60 s from beginning:



16. Now, let us draw Temp vs Time: Choose View-Temp vs Time:

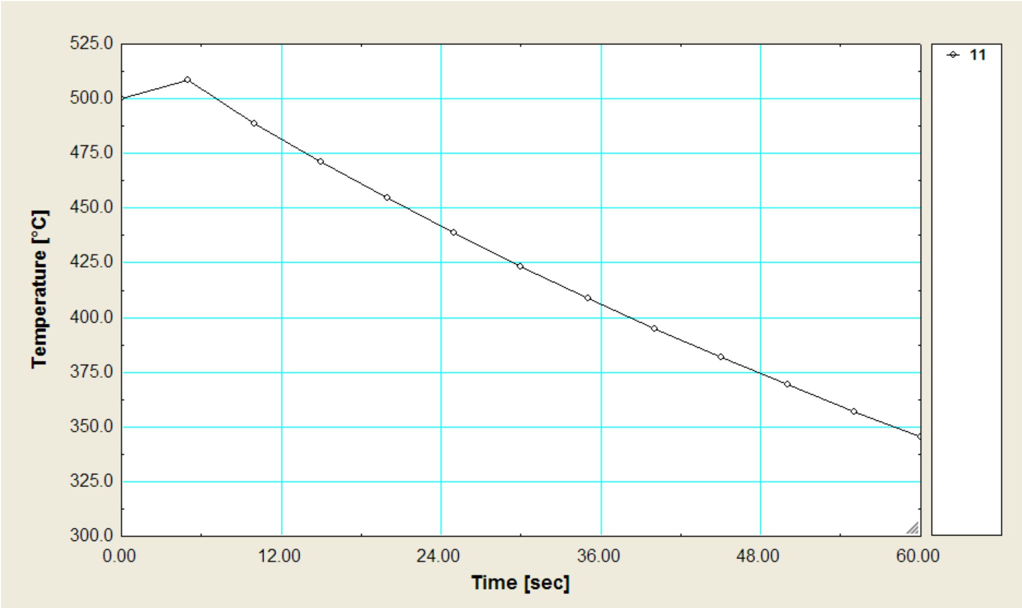


We get the following screen: For Centre Node, fill up Node No. as 11:

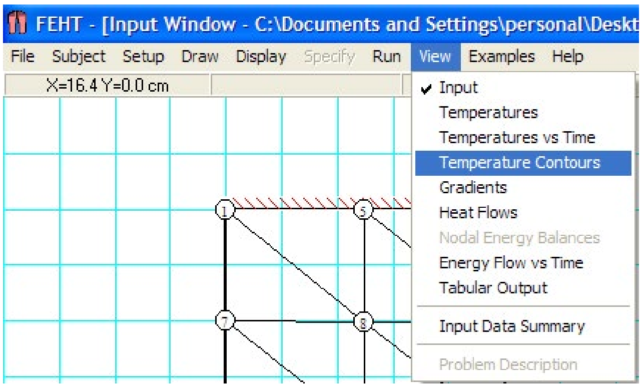


Note that we can adjust Nodes, Temp and Time scales etc.

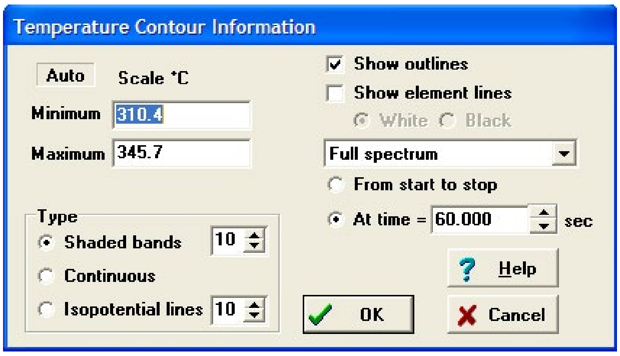
Clicking OK gives:



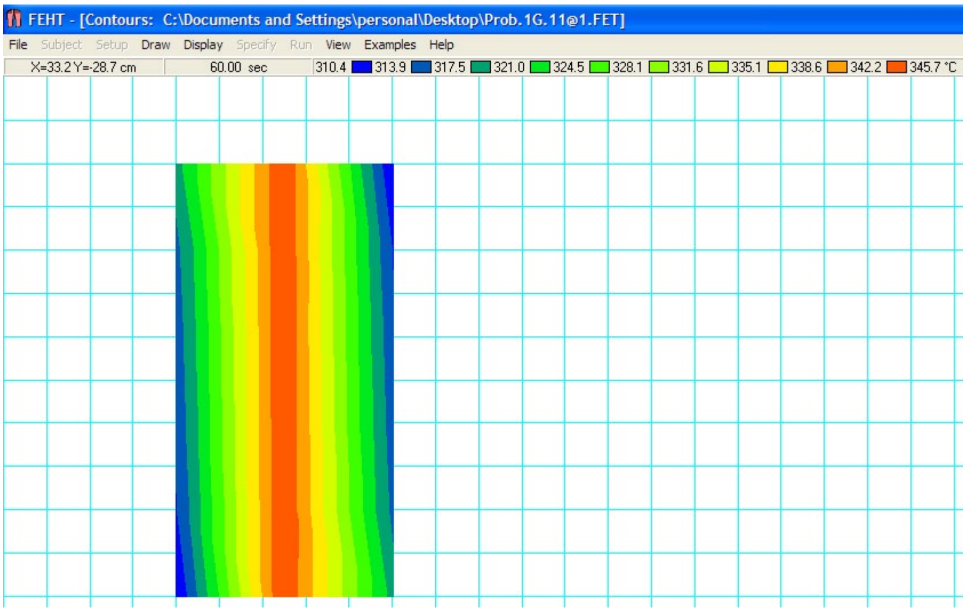
17. Temp contours can be drawn. Choose View-Temp contours:



We get the following screen. Accept the default values, click OK:



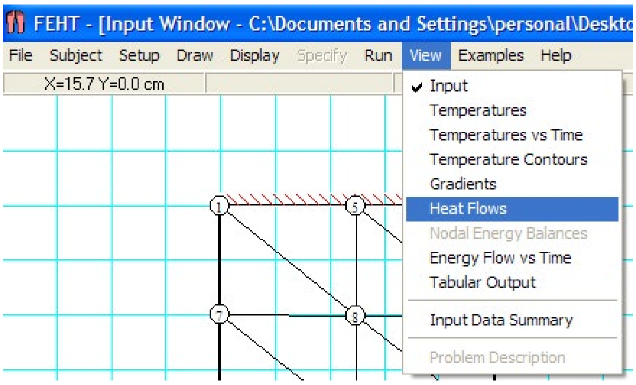
Following colour contour is presented:



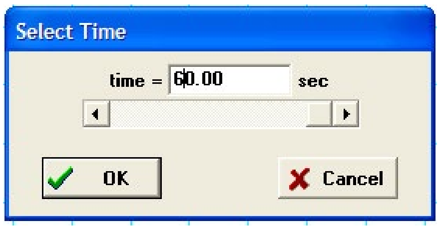
On the top of the screen, colour code for Temps is shown.

18. Heat flows for the given boundary can be calculated at any given time:

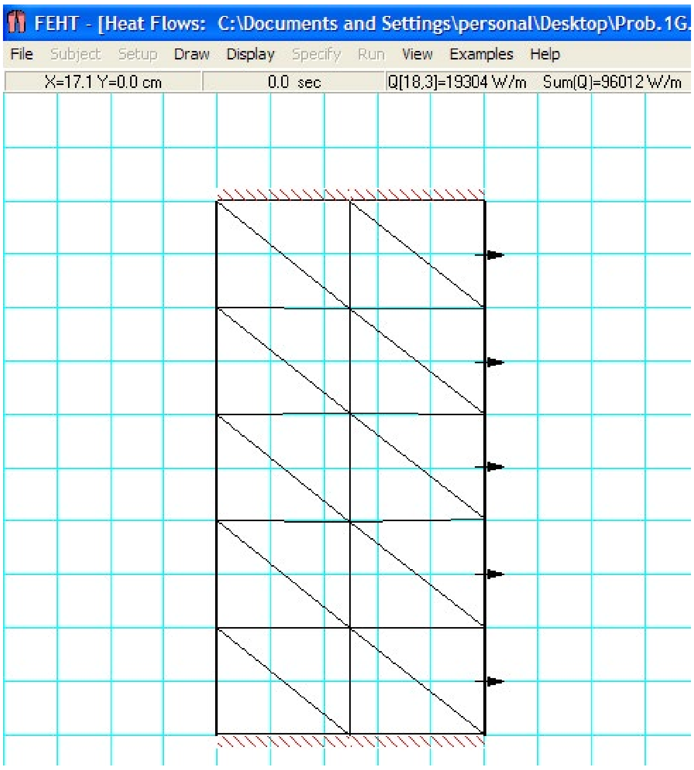
Choose View- Heat Flows:



We get following screen. Enter Time = 60 s:

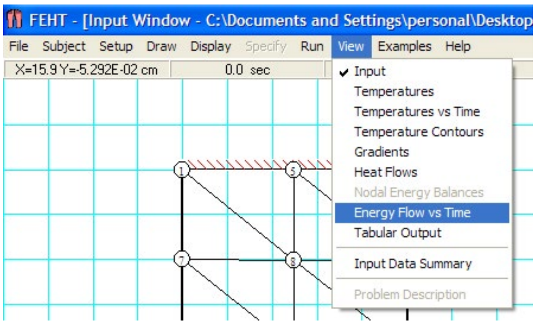


Click OK. We get:

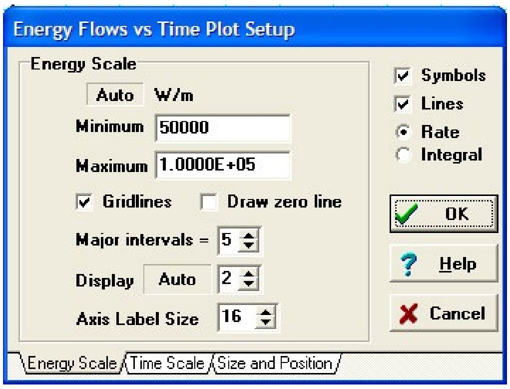


Note that Q = 96012 W/m for RHS.

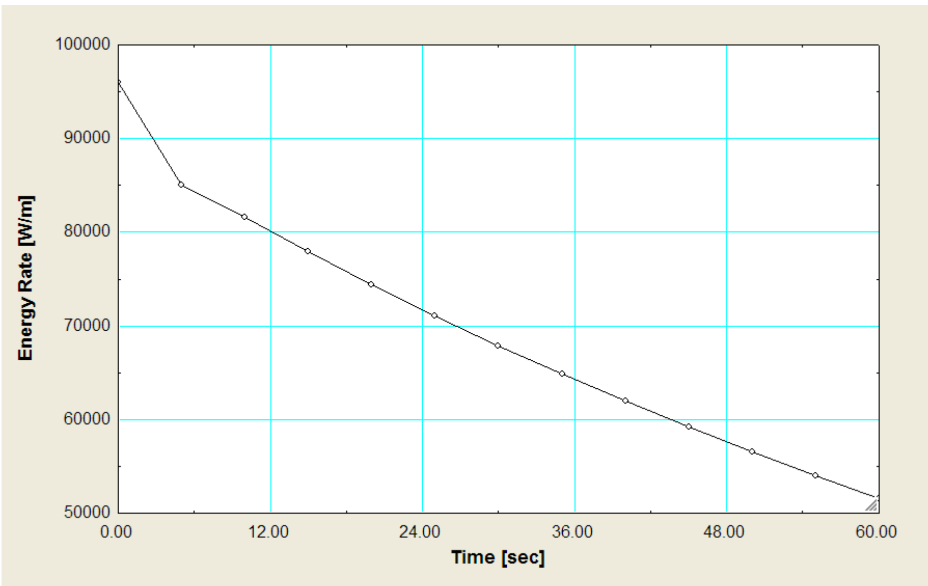
19. Choose View-Energy Flow vsTime, after selecting the RHS boundary:



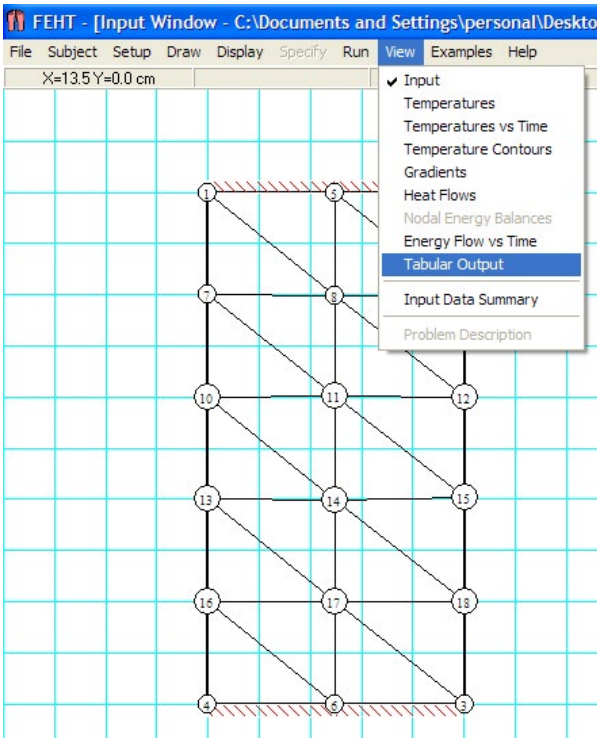
We get following screen. Accept default values.:



Clicking on OK gives:



20. Next, for accurate values, we can get the Tabular output from View-Tabular output:



Part of the Table obtained is shown below:

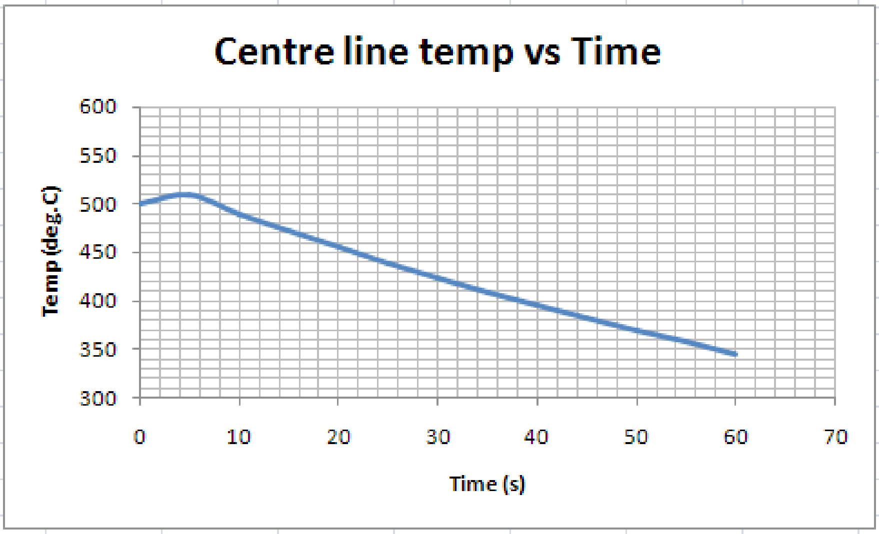
Tabular Nodal Results

Close | Print | Select All | Copy | Save as

Node	X [m]	Y [m]	T (#0) [°C]	T (#1) [°C]
Time [sec]			0	5
1	0.08	-0.04022	500	459.8
2	0.1804	-0.04022	500	447
3	0.1804	-0.2402	500	460.1
4	0.08	-0.2402	500	446.5
5	0.1296	-0.04022	500	508.6
6	0.1296	-0.2402	500	508.5
7	0.08	-0.0799	500	454.3
8	0.1296	-0.08043	500	508.1
9	0.1804	-0.08043	500	454.3
10	0.08	-0.1201	500	454.3
11	0.1296	-0.1196	500	508.5
12	0.1804	-0.1201	500	454.6

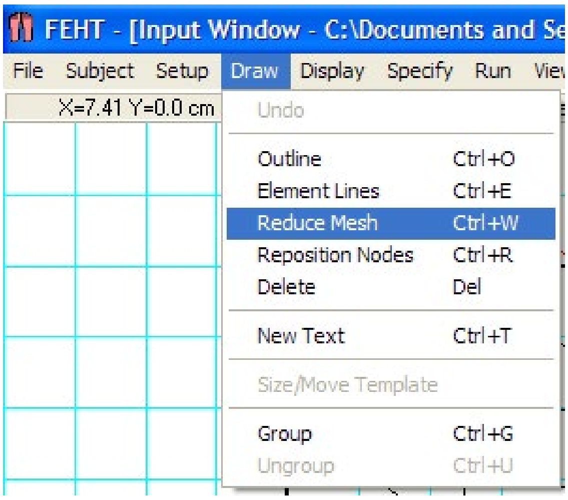
21. However, it can be copied into Excel and processed to draw better graphs:

Node	X	Y	T0 (deg.C)	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12
Time(s)-->			0	5	10	15	20	25	30	35	40	45	50	55	60
1	0.08	-0.04022	500	459.8	445.9	431	416.4	402.3	388.8	375.9	363.5	351.7	340.4	329.6	319.3
2	0.1804	-0.04022	500	447	434.1	417.8	403.9	390	377	364.6	352.7	341.4	330.6	320.2	310.4
3	0.1804	-0.2402	500	460.1	446.2	431.1	416.4	402.3	388.7	375.8	363.5	351.7	340.4	329.6	319.3
4	0.08	-0.2402	500	446.5	434.1	417.8	403.9	390	377.1	364.6	352.8	341.5	330.6	320.3	310.4
5	0.1296	-0.04022	500	508.6	488.5	471.3	454.5	438.7	423.5	409	395.1	381.9	369.3	357.2	345.7
6	0.1296	-0.2402	500	508.5	488.2	471.1	454.3	438.5	423.3	408.8	395	381.8	369.1	357.1	345.5
7	0.08	-0.0799	500	454.3	441.3	426.2	411.8	397.9	384.5	371.8	359.6	348	336.9	326.3	316.1
8	0.1296	-0.08043	500	508.1	488.6	471	454.5	438.6	423.4	408.9	395.1	381.8	369.2	357.2	345.6
9	0.1804	-0.08043	500	454.3	438.2	423	408.2	394.4	381.1	368.5	356.5	345	334	323.5	313.5
10	0.08	-0.1201	500	454.3	440.2	425	410.5	396.6	383.3	370.6	358.5	346.9	335.8	325.3	315.2
11	**0.1296**	**-0.1196**	**500**	**508.5**	**488.4**	**471.1**	**454.4**	**438.6**	**423.4**	**408.9**	**395**	**381.8**	**369.2**	**357.1**	**345.6**
12	0.1804	-0.1201	500	454.6	439.9	424.2	409.7	395.7	382.5	369.8	357.7	346.1	335.1	324.6	314.5
13	0.08	-0.1598	500	454.1	439.8	424.2	409.7	395.8	382.5	369.8	357.7	346.2	335.2	324.6	314.5
14	0.1296	-0.1603	500	508.5	488.4	471.1	454.5	438.6	423.4	408.9	395.1	381.8	369.2	357.1	345.6
15	0.1804	-0.1593	500	454.7	440.4	425.1	410.5	396.6	383.3	370.6	358.5	346.9	335.8	325.2	315.1
16	0.08	-0.2	500	453.8	438	423	408.2	394.4	381.2	368.6	356.5	345	334.1	323.6	313.5
17	0.1296	-0.2	500	508.1	488.5	471	454.5	438.5	423.4	408.9	395.1	381.8	369.2	357.1	345.6
18	0.1804	-0.2	500	454.7	441.5	426.3	411.8	397.9	384.5	371.8	359.6	348	336.8	326.2	316.1

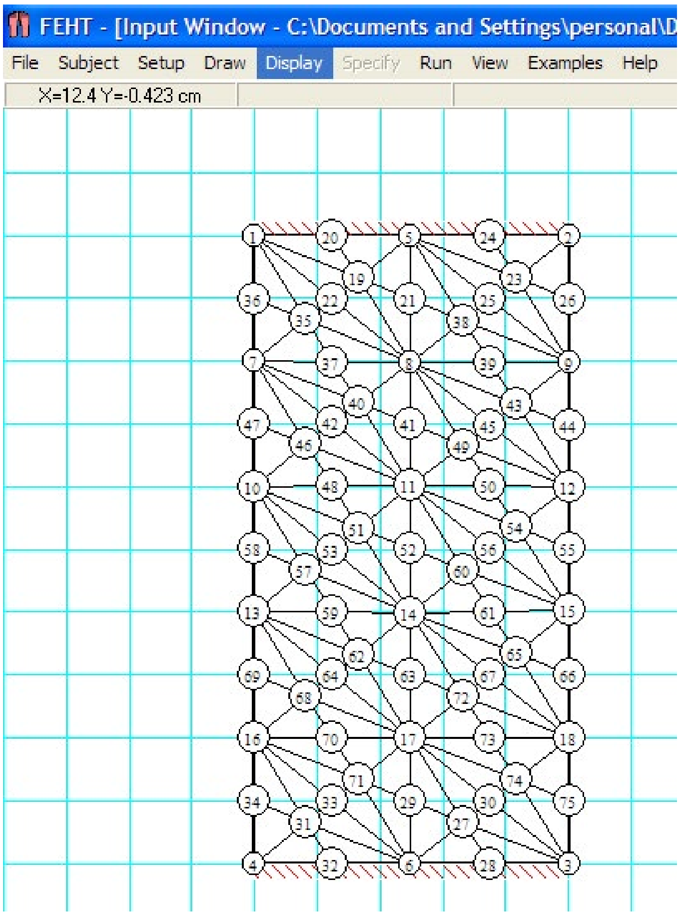


22. Now, we can reduce the mesh, if we wish, and see the results:

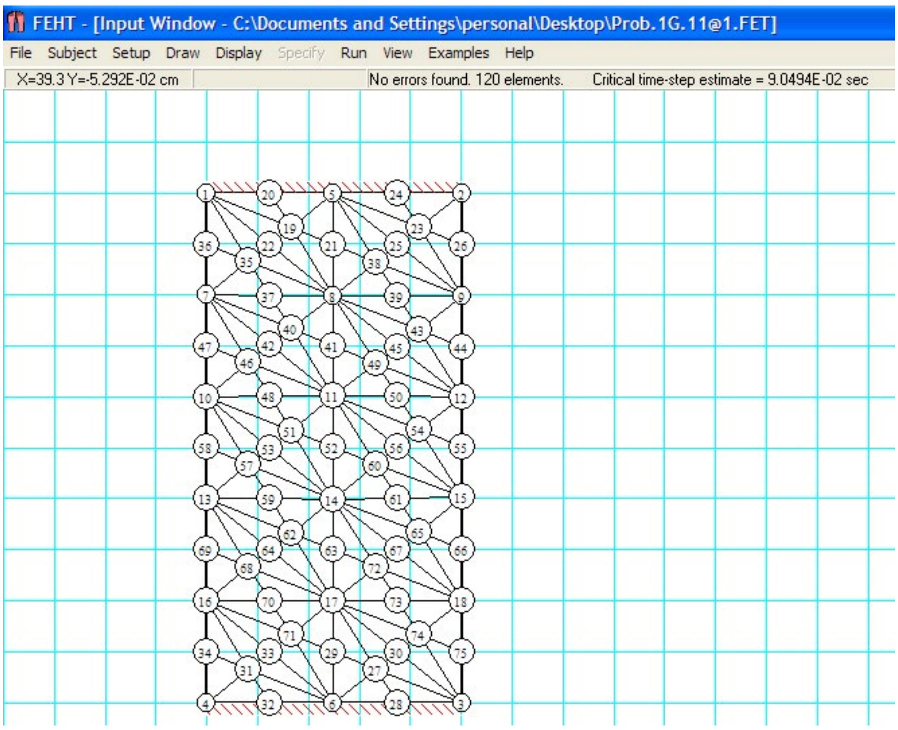
Click on Draw-Reduce Mesh:



We get:

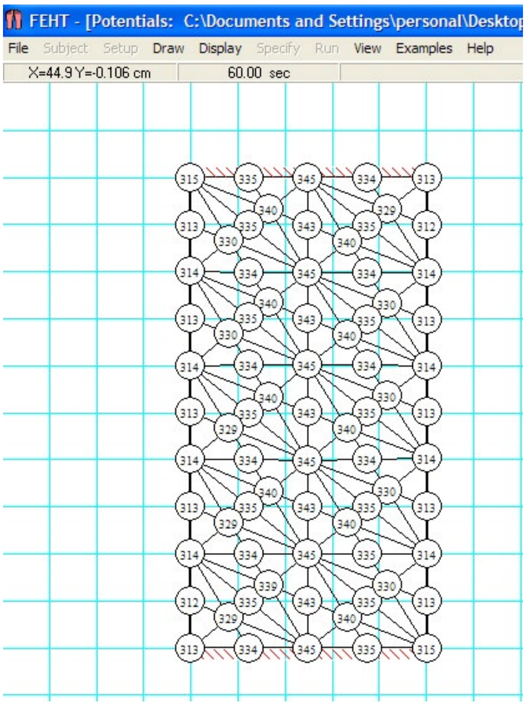


23. Click on Run – Check:



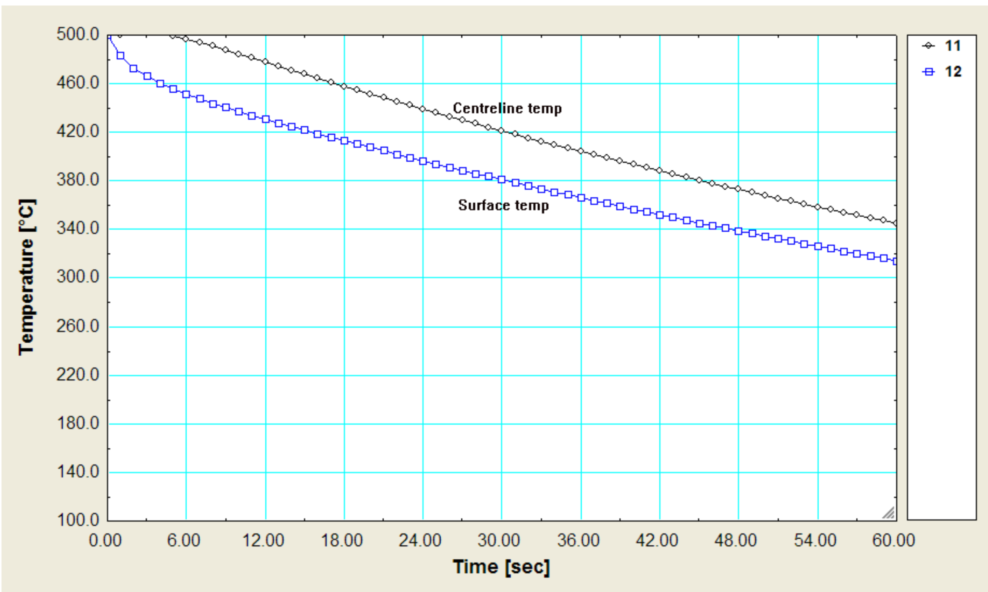
We see on top of above screen that there are now 120 elements, and no errors.

24. Run-Calculate; and, View-Temperatures:



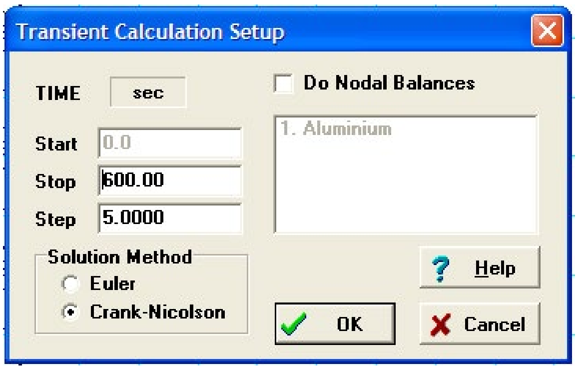
Note that Centre temp (Node 11) is 345 C and Surface temp (Node 12) is 314 C.

25. Click on View-Temp vs Time, and we get:



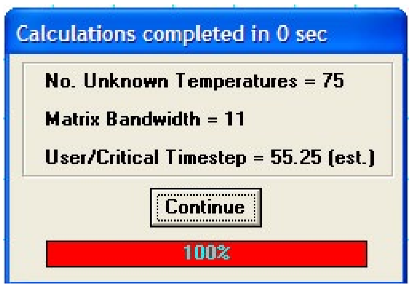
26. **When is Steady state temp. reached?**

Go to Run-Calculate and we get the following screen: Let us calculate for 600 s with a step of 5 s;



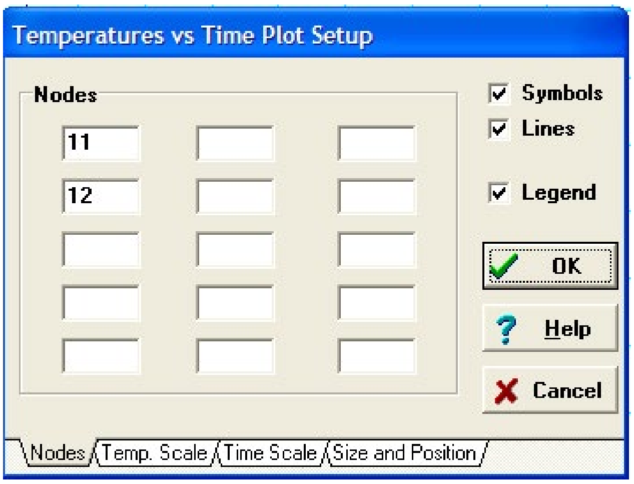
Hit OK:

We get the following screen:



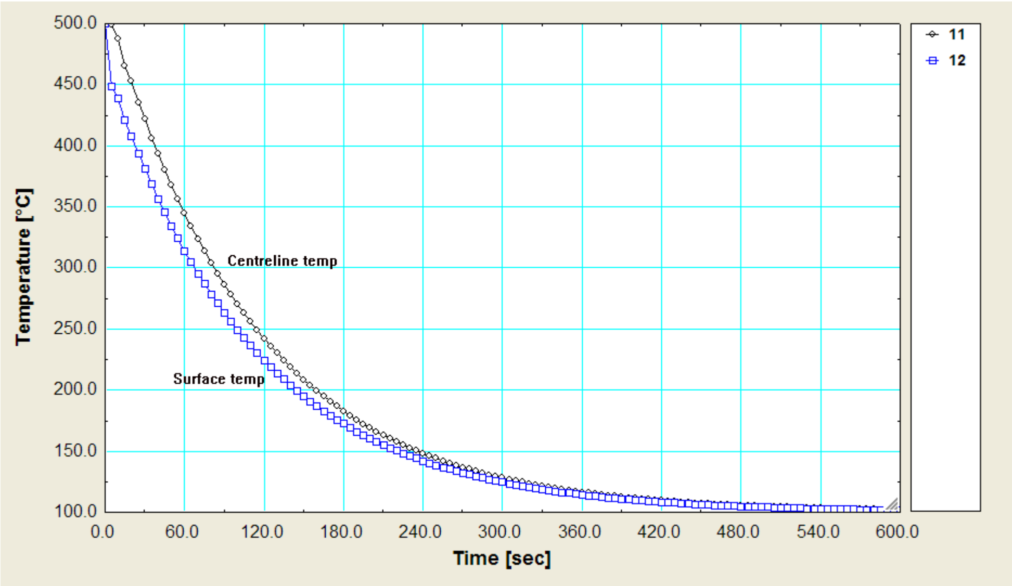
Hit Continue.

Now, choose View – Temp vs Time. We get:



Draw the plot for Node 11 (centre line) and Node 12 (Surface node): Press OK:

We get:



Thus, we see that steady state is reached after about 540 s when the surface temp is almost equal to the surrounding fluid temp (i.e. 100 C).

"**Prob. 1G.12.** A solid iron rod of dia 60 mm, initially at a temp of 800 C, is suddenly dropped into an oil bath at 50 C. The heat transfer coeff between the fluid and the surface is 400 W/m^2.K. The properties of iron rod are as follows: alpha = 2 × 10^5 m^2/s, k = 60 W/m.C. (i) Calculate the centre line temp 10 min after immersion in fluid (ii) how long will it take the centre line temp to reach 100 C? (iii) determine the energy removed from the rod during 10 min time. [VTU – VI Sem. B.E. – May–June 2006]:"

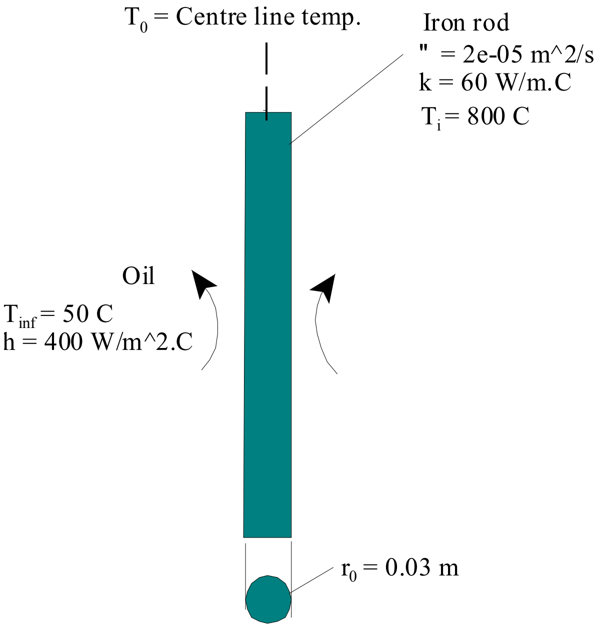


Fig.Prob.1G.12

EES Solution:

"Data:"

r_o = 0.03[m]
T_i = 800[C]
T_inf = 50[C]
h = 400[W/m^2-C]
T_o = 100[C]
alpha = 2e-05[m^2/s]
k = 60[W/m-C]

"Calculations:"

Biot = (h * r_o / 2) / k "....Biot No."

"Biot = 0.1; anyway, use Heisler charts or one – term solution:"

Biot_star = h * r_o / k "...**this is the Biot No. to be used either with Heisler charts or with one term solutions"**

"Find Lambda_1 and A_1 based on Biot_star:"

$$\lambda_1 \cdot \frac{J(1, \lambda_1)}{J(0, \lambda_1)} = Biot_{star}$$

$$A_1 = 2 \cdot \frac{J(1, \lambda_1)}{\lambda_1 \cdot \left(J(0, \lambda_1)^2 + J(1, \lambda_1)^2\right)}$$

"In EES, above eqns are entered as:"

Lambda_1 * BesselJ(1,Lambda_1) / BesselJ(0,Lambda_1) = Biot_star

A_1 = 2 * BesselJ(1,Lambda_1) / (Lambda_1 * ((BesselJ(0,Lambda_1))^2 + (BesselJ(1,Lambda_1))^2))

"Time to reach a Centre temp of 100 C:"

Theta_o =(T_o – T_inf) / (T_i – T_inf) "For Heisler chart verification"

$$\frac{T_o - T_{inf}}{T_i - T_{inf}} = A_1 \cdot \exp\left(-\lambda_1^2 \cdot Fo\right)$$

"Above eqn. is entered as:"

(T_o – T_inf) / (T_i – T_inf) = A_1 * exp(-Lambda_1^2 * Fo) "Finds Fourier no, Fo"

Fo = alpha * tau / (r_o)^2 "Finds tau … required to reach 100 C at the centre"

"What is the centre temp after 10 min?"

tau_2 = 600[s]

Fo_2 = alpha * tau_2 / (r_o)^2 "…new Fourier no. Fo_2"

(T_o_2 – T_inf) / (T_i – T_inf) = A_1 * exp(-Lambda_1^2 * Fo_2) "Finds the centre temp. T_o_2"

"Energy removed after 10 min:"

Qmax = V * (k / alpha) * (T_i – T_inf) " max heat transferred … since (rho * cp) = (k / alpha)"

V = pi * r_o^2 * 1"..volume of 1 m long cyl."

$$QbyQmax = 1 - 2 \cdot \left[\frac{T_{o,2} - T_{inf}}{T_i - T_{inf}}\right] \cdot \frac{J(1, \lambda_1)}{\lambda_1}$$

"Above eqn. is entered in EES as:"

QbyQmax = 1 – 2 * (T_o_2 – T_inf) / (T_i – T_inf) * BesselJ(1,Lambda_1)/Lambda_1 "…ratio of Q / Q_max"

Q = Qmax * QbyQmax "[W]… finds actual heat transferred, Q"

Results:

Unit Settings: SI C kPa J mass deg

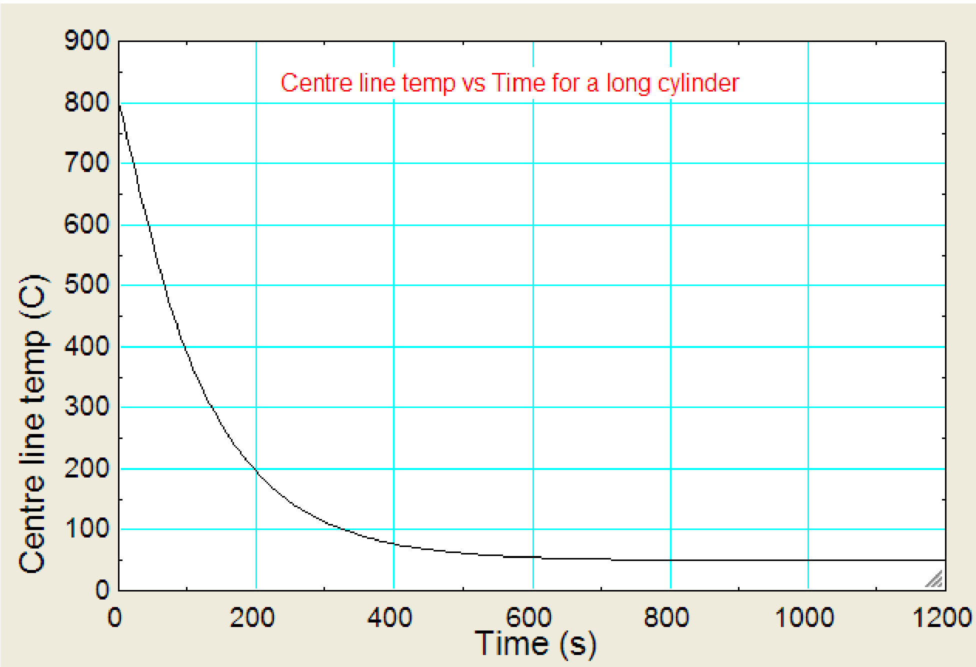
α = 0.00002 [m^2/s]	A_1 = 1.048	Biot = 0.1
$Biot_{star}$ = 0.2	Fo = 7.238	Fo_2 = 13.33
h = 400 [W/m^2-C]	k = 60 [W/m-C]	λ_1 = 0.617
Q = 6.322E+06 [J]	QbyQmax = 0.9938	Qmax = 6.362E+06 [J]
r_o = 0.03 [m]	τ = 325.7 [s]	τ_2 = 600 [s]
θ_o = 0.06667	T_i = 800 [C]	T_{inf} = 50 [C]
T_o = 100 [C]	$T_{o,2}$ = 54.91 [C]	V = 0.002827 [m^3]

Thus:

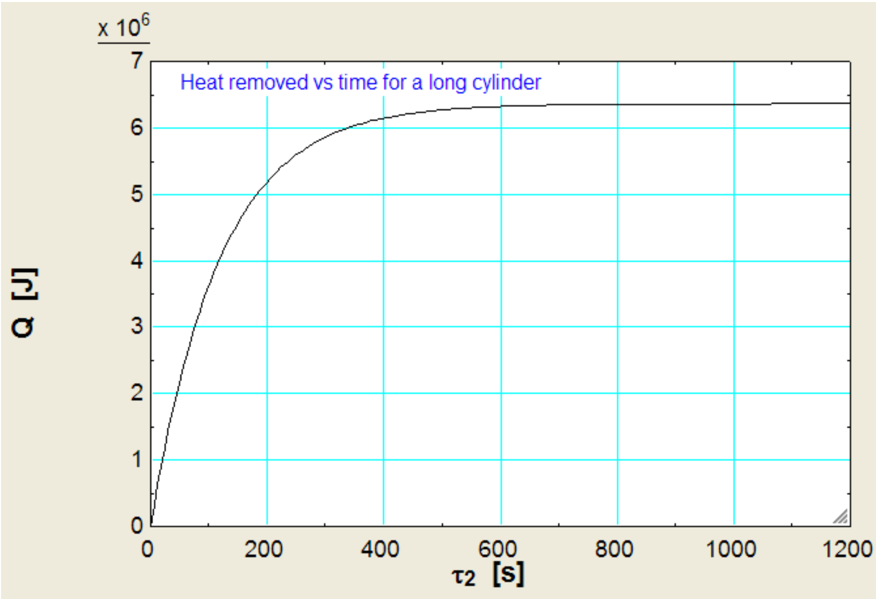
tau = 325.7 s …. Time required for the centre temp, T_0, to reach 100 C … Ans.

T_02 = 54.91 C …. Temp at the centre after tau2 = 10 min (= 600 s) …. Ans.

Plot centre temp against time required:



Also plot heat removed Q against time elapsed:



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Prob. 1G.13. A long cylinder 12 cm in dia and initially at 20 C is placed into a furnace at 820 C with a local heat transfer coeff of 140 W/m^2.K. Calculate the time required for the axis temp to reach 800 C. Also calculate the corresponding temp. at a radius of 5.4 cm at that time. Take α = 6.11 × 10^-6 m^2/s, k = 21 W/m.K. [VTU – VI Sem. B.E. – June–July 2011]

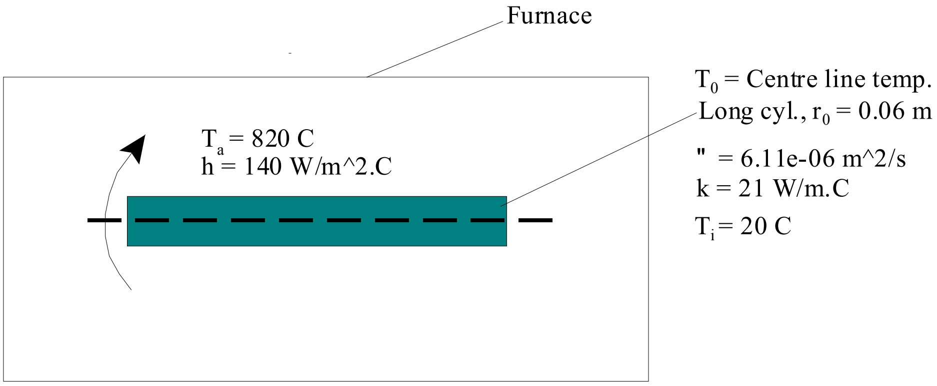


Fig.Prob.1G.13

Mathcad Solution:

Data:

$L := 1$ m...length of cyl....assumed

$r_0 := 0.06$ m...outer radius of cyl.

$\alpha := 6.11 \cdot 10^{-6}$ m²/s....thermal diffusivity

$k := 21$ W/(m.C)....thermal cond.

$T_i := 20$ C...initial temp. of cyl.

$T_a := 820$ C....temp. of surroundings

$h := 140$ W/(m².C).....heat transfer coeff. between the surface and the surroundings

$T_0 := 800$ C....centre temp after a time period of τ seconds.

To calculate: (i) the time τ for the centre to reach 800 C, (ii) temp. at a radius of 5.4 cm at that time and, in addition, (iii) calculate the amount of heat transferred during this period.

First check if lumped system analysis is applicable:

$$Bi := \frac{h \cdot \frac{r_0}{2}}{k}$$ ...define Biot number...for a cylinder, L_c = (V/A) = r0/2

i.e. Bi = 0.2 ...Biot number.

It is noted that Biot number is > 0.1; so, **lumped system analysis is not applicable**.

We have to adopt Heisler chart solution, or **one term approximation solution.**

For using the charts, or one term solution, now, remember that Bi is defined as:

$$Bi\_star := \frac{h \cdot r_0}{k}$$ ...define new Biot number

i.e. Bi_star = 0.4 ...Biot number

Fourier number: $$Fo = \frac{\alpha \cdot \tau}{r_0^2}$$ ....define Fourier number

Solution by one term approximation:

Time required for the centre temp. to reach 800 C:

We have:

Centre of long cylinder:
(r = 0)

$$\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo}$$ ...eqn..(A)

A_1 and λ_1 have to be found from Table against Bi.

OR: by using the eqns as follows:

$$\lambda_1 \cdot \frac{J1(\lambda_1)}{J0(\lambda_1)} = Bi\_star$$ and, $$A_1 = 2 \cdot \frac{J1(\lambda_1)}{\lambda_1 \cdot \left(J0(\lambda_1)^2 + J1(\lambda_1)^2\right)}$$

Use Solve Block of Mathcad to get λ1: Start with a guess value for λ1:

$\lambda_1 := 1$ ....guess value

Given

$$\lambda_1 \cdot \frac{J1(\lambda_1)}{J0(\lambda_1)} = Bi\_star$$

$\lambda_1 := Find(\lambda_1)$

i.e. $\lambda_1 = 0.852$ ....value of λ1

Therefore:

$$A_1 := 2 \cdot \frac{J1(\lambda_1)}{\lambda_1 \cdot \left(J0(\lambda_1)^2 + J1(\lambda_1)^2\right)}$$

i.e. $A_1 = 1.093$ ....value of A1

Now use eqn. (A) and Solve Block of Mathcad to get time τ required for the centre to reach 800 C:

$\tau := 100$ s .... trial value

Given

$$\frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot \left(\frac{\alpha \cdot \tau}{r_0^2}\right)}$$ ....since Fourier no., Fo = (α*τ / r0^2)

$\tau := \text{Find}(\tau)$

i.e. $\tau = 3.069 \cdot 10^3$ **s .... time reqd. for centre to reach 800 C .... Ans.**

Note: Now, Fourier no. is: $Fo := \frac{\alpha \cdot \tau}{r_0^2}$

i.e. $Fo = 5.21$ **....which is > 0.2. Therefore use of one term solution is justified.**

Temp. at a radius of 5.4 cm at this time:

We have:

Long cylinder: $$\theta(r,\tau) = \frac{T(r,\tau) - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \cdot J0\left(\frac{\lambda_1 \cdot r}{r_0}\right)$$ ...Fo > 0.2...eqn. (B)

In eqn. (B), J0 is the zeroth order Bessel function of the first kind. Its value can be read from Table. However, while using Mathcad, J0 can be got directly by typing 'J0 (λ1)='.

i.e. $J0(\lambda_1) = 0.827$

But, while using Mathcad, it is not even necessary to separately obtain the value of $J0(\lambda_1)$ See below the expression for T. While calculating the expression for T, value of $J0(\lambda_1)$ is returned and substituted automatically, and we get the final value of T as shown.

Here, r/r0 = (5.24 / 6), at the r = 5.4 cm. So, we get:

$r := 0.054$ m ... radius at which temp is required

$$T := (T_i - T_a) \cdot \left(A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \cdot J0\left(\frac{\lambda_1 \cdot r}{r_0}\right)\right) + T_a$$ ...define T(r,τ)

i.e. $T = 802.831$ **C...temp. at r = 5.4 cm...Ans.**

Amount of heat transferred, Q:

For heat transfer, we have:

Cylinder: $$\frac{Q}{Q_{max}} = 1 - 2 \cdot \theta_0 \cdot \frac{J_1(\lambda_1)}{\lambda_1}$$ ....eqn.(C)

i.e. $$Fraction := 1 - 2 \cdot \frac{T_0 - T_a}{T_i - T_a} \cdot \frac{J1(\lambda_1)}{\lambda_1}$$ ....define Fraction, Q/Q_{max}

i.e. $Fraction = 0.977$

Now, $$Q_{max} := \left(\frac{k}{\alpha}\right) \cdot \left(\pi \cdot r_0^2 \cdot L\right) \cdot \left(T_a - T_i\right)$$ J....max. heat transfer

i.e. $Q_{max} = 3.11 \cdot 10^7$ J

To draw radial temp. distribution at different times:

Let us draw radial temp. distribution at τ = 15 min., 25 min., and 1 h.

We have, for temp. distribution at any location:

Long cylinder: $$\theta(r,\tau) = \frac{T(r,\tau) - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \cdot J_0\left(\frac{\lambda_1 \cdot r}{r_0}\right)$$ ...Fo > 0.2...

Centre of long cylinder:
(r = 0) $$\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo}$$

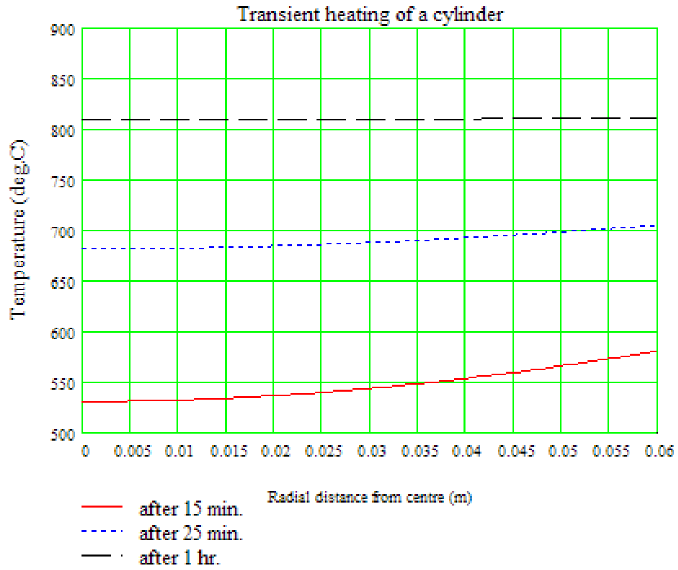
Fourier number as a function of τ: $$Fo(\tau) := \frac{\alpha \cdot \tau}{r_0^2}$$ ...for cylinder

Then, $$T(r,\tau) := \begin{cases} T_a + \left(T_i - T_a\right) \cdot \left(A_1 \cdot e^{-\lambda_1^2 \cdot Fo(\tau)}\right) & \text{if } r=0 \\ T_a + \left(T_i - T_a\right) \cdot \left(A_1 \cdot e^{-\lambda_1^2 \cdot Fo(\tau)} \cdot J0\left(\frac{\lambda_1 \cdot r}{r_0}\right)\right) & \text{otherwise} \end{cases}$$ .....eqn. (D)

For a given τ, we will plot eqn.(D) against r; then, we will repeat for different times, τ:

We use Mathcad to draw the graph. First, define a range variable r, varying from 0 to say, 0.06 m, with an increment of 0.0005.

r := 0, 0.0005 .. 0.06 ....define a range variable r varying from zero to 0.06 m, with an increment of 0.001 m



Note:

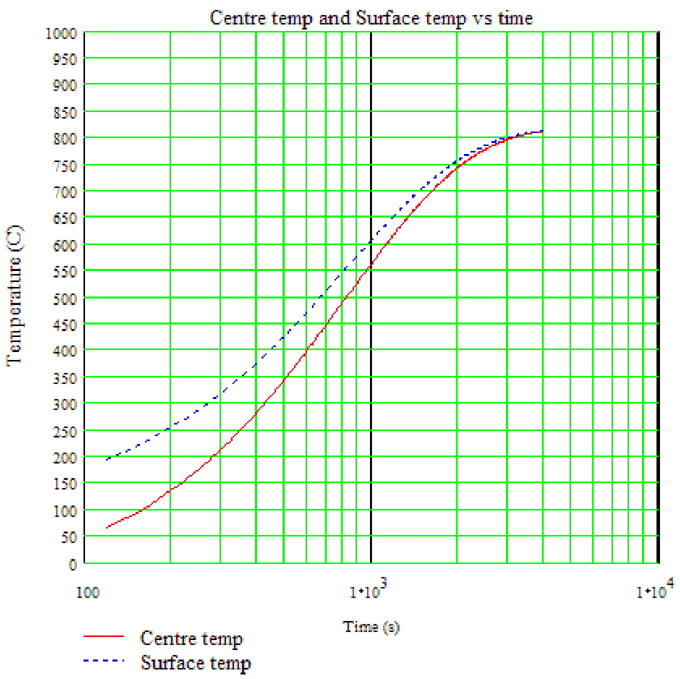
1) see from the above fig. how cooling progresses with time. After a time period of 1 hr. the temperatures along the radius are almost uniform.
2) eqn. (D) illustrates a small piece of Mathcad programming. It uses the "if…otherwise" condition, i.e. if r0 = 0, the temp. at the centre is given by eqn.(A); otherwise, temp. distribution is given by eqn. (B).

Plot centre temp and surface temp as a function of time:

Centre temp and surface temp are already defined as functions of r and τ in eqn. (D):

$$T(r,\tau) := \begin{cases} T_a + (T_i - T_a)\cdot\left(A_1\cdot e^{-\lambda_1^2\cdot Fo(\tau)}\right) & \text{if } r=0 \\ T_a + (T_i - T_a)\cdot\left(A_1\cdot e^{-\lambda_1^2\cdot Fo(\tau)}\cdot J0\left(\frac{\lambda_1\cdot r}{r_0}\right)\right) & \text{otherwise} \end{cases}$$ .....eqn. (D)

$\tau := 120, 130 .. 4000$ ...define a range variable, τ



Note: We have taken the lower limit of τ as 120 s since, to have Fo > 0.2, τ should have a value of 117.84 s or more. And, for the one term solutions to be valid, therefore, we should have τ > 120.

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Prob. 1G.14: An ordinary egg can be approximated as a 5.5 cm dia sphere whose properties are: k = 0.6 W/m.C, α = 0.14 × 10^-6 m^2/s. The egg is initially at a uniform temp of 8 C and is dropped into boiling water at 97 C. taking h = 1400 W/m^2.C, determine how long it will take for the centre of the egg to reach 70 C. Also plot Time against different temperatures at centre T_c (varying from 50 to 95 C) [Ref: 2]

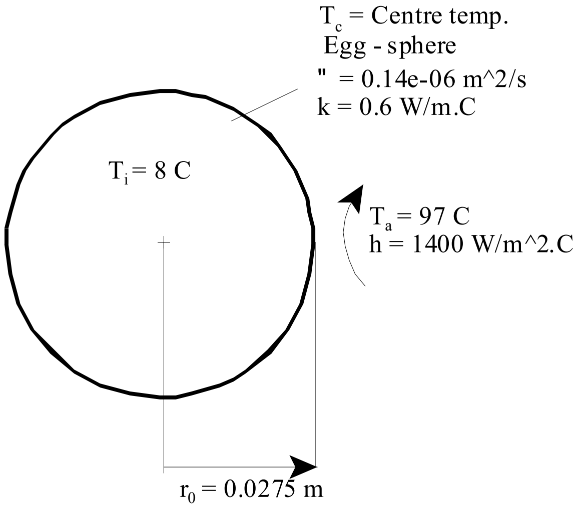


Fig.Prob.1G.14

Mathcad Solution:

Data:

$r_0 := 0.0275$ m $\quad k := 0.6$ W/m.C $\quad \alpha := 0.14 \cdot 10^{-6}$ m^2/s

$Ti := 8$ C $\quad Ta := 97$ C $\quad h := 1400$ W/m^2.C

$Tc := 70$ C ... temp at centre

Calculations:

First, calculate the Biot No. to see if It is less than or more than 0.1:

$$Bi := \frac{h \cdot \left(\frac{r_0}{3}\right)}{k} \qquad Bi = 21.389$$ ...Biot No. .. Remember: for sphere Lc = r0 / 3

Since Bi >> 0.1, use one term solution. or Heisler charts.

For this, Biot No. is defined differently:

Find new Bi: $Bi\_star := \frac{h \cdot r_0}{k}$

i.e. $Bi\_star = 64.167$

Find λ1: Use Solve Block of Mathcad: Start with a guess value for λ1:

$\lambda 1 := 0.1$ ....trial value

Given

$1 - \lambda 1 \cdot \cot(\lambda 1) = Bi\_star$

$\lambda 1 := Find(\lambda 1)$

i.e. $\lambda 1 = 3.093$

Find A1:

$$A1 := \frac{4 \cdot (\sin(\lambda 1) - \lambda 1 \cdot \cos(\lambda 1))}{2 \cdot \lambda 1 - \sin(2 \cdot \lambda 1)}$$

i.e. $A1 = 1.998$

Time required to reach a temp of 70 C at the centre:

We have, at the centre of sphere:

$$\frac{Tc - Ta}{Ti - Ta} = 0.303$$

Therefore:

$Fo := 0.3$ ..trial value

Given

$$\frac{Tc - Ta}{Ti - Ta} = A1 \cdot \exp\left(-\lambda 1^2 \cdot Fo\right)$$ **...eqn. (A) ...for temp at centre of sphere**

$Fo := \mathrm{Find}(Fo)$

i.e. $Fo = 0.197$ **....Fourier No.**

Now, $Fo = \frac{\alpha \cdot \tau}{r_0^{\,2}}$

Therefore: $\tau := \frac{Fo \cdot r_0^{\,2}}{\alpha}$ $\tau = 1.064 \cdot 10^3$ **s...time reqd to reach 70 C at centre....Ans.**

To draw the graph of Time vs. centre temp:

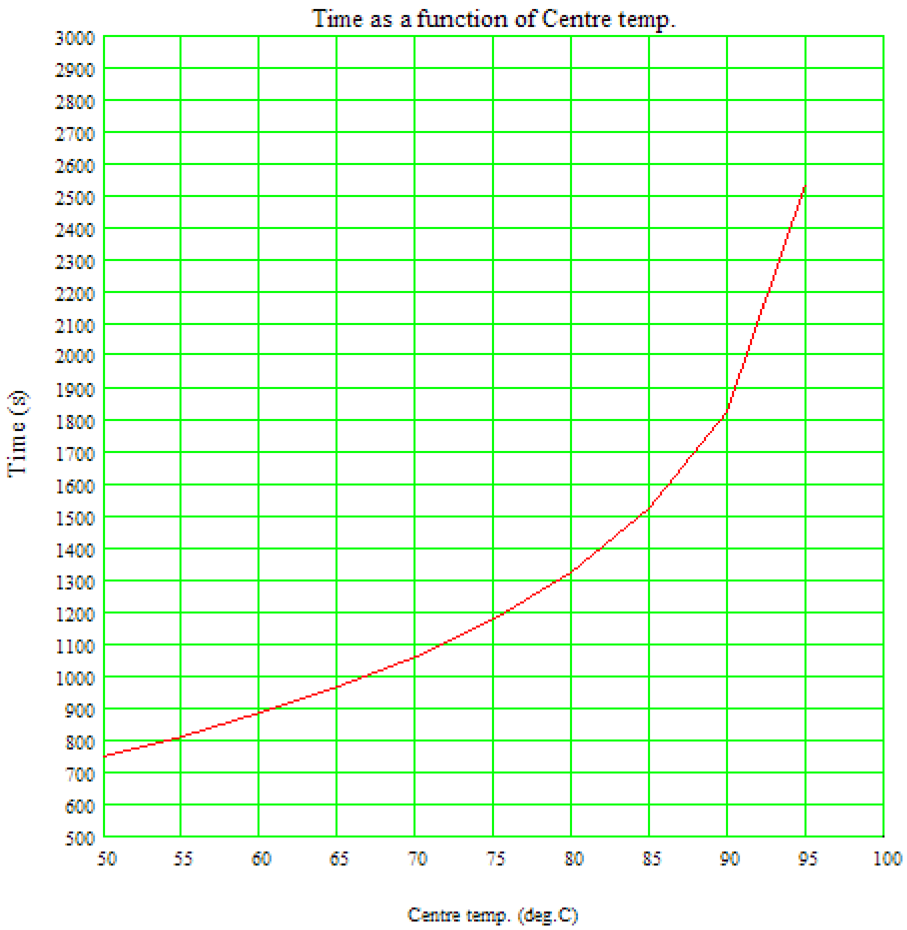
From eqn (A), we have:

$$\ln\left[\frac{(Tc - Ta)}{A1 \cdot (Ti - Ta)}\right] = -\lambda 1^2 \cdot \frac{\alpha \cdot \tau}{r_0^{\,2}}$$

Then: $$\tau(Tc) := \frac{\ln\left[\frac{(Tc - Ta)}{A1 \cdot (Ti - Ta)}\right] \cdot r_0^{\,2}}{-\alpha \cdot \lambda 1^2}$$ ....define τ as a function of Tc

$Tc := 50, 55 .. 95$ ...Let Tc vary from 50 C to 95 C with increment of 5 deg.

Tc	τ(Tc)
50	751.41
55	814.934
60	886.519
65	968.513
70	1064.467
75	1180.128
80	1325.742
85	1522.454
90	1826.862
95	2534.382



In addition:

Find the temp at the surface when centre temp is 70 C. Also plot the centre temp and surface temp against timse for τ = 1100 to 3600 s (since τ required for Fo = 0.2 is 1080 s)

Surface temp when centre temp is 70 C:

For surface temp., we have:

sphere:

$$\theta(x,\tau) = \frac{T(r,\tau) - Ta}{Ti - Ta} = A1 \cdot e^{-\lambda_1^{2} \cdot Fo} \cdot \frac{\sin\left(\frac{\lambda 1 \cdot r}{r_0}\right)}{\frac{\lambda 1 \cdot r}{r_0}} \qquad \text{...Fo > 0.2....(eqn. B)}$$

Here, r/r0 = 1, at the surface of the sphere. So, we get:

$$Ts := (Ti - Ta) \cdot \left(A1 \cdot e^{-\lambda 1^{2} \cdot Fo} \cdot \frac{\sin(\lambda 1)}{\lambda 1} \right) + Ta \qquad \text{...define T(r,τ)}$$

i.e. $Ts = 96.573$ **C...temp. at the surface...Ans.**

Heat transferred during this period:

For heat transfer, we have:

Sphere: $$\frac{Q}{Q_{max}} = 1 - 3 \cdot \theta_0 \cdot \left(\frac{\sin(\lambda 1) - \lambda 1 \cdot \cos(\lambda 1)}{\lambda 1^3} \right)$$ .....(eqn. C)

Now: $Tc := 70$ C

i.e. $$Fraction := 1 - 3 \cdot \frac{Tc - Ta}{Ti - Ta} \cdot \left(\frac{\sin(\lambda 1) - \lambda 1 \cdot \cos(\lambda 1)}{\lambda 1^3} \right)$$ ....define Fraction, Q/Q_{max}

i.e. $Fraction = 0.903$

Now, $Q_{max} = \rho \cdot V \cdot C_p \cdot (Ta - Ti)$ ....max. heat transfer possible

i.e. $$Q_{max} := \frac{k}{\alpha} \cdot \left(\frac{4}{3} \cdot \pi \cdot r_0^{\,3} \right) \cdot (Ta - Ti)$$ ...J ... since $(\rho.cp) = (k / \alpha)$

i.e. $Q_{max} = 3.323 \cdot 10^4$ J....

Therefore, $Q := Q_{max} \cdot Fraction$ J....heat transferred in 1064 s .

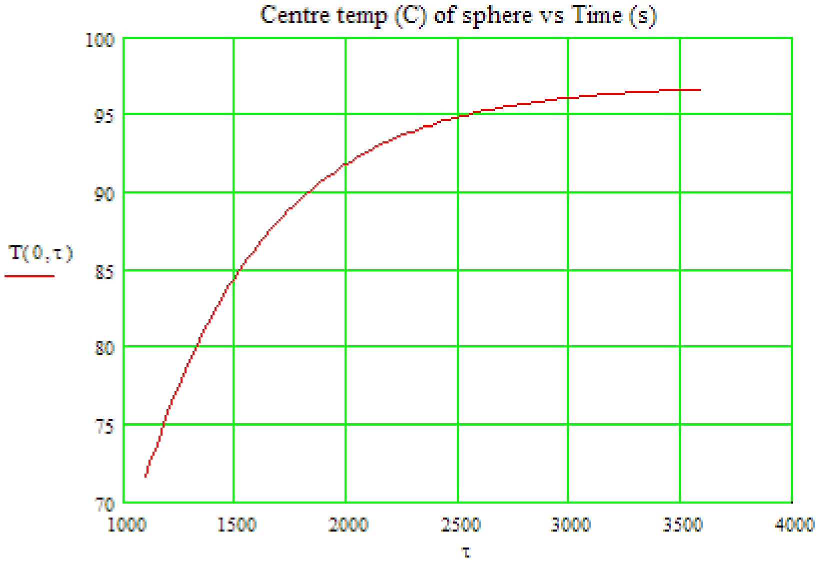
i.e. $Q = 3.002 \cdot 10^4$ **J....heat transferred in 1064 s....Ans.**

To plot Tc against time:

Fourier number as a function of τ: $$Fo(\tau) := \frac{\alpha \cdot \tau}{r_0^{\,2}}$$ ...for sphere

Then, $$T(r,\tau) := \begin{cases} Ta + (Ti - Ta) \cdot \left(A1 \cdot e^{-\lambda 1^2 \cdot Fo(\tau)} \right) & \text{if } r = 0 \\ Ta + (Ti - Ta) \cdot \left[A1 \cdot e^{-\lambda 1^2 \cdot Fo(\tau)} \cdot \dfrac{\sin\left(\dfrac{\lambda 1 \cdot r}{r_0}\right)}{\dfrac{\lambda 1 \cdot r}{r_0}} \right] & \text{otherwise} \end{cases}$$ .....eqn. (D)

$\tau := 1100, 1120 .. 3600$ ....define a range variable τ

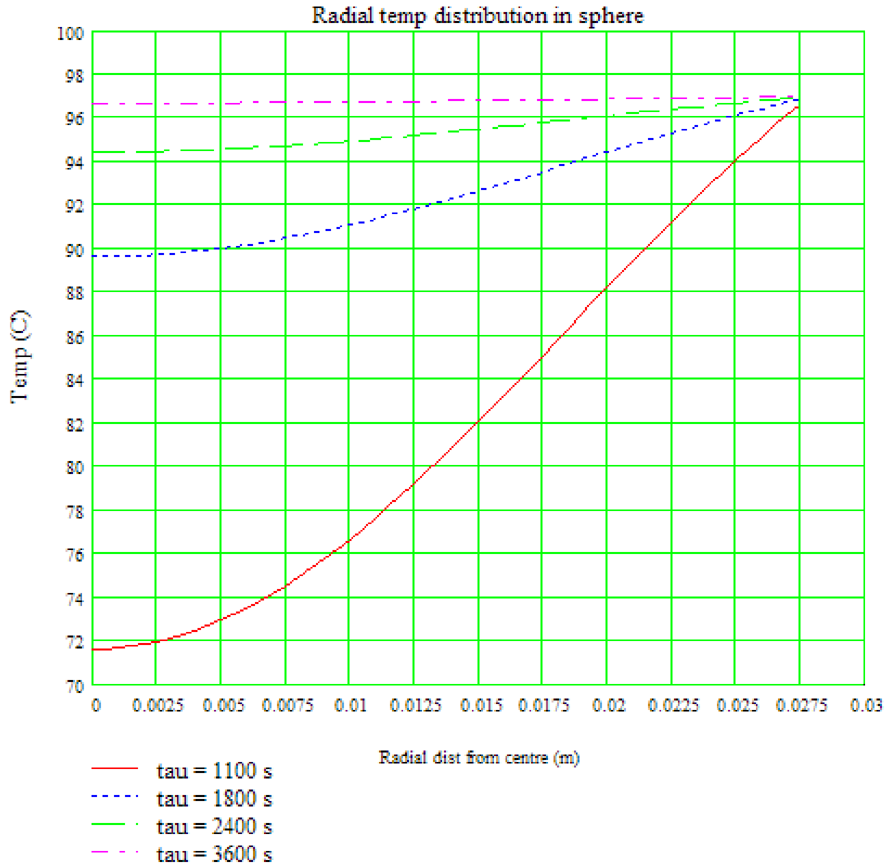


It is observed that after about 3600 s, centre temp is almost equal to 97 C.

To plot radial temp distribution at different times:

Then, $T(r,\tau) := \begin{cases} Ta + (Ti - Ta)\cdot\left(A1\cdot e^{-\lambda 1^2\cdot Fo(\tau)}\right) & \text{if } r=0 \\ Ta + (Ti - Ta)\cdot\left[A1\cdot e^{-\lambda 1^2\cdot Fo(\tau)}\cdot\dfrac{\sin\left(\dfrac{\lambda 1\cdot r}{r_0}\right)}{\dfrac{\lambda 1\cdot r}{r_0}}\right] & \text{otherwise} \end{cases}$ .....eqn. (D)

$r := 0, 0.0005 .. 0.0275$ ....define a range variable r



It may be noted that after about 3600 s, the temperatures at the centre and surface are almost same.

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"**Prob. 1G.15.** An Iron sphere (k = 60 W/m.C, cp = 460 J/kg.C, rho = 7850 kg/m^3, and alpha = 1.6 × 10^-5 m^2/s) of diameter d = 5 cm, is initially at uniform temp T_i = 225 C. Suddenly, the surface of the sphere is exposed to an ambient at T_inf = 25 C with a heat transfer coeff h = 500 W/m^2.C. (i) Calculate the centre temp at time tau = 2 min after start of cooling, (ii) Calculate the temp at a depth of 1 cm from the surface at time tau = 2 min after start of cooling, and (iii) Calculate the energy removed from the sphere during this period. [VTU – VI Sem. B.E. – Dec. 07–Jan. 2008]:"

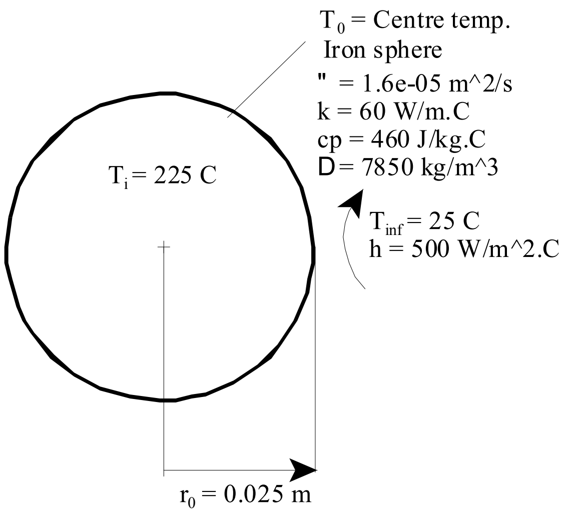


Fig.Prob.1G.15

EES Solution:

"Data:"

r_0 = 0.025[m] "…outside radius of sphere"

T_i = 225[C]

T_inf = 25[C]

h = 500[W/m^2-C]

tau = 120[s]

alpha = 1.6e-05[m^2/s]

k = 60[W/m-C]

cp = 460 [J/kg-C]

rho=7850 [kg/m^3]

"Calculations:"

"First of all, calculate the Biot No. to see if it is more than or less than 0.1:"

Biot = (h * r_0 / 3) / k "…Biot No. for a sphere; remember Lc = r_0 /3 for a sphere"

"We see that Bi = 0.06944 i.e. Bi < 0.1. So, lumped system analysis is applicable and temp anywhere within the sphere will be within 5 %.. But, Fo = 3.072 i.e. > 0.2 for this problem.

So, let us use One term solution (or Heisler charts) since they have asked for centre temp and temp at another radius separately.:

For One term solution, we take characteristic dimension as Lc = r_0 in the new definition of Biot No. as follows:"

Biot_star = h * r_0 / k "….Biot No. for use in one term solution or Heisler charts"

"We need to find out Lambda1 and A1:"

"Find Lambda_1 and A_1:"

$$1 - \lambda_1 \cdot \frac{1}{\tan(\lambda_1)} = \text{Biot}_{star} \quad \text{... eqn which Finds Lambda}_1 \text{ for a sphere}$$

$$A_1 = 4 \cdot \left[\frac{\sin(\lambda_1) - \lambda_1 \cdot \cos(\lambda_1)}{2 \cdot \lambda_1 - \sin(2 \cdot \lambda_1)} \right] \quad \text{...Finds A1}$$

Above equations are entered in EES as follows:

1- Lambda_1 * (1 / tan(Lambda_1)) = Biot_star "… eqn which Finds Lambda_1 for a sphere"

A_1 = 4 * (sin(Lambda_1) – Lambda_1 * cos(Lambda_1)) / ((2 * Lambda_1 – sin(2 * Lambda_1))) "…Finds A1"

"Centre temp at tau = 2 min:"

Fo = alpha * tau / r_0^2 "Finds Fourier No, Fo"

Theta_0 =(T_0 – T_inf) / (T_i – T_inf) "For Heisler chart verification"

$$\frac{T_0 - T_{inf}}{T_i - T_{inf}} = A_1 \cdot \exp\left(-\lambda_1^2 \cdot Fo\right) \quad \text{Finds } T_0$$

Above eqn. for T_0 is entered in EES:

(T_0 – T_inf) / (T_i – T_inf) = A_1 * exp(- Lambda_1^2 * Fo) "Finds T_0"

"What is the centre temp at 1 cm from surface (i.e. r = 1.5 cm) after 2 min?"

$$\frac{T_r - T_{inf}}{T_i - T_{inf}} = A_1 \cdot \exp\left(-\lambda_1^2 \cdot Fo\right) \cdot \frac{\sin\left[\lambda_1 \cdot \frac{r}{r_0}\right]}{\lambda_1 \cdot \frac{r}{r_0}} \quad \text{Finds } T_r$$

Above eqn for T_r is entered in EES:

(T_r – T_inf) / (T_i – T_inf) = A_1 * exp(-Lambda_1^2 * Fo) * sin(Lambda_1 *(r / r_0)) / (Lambda_1 * (r / r_0)) "Finds T_r"

"For Heisler chart verification:"

rbyro = (r / r_0)

Theta = (T_r – T_inf) / (T_0 – T_inf)

"Energy removed after 2 min:"

Qmax = V * rho * cp * (T_i – T_inf) "[J] …max energy removed"

$$QbyQmax = 1 - \left[3 \cdot \left(\frac{T_0 - T_{inf}}{T_i - T_{inf}}\right) \cdot \left(\frac{\sin(\lambda_1) - \lambda_1 \cdot \cos(\lambda_1)}{\lambda_1^3}\right)\right]$$ ....ratio of Q / Q_{max}

Above eqn for ratio of Q / Qmax is now entered in EES:

QbyQmax = 1 – 3 * (T_0 – T_inf) / (T_i – T_inf) * (sin(Lambda_1)-Lambda_1 * cos(Lambda_1)) / Lambda_1^3 "….ratio of Q / Q_max"

Q = Qmax * QbyQmax "[J] … actual energy removed from the sphere in time tau"

Results:

Unit Settings: SI C kPa kJ mass rad

α = 0.000016 [m²/s]	A_1 = 1.062	Biot = 0.06944
$Biot_{star}$ = 0.2083	cp = 460 [J/kg-C]	Fo = 3.072
h = 500 [W/m²-C]	k = 60 [W/m-C]	λ_1 = 0.7743
Q = 39781 [J]	QbyQmax = 0.8416	Qmax = 47268 [J]
r = 0.015 [m]	rbyro = 0.6	ρ = 7850 [kg/m³]
r_0 = 0.025 [m]	τ = 120 [s]	θ = 0.9644
θ_0 = 0.1683	T_0 = 58.66 [C]	T_i = 225 [C]
T_{inf} = 25 [C]	T_r = 57.46 [C]	V = 0.00006545 [m³]

Thus:

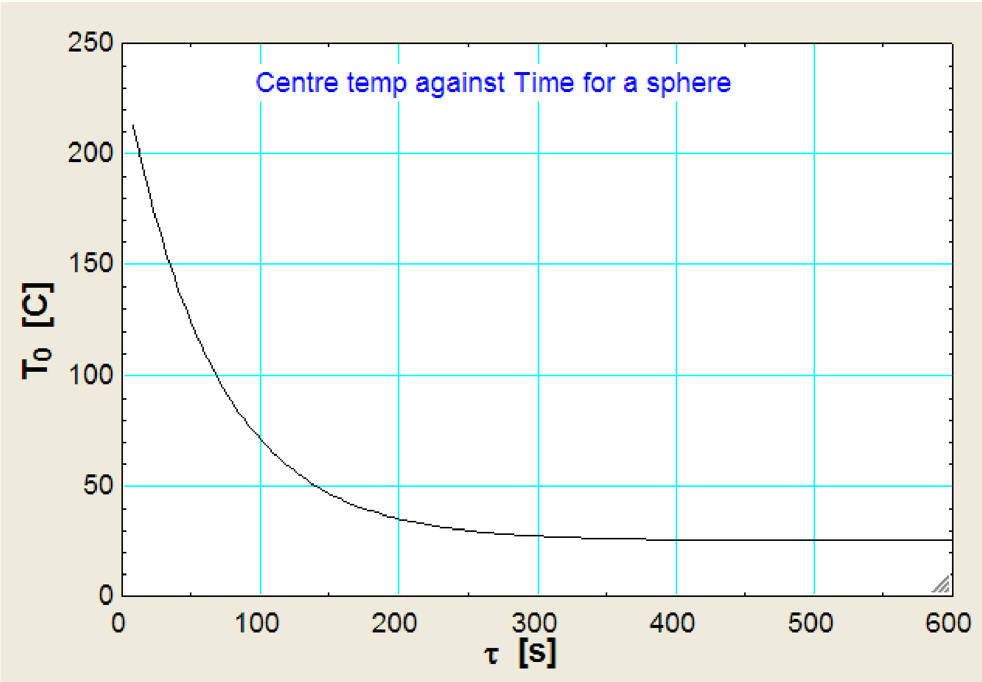
T_0 = 58.66 C … centre temp after tau = 120 s … Ans.

T_r = 57.46 C … temp at a depth of 1 cm (i.e. r = 1.5 cm) after tau = 120 s …. Ans.

Q = 39781 J …. Energy removed during the time 120 s …. Ans.

Note: Verify the results from Heisler charts. To help you in this, values of theta, theta_0 and rbyr0 are readily calculated with EES. (See Results above).

Plot the centre temp against the time required to reach that temp:



Here the graph is plotted for the time tau = 8 s to 600 s. Min value of 8 sec is chosen since for the one term solution to be valid, we should have Fo > 0.2, i.e. tau should be more than 7.8 s.
It may be observed from the plot that after about 360 s, the temp at centre is almost the same as that of the surroundings.

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"**Prob. 1G.16.** A large slab of wrought iron is at a uniform temp of 375 C. The temp of one surface of this slab is suddenly changed to 75 C. Calculate the time required for the temp to reach 275 C at a depth of 5 cm from the surface and the quantity of energy transferred per unit area of the surface during this period. Take k = 60 W/m.K and alpha = 1.626 × 10^ -5 m^2 / s. [VTU – VI Sem. B.E. – May–June 2010]:"

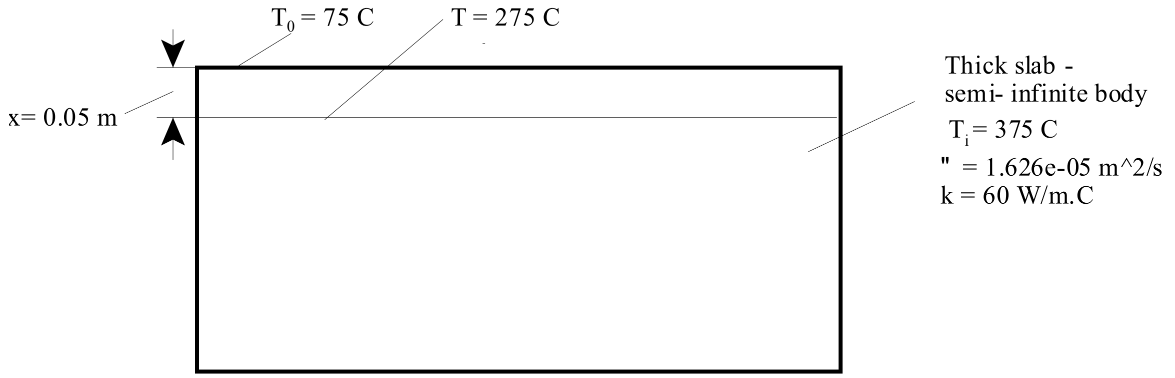


Fig.Prob.1G.16

EES Solution:

"Data:"

T_i = 375[C]

T_0 = 75[C]

x = 0.05[m]

T = 275[C]

alpha = 1.626e-05

k = 60[W/m-C]

A = 1[m^2]

"Calculations:"

"Here, the thick slab is treated as a semi – infinite body. Then:"

$$\frac{T - T_0}{T_i - T_0} = \text{Erf}\left[\frac{x}{2 \cdot \sqrt{\alpha \cdot \tau}}\right] \quad \text{Finds tau}$$

Above eqn to find out tau is entered in EES as follows:

(T – T_0) / (T_i – T_0) = erf(x / (2 * sqrt(alpha*tau))) "Finds tau"

"And energy transferred is given by:"

"And energy transferred is given by:"

$$Q_{tot} = 1.13 \cdot k \cdot A \cdot (T_0 - T_i) \cdot \sqrt{\frac{\tau}{\alpha}} \quad J \text{ ... energy transferred}$$

"i.e."

Q_tot = 1.13 * k * A * (T_0 – T_i)*sqrt(tau/alpha) "[J] … energy transferred"

Results:

Unit Settings: SI C kPa kJ mass deg

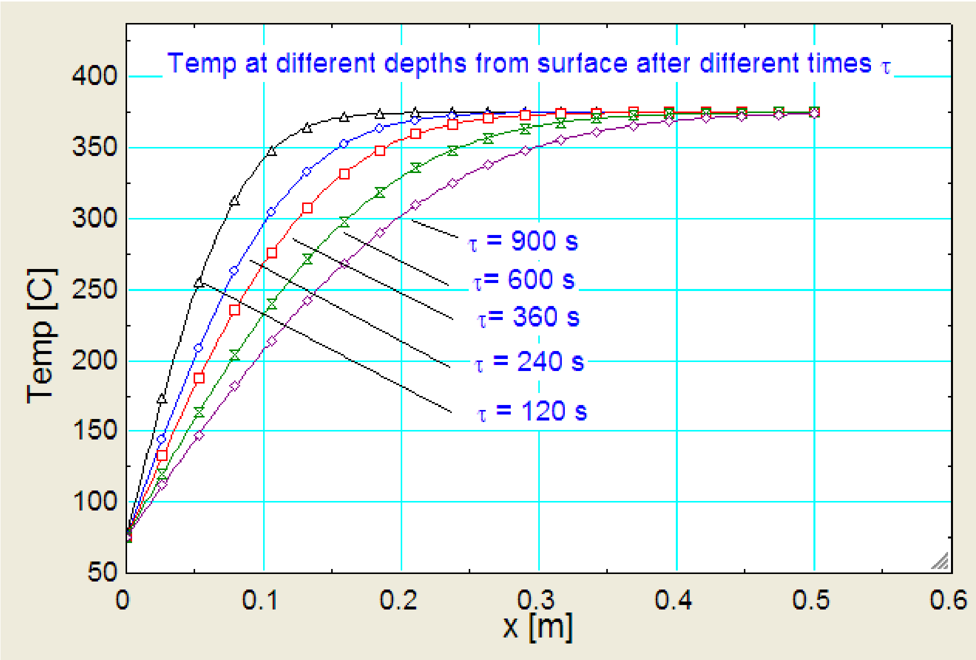
$A = 1$ [m^2]	$\alpha = 0.00001626$ [m^2/s]	$k = 60$ [W/m-C]
$Q_{tot} = -4.572E+07$ [J/m^2]	$T = 275$ [C]	$\tau = 82.14$ [s]
$T_0 = 75$ [C]	$T_i = 375$ [C]	$x = 0.05$ [m]

Thus:

tau = 82.14 s …. Time reqd for temp to reach 275 C at a depth x = 5 cm from surface … Ans.

Q_tot = – 4.572E07 J … heat transferred during this period … Ans. (-ve sign indicates that energy has flowed out of the slab).

Plot the temperatures at various depths from the surface after different time periods:



Temperatures at various depths upto 0.5 m are plotted for different time periods.

One can observe the progress of cooling of the slab with time.

==

Prob. 1G.17. In areas where ambient temperature drops to subzero temperatures and remains so for prolonged periods, freezing of water in underground pipelines is a major concern. It is of interest to know at what depth the water pipes should be buried so that the water does not freeze.

At a particular location, the soil is initially at an uniform temperature of 12 C and the soil is subjected to a subzero temperature of -10 C continuously for 50 days.

1) What is the minimum burial depth required to ensure that the water in the pipes does not freeze? i.e. pipe surface temperature should not fall below 0 C.
2) Plot the temp. distributions in the soil for different times i.e. after 1 day, 2 days etc.

Properties of soil may be taken as: $\alpha = 0.138 \times 10^{-6}$ m²/s, $\rho = 2050$ kg/m³, k = 0.52 W/(m.K), cp = 1840 J/kg.K.

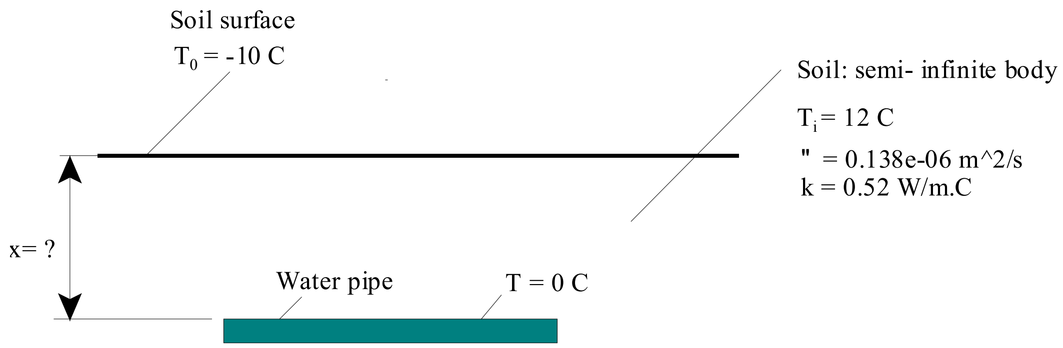


Fig.Prob.1G.17

Mathcad Solution:

Data:

$\alpha := 0.138 \cdot 10^{-6}$ m²/s....thermal diffusivity of soil

$k := 0.52$ W/(m.C)....thermal cond. of soilr

$T_i := 12$ C...initial temp. of soil

$T_0 := -10$ C....temp. of surface

$T := 0.0$ C....temp. of freezing of water

$\tau := 50 \cdot 24 \cdot 3600$ s... for 50 days

i.e. $\tau = 4.32 \cdot 10^6$ s....time duration of exposure of soil to subzero temp.

To find: the depth x reqd. to reach 0 C under these conditions:

We will consider earth's surface as a semi-infinite medium, with the surface suddenly brought to and maintained at a constant temperature, To.

So, eqn. (A) is applicable, to get temperature variation as function of position and time, i.e.

$$\frac{T(x,\tau) - T_0}{T_i - T_0} = \mathrm{erf}\left(\frac{x}{2 \cdot \sqrt{\alpha \cdot \tau}}\right) \quad \text{....eqn..(A)}$$

Now, we get: $\frac{T - T_0}{T_i - T_0} = 0.455$ since all temperatures are given.

Use Solve Block of Mathcad to find the min. burial depth, x. Start with a guess value for x:

$x := 0.3$ m....guess value for x

Given

$$\frac{T - T_0}{T_i - T_0} = \mathrm{erf}\left(\frac{x}{2 \cdot \sqrt{\alpha \cdot \tau}}\right)$$

$x := \mathrm{Find}(x)$

i.e. $x = 0.66$ **m ....min. burial depth to prevent freezing of water line....Ans.**

To plot the temp. distributions in the soil at a depth of 1 m for different times, t:

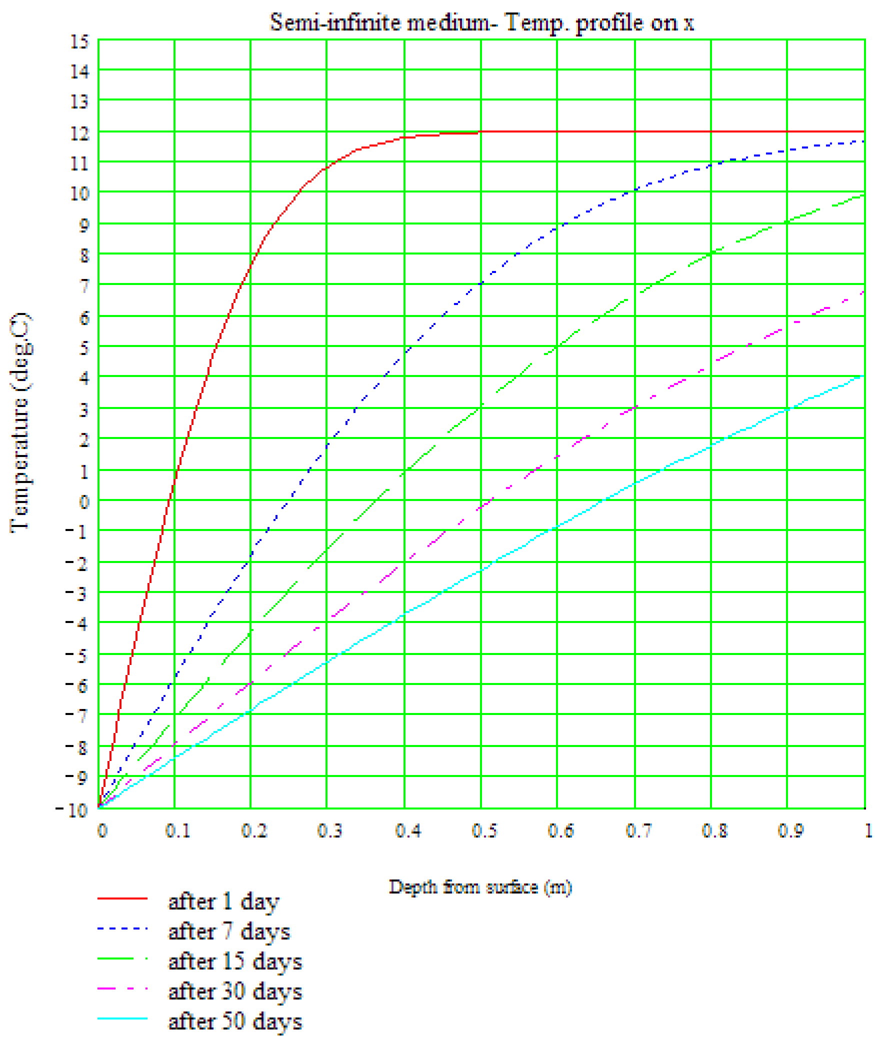
Again, we use eqn. (A). From this eqn. temperature as a function of x and τ is written as:

$$T(x,\tau) := T_0 + (T_i - T_0)\cdot \text{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}\right) \qquad \text{....eqn. (B)}$$

To plot eqn.(B) against x for different τ, in Mathcad:

First of all, define a range variable x varying from 0 to 1 m at an interval of, say, 0.01 m:

$x := 0, 0.01 .. 1$ ...define a range variable x , varying from 0 to 1 m, with an increment of 0.01 m.



Note from the above fig. that:

1) **even after a period of 50 days of exposure of the surface to an ambient at -10 C, temp. at a depth o 1 m has reached only about 4 deg. C.**
2) **after 50 days, freezing temp. of 0 deg. C is reached at depth of 0.66 m, as calculated.**
3) **slope of the temperature curve, dT/dx, at the surface (i.e. at x = 0) decreases as time increases; that means that heat extracted from the surfaces decreases as time increases.**

===

Prob. 1G.18. A thick Aluminium block initially at 27 C is subjected to a constant heat flux of 3500 W/m^2 by an electrical resistance heater whose top surface is insulated. Determine how much the surface temp of the block will rise after 30 min.

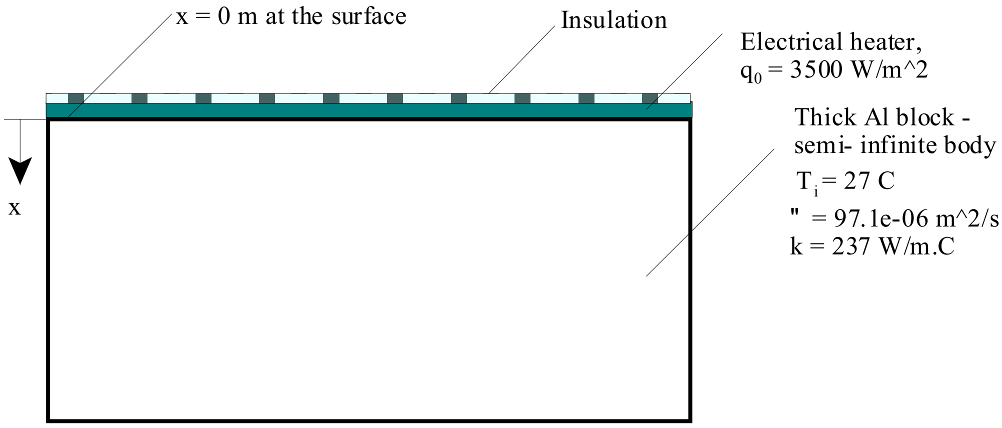


Fig.Prob.1G.18

Mathcad Solution:

Data:

$\alpha := 97.1 \cdot 10^{-6}$ m^2/s....thermal diffusivity of slab

$k := 237$ W/(m.C)....thermal cond. of Al

$T_i := 27$ C...initial temp. of slab

$q_0 := 3500$ W/m^2....const. heat flux on the surface

$x := 0$ m....at the surface

$\tau := 1800$ s....time period

To find: the surface temp. after a period of time τ = 1800 s.

Temp. at the surface after a time period of 1800 s:

This is the case of a semi-infinite slab, with constant heat flux conditions at its exposed surface. So, this is case, eqn. (A) given below is applicable:.

Eqn (A) to get temperature variation as function of position and time:

$$T(x,\tau) := T_i + \frac{2 \cdot q_0 \cdot \sqrt{\frac{\alpha \cdot \tau}{\pi}}}{k} \cdot \exp\left(\frac{-x^2}{4 \cdot \alpha \cdot \tau}\right) - \frac{q_0 \cdot x}{k} \cdot \left(1 - \mathrm{erf}\left(\frac{x}{2 \cdot \sqrt{\alpha \cdot \tau}}\right)\right) \quad .....(A)$$

Substituting and calculating, we get,

i.e. $T(x,\tau) = 33.967$ C...**temp. at the surface after a time period of 1800 s..Ans.** .

Note: In Mathcad, there is no need to separately find out erf() and substitute etc. All calculations are done in one step, since error function is one of the built-in functions in Mathcad.

==

Prob. 1G.19. A very thick slab ($\alpha = 5.6 \times 10^{-6}$ m²/s, k = 20 W/(m.C)) is initially at an uniform temperature of 325 C. Suddenly, its surface is subjected to convective cooling with a heat transfer coeff. h = 100 W/(m².C) into a coolant at 15 C. Calculate the temperature at the surface and and at a depth of 4.5 cm from the surface 3 min. after the start of cooling.

(b) Also, plot the temp histories at the surface and at a depth of 4.5 cm at various times for:

1) α = 5.6 × 10^-7, 5.6 × 10^-6 and 5.6 × 10^-5 m^2/s
2. k = 2, 20 and 200 W/m.C [Ref. 3]

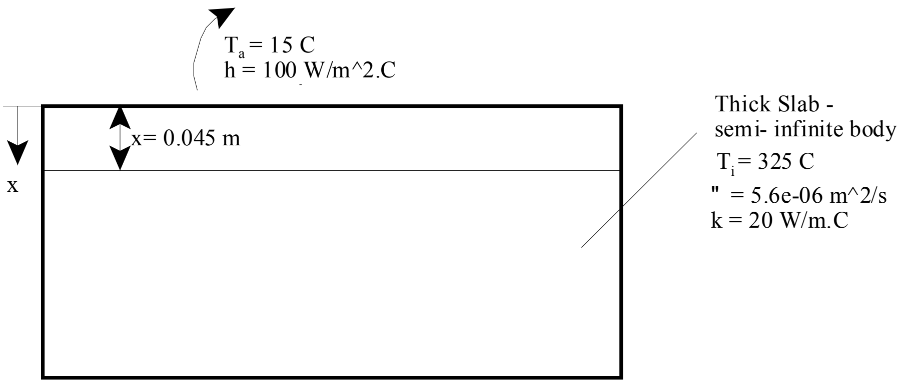


Fig.Prob.1G.19

Mathcad Solution:

Data:

$\alpha := 5.6 \cdot 10^{-6}$ m²/s....thermal diffusivity of slab

$k := 20$ W/(m.C)....thermal cond.

$T_i := 325$ C...initial temp. of slab

$T_a := 15$ C...coolant temp.

$h := 100$ W/(m^2.C)...heat transfer coeff. between ambient and slab surface

$x := 0.045$ m....depth from the surface

$\tau := 180$ s....time period

To find: the temp. after a period of time τ = 180 s, at the surface and at a a depth of 0.045 m.

Temp. at the surface, i.e. x = o, after a time period of 180 s:

This is the case of a semi-infinite slab, with convection conditions at its exposed surface. So, for this cae, the applicable eqn. is:

$$\frac{T(x,\tau)-T_i}{T_a-T_i}=1-\mathrm{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}\right)-\left(\exp\left(\frac{h\cdot x}{k}+\frac{h^2\cdot\alpha\cdot\tau}{k^2}\right)\right)\cdot\left(1-\mathrm{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}+\frac{h\cdot\sqrt{\alpha\cdot\tau}}{k}\right)\right) \quad \text{.eqn..(A)}$$

At the surface: $x := 0$

Use the Solve Block of Mathcad to solve for Ts. Start with a guess value for Ts, say 50 C:

$Ts := 50$ C.... guess value for Ts

Given

$$\frac{Ts-T_i}{T_a-T_i}=1-\mathrm{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}\right)-\left(\exp\left(\frac{h\cdot x}{k}+\frac{h^2\cdot\alpha\cdot\tau}{k^2}\right)\right)\cdot\left(1-\mathrm{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}+\frac{h\cdot\sqrt{\alpha\cdot\tau}}{k}\right)\right)$$

$Ts := \mathrm{Find}(Ts)$

i.e. $Ts = 276.44$ **C .... temp at the surface after 180 s ... Ans.**

Temp. Tx, at a depth of 4.5 cm, after a time period of 180 s:

Again, eqn. (A) is applicable, but with x = 0.045 m:

$x := 0.045$ m ... at a depth of 4.5 cm from surface

Again, use the Solve Block of Mathcad to solve for Tx. Start with a guess value for Tx, say 50 C:

$Tx := 50$ C.... guess value for Tx

Given

$$\frac{Tx-T_i}{T_a-T_i}=1-\mathrm{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}\right)-\left(\exp\left(\frac{h\cdot x}{k}+\frac{h^2\cdot\alpha\cdot\tau}{k^2}\right)\right)\cdot\left(1-\mathrm{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}+\frac{h\cdot\sqrt{\alpha\cdot\tau}}{k}\right)\right)$$

$Tx := \mathrm{Find}(Tx)$

i.e. $Tx = 314.526$ **C .... temp at a depth of 4.5 cm from the surface after 180 s ... Ans.**

Note the ease with which above expression is calculated in Mathcad.

(b):

To plot the temp histories at the surface and at a depth of 4.5 cm at various times for:
(i) α = 5.6 x 10^-7, 5.6 x 10^-6 and 5.6 x 10^-5 m^2/s
(ii) k = 2, 20 and 200 W/m.C

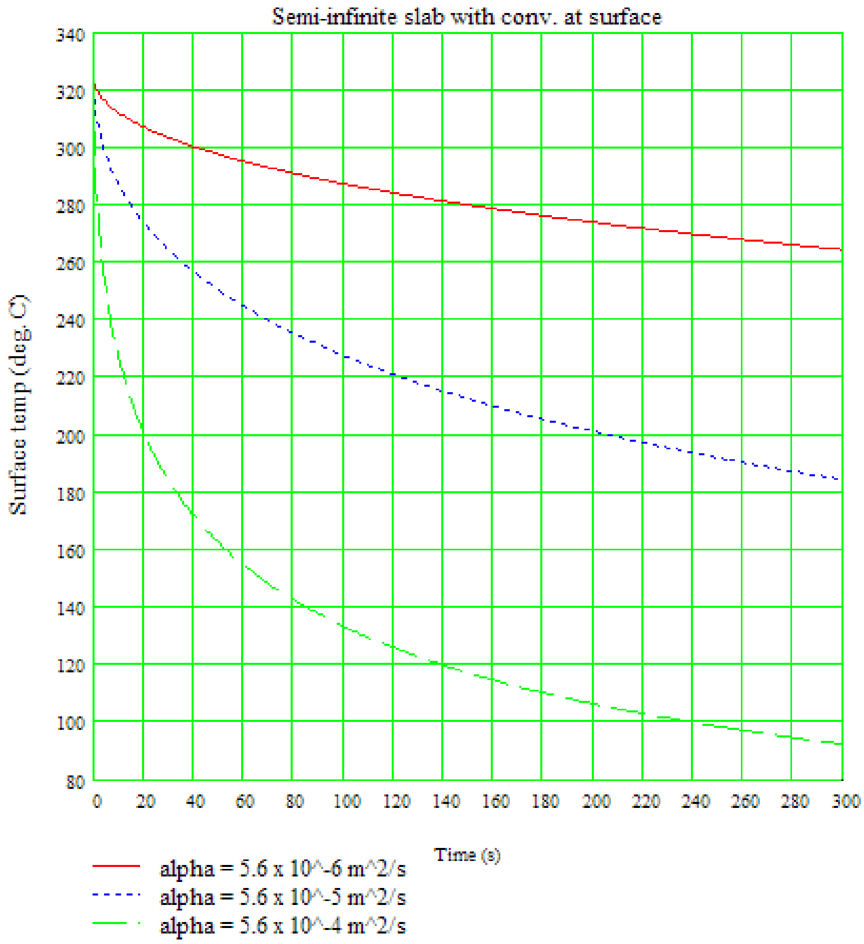
We have, from eqn. (A), writing T as a function of x, α, k and τ:

$$T(x,\alpha,k,\tau) := T_i + (T_a - T_i)\cdot\left[1 - \text{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}\right) - \left(\exp\left(\frac{h\cdot x}{k} + \frac{h^2\cdot\alpha\cdot\tau}{k^2}\right)\right)\cdot\left(1 - \text{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}} + \frac{h\cdot\sqrt{\alpha\cdot\tau}}{k}\right)\right)\right]$$

First, plot T vs τ for x = 0 and k = 20 W/m.C and three different values of α:

$k := 20$ W/m.C $\quad x := 0$ ...at the surface

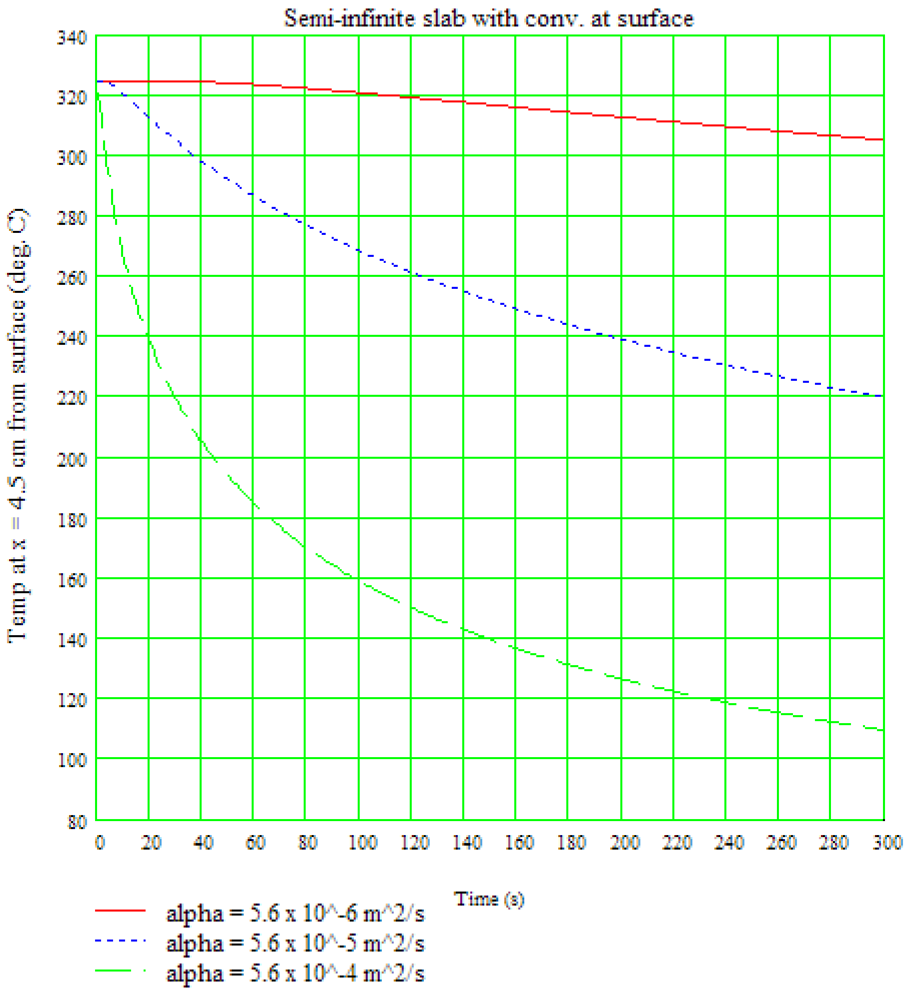
$\tau := 0, 1 .. 300$ ...define a range variable τ, varying from 0 to 300 s. at an interval of 1 s.



Next, plot T vs τ for x = 4.5 cm and k = 20 W/m.C and three different values of α:

$k := 20$ W/m.C $x := 0.045$ ...at a depth of 4.5 cm from the surface

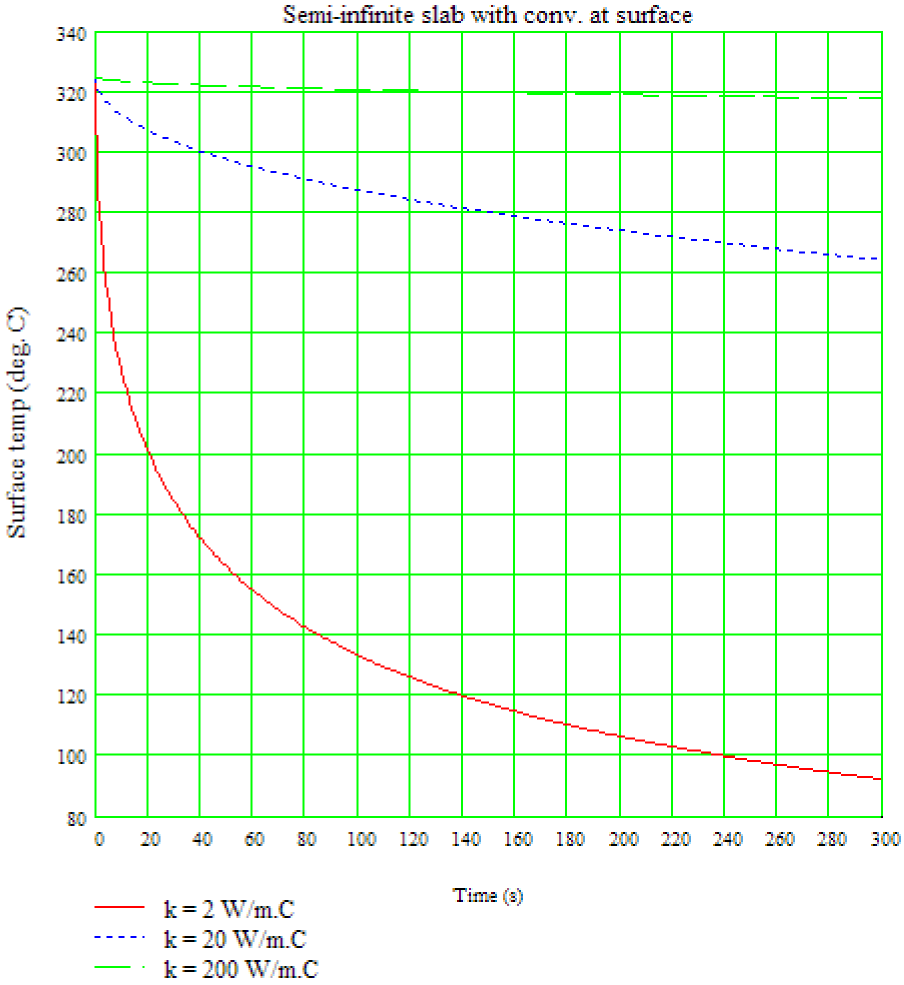
$\tau := 0, 1 .. 300$ ...define a range variable τ, varying from 0 to 300 s. at an interval of 1 s.



Next, plot T vs τ for surface and alpha = 5.6 x 10^-6 W/m.C and three different values of k:

$\alpha := 5.6 \cdot 10^{-6}$ m^2/s $x := 0$ ...at the surface

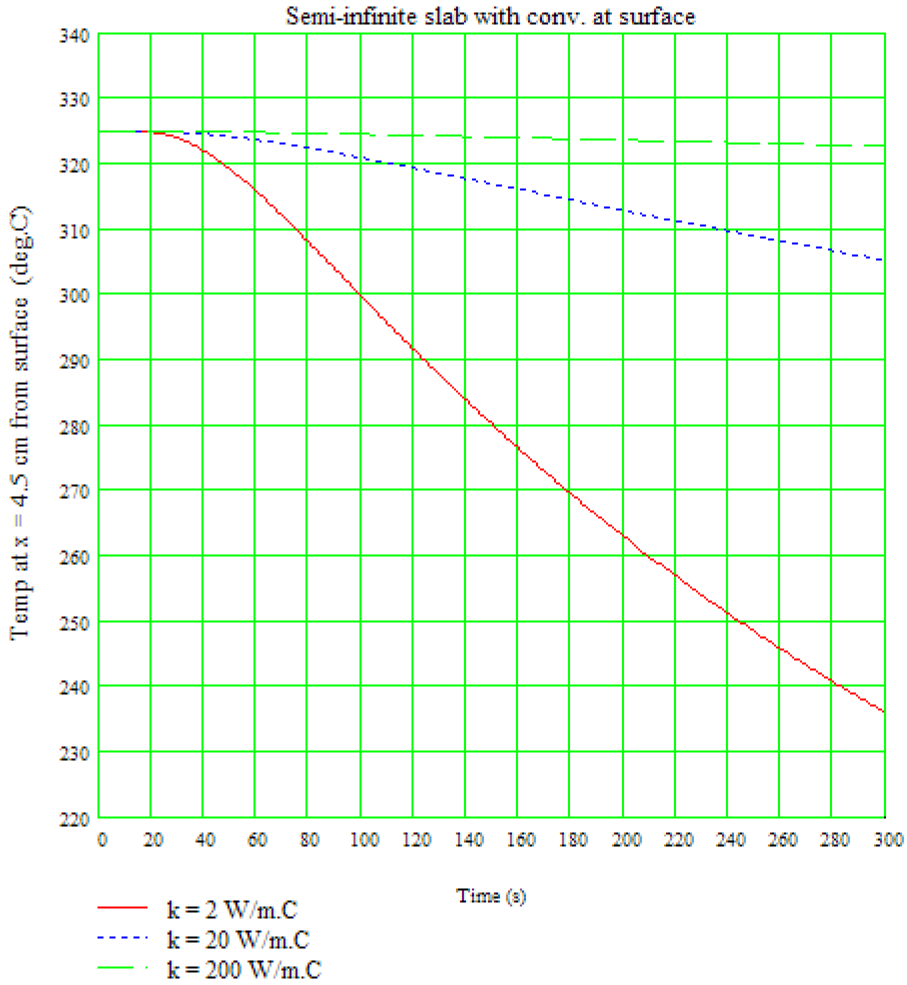
$\tau := 0, 1..300$ ...define a range variable τ, varying from 0 to 300 s. at an interval of 1 s.



Next, plot T vs τ at x = 4.5 cm and alpha = 5.6 x 10^-6 W/m.C and three different values of k:

$\alpha := 5.6 \cdot 10^{-6}$ m^2/s $x := 0.045$ m...at a depth of 4.5 cm from the surface

$\tau := 0, 1..300$ ...define a range variable τ, varying from 0 to 300 s. at an interval of 1 s.



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Prob. 1G.20. A solid copper sphere of 10 cm dia (density = 8954 kg/m^3, cp = 383 J/kg.C, k = 386 W/m.C), initially at a uniform temp of 250 C is suddenly immersed in a well stirred fluid maintained at a uniform temp of 50 C. Heat transfer coeff between the sphere and the fluid is 200 W/m^2.C. Determine the temp of the copper block at 5 min after the immersion. [VTU – VI Sem. B.E. – June 2012]. Also, plot the Temp vs time curve for different values of h:

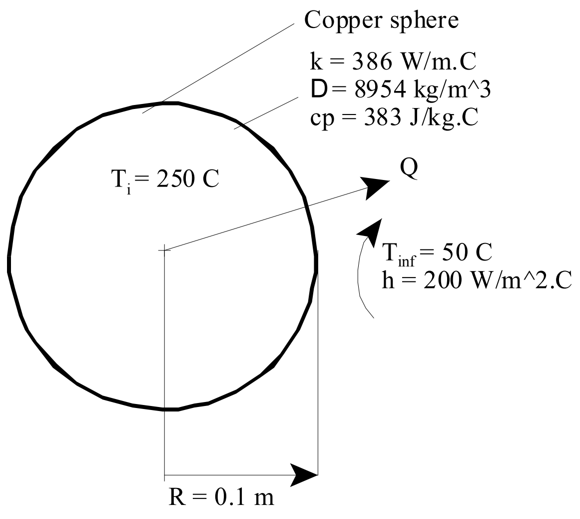


Fig.Prob.1G.20

This is the same as Prob.1G.2.

However, we will solve this problem with EXCEL:

In Lumped system analysis (i.e. Bi < 0.1), for temp distribution, we have:

$$\frac{\theta}{\theta_i}=\frac{T(\tau)-T_a}{T_i-T_a}=\exp\left(\frac{-h\cdot A\cdot \tau}{\rho\cdot C_p\cdot V}\right) \qquad \text{if Bi} < 0.1....$$

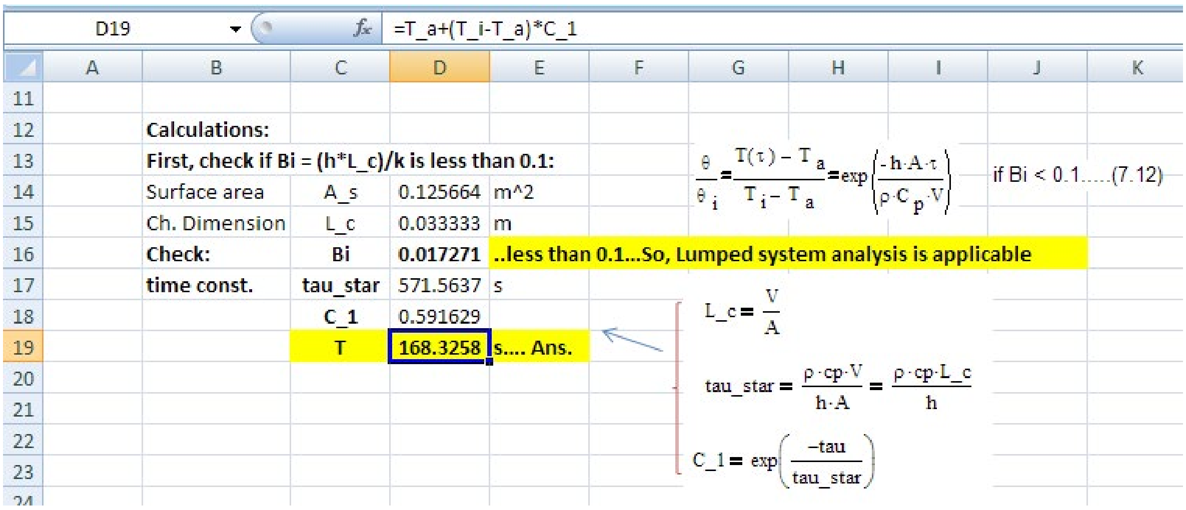
Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data and name the cells:

tau | f_x 300

	A	B	C	D	E
1					
2		Data:			
3			Rad	0.1	m
4			rho	8954	kg/m^3
5			c	383	J/kg.C
6			k	386	W/m.C
7			h	200	W/m^2.C
8			T_i	250	C
9			T_a	50	C
10			tau	300	s

2. Enter the calculations for Temp. Note that first, we check if Biot No is less than 0.1. If so, Lumped system analysis is applicable. Then, time constant tau_star is also calculated. Finally, temp after 300 s is calculated. Note how the long eqn is calculated in parts:



Thus, temp of the sphere after 300 s is 168.326 deg.C …. Ans.

3. Now, draw temp vs time for h = 20, 100 and 200 W/m^2.C:

Set up the worksheet as shown:

D31 | fx | =T_a+(T_i-T_a)*EXP(-D29*C31/(rho*cp*L_

	A	B	C	D	E	F	G
25							
26		Plot Temp vs time for h = 20, 100 and 200 W/m^2.C:					
27							
28				Temperature of Sphere			
29			h (W/m^2.C)=	20	100	200	
30			Time (s)				
31			0	250	250	250	
32			100				
33			200				
34			300				
35			400				

This Table will calculate temps at various times for three different values of 'h'. Time varies from 0 to 40000 s. In cell D31, the eqn for Temp is entered. See the eqn in the Formula bar. Take care to see that for h = 20, i.e. cell D29 is entered as 'absolute reference' and time in cell C31 is entered as 'relative reference' so that when you drag-copy cell D31 downwards, calculation for each row is automatically adjusted. Similarly, enter the eqns for Temp with h = 100 and h = 200 W/m^2.C in cells E31 and E32 respectively. Note that the cells D31, E31 and F31 calculate to a result of 250 C, as it should be, since these are the initial temperatures, i.e. at tau = 0.

4. Now, select cells D31 to F31 and drag-copy to the end of the Table. All calculations are immediately completed: Part of the Table only is shown below, to conserve space:

F31 | fx | =T_a+(T_i-T_a)*EXP(-F29*C31/(rho*cp*L_c))

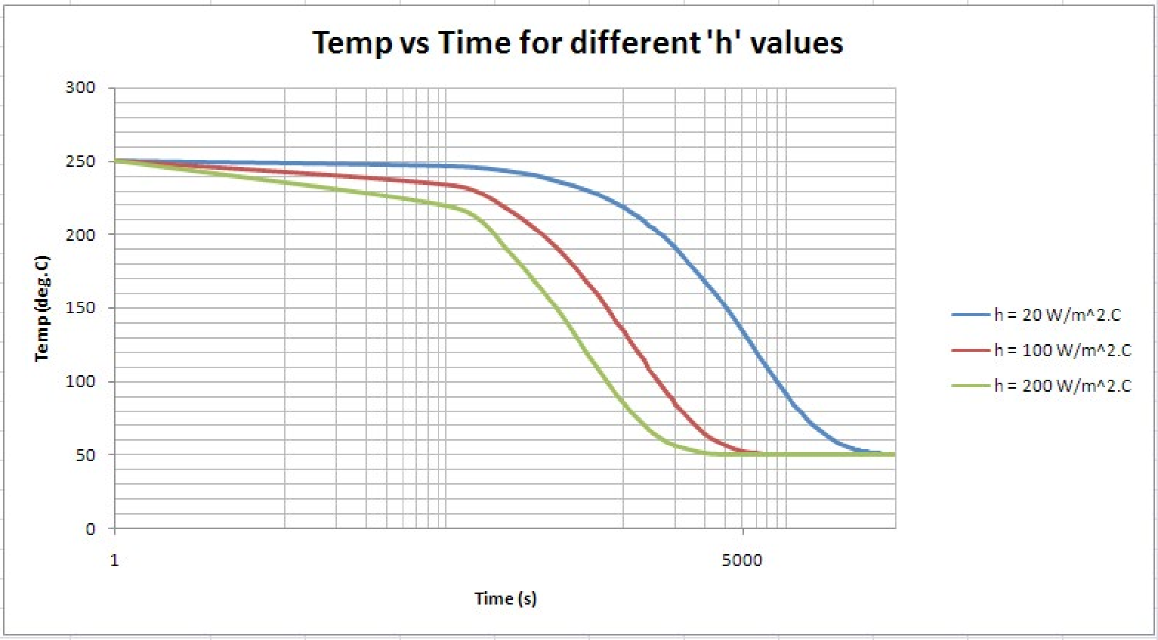
	A	B	C	D	E	F	G
26		Plot Temp vs time for h = 20, 100 and 200 W/m^2.C:					
27							
28				Temperature of Sphere			
29			h (W/m^2.C)=	20	100	200	
30			Time (s)				
31			0	250	250	250	
32			100	246.5313	233.2476	217.8983	
33			200	243.1227	217.8983	190.9493	
34			300	239.7732	203.8348	168.3258	
35			400	236.4818	190.9493	149.3335	
36			500	233.2476	179.1431	133.3896	
37			600	230.0694	168.3258	120.0049	
38			700	226.9463	158.4145	108.7686	
39			800	223.8774	149.3335	99.33572	
40			900	220.8617	141.0131	91.41693	
41			1000	217.8983	133.3896	84.76917	
42			1100	214.9864	126.4047	79.18843	
43			1200	212.1249	120.0049	74.50345	
44			1300	209.3131	114.1412	70.57044	
45			1400	206.55	108.7686	67.26871	
46			1500	203.8348	103.846	64.49694	
47			1600	201.1668	99.33572	62.17006	
48			1700	198.545	95.20325	60.21667	
49			1800	195.9686	91.41693	58.57681	
50			1900	193.437	87.94775	57.20016	

F89 =T_a+(T_i-T_a)*EXP(-F29*C89/(rho*cp*L_c))

	A	B	C	D	E	F	G
66			17000	60.21667	50.00007	50	
67			18000	58.57681	50.00003	50	
68			19000	57.20016	50.00001	50	
69			20000	56.04447	50.00001	50	
70			21000	55.07429	50	50	
71			22000	54.25982	50	50	
72			23000	53.57609	50	50	
73			24000	53.00209	50	50	
74			25000	52.52023	50	50	
75			26000	52.11571	50	50	
76			27000	51.77613	50	50	
77			28000	51.49104	50	50	
78			29000	51.25172	50	50	
79			30000	51.05081	50	50	
80			31000	50.88214	50	50	
81			32000	50.74055	50	50	
82			33000	50.62169	50	50	
83			34000	50.5219	50	50	
84			35000	50.43813	50	50	
85			36000	50.36781	50	50	
86			37000	50.30877	50	50	
87			38000	50.25921	50	50	
88			39000	50.21761	50	50	
89			40000	50.18268	50	50	

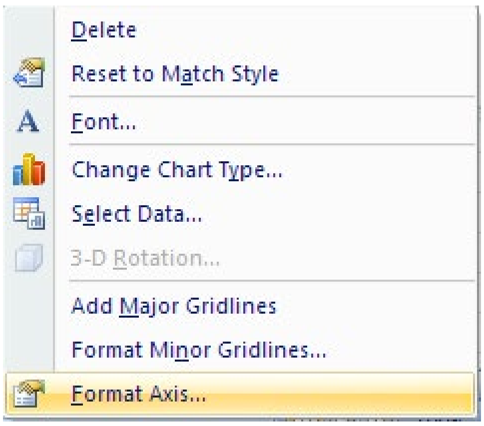
Note that the sphere cools to the ambient temp of 50 C after 11000 s, 21000 s and beyond 40000 s for h = 200, 100 and 20 W/m^2.C respectively.

5. Now, plot the graphs:

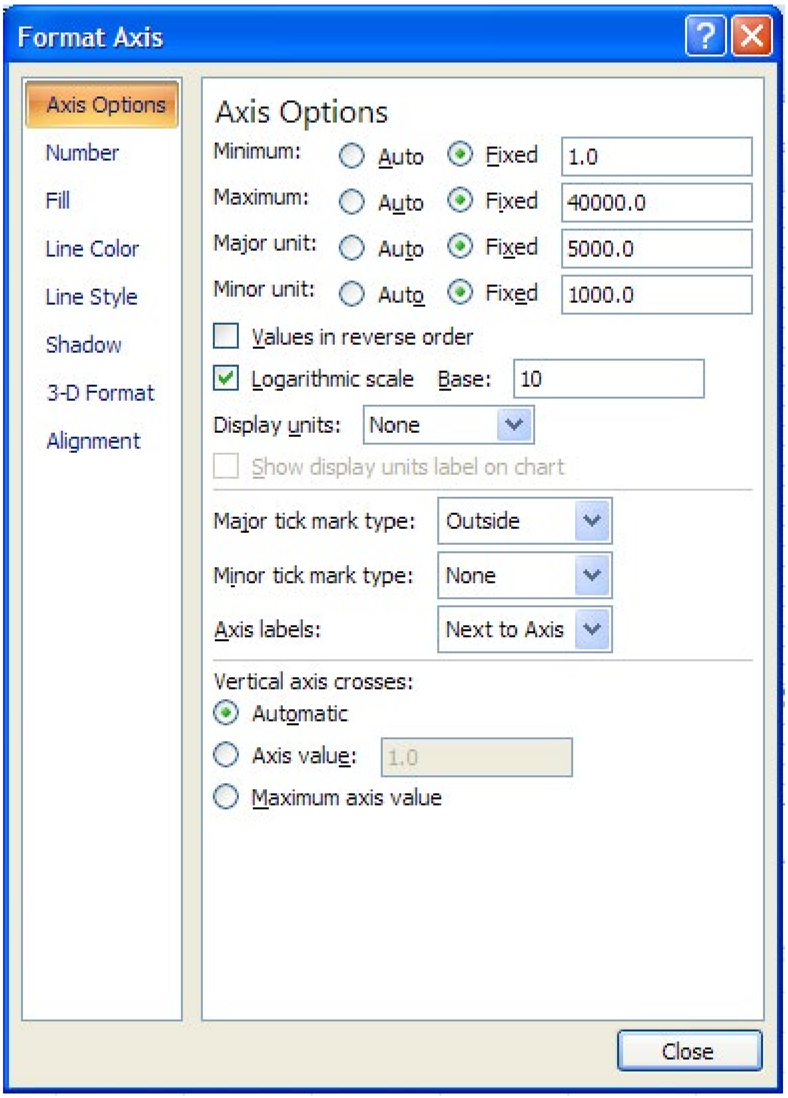


Note that in the above plot, x-axis is in log scale for clarity since time varies through a large range.

To change to log scale, simply select the axis concerned, and right-click; we get:



Click on Format Axis to get:



Put a check mark on 'Logarithmic Scale' (of course, while using logarithmic scale, minimum of axis should be 1 and not zero), click Close. And the Log scale is effected.

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Prob.1G.21. A 50 cm × 50 cm copper slab, 6 mm thick, at an uniform temperature of 350 C, suddenly has its surface temperature lowered to 30 C. Find the time at which the slab temperature becomes 100 C. Given: **ρ** = 9000 kg/m3, cp = 0.38 kJ/(kg.K), k = 370 W/(m.K), h = 100 W/(m2.K). Also find out the rate of cooling after 60 seconds.

EXCEL Solution:

$$\frac{\theta}{\theta_i} = \frac{T(\tau) - T_a}{T_i - T_a} = \exp\left(\frac{-h \cdot A \cdot \tau}{\rho \cdot C_p \cdot V}\right)$$ if Bi < 0.1.... 1), temp distribution is given by:

Following are the steps:

1. Set up the EXCEL worksheet, enter data and name the cells:

T_a | fx | 30

	A	B	C	D	E	F
1						
2		**Data:**				
3		Length	**L**	**0.05**	m	
4		Breadth	**B**	**0.05**	m	
5		thickness	**delta**	**0.006**	m	
6		density	**rho**	**9000**	kg/m^3	
7		sp.heat	**cp**	**380**	J/kg.C	
8		th. Cond.	**k**	**370**	W/m.C	
9		heat tr coeff	**h**	**100**	W/m^2.C	
10		Initial temp	**T_i**	**350**	C	
11		Ambient temp	**T_a**	**30**	C	
12		Final temp	**T**	**100**	C...to be attained	
13		**Find time tau required for the slab to reach 100 C.**				

2. Do the preliminary calculations, verify if Bi < 0.1:

D21 | fx | =h*L_c/k

A	B	C	D	E	F
	Initial temp	**T_i**	**350**	C	
	Ambient temp	**T_a**	**30**	C	
	Final temp	**T**	**100**	C...to be attained	
	Find time tau required for the slab to reach 100 C.				
	Calculations:				
	First, check if Bi = (h*L_c)/k is less than 0.1:				
	Surface area	A_s	0.005	m^2	
	Volume	V	0.000015	m^3	
	Ch. Dimension	L_c	0.0030	m	
	Check:	**Bi**	**0.000811**	**..less than 0.1...So. Lumped system analysis is applicable**	

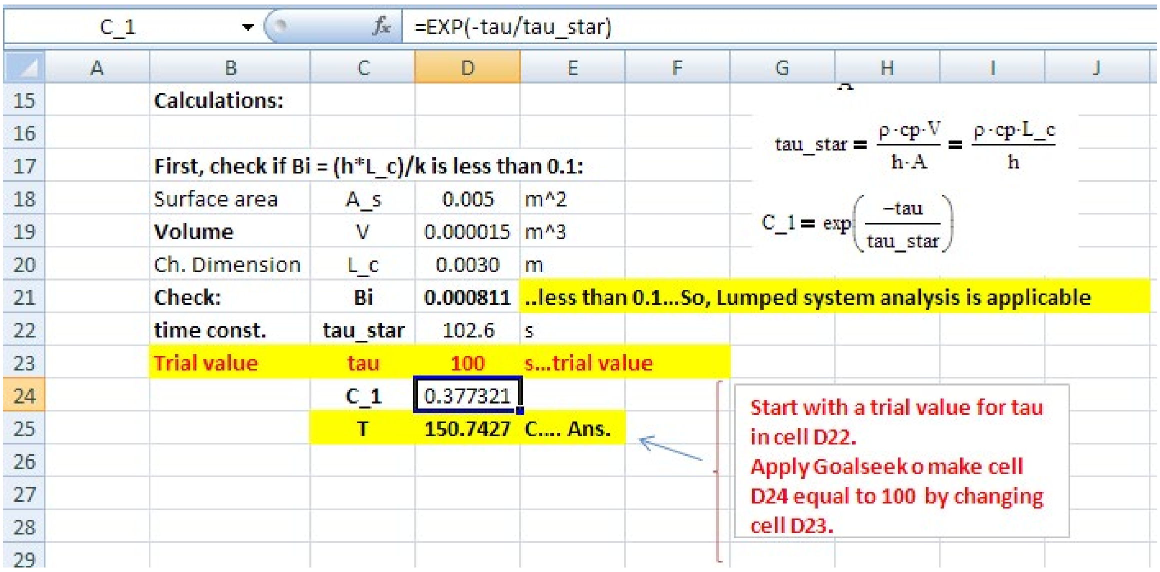
$$\frac{\theta}{\theta_i} = \frac{T(\tau) - T_a}{T_i - T_a} = \exp\left(\frac{-h \cdot A \cdot \tau}{\rho \cdot C_p \cdot V}\right)$$ if Bi < 0.1....

$$L\_c = \frac{V}{A}$$

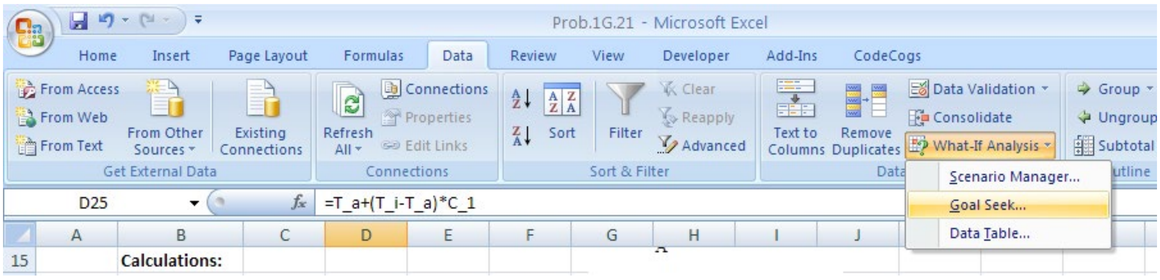
$$tau\_star = \frac{\rho \cdot cp \cdot V}{h \cdot A} = \frac{\rho \cdot cp \cdot L\_c}{h}$$

$$C\_1 = \exp\left(\frac{-tau}{tau\_star}\right)$$

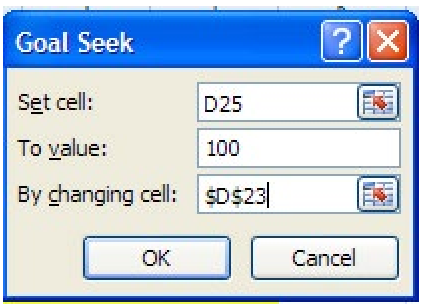
3. To calculate the time at which the plate will reach 100 deg. C, start with a trial value for time tau, do the calculations to find the temp, T. Obviously, this will be different from the desired value of 100deg.C



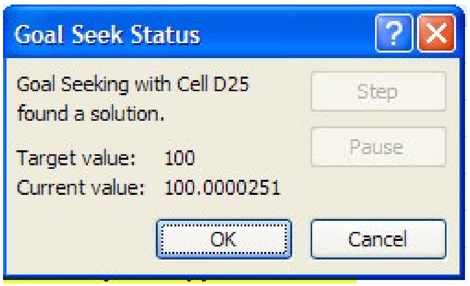
4. Note that cell D25 shows 150.7427 C when the assumed value of tau in cell D23 is 100 s. So, apply Goal seek in EXCEL to make cell D25 = 100 by changing cell D23. Go to Data-What If Analysis – Goal seek:



Click on Goal seek. Fill up the cell values in the window that pops up:



Press OK. We get:



Press OK, and see the result in cell D23:

tau | f_x 155.934085587027

	A	B	C	D	E	F	G	H	I	J
18		Surface area	A_s	0.005	m^2					
19		**Volume**	V	0.000015	m^3		$C\_1 = \exp\left(\frac{-tau}{tau\_star}\right)$			
20		Ch. Dimension	L_c	0.0030	m					
21		**Check:**	**Bi**	**0.000811**	**..less than 0.1...So, Lumped system analysis is applicable**					
22		**time const.**	**tau_star**	102.6	s					
23		Trial value	tau	155.9341	s...trial value					
24			**C_1**	0.21875						
25			**T**	**100**	**C.... Ans.**		Start with a trial value for tau in cell D22. Apply Goalseek o make cell D24 equal to 100 by changing cell D23.			
26										
27										
28										
29										

5. To find dT/dtau at time = 60 s:

First, construct a Table of Time vs Temp, as we did in the previous example:

D37 | =T_a+(T_i-T_a)*C37

	A	B	C	D	E
29					
30					
31					
32		To find rate of cooling after 60 S:			
33					
34		First, condtruct a Table of Temp vs Time:			
35					
36		tau (s)	C_1	T (deg.C)	
37		0	1	350	
38		5			
39		10			
40		15			
41		20			

C_1 is calculated for each value of tau, in column C. See how the formula is entered for Temp in cell D37. Take care to enter 'relative reference' for C_1, so that all values will be correctly calculated when you drag-copy cellsC37 and D37 downwards to the required time limit. Part of the Table produced is shown below:

D51 | =T_a+(T_i-T_a)*C51

	A	B	C	D	E	F	G	H
35								
36		tau (s)	C_1	T (deg.C)				
37		0	1	350				
38		5	0.952435	334.7793		At 60 s:	dT/dtau	
39		10	0.907133	320.2827			-1.73792	C/s
40		15	0.863986	306.4755				
41		20	0.822891	293.3251				
42		25	0.78375	280.8001				
43		30	0.746472	268.8709				
44		35	0.710966	257.5091				
45		40	0.677149	246.6878				
46		45	0.644941	236.3811				
47		50	0.614265	226.5647				
48		55	0.585047	217.2152				
49		60	0.55722	208.3104				
50		60.000001	0.55722	208.3104				
51		65	0.530716	199.8291				

Note that at tau = 60 s, we have given a very small increment of 0.000001 s to time and found out the corresponding decrease in Temp.

We see that thus calculated value of dT/dtau (at 60 s) is -1.73792 deg.C/s … Ans.

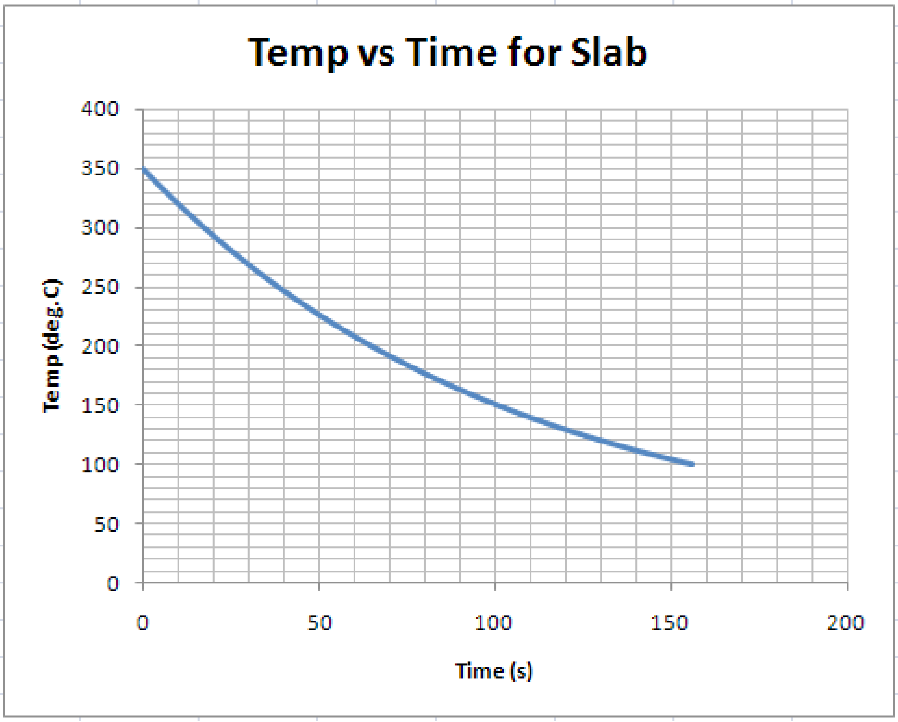
Here, -ve sign indicates that temp decreases as time increases.

6. Now, extend the Table up to time = 155.9341 s and observe that at that time, we get a temp of 100 C, as we should.

D70 | fx =T_a+(T_i-T_a)*C70

	A	B	C	D	E
48		55	0.585047	217.2152	
49		60	0.55722	208.3104	
50		60.000001	0.55722	208.3104	
51		65	0.530716	199.8291	
52		70	0.505473	191.7513	
53		75	0.48143	184.0576	
54		80	0.458531	176.73	
55		85	0.436721	169.7508	
56		90	0.415949	163.1036	
57		95	0.396164	156.7726	
58		100	0.377321	150.7427	
59		105	0.359374	144.9997	
60		110	0.34228	139.5298	
61		115	0.326	134.32	
62		120	0.310494	129.3581	
63		125	0.295726	124.6322	
64		130	0.281659	120.131	
65		135	0.268262	115.844	
66		140	0.255503	111.7609	
67		145	0.24335	107.8719	
68		150	0.231775	104.168	
69		155	0.220751	100.6402	
70		155.9341	0.21875	100	

7. And, now plot Temp vs Time:



==

Prob.1G.22. Plot a graph of Temp vs Time, in dimensionless coordinates, applicable to Lumped parameter analysis of all the three geometries, viz. plane slab, cylinder and sphere.

EXCEL Solution:

When lumped parameter analysis is applicable (i.e. when Bi < 0.1), for temp distribution, we have the relation:

$$\frac{\theta}{\theta_i} = \frac{T(\tau) - T_a}{T_i - T_a} = \exp\left(\frac{-h \cdot A \cdot \tau}{\rho \cdot C_p \cdot V}\right) \quad \text{if Bi < 0.1.....(7.12)}$$

Using dimensionless numbers Biot No (Bi) and Fourier No. (Fo), this can be written as:

$$\frac{\theta}{\theta_i} = \frac{T(\tau) - T_a}{T_i - T_a} = \exp(-Bi \cdot Fo) \quad \text{if Bi < 0.1.....(7.13)}$$

1. Now, let us set up the EXCEL worksheet to calculate (θ/θi) for different values of (Bi * Fo):

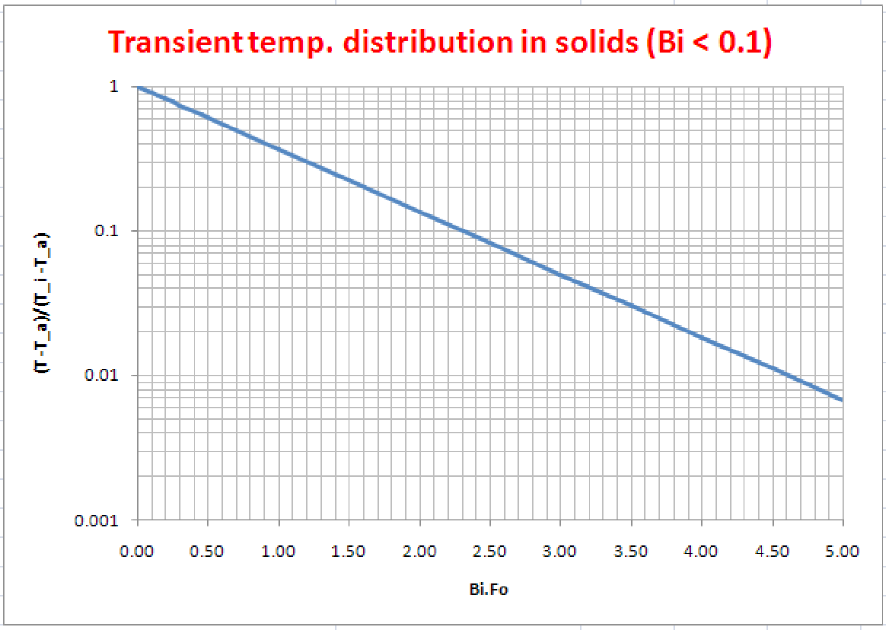
I69

	A	B	C	D	E
4		**Lumped analysis or Newtonian heating or cooling:**			
5		**Bi**	**less than 0.1**		
6		Theta=	T-Ta		
7		Theta_i =	T_i-Ta		
8		Theta/Theta_i=	exp(-h*A*tau/(rho*cp*V)		
9		tau =	time of cooling/heating		
10		tau_star=	(rho*cp*V)/(h*A)	time const.	
11		Theta/Theta_i=	exp(-tau/tau_star)		
12		Bi=	h*Lc/k	**Biot No.**	
13		**Lc=**	**half thickness of Slab**		
14			**R/2 for cyl**	**Ch. Dimension**	
15			**R/3 for Sphere**		
16			**L/6 for cube of side L**		
17					
18		Fo=	alpha*tau/Lc^2	**Fourier No.**	
19		alpha=	k/(rho*cp)	Th. diffusivity	
20					
21		**Theta/Theta_i=**	**exp(-Bi*Fo)**		

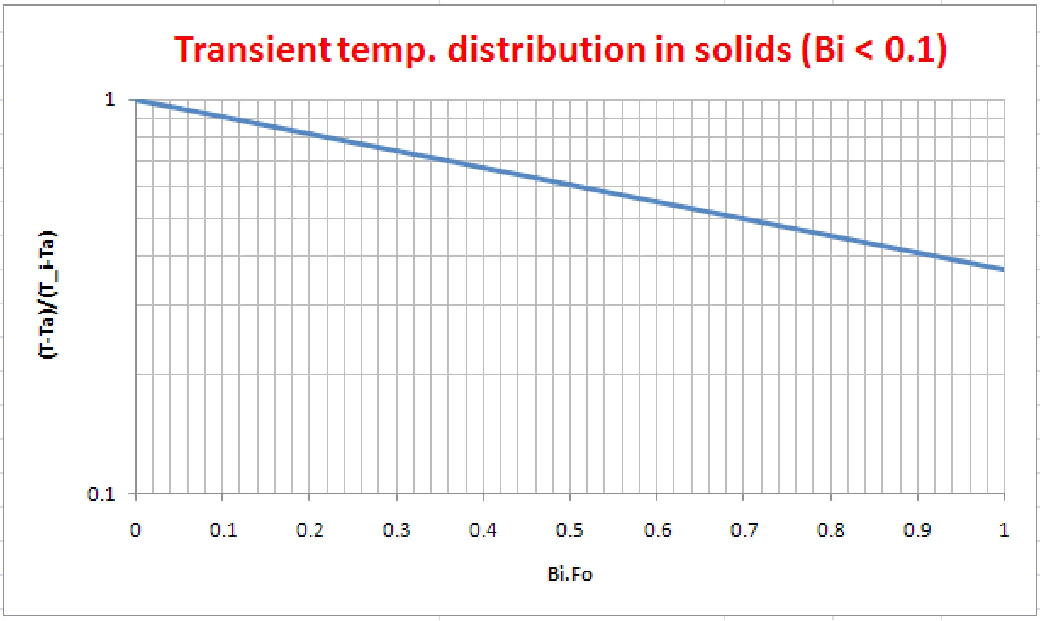
2. Next, generate the Table:

To draw graph Theta/Theta_i vs. Bi.Fo:	
Bi*Fo	Theta/Theta_i =exp(-Bi.Fo)
0.00	1
0.05	0.951229
0.10	0.904837
0.15	0.860708
0.20	0.818731
0.25	0.778801
0.30	0.740818
0.35	0.704688
0.40	0.670320
0.45	0.637628
0.50	0.606531
1.00	0.367879
1.50	0.223130
2.00	0.135335
2.50	0.082085
3.00	0.049787
3.50	0.030197
4.00	0.018316
4.50	0.011109
5.00	0.006738

3. And plot the graphs:



And for a smaller range of Bi.Fo:



Above Table/graphs can be used to:

1) Calculate the time (tau) when the temp to be attained (T) is known, or
2) Calculate the Temp when the time (tau) is given.

==

Prob.1G.23. A steel plate ($\alpha = 1.2 \times 10^{-5}$ m^2/s, k = 43 W/(m.C)), of thickness 2L = 10 cm, initially at an uniform temperature of 250 C is suddenly immersed in an oil bath at Ta = 45 C. Convection heat transfer coeff. between the fluid and the surfaces is 700 W/(m2.C).

1) How long will it take for the centre plane to cool to 100 C?
2) What fraction of the energy is removed during this time?
3) Draw the temp. profile in the slab at different times.

EXCEL Solution:

Following are the steps:

1. Set up the worksheet, enter data and name the cells, calculate Biot No as the first step:

C33 =h*L_c/k

	A	B	C	D	E	F	G	H
24	Data:	L_c	0.05	m				
25		alpha	1.20E-05	m2/s				
26		k	43	W/m.C				
27		T_i	250	C				
28		Ta	45	C				
29		h	700	W/m2.C				
30		T_o	100	C				
31								
32								
33		Bi	0.81395349	...Greater than 0.1. So, adopt 1 Term approximation solution.				

We see that Bi > 0.1.

So, Lumped system analysis is not applicable.

We can use *Heisler's charts* or *One-term solution* for temp distribution.

Heisler's charts are difficult to read since logarithmic scales are used and the curves are rather crowded.

So, when a computer is available, it is always recommended to use One-term solution.

Note one important difference in defining Biot number now, while using the tabular or chart solution or one-term solution:

Characteristic length in Biot number is taken as half thickness L for a plane wall, Radius R for a long cylinder and sphere, instead of being calculated as V/A, as done in lumped system analysis.

To differentiate from Lumped analysis, we now call this Biot No. as 'Biot_star'.

For all these three geometries, as mentioned earlier, the solution involves infinite series, which are difficult to deal with. However, it is observed that for Fo > 0.2, considering only the first term of the series and neglecting other terms, involves an error of less than 2%. Generally, we are interested in times, Fo > 0.2. So, it becomes very useful and convenient to use one term approximation solution, for all these three cases, as follows:

Plane wall:

$$\theta(x,\tau) = \frac{T(x,\tau) - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \cdot \cos\left(\frac{\lambda_1 \cdot x}{L}\right) \quad \text{...Fo > 0.2....(7.24, a)}$$

Long cylinder:

$$\theta(x,\tau) = \frac{T(r,\tau) - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \cdot J_0\left(\frac{\lambda_1 \cdot r}{R}\right) \quad \text{...Fo > 0.2....(7.24, b)}$$

sphere: $$\theta(x,\tau) = \frac{T(r,\tau) - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \cdot \frac{\sin\left(\frac{\lambda_1 \cdot r}{R}\right)}{\frac{\lambda_1 \cdot r}{R}}$$ ...Fo > 0.2....(7.24, c)

In the above equations, A_1 and 8_1 are functions of Biot number only.

A_1 and 8_1 are calculated from the following relations:

For Plane wall:

$$\lambda_1 \cdot \tan(\lambda_1) = Bi$$

$$A_1 := \frac{4 \cdot \sin(\lambda_1)}{2 \cdot (\lambda_1) + \sin[2 \cdot (\lambda_1)]}$$

For Long cylinder:

$$\lambda_1 \cdot \frac{J1(\lambda_1)}{J0(\lambda_1)} = Bi$$

$$A_1 := \frac{2 \cdot J1(\lambda_1)}{(\lambda_1) \cdot \left(J0(\lambda_1)^2 + J1(\lambda_1)^2\right)}$$

For Sphere:

$$1 - \lambda_1 \cdot \cot(\lambda_1) = Bi$$

$$A_1 := \frac{4 \cdot [\sin(\lambda_1) - (\lambda_1) \cdot \cos(\lambda_1)]}{2 \cdot (\lambda_1) - \sin[2 \cdot (\lambda_1)]}$$

Values of A_1 and 8^1 are given in Table 1G.2. Function J_0 is the zeroth order Bessel function of the first kind and its values can be read from Table 1G.3. (See Tables in the beginning of this chapter).

Now, at the centre of a plane wall, cylinder and sphere, we have the condition x = 0 or r = 0. Then, noting that cos(0) = 1, J0 (0) = 1, and limit of sin(x)/x is also 1, eqns. (7.24) become:

At the centre of plane wall, cylinder and sphere:

Centre of plane wall:
(x = 0)

$$\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \quad (7.25a)$$

Centre of long cylinder:
(r = 0)

$$\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \quad (7.25b)$$

Centre of sphere:
(r = 0)

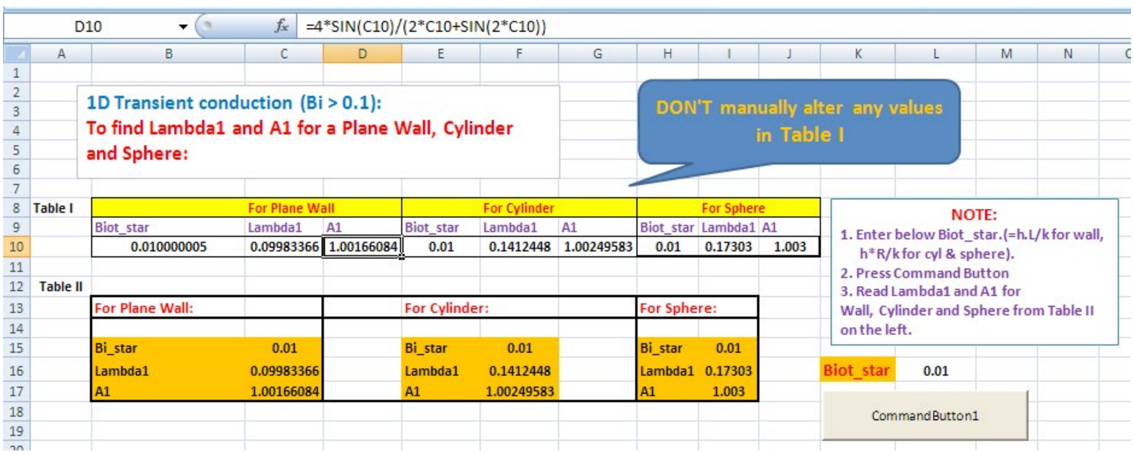
$$\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \quad (7.25b)$$

Therefore, first step in the solution is to calculate the Biot number; once the Biot number is known, constants A_1 and 8_1 are found out using Tables (1G.2) and (1G.3), and then use relations given in eqns. (7.24) and (7.25) to calculate the temperature at any desired location.

To find constants A1 and 81 when Biot No. is known:

We can use the Tables as mentioned earlier. However, while using EXCEL, it is very convenient if we could quickly calculate these constants in the worksheet itself.

Following is the scheme:

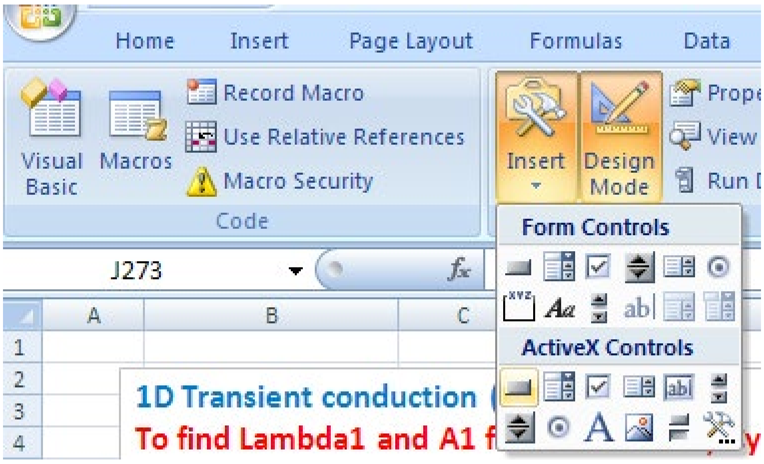


In this scheme, Biot_star is entered in cell L16. Then, click the 'command button1' shown below Biot_star in worksheet above. Table-I calculates the parameters in background, and the **user has to see only the Table-II to get results.** A NOTE in the box above Biot_star in the worksheet gives the procedure. (Remember the definition of Biot_star).

Now, eqns for λ1 are non-linear and have to be found out by trial and error, i.e. GoalSeek in EXCEL.

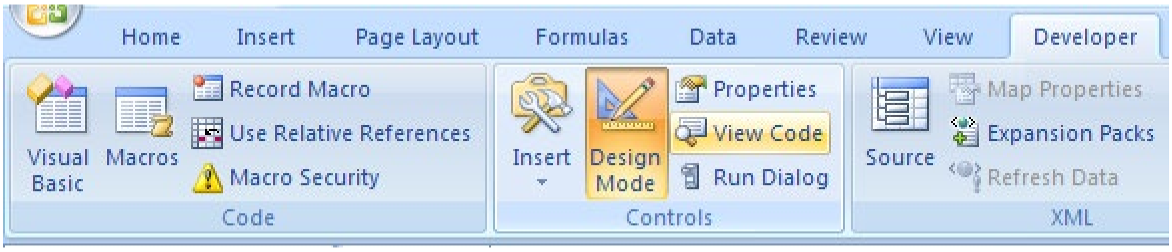
So, we shall write a small VBA program to do this job:

Now, to run this program, we will have the above mentioned 'command button 1', and link that command button to this program. To do that, go to Developer-Insert-ActiveX controls:



Press on first button in ActiveX controls and *draw the button on the worksheet to a suitable size and position it where you need.*

Then, press 'View code':




```
Private Sub CommandButton1_Click()

End Sub
```

Now, write the code in between 'Private Sub …. End Sub', as shown below:

```
Option Explicit
Private Sub CommandButton1_Click()
Dim N As Double
N = Range("L16").Value

If (N < 0.1) Then
  Range("C10") = 0.24
  Range("F10") = 0.34
  Range("I10") = 0.42
ElseIf (N >= 0.1) And (N < 1#) Then
  Range("C10") = 0.65
  Range("F10") = 0.94
  Range("I10") = 1.6
ElseIf (N >= 1) And (N < 10) Then
  Range("C10") = 1.38
  Range("F10") = 2.09
  Range("I10") = 2.72
ElseIf (N >= 10) And (N < 901) Then
  Range("C10") = 1.54
  Range("F10") = 2.36
  Range("I10") = 3.07
Else
  Range("C10") = 1.57
  Range("F10") = 2.4
  Range("I10") = 3.14
End If

Range("B10").GoalSeek Goal:=N, ChangingCell:=Range("C10")
Range("E10").GoalSeek Goal:=N, ChangingCell:=Range("F10")
Range("H10").GoalSeek Goal:=N, ChangingCell:=Range("I10")

End Sub
```

In the above program:

Line 4: reads the value of N from cell L16

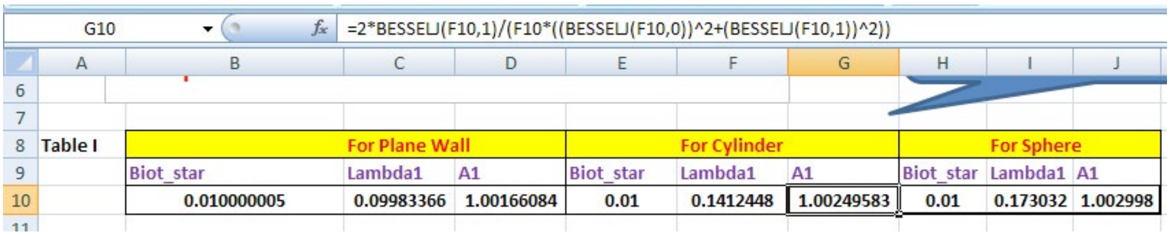
Lines 5 to 25: contains the 'If .. End If' construct where the guess values for λ1 are assigned to respective cells in Table-I for slab, cylinder and sphere. *It is important that proper guess values are allotted since there are more than one roots for these eqns.*

Lines 26, 27, 28: Goal Seek operation to determine λ1 for all the three cases.

After getting λ1, A1 is calculated for all the three cases using the eqns for A1 in cells D10, G10 and J10 for slab, cylinder and sphere respectively.

In the above screen shot, observe the eqn for A1 for slab, in cell D10, in the Formula bar.

Screen shot given below shows in the Formula bar, the eqn for A1 for the case of cylinder, in cell G10:

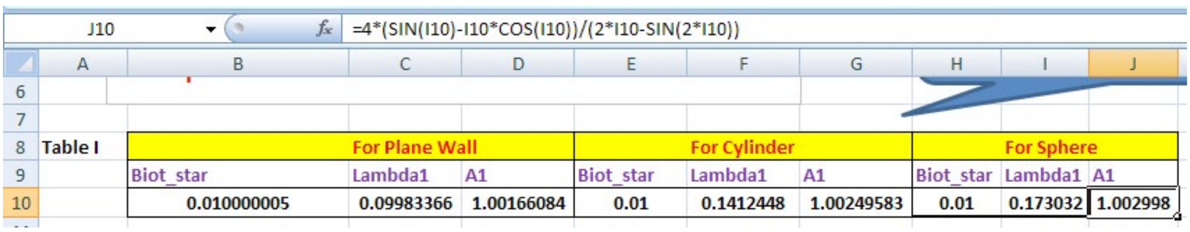


G10 =2*BESSELJ(F10,1)/(F10*((BESSELJ(F10,0))^2+(BESSELJ(F10,1))^2))

	A	B	C	D	E	F	G	H	I	J
8	Table I	For Plane Wall			For Cylinder			For Sphere		
9		Biot_star	Lambda1	A1	Biot_star	Lambda1	A1	Biot_star	Lambda1	A1
10		0.010000005	0.09983366	1.00166084	0.01	0.1412448	1.00249583	0.01	0.173032	1.002998

Note that the eqn involves Bessel functions of the first kind, of order 0 and 1. They are built-in Functions in EXCEL.

Similarly, the screen shot below gives the eqn for A1 for sphere, as entered in cell J10:

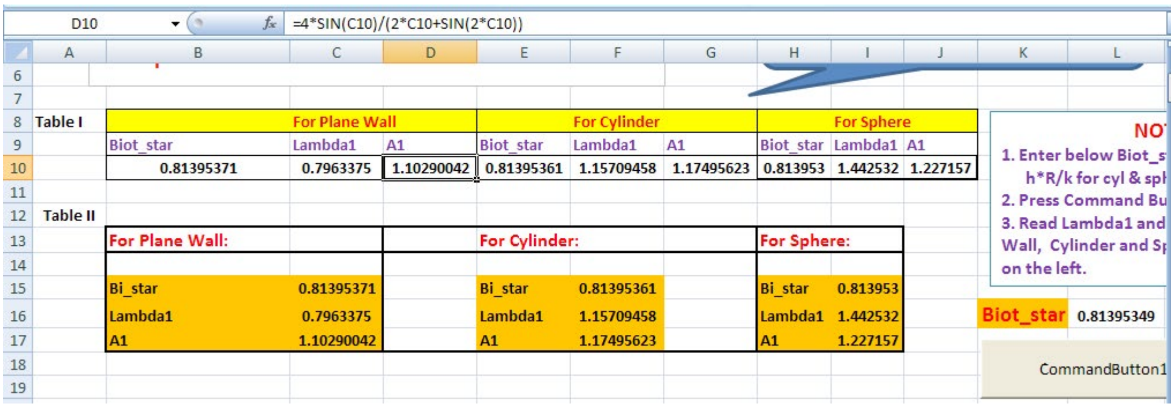


J10 =4*(SIN(I10)-I10*COS(I10))/(2*I10-SIN(2*I10))

	A	B	C	D	E	F	G	H	I	J
8	Table I	For Plane Wall			For Cylinder			For Sphere		
9		Biot_star	Lambda1	A1	Biot_star	Lambda1	A1	Biot_star	Lambda1	A1
10		0.010000005	0.09983366	1.00166084	0.01	0.1412448	1.00249583	0.01	0.173032	1.002998

In Table-II, the values of Biot_star, λ1 and A1 are grouped together for slab, cylinder and sphere, for convenience in reading and using while solving problems.

2. Now, for the present problem, we have calculated Biot No. as: Biot_star = **0.81395349. Run the program by clicking on 'Command button 1',** and we get λ1 and A1: We get:



D10 =4*SIN(C10)/(2*C10+SIN(2*C10))

Table I

For Plane Wall			For Cylinder			For Sphere		
Biot_star	Lambda1	A1	Biot_star	Lambda1	A1	Biot_star	Lambda1	A1
0.81395371	0.7963375	1.10290042	0.81395361	1.15709458	1.17495623	0.813953	1.442532	1.227157

Table II

For Plane Wall:		For Cylinder:		For Sphere:	
Bi_star	0.81395371	Bi_star	0.81395361	Bi_star	0.813953
Lambda1	0.7963375	Lambda1	1.15709458	Lambda1	1.442532
A1	1.10290042	A1	1.17495623	A1	1.227157

Note values of λ1(= 0.7963375) and A1 (= 1.1029). (Check these values with reference to the Table 1G.2.)

3. To find time required for the centre temp to reach 100 deg.C:

We have, for centre temp:

Centre of plane wall: (x = 0)
$$\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \quad (7.25,\ a)$$

In the above eqn LHS is now known. In the RHS, only Fo is unknown. So, calculate Fo.

And, since Fo = (α.τ / L_c^2), we can calculate τ. This is done in the following portion of worksheet:

Fo | f_x =-LN(C39/A_1)/Lambda1^2

	A	B	C	D	E	F	G	H
33		Bi_star	0.81395349	...Greater than 0.1. So, adopt 1 Term approximation solution.				
34		Lambda1	0.796337	From GoalSeek Table II above.				
35		A_1	1.1029					
36		Centre temp is 100 C. i.e. (T_o - Ta)/(T_i - Ta) = 0.26829 = A1.exp(-Fo.Lambda1^2) ... eqn(7.25, a)						
37		Calculate Fo:						
38								
39		(T_0 - Ta)/(T_i - Ta) =	0.26829268					
40		Fo	2.22914773					
41		Therefore, tau=	464.406	s...Ans.				

Thus, tau = 464.406 s … Ans.

4. To find surface temp, we have:

Plane wall:
$$\theta(x,\tau) = \frac{T(x,\tau) - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \cdot \cos\left(\frac{\lambda_1 \cdot x}{L}\right) \quad \text{...Fo > 0.2} \quad (7.26,\ a)$$

Here, at the surface: (x/L) = 1. This is calculated below:

C45 | f_x =(T_i-Ta)*(A_1*EXP(-Fo*Lambda1^2)*COS(Lambda1))+Ta

	A	B	C	D	E	F
42						
43		At the surface, x/L = 1. Then,				
44		T = (T_i - Ta)*(A1.exp(-Fo.Lambda1^2).cos(Lambda1)) + Ta				
45		i.e. T_surface=	83.463	deg.C....Ans.		

Thus, surface temp when the centre has reached 100 C is: 83.463 C….Ans.

Eqn entered in cell C45 for T_surface can be seen in the Formula bar.

5. Fraction of heat removed during this time:

This is given by:

Plane wall: $$\frac{Q}{Q_{max}} = 1 - \theta_0 \cdot \frac{\sin(\lambda_1)}{\lambda_1} \qquad (7.26, a)$$

Where

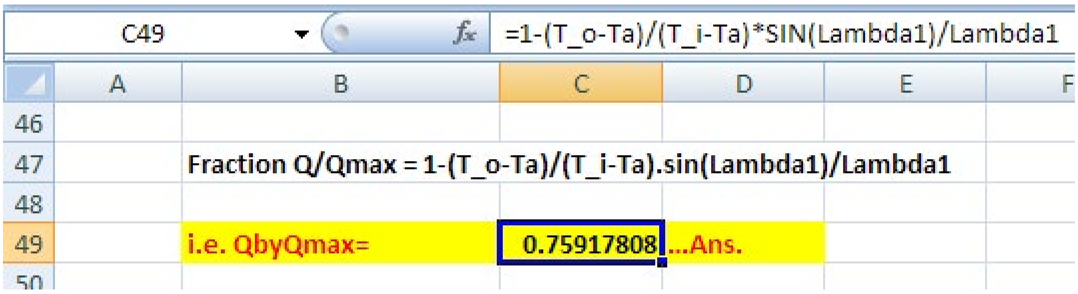
$$Q_{max} = \rho \cdot V \cdot C_p \cdot (T_i - T_a) = m \cdot C_p \cdot (T_i - T_a) \qquad (7.26, a)$$

Also, for cylinder and sphere:

Cylinder: $$\frac{Q}{Q_{max}} = 1 - 2 \cdot \theta_0 \cdot \frac{J_1(\lambda_1)}{\lambda_1} \qquad .....(7.27, b)$$

Sphere: $$\frac{Q}{Q_{max}} = 1 - 3 \cdot \theta_0 \cdot \left(\frac{\sin(\lambda_1) - \lambda_1 \cdot \cos(\lambda_1)}{\lambda_1^3} \right) \qquad (7.27, c)$$

Now, for the present case:



Thus, Q/Q_max = 0.759 …. Ans.

Again, note the eqn for (Q/Qmax) in the Formula bar.

6. Prepare a Table of temp distribution in the plate at different times:

Use the following eqn.

Centre of plane wall: (x = 0) $$\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \qquad (7.25, a)$$

Therefore,

At the centre, i.e. at x = 0:

$$T_a + (T_i - T_a) \cdot \left(A_1 \cdot e^{-\lambda_1^2 \cdot Fo(\tau)} \right) \quad \text{if } x = 0$$

And, at any other x:

$$T_a + (T_i - T_a) \cdot \left(A_1 \cdot e^{-\lambda_1^2 \cdot Fo(\tau)} \cdot \cos\left(\frac{\lambda_1 \cdot x}{L} \right) \right)$$

Now, in EXCEL, construct the following Table:

C56 =(T_i-Ta)*(A_1*EXP(-C54*Lambda1^2))+Ta

To draw temp. profile at different times:

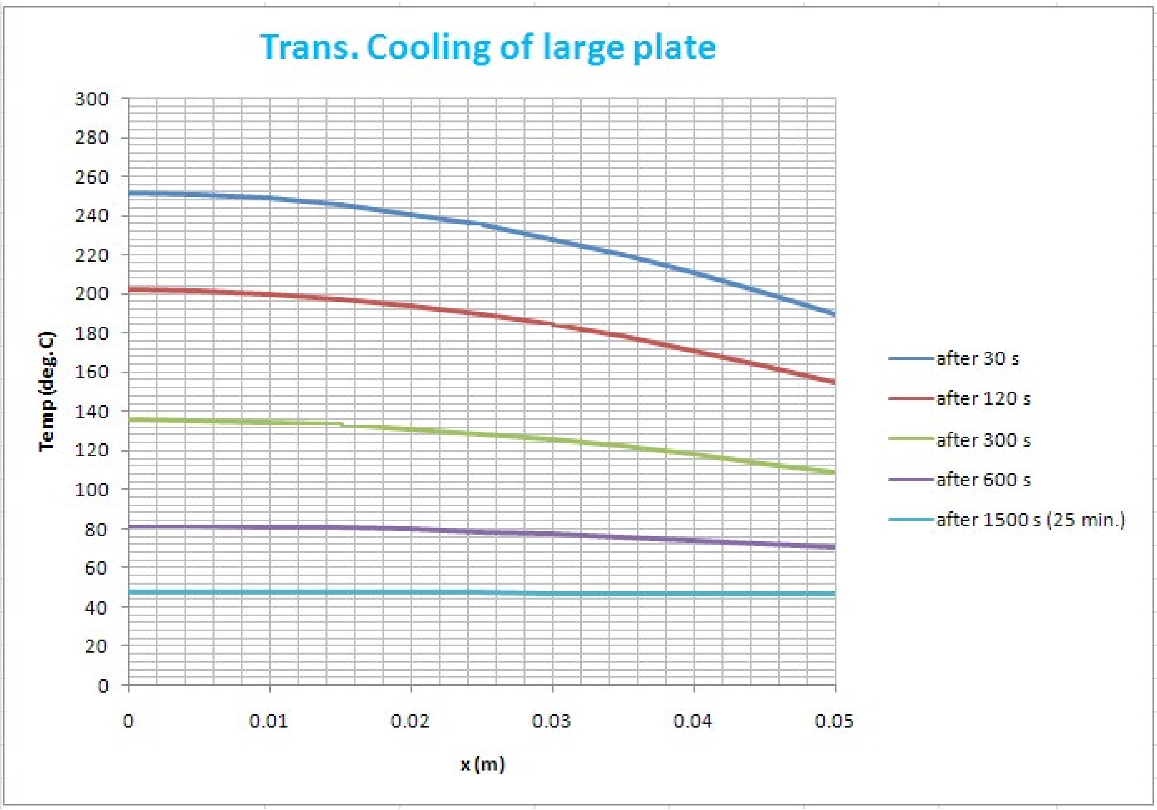
tau (s)=	30	120	300	600	1500	
Fo=	0.144	0.576	1.44	2.88	7.2	
x	Tx	Tx	Tx	Tx	Tx	
0	251.363	201.911	135.720	81.401	47.352	..At centre
0.005	250.709	201.414	135.432	81.286	47.344	
0.01	248.751	199.925	134.571	80.940	47.322	
0.015	245.502	197.455	133.143	80.367	47.285	
0.02	240.982	194.018	131.156	79.570	47.233	
0.025	235.219	189.637	128.623	78.553	47.168	
0.03	228.252	184.339	125.560	77.324	47.088	
0.035	220.122	178.157	121.986	75.890	46.996	
0.04	210.883	171.132	117.924	74.261	46.890	
0.045	200.592	163.307	113.400	72.445	46.773	
0.05	189.316	154.733	108.443	70.456	46.644	

$T_a + (T_i - T_a)\cdot\left(A_1\cdot e^{-\lambda_1^2\cdot Fo(\tau)}\right)$ if x=0

$T_a + (T_i - T_a)\cdot\left(A_1\cdot e^{-\lambda_1^2\cdot Fo(\tau)}\cdot\cos\left(\frac{\lambda_1 x}{L}\right)\right)$

Note: For each tau, Fo is calculated. For centre and at x = L, (i.e. at the edge of slab), the eqn for Temps are shown separately in the worksheet, for clarity. It is very easy in EXCEL to drag-copy and fill up the Table.

7. Now, plot the curves of Temp vs x for different times:



Prob.1G.24. An apple, which can be considered as a sphere of 8 cm diameter, is initially at an uniform temperature of 25 C. It is put into a freezer at -15 C and the heat transfer coeff. between the surface of the apple and the surroundings in the freezer is 15 W/(m2.C). If the thermo-physical properties of apple are given to be: ρ = 840 kg/m3, cp = 3.6 kJ/(kg.C), k = 0.513 W/(m.C), and α = 1.3 × 10-7 m2/s, determine:

1) centre temp. of the apple after 1 hour,
2) surface temp. of apple at that time, and
3) amount of heat transferred from the apple.
4) draw the temp. profile along the radius for different times

EXCEL Solution:

For a sphere, we have:

For temp distribution:

sphere: $$\theta(x,\tau) = \frac{T(r,\tau) - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo} \cdot \frac{\sin\left(\frac{\lambda_1 \cdot r}{R}\right)}{\frac{\lambda_1 \cdot r}{R}}$$ ...Fo > 0.2....(7.24, c)

Centre of sphere: $\theta_0 = \frac{T_0 - T_a}{T_i - T_a} = A_1 \cdot e^{-\lambda_1^2 \cdot Fo}$ ...(7.25, c)
(r = 0)

λ1 and A1 are determined from:

$$1 - \lambda_1 \cdot \cot(\lambda_1) = Bi$$

$$A_1 := \frac{4 \cdot [\sin(\lambda_1) - (\lambda_1) \cdot \cos(\lambda_1)]}{2 \cdot (\lambda_1) - \sin[2 \cdot (\lambda_1)]}$$

Following are the steps in EXCEL Solution:

1. Set up the worksheet, enter data and name the cells, calculate Biot No in the usual way (i.e. taking L_c = R/3) to determine if Lumped parameter analysis is applicable, as the first step:

D31 | f_x =h*(R_sph/3)/k

	A	B	C	D	E	F	G	H	I
16			Data:						
17									
18			R_sph	0.04	m				
19			alpha	1.30E-07	m2/s				
20			k	0.513	W/m.C				
21			cp	3600	J/kg.C				
22			rho	840	kg/m3				
23			T_i	25	C				
24			Ta	-15	C				
25			h	15	W/m2.C				
26			tau	3600	s				
27			Fo_star	2.9250E-01					
28									
29			First, check Biot No.:						
30									
31			Bi	0.3898635	>0.1, so, use 1 term appx. Soln. with Lc = Rsph!				

We see that Bi > 0.1.

So, Lumped system analysis is not applicable.

We will use *One-term solution* for temp distribution.

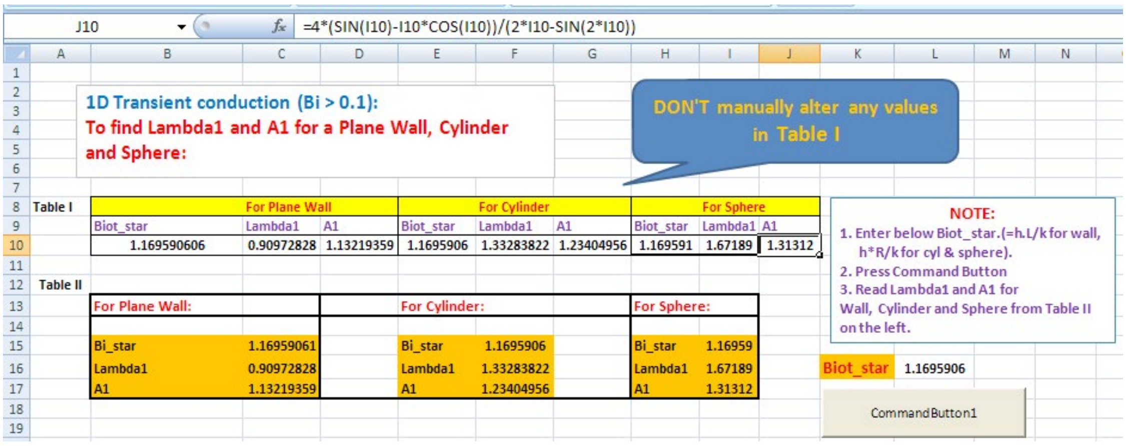
And, when a computer is available, it is always recommended to use One-term solution.

Note that for all further calculations, i.e. to find out Biot_star, Fo etc., we use L_c = R_sph

2. **To find constants A_1 and 8_1 when Biot No. is known:**

We can use the Tables 1G.2 & 3 given earlier. However, while using EXCEL, it is very convenient if we could quickly calculate these constants in the worksheet itself.

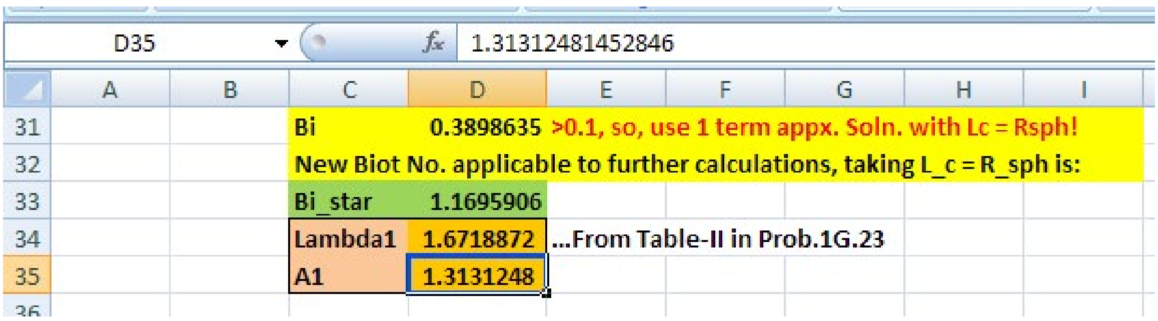
See the following is the scheme, which was described and used in solving the previous problem:



Note that after entering the value of Biot_star and pressing command button1, we got:

Λ1 = 1.67189, and A1 = 1.31312. Eqn. for A1 can be seen in the Formula bar.

So, they are entered in the worksheet:



3. Now, to calculate the temperatures in the centre and also at any radius, since the eqns are a little complicated, let us write VBA Functions: The procedure has been described in detail in Prob.1F.23. Briefly, we proceed as follows:

Click: Developer-Visual Basic-Insert-Module. A blank window presents itself, and type the code therein:

```
Option Explicit

Function OneDTransCondn_sph_Tcentre(T_i As Double, Ta As Double, A1 As Double, _
 Lambda1 As Double, Fo_star As Double) As Double
 'Finds centre temp of sphere when A1, Lambda1 and Fo_star are given, i.e. when time tau is given.
 'Remember: Fo_star = alpha*tau/R_sph^2
 Dim RHS As Double
 RHS = A1 * Exp(-Fo_star * Lambda1 ^ 2)
 OneDTransCondn_sph_Tcentre = (T_i - Ta) * RHS + Ta
 End Function
 Function OneDTransCondn_sph_tauforT0(T_0 As Double, T_i As Double, Ta As Double, A1 As Double, _
 Lambda1 As Double, Alpha As Double, R_sph As Double) As Double
 'Finds time tau for centre temp of sphere to reach T0.
 'Remember: Fo_star = alpha*tau/R_sph^2
  Dim AA As Double, Fo_star As Double
  AA = (T_0 - Ta) / (T_i - Ta)
  Fo_star = -Application.Ln(AA / A1) / Lambda1 ^ 2
  OneDTransCondn_sph_tauforT0 = Fo_star * R_sph ^ 2 / Alpha
End Function

Function OneDTransCondn_sph_Tr(T_i As Double, Ta As Double, A1 As Double, _
 Lambda1 As Double, Fo_star As Double, r As Double, R_sph As Double) As Double
 'Finds temp of sph at given (r/R_sph)
 'Remember: Fo_star = alpha*tau/R_sph^2
 Dim AA As Double, BB As Double
 AA = A1 * Exp(-Fo_star * Lambda1 ^ 2)
 BB = Sin(Lambda1 * r / R_sph) / (Lambda1 * r / R_sph)
 OneDTransCondn_sph_Tr = Ta + (T_i - Ta) * AA * BB
 End Function
Function OneDTransCondn_sph_QbyQmax(T_i As Double, Ta As Double, A1 As Double, _
 Lambda1 As Double, Fo_star As Double) As Double
'Finds QbyQmax for sphere
Dim Tcentre As Double, AA As Double, BB As Double
Tcentre = OneDTransCondn_sph_Tcentre(T_i, Ta, A1, _
 Lambda1, Fo_star)
 AA = 3 * (Tcentre - Ta) / (T_i - Ta)
 BB = (Sin(Lambda1) - Lambda1 * Cos(Lambda1)) / Lambda1 ^ 3
 OneDTransCondn_sph_QbyQmax = 1 - AA * BB
 End Function
```

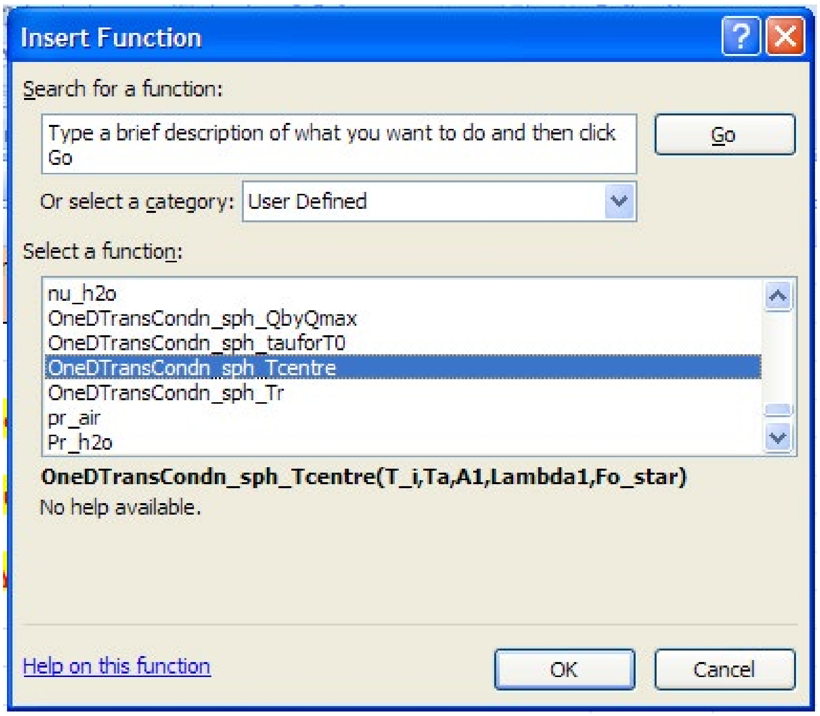
Above screen shows four Functions written in VBA. They are, in order, from top:

a) Function to determine the centre temp of sphere
b) Function to find time required for the centre temp to reach T0
c) Function to determine the temp of sphere at any given radius, and
d) Function to determine (Q/Qmax)

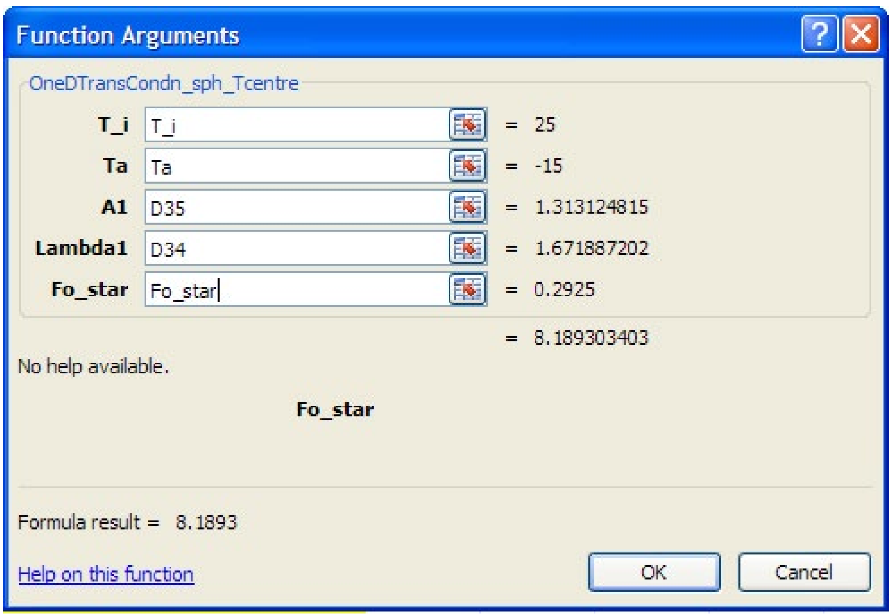
Read the comments given in the code.

Save the worksheet as ‘Macro enabled worksheet’. Now, these Functions are available in the worksheet like any other built-in Functions of EXCEL. You can go to Formulas tab and click on ‘Insert Function’ and use the Functions.

4. Now, proceed with the calculations to calculate the centre temp: When you click on Formulas-Insert Function, we get the following screen. Select the ‘User Defined’ category, and choose OneDTransCondn_sph_Tcentre from the Function list:



5. Click OK, and we get the following screen. Fill up the data required (by point and click to avoid errors)



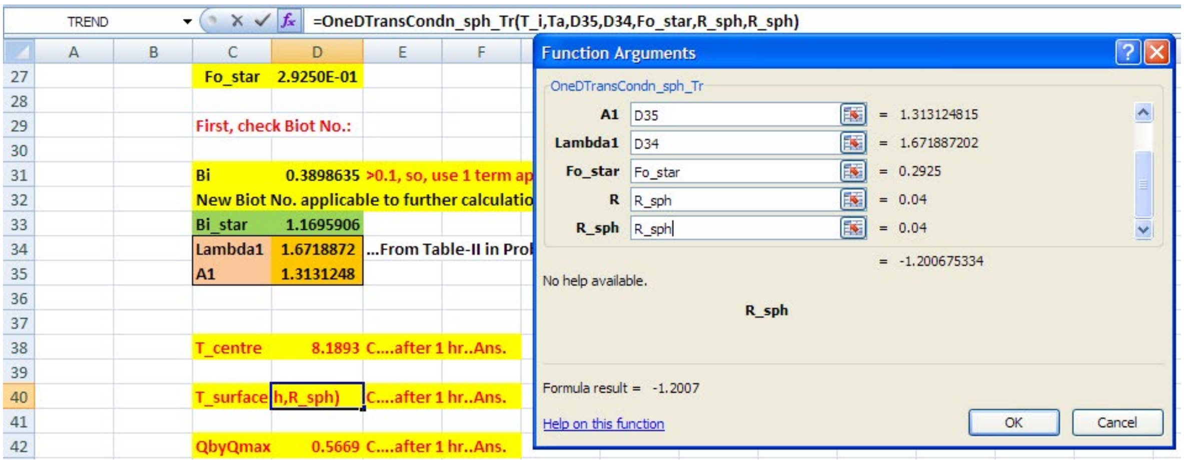
6. Press OK. We get:

D38 =OneDTransCondn_sph_Tcentre(T_i,Ta,D35,D34,Fo_star)

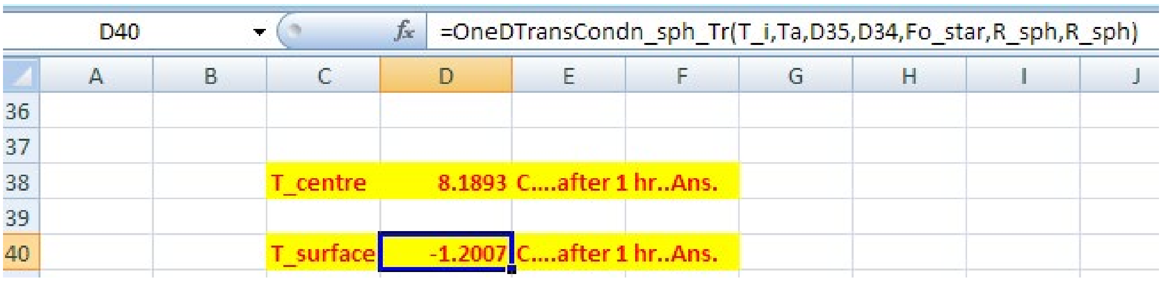
	A	B	C	D	E	F	G	H	I
28									
29			First, check Biot No.:						
30									
31			Bi	0.3898635	>0.1, so, use 1 term appx. Soln. with Lc = Rsph!				
32			New Biot No. applicable to further calculations, taking L_c = R_sph is:						
33			Bi_star	1.1695906					
34			Lambda1	1.6718872	...From Table-II in Prob.1G.23				
35			A1	1.3131248					
36									
37									
38			T_centre	8.1893	C....after 1 hr..Ans.				

i.e. we get: T_centre = 8.189 deg.C …. Ans.

7. Similarly, get T_surface (i.e. at r = R_sph):

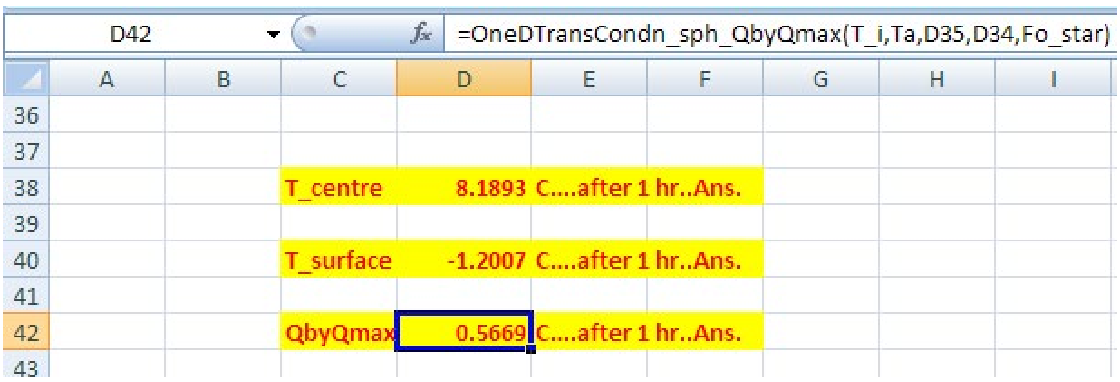


Press OK. We find:



i.e. we get: T_surface = -1.2007 deg.C …. Ans.

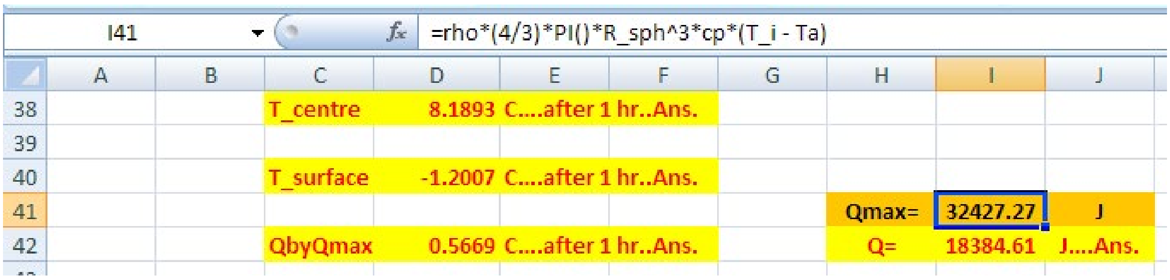
8. And, similarly, get Q/Qmax:



i.e. Q/Qmax = 0.5669 … Ans.

Formula entered for Q/Qmax in cell D42 can be seen in the Formula bar.

Now, Qmax = rho * Vol * cp * (T_i – Ta). Then, heat transferred during this time duration is obtained as: Q = (Q/Qmx) * Qmax. This calculation is shown in the screen shot below:


I41 =rho*(4/3)*PI()*R_sph^3*cp*(T_i - Ta)

	A	B	C	D	E	F	G	H	I	J
38			T_centre	8.1893	C....after 1 hr..Ans.					
39										
40			T_surface	-1.2007	C....after 1 hr..Ans.					
41								Qmax=	32427.27	J
42			QbyQmax	0.5669	C....after 1 hr..Ans.			Q=	18384.61	J....Ans.

Note the eqn for Qmax entered in cell I41, in the Formula bar above.

9. Next, let us draw the temp profile along the radius of the sphere at different times:

Set up the worksheet as shown below:

D49 =alpha*D48/R_sph^2

	A	B	C	D	E	F	G	H
46			To draw temp. profile at different times:					
47								
48			tau (s)=	1800	3600	7200	10800	18000
49			Fo_star=	0.14625	0.2925	0.585	0.8775	1.4625
50			r	Tr	Tr	Tr	Tr	Tr
51			0					
52			0.005					
53			0.01					
54			0.015					
55			0.02					
56			0.025					
57			0.03					
58			0.035					
59			0.04					

Here, for each tau, Fo_star is different, and tau and Fo_star are entered in rows 48 and 49. Eqn for Fo_atar entered in cell D49 can be seen in the Formula bar. Let 'r' vary from 0 to 0.04 m as shown.

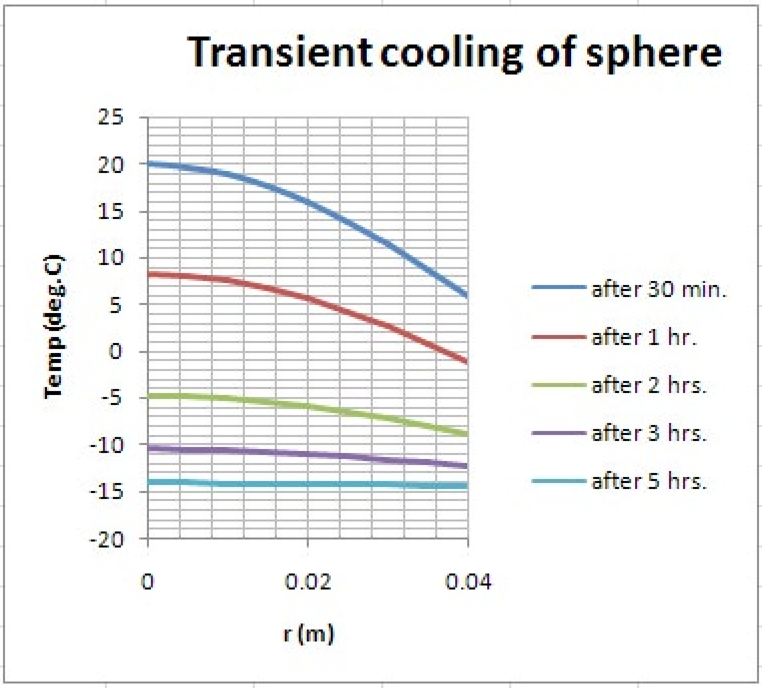
Enter in cell D51 the eqn for T_centre since r = 0. Then, in cell D52, enter the eqn for Tr, taking care to refer to cell C52 (i.e. radius, r) by 'relative reference' so that when we drag-copy downwards up to cell D59, the calculations automatically adjust themselves. Repeat the procedure for other times. Completed Table is shown below.

D52 =OneDTransCondn_sph_Tr(T_i,Ta,D35,Lambda1,D49,C52,R_sph)

	A	B	C	D	E	F	G	H	I	J
47										
48			tau (s)=	1800	3600	7200	10800	18000		
49			Fo_star=	0.14625	0.2925	0.585	0.8775	1.4625		
50			r	Tr	Tr	Tr	Tr	Tr		
51			0	19.900	8.189	-4.762	-10.480	-14.119	...at the centre	
52			0.005	19.647	8.021	-4.836	-10.513	-14.125		
53			0.01	18.893	7.520	-5.058	-10.611	-14.144		
54			0.015	17.658	6.700	-5.420	-10.770	-14.176		
55			0.02	15.975	5.581	-5.914	-10.988	-14.218		
56			0.025	13.887	4.194	-6.526	-11.259	-14.271		
57			0.03	11.447	2.573	-7.242	-11.575	-14.332		
58			0.035	8.718	0.759	-8.042	-11.928	-14.401		
59			0.04	5.768	-1.201	-8.908	-12.310	-14.476		

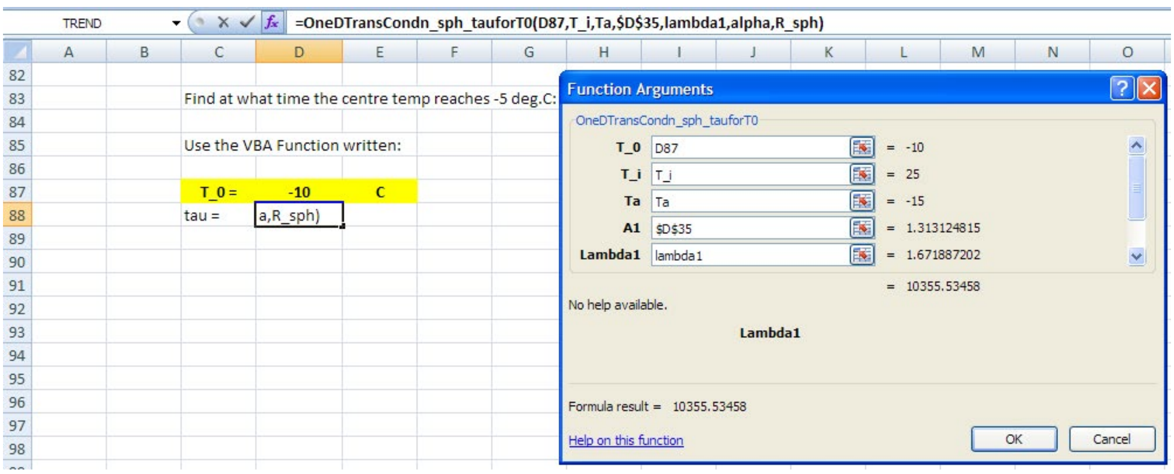
Check: for time = 3600 s, check the centre temp and surface temp. They match with the values obtained above.

10. Plot the graphs of Temp vs radius for different times:

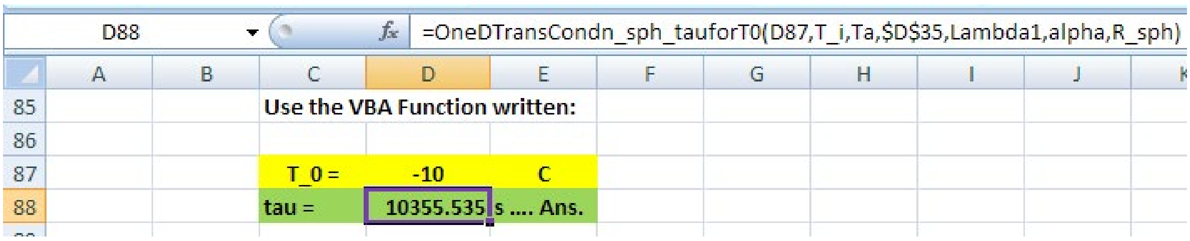


Note that after about 5 hrs, the steady state temp is almost reached.

11. Finally, let **us find at what time the centre temp will reach -10 deg.C.** We will use the VBA Function that was written for this purpose. (See the VBA code above). Following is the screen shot where we entered the Formula for tau:



Click OK. And, we get the calculated value of tau for the centre to reach -10 C:



i.e. tau = 10355.535 s … Ans.

==

Prob.1G.25. A thick copper slab (α = 1.1 × 10-4 m2/s, k = 380 W/(m.C)) is initially at an uniform temperature of 250 C. Suddenly, its surface temperature is lowered to 60 C.

1) How long will it take the temperature at a depth of 3 cm to reach 100 C?
2) What is the heat flux at the surface at that time?
3) What is the total amount of heat removed from the slab per unit surface area till that time?

EXCEL Solution:

Since this is a very large slab, we will consider it as a **semi-infinite medium**, with the surface suddenly brought to and maintained at a constant temperature, To.

Then, from Ref.[1]:

Temperature variation as function of position and time:

$$\frac{T(x,\tau) - T_0}{T_i - T_0} = \operatorname{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}\right) \quad \text{.......(7.29)}$$

Heat flux at the surface:

This is obtained from eqn. (7.33), i.e.

$$Q_{surface} = k\cdot A\cdot\frac{(T_0 - T_i)}{\sqrt{\pi\cdot\alpha\cdot\tau}} \quad \text{W......(7.33)}$$

Total amount of heat removed, per unit surface area:

$$Q_{total} := 1.13\cdot k\cdot A\cdot(T_0 - T_i)\cdot\sqrt{\frac{\tau}{\alpha}} \quad \text{J......(7.34)}$$

Following are the steps in EXCEL Solution:

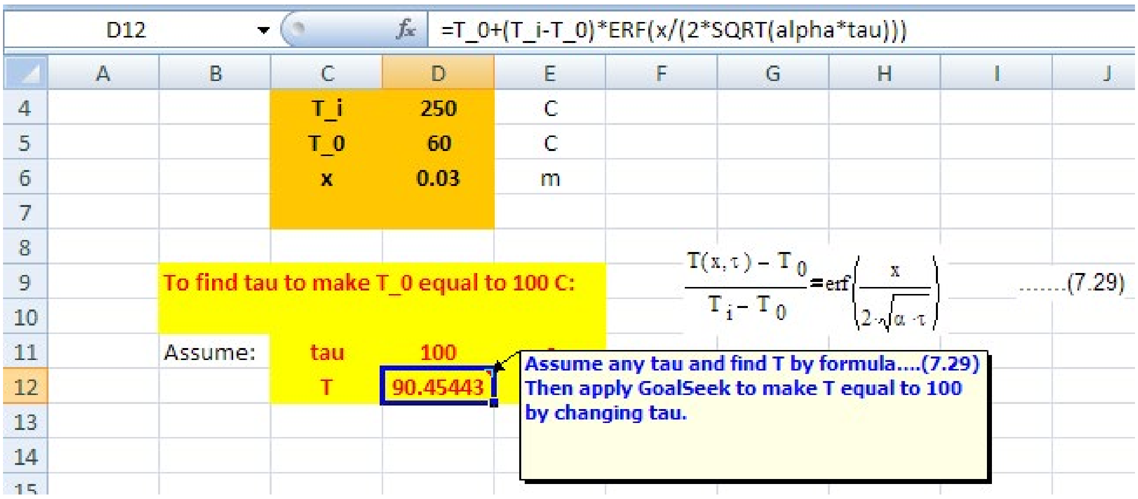
1. Set up the worksheet, enter data and name the cells:

C8 | fx | tau

	A	B	C	D	E
1					
2		Data:	alpha	1.10E-04	m2/s
3			k	380	W/m.C
4			T_i	250	C
5			T_0	60	C
6			x	0.03	m

2. How long will it take the temperature at a depth of 3 cm to reach 100 C?

Here, we use eqn. (7.29) shown above. Now, x = 0.03 m. First, assume a value of tau. Then calculate T with eqn (7.29). Obviously, the result will be different from 100 C. Note that the 'error function' ERF() is a built-in Function in EXCEL:

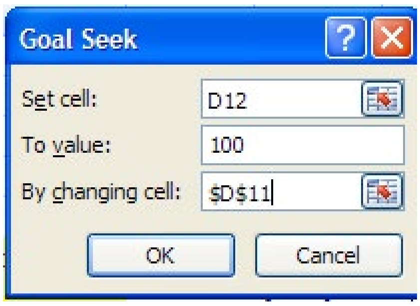


See the eqn entered for T (cell D12) in the Formula bar.

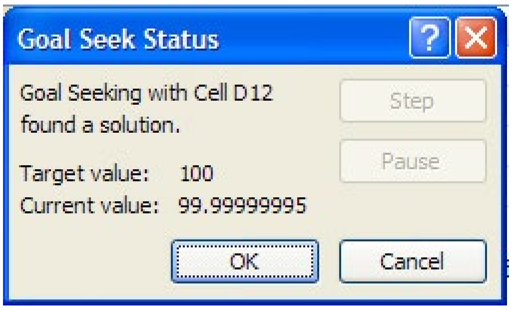
Now, apply Goal Seek in EXCEL to make T equal to 100 by changing tau:

Go to Data-What If Analysis – Goal Seek:

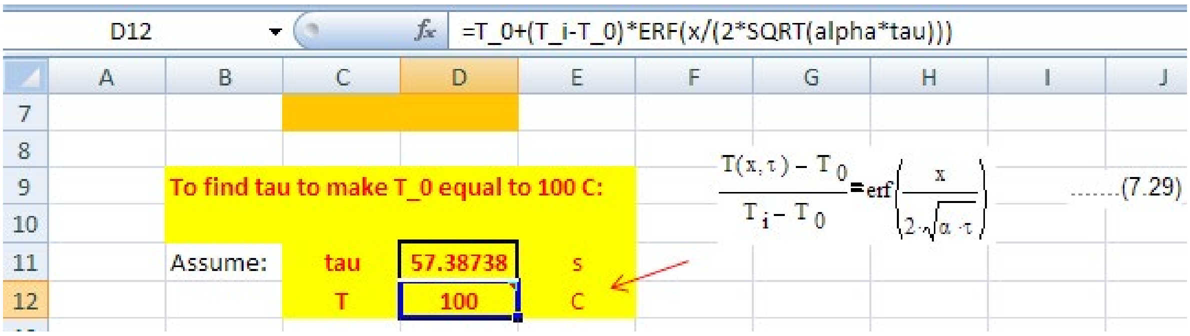
Click on Goal Seek. We get the pop up:



Fill it up as shown, and click OK. We get:

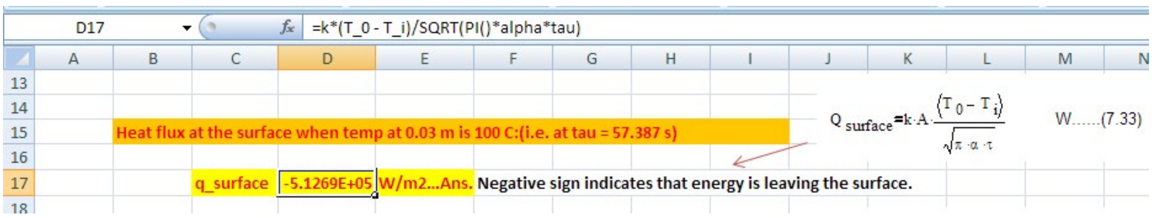


Click OK and see the resulting tau in cell D11:



Thus, tau = 57.3874 s … Ans.

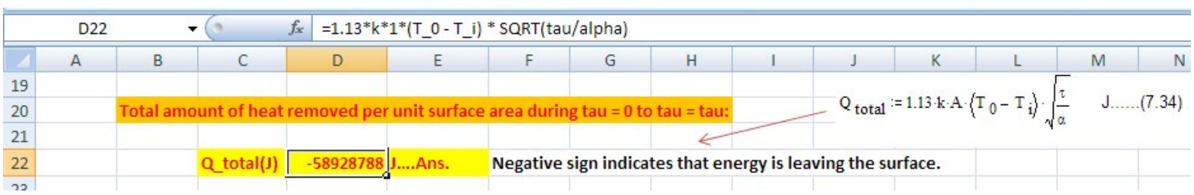
3. Find out the heat flux at the surface at that time:



See the eqn for q_surface in the Formula bar.

i.e. q_surface = -5.1269E05 W/m^2 … Ans.

4. And, the total amount of heat removed:



See the eqn for Q_total in the Formula bar.

Thus: Q_total = 5.893E07 J … Ans.

5. Plot Temp at a depth of 3 cm vs tau: First, prepare a Table as shown below:

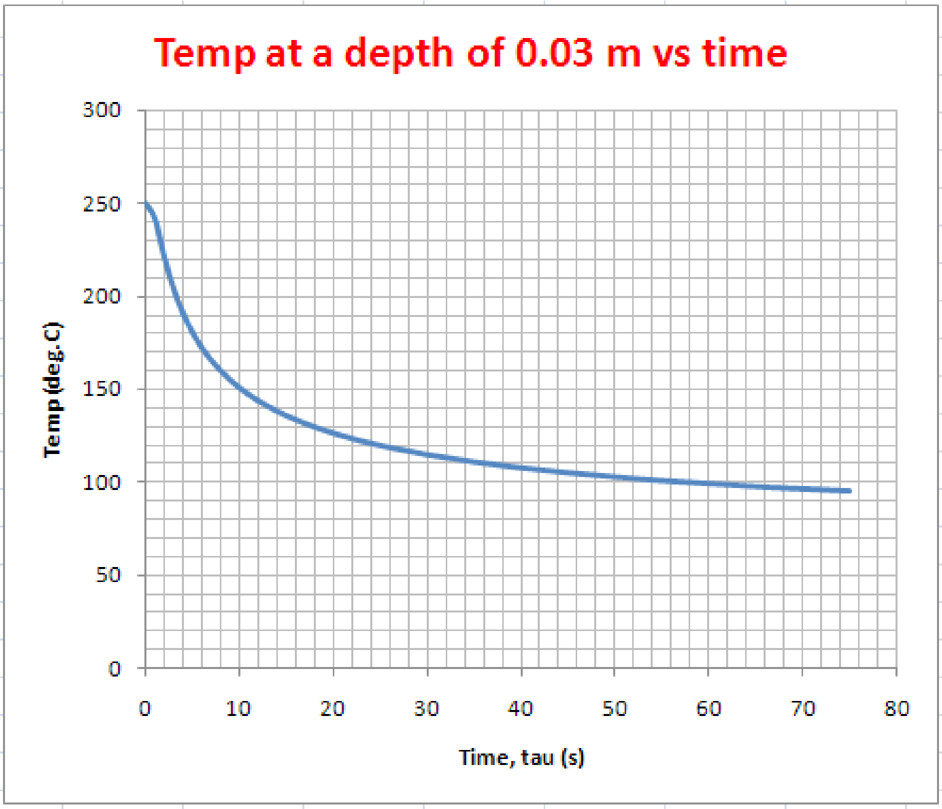
D28 =T_0+(T_i-T_0)*ERF(x/(2*SQRT(alpha*C28)))

To draw graph of T (x,tau) vs tau:

	tau (s)	T(x,tau), deg.C
27	0	250
28	1	241.8083
29	2	220.9943
30	3	203.8474
31	4	190.7441
32	5	180.5147
33	6	172.2973
34	7	165.5285
35	8	159.8357
36	9	154.9650
37	10	150.7381
38	11	147.0257
39	12	143.7321
40	13	140.7843
41	14	138.1263
42	15	135.7135
43	16	133.5105
44	17	131.4888
45	18	129.6248
46	19	127.8991
47	20	126.2954
48	21	124.8001
49	22	123.4014
50	23	122.0893
51	24	120.8555
52	25	119.6922
53	30	114.7346
54	35	110.8364
55	40	107.6674
56	45	105.0253
57	50	102.7788
58	55	100.8381
59	57.387	100.0001
60	65	97.6366
61	70	96.2945
62	75	95.0864

Note tht tau is entered in the eqn with' relative reference' so that drag-copy downwards gives correct results

And, plot the graph:



6. **Finally, plot the variation in temp in the plate with time:**

Set up the Table as shown. Initial temp is, of course, 250 deg.C. Only part of the Table is shown below.

E67 | f_x =T_0+(T_i-T_0)*ERF(C67/(2*SQRT(alpha*20)))

	A	B	C	D	E	F	G	H	I	J	K	L	M
63													
64		To draw Temp vs distance at different times:											
65				tau = 0 s	tau = 20 s	tau = 30 s	tau = 40 s	tau = 50 s	tau = 60 s	tau = 70 s	tau = 80 s	tau = 90 s	tau = 100 s
66			x(m)	T(x,0), deg.C	T(x,20), deg.C	T(x,30), deg.C	T(x,40), deg.C	T(x,50), deg.C	T(x,60), deg.C	T(x,70), deg.C	T(x,80), deg.C	T(x,90), deg.C	T(x,100), deg.C
67			0	250.0	60.0								
68			0.01	250.0									
69			0.02	250.0									
70			0.03	250.0									
71			0.04	250.0									

....

	A	B	C	D	E	F	G	H	I	J	K	L	M
111			0.44	250.0									
112			0.45	250.0									
113			0.46	250.0									
114			0.47	250.0									
115			0.48	250.0									
116			0.49	250.0									
117			0.5	250.0									

Eqn for temp at x = 0.01 m and tau = 20 s, in cell E67, can be seen in the Formula bar. Now, drag-copy the same downwards to complete the calculations for tau = 20 s:

E73 | f_x =T_0+(T_i-T_0)*ERF(C73/(2*SQRT(alpha*20)))

	A	B	C	D	E	F	G	H	I	J	K	L	M
64		To draw Temp vs distance at different times:											
65				tau = 0 s	tau = 20 s	tau = 30 s	tau = 40 s	tau = 50 s	tau = 60 s	tau = 70 s	tau = 80 s	tau = 90 s	tau = 100 s
66			x(m)	T(x,0), deg.C	T(x,20), deg.C	T(x,30), deg.C	T(x,40), deg.C	T(x,50), deg.C	T(x,60), deg.C	T(x,70), deg.C	T(x,80), deg.C	T(x,90), deg.C	T(x,100), deg.C
67			0	250.0	60.0								
68			0.01	250.0	82.8								
69			0.02	250.0	105.0								
70			0.03	250.0	126.3								
71			0.04	250.0	146.2								
72			0.05	250.0	164.3								
73			0.06	250.0	180.5								
74			0.07	250.0	194.7								

E117 =T_0+(T_i-T_0)*ERF(C117/(2*SQRT(alpha*20)))

	A	B	C	D	E
88			0.21	250.0	249.7
89			0.22	250.0	249.8
90			0.23	250.0	249.9
91			0.24	250.0	249.9
92			0.25	250.0	250.0
93			0.26	250.0	250.0
94			0.27	250.0	250.0
95			0.28	250.0	250.0
96			0.29	250.0	250.0
97			0.3	250.0	250.0
98			0.31	250.0	250.0
99			0.32	250.0	250.0
100			0.33	250.0	250.0
101			0.34	250.0	250.0
102			0.35	250.0	250.0
103			0.36	250.0	250.0
104			0.37	250.0	250.0
105			0.38	250.0	250.0
106			0.39	250.0	250.0
107			0.4	250.0	250.0
108			0.41	250.0	250.0
109			0.42	250.0	250.0
110			0.43	250.0	250.0
111			0.44	250.0	250.0
112			0.45	250.0	250.0
113			0.46	250.0	250.0
114			0.47	250.0	250.0
115			0.48	250.0	250.0
116			0.49	250.0	250.0
117			0.5	250.0	250.0

7. Similarly, enter eqns for temp at tau = 30, 40, 50, … 100 s, in cells F67 to M67, and drag-copy them downwards to complete the calculations:

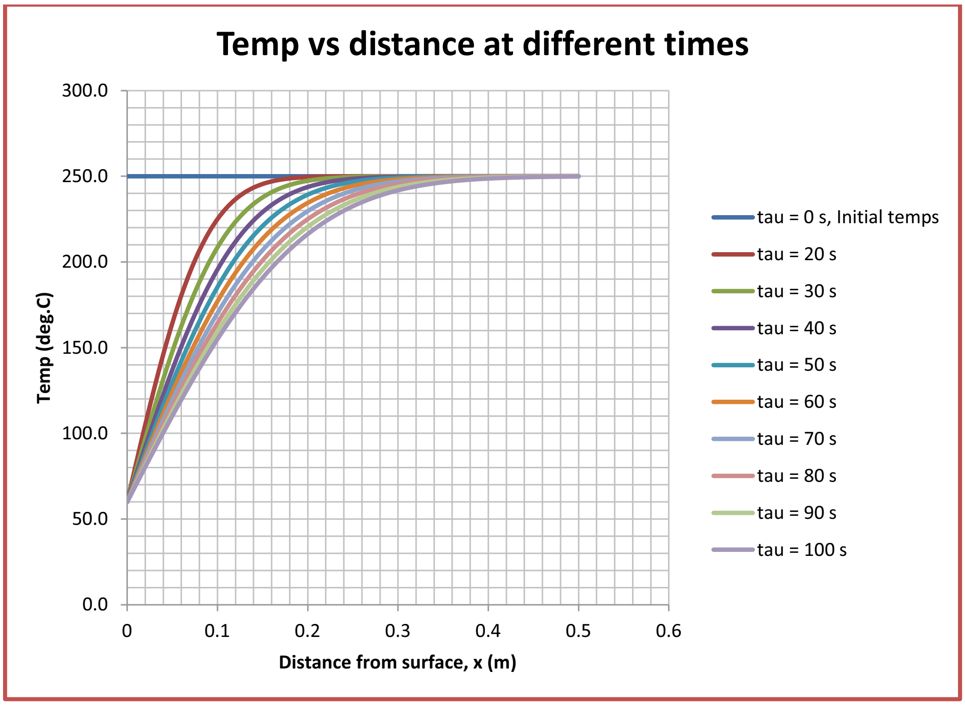
M67 | f_x =T_0+(T_i-T_0)*ERF(C67/(2*SQRT(alpha*100)))

	A	B	C	D	E	F	G	H	I	J	K	L	M
63													
64		To draw Temp vs distance at different times:											
65				tau = 0 s	tau = 20 s	tau = 30 s	tau = 40 s	tau = 50 s	tau = 60 s	tau = 70 s	tau = 80 s	tau = 90 s	tau = 100 s
66			x(m)	T(x,0), deg.C	T(x,20), deg.C	T(x,30), deg.C	T(x,40), deg.C	T(x,50), deg.C	T(x,60), deg.C	T(x,70), deg.C	T(x,80), deg.C	T(x,90), deg.C	T(x,100), deg.C
67			0	250.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
68			0.01	250.0	82.768	78.613	76.130	74.432	73.178	72.203	71.416	70.765	70.213
69			0.02	250.0	105.025	96.947	92.078	88.734	86.257	84.327	82.768	81.475	80.380
70			0.03	250.0	126.295	114.735	107.667	102.779	99.139	96.295	93.991	92.078	90.454
71			0.04	250.0	146.166	131.732	122.735	116.446	111.733	108.031	105.025	102.521	100.393
72			0.05	250.0	164.313	147.732	137.134	129.625	123.950	119.467	115.811	112.756	110.152
73			0.06	250.0	180.515	162.566	150.738	142.219	135.713	130.538	126.295	122.735	119.692

M117 | f_x =T_0+(T_i-T_0)*ERF(C117/(2*SQRT(alpha*100)))

	A	B	C	D	E	F	G	H	I	J	K	L	M
102			0.35	250.0	250.000	249.997	249.964	249.839	249.560	249.089	248.417	247.555	246.525
103			0.36	250.0	250.000	249.998	249.976	249.886	249.672	249.293	248.735	248.002	247.108
104			0.37	250.0	250.000	249.999	249.985	249.920	249.757	249.455	248.995	248.375	247.604
105			0.38	250.0	250.000	249.999	249.990	249.945	249.821	249.582	249.206	248.685	248.022
106			0.39	250.0	250.000	250.000	249.994	249.962	249.869	249.682	249.376	248.940	248.375
107			0.4	250.0	250.000	250.000	249.996	249.974	249.905	249.759	249.512	249.150	248.670
108			0.41	250.0	250.000	250.000	249.998	249.982	249.932	249.819	249.620	249.321	248.916
109			0.42	250.0	250.000	250.000	249.999	249.988	249.951	249.864	249.706	249.461	249.120
110			0.43	250.0	250.000	250.000	249.999	249.992	249.965	249.899	249.774	249.574	249.289
111			0.44	250.0	250.000	250.000	249.999	249.995	249.976	249.926	249.827	249.664	249.428
112			0.45	250.0	250.000	250.000	250.000	249.997	249.983	249.945	249.868	249.737	249.541
113			0.46	250.0	250.000	250.000	250.000	249.998	249.988	249.960	249.900	249.795	249.634
114			0.47	250.0	250.000	250.000	250.000	249.999	249.992	249.971	249.925	249.841	249.709
115			0.48	250.0	250.000	250.000	250.000	249.999	249.994	249.979	249.944	249.877	249.770
116			0.49	250.0	250.000	250.000	250.000	249.999	249.996	249.985	249.958	249.906	249.819
117			0.5	250.0	250.000	250.000	250.000	250.000	250.000	249.997	249.989	249.969	249.928	249.858

8. Now, plot the results:



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Prob.1G.26. A thick concrete slab (α = 7 × 10^{-7} m^2/s, k = 1.37 W/(m.C)) is initially at an uniform temperature of 350 C. Suddenly, its surface is subjected to convective cooling with a heat transfer coeff. h = 100 W/(m2.C) into an ambient at 30 C. Calculate the temperature 8 cm from the surface, 1 h after the start of cooling.

EXCEL Solution:

Treating this as a semi-infinite medium subjected to convection at the surface, we have the following eqn for temp at given x and time, T(x,τ):

$$\frac{T(x,\tau)-T_i}{T_a-T_i}=1-\operatorname{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}\right)-\left(\exp\left(\frac{h\cdot x}{k}+\frac{h^2\cdot\alpha\cdot\tau}{k^2}\right)\right)\cdot\left(1-\operatorname{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}+\frac{h\cdot\sqrt{\alpha\cdot\tau}}{k}\right)\right) \tag{7.36}$$

i.e.

$$T(x,\tau):=T_i+\left(T_a-T_i\right)\cdot\left[1-\operatorname{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}\right)-\left(\exp\left(\frac{h\cdot x}{k}+\frac{h^2\cdot\alpha\cdot\tau}{k^2}\right)\right)\cdot\left(1-\operatorname{erf}\left(\frac{x}{2\cdot\sqrt{\alpha\cdot\tau}}+\frac{h\cdot\sqrt{\alpha\cdot\tau}}{k}\right)\right)\right]$$

Following are the steps in EXCEL calculations:

1. Set up the worksheet, enter data, and name the cells. Calculation for temp at tau = 3600 s and x = 0.08 m, will be done in row 18 as shown:

tau | fx 3600

	A	B	C	D	E
8					
9					
10		Data:	alpha	7.00E-07	m2/s
11			k	1.37	W/m.C
12			T_i	350	C
13			Ta	30	C
14			h	100	W/m2.C
15			x	0.08	m
16			tau	3600	s
17					
18		Temp. at (T(x,tau)=			C....Ans

2. Now, to calculate the temperatures according to eqn. (7.35) shown above, since the eqn is a little complicated, let us write VBA Functions: The procedure has been described in detail in Prob.1F.23. Briefly, we proceed as follows:

Click: Developer-Visual Basic-Insert-Module. A blank window presents itself, and type the code therein:

```
Option Explicit

Function Semi_infinite_Solid_Convectionatsurface_Tempxtau(T_i As Double, Ta As Double, _
Alpha As Double, tau As Double, k As Double, x As Double, h As Double) As Double
'Finds temp at given (x,tau)when the surface is subjected to convection
Dim AA As Double, BB As Double, CC As Double, DD As Double
If tau = 0 Then
   tau = 0.000000001
End If
AA = x / Sqr((4 * Alpha * tau))
BB = Application.Erf(AA)
CC = h * x / k + h ^ 2 * Alpha * tau / k ^ 2
DD = Application.Erf(AA + (h / k) * Sqr((Alpha * tau)))
    Semi_infinite_Solid_Convectionatsurface_Tempxtau = T_i + (Ta - T_i) * _
    (1 - BB - Exp(CC) * (1 - DD))
End Function
```

In the above code:

Line 1, 2: defines the Function name and dimension of variables used

Line 3: says what this program will do

Line 4: dimensions of internally defined variables

Lines 5, 6, 7: If –End If construct which ensures that for tau = 0 the program does not give an error "Div by 0!!". Here, if tau = 0, then, tau is reset to some very small value almost equal to zero (of say 1E-09), so that the program continues.

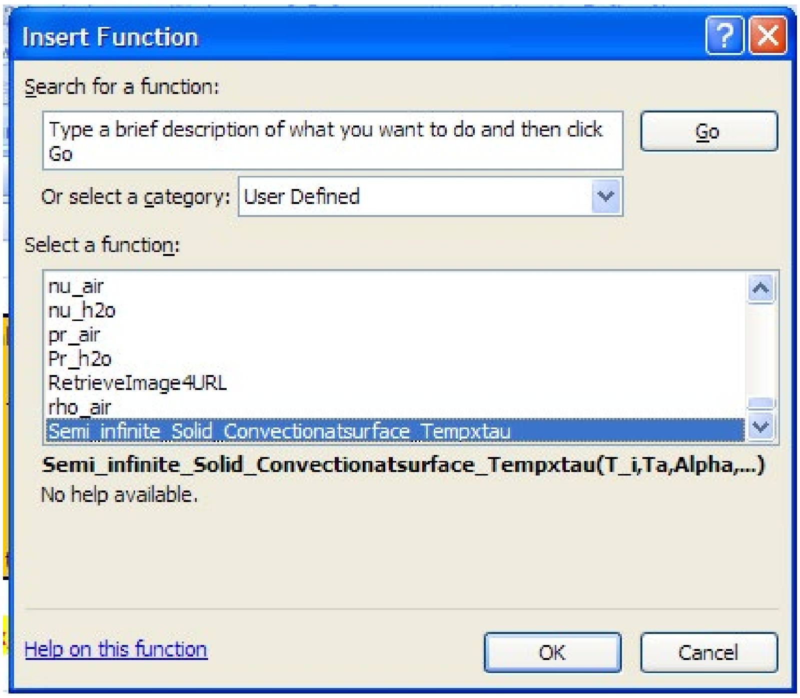
Lines 8 to 13: actual calculations. Note that the complex eqn is calculated in part to avid errors.

Line 14: End statement of Function.

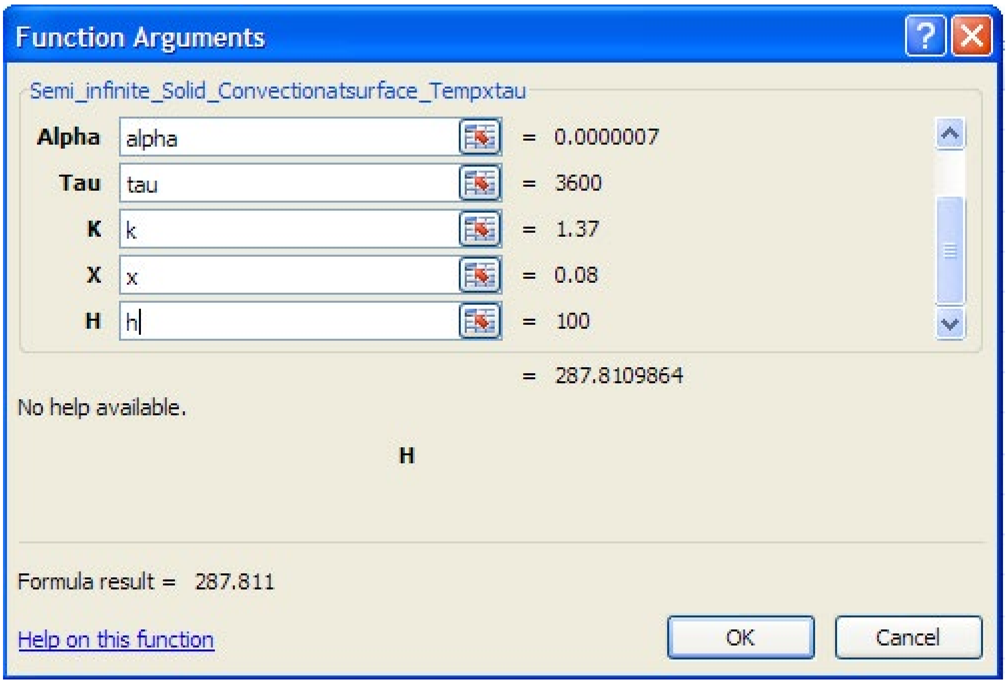
Read the comments given in the code.

Save the worksheet as *'Macro enabled worksheet'*. Now, this Function is available in the worksheet like any other built-in Functions of EXCEL.

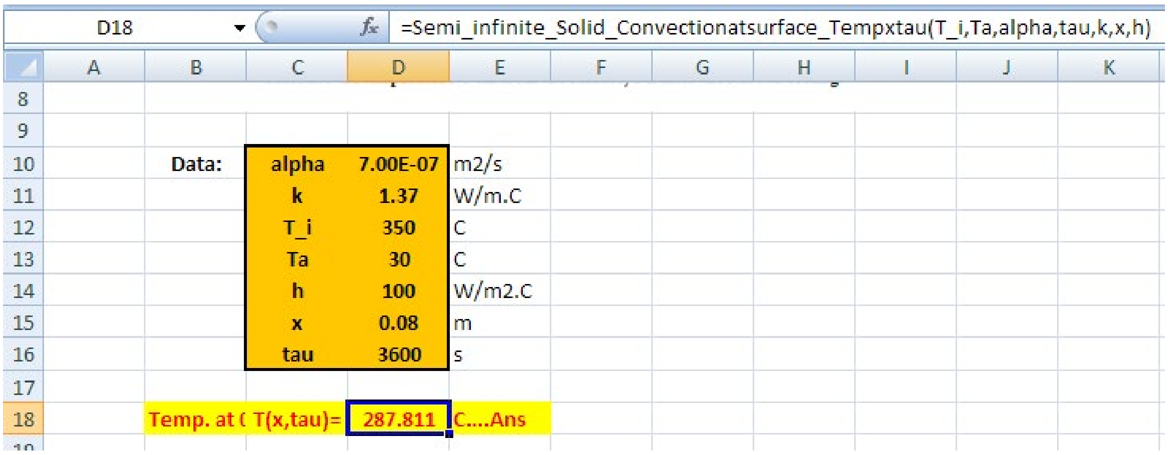
a) Go to Formulas tab and click on 'Insert Function' and select the 'User Defined' category, and choose **Semi_infinite_Solid_Convectionatsurface_Tempxtau** from the Function list:



b) Click OK, and we get the following screen. Fill up the data required (by 'point and click' to avoid errors)



c) Press OK. We get:



i.e. we get: T(0.08, 3500) = 287.811 deg.C …. Ans.

In the screen shot above, we can see the eqn entered in cell D18 for Temp, in the Formula bar.

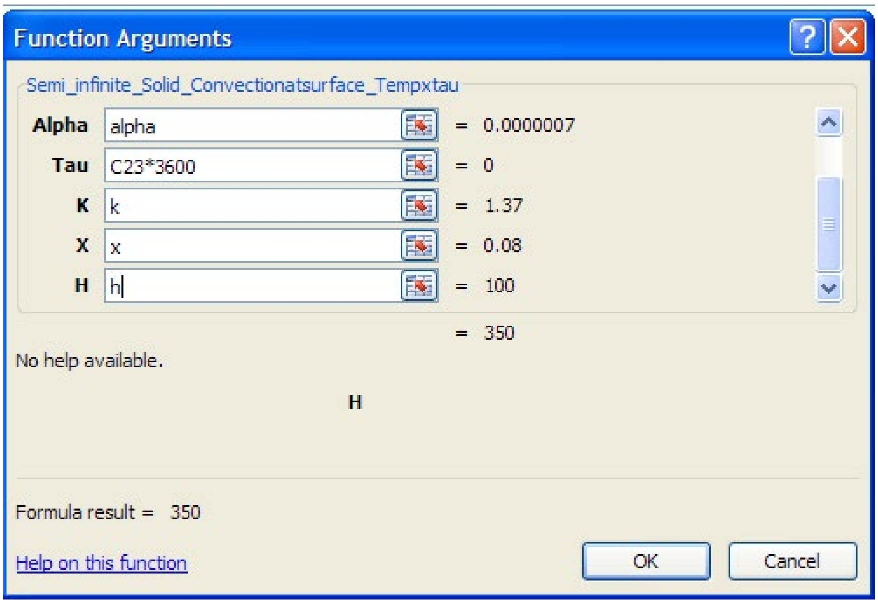
4. Now, let us plot the variation of temp at x = 0.08 m for different times.

   a) First, build a Table as shown:

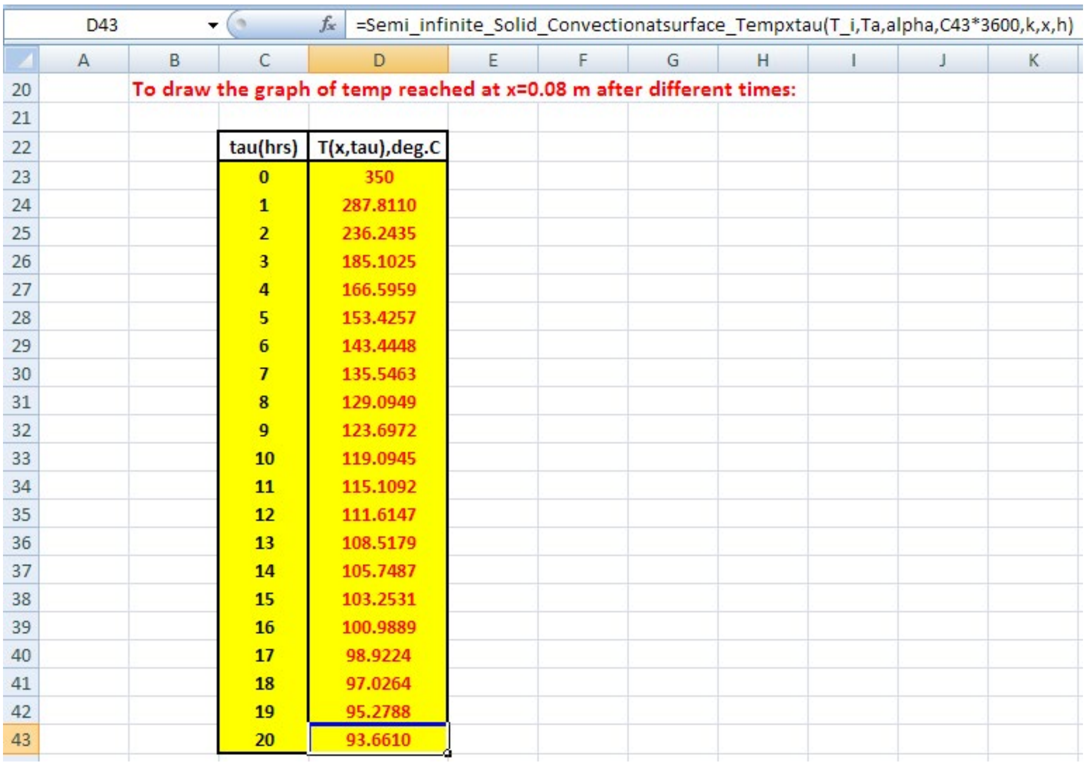
To draw the graph of temp reached at x=0.08 m after different times:

tau(hrs)	T(x,tau),deg.C
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

b) Now, enter the eqn for Temp in cell D23, taking care to refer to tau (in cell C23) by *'relative reference'* so that we can complete the Table by drag-copy up to cell D43. Also, note that in the Table, tau is in hrs, and to use the Function, we need to have tau in seconds; so, multiply the Table value in cellC23 by 3600 as shown. See the screen shot of entering the Function variables below:



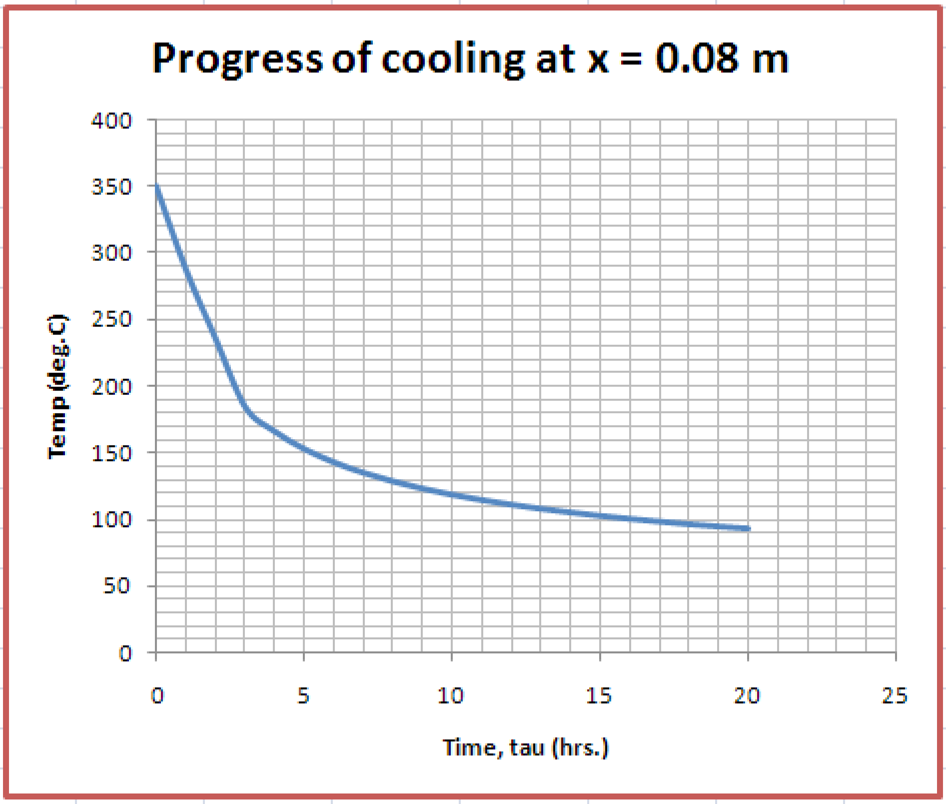
c) Now, drag- copy the cell D23 up to cell D43 to complete the calculations:



tau(hrs)	T(x,tau),deg.C
0	350
1	287.8110
2	236.2435
3	185.1025
4	166.5959
5	153.4257
6	143.4448
7	135.5463
8	129.0949
9	123.6972
10	119.0945
11	115.1092
12	111.6147
13	108.5179
14	105.7487
15	103.2531
16	100.9889
17	98.9224
18	97.0264
19	95.2788
20	93.6610

See in the above screen shot the Function entered in cell D43.

d) Now, plot the results:



Note that the plate has not cooled to the ambient temp of 30 C, even after 20 hrs.

==

1H Two-dimensional conduction Shape factor:

Learning objectives:

10. Applying **'2D conduction shape factor'** is a simple method to analyse a particular type of 2-D conduction problems where *steady state heat transfer occurs between two surfaces at fixed temperatures, T1 and T2, with an intervening solid medium in between.*
11. ***If Q is the rate of heat transfer between two temperature potentials T1 and T2, with the thermal conductivity of intervening material being k, with no heat generation in the medium, we write:***
    Q = k S (T1 – T2) (1)
    where **S is known as 'Shape factor'** and has dimension of length.
12. Note that eqn.(1) is applicable only when there is conduction i.e. in solids.
13. Immediately it follows that thermal resistance of the medium is given by:
    Rth = 1/(k S) (2)

14. Since we can write: S = 1/(Rth.k), we get:

$$S_{wall} = \frac{A}{L}$$

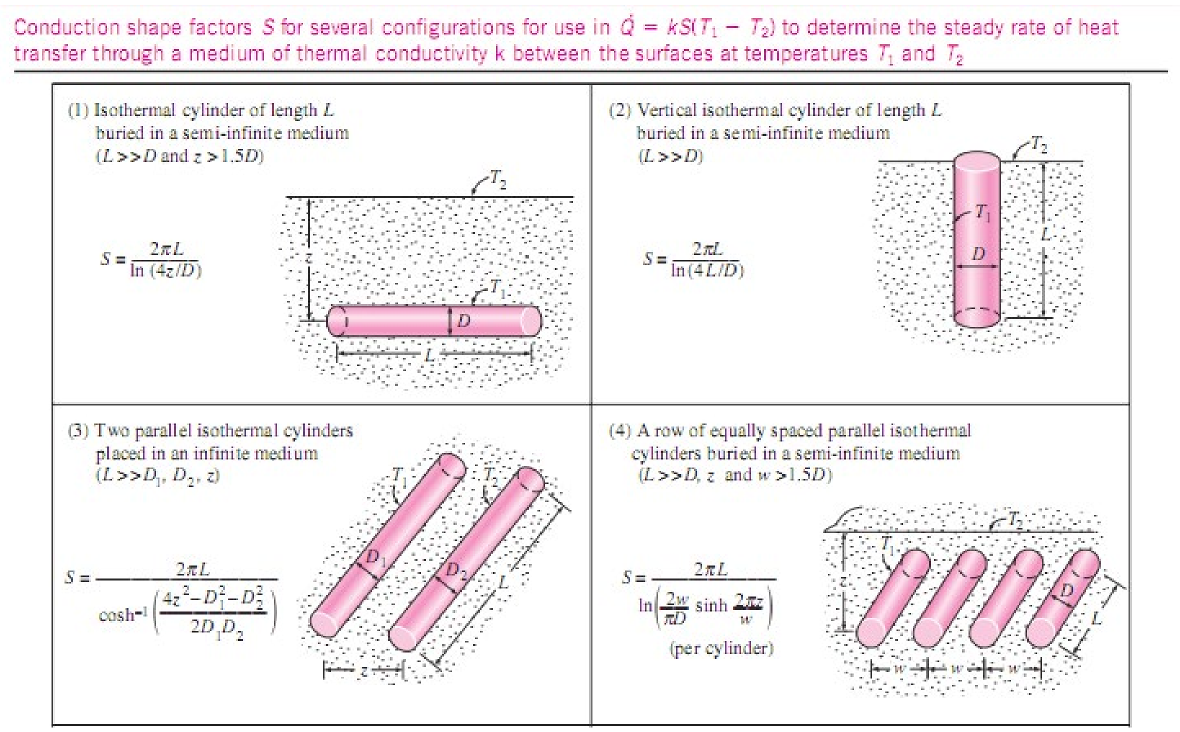
$$S_{cyl} = \frac{2 \cdot \pi \cdot L}{\ln\left(\frac{r_o}{r_i}\right)}$$

and,

$$S_{sph} = \frac{4 \cdot \pi \cdot r_i \cdot r_o}{r_o - r_i}$$

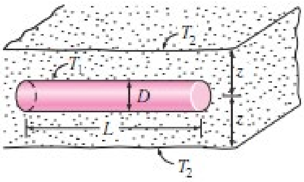
15. S has been computed by researchers using electrical analogy or numerical methods, for several cases of practical interest.
16. Tables below give conduction shape factors for a few selected two dimensional systems.
17. Some popular types of problems are solved below.

Table 1H.1.Conduction Shape factors:[Ref. 2]



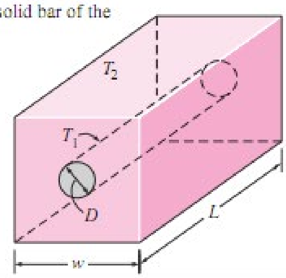
(5) Circular isothermal cylinder of length L in the midplane of an infinite wall ($z > 0.5D$)

$$S = \frac{2\pi L}{\ln(8z/\pi D)}$$



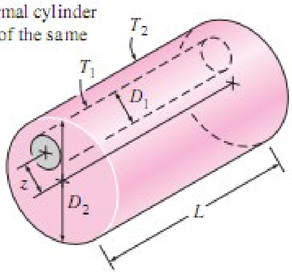
(6) Circular isothermal cylinder of length L at the center of a square solid bar of the same length

$$S = \frac{2\pi L}{\ln(1.08\, w/D)}$$



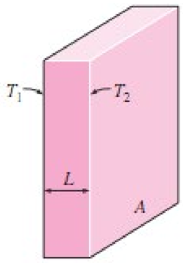
(7) Eccentric circular isothermal cylinder of length L in a cylinder of the same length ($L > D_2$)

$$S = \frac{2\pi L}{\cosh^{-1}\left(\frac{D_1^2 + D_2^2 - 4z^2}{2D_1D_2}\right)}$$



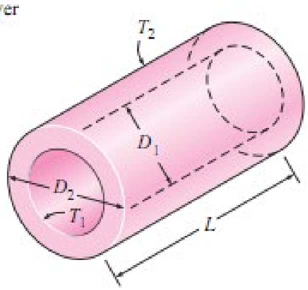
(8) Large plane wall

$$S = \frac{A}{L}$$



(9) A long cylindrical layer

$$S = \frac{2\pi L}{\ln(D_2/D_1)}$$



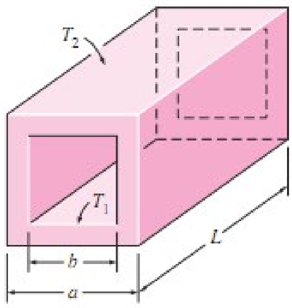
(10) A square flow passage

(a) For $a/b > 1.4$,

$$S = \frac{2\pi L}{0.93 \ln(0.948\, a/b)}$$

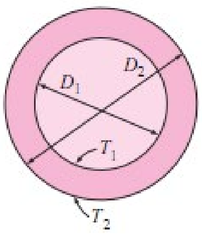
(b) For $a/b < 1.41$,

$$S = \frac{2\pi L}{0.785 \ln(a/b)}$$



(11) A spherical layer

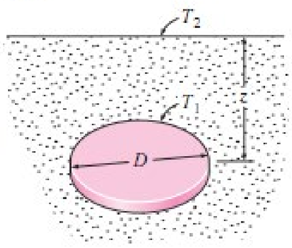
$$S = \frac{2\pi D_1 D_2}{D_2 - D_1}$$

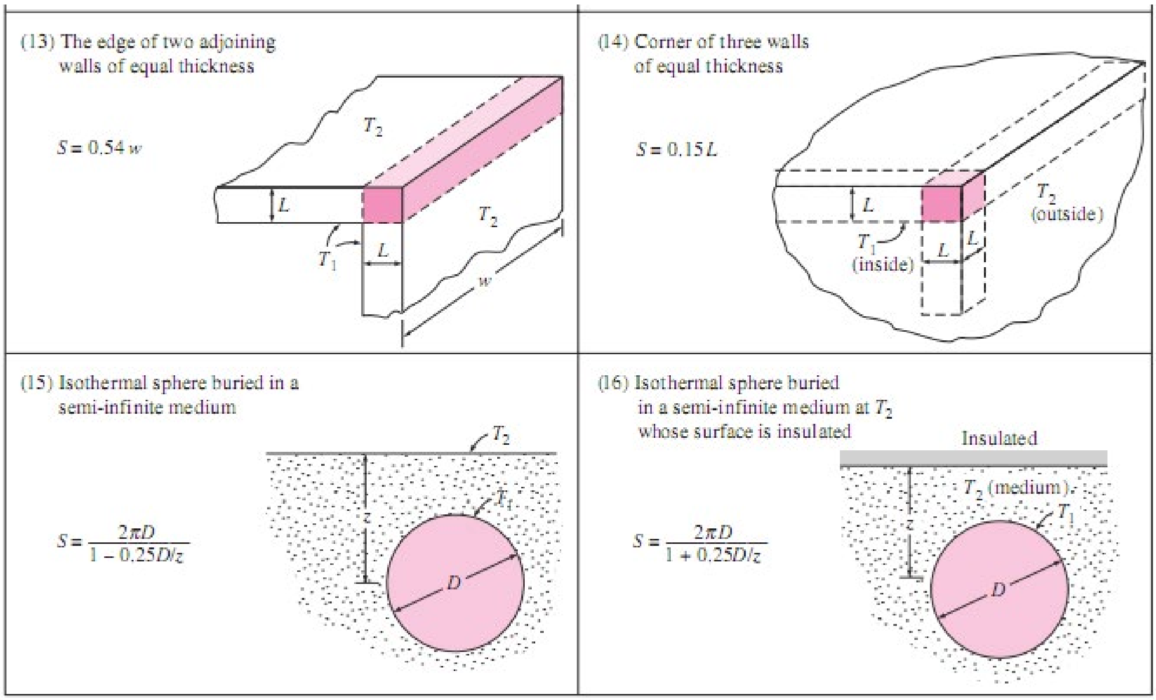


(12) Disk buried parallel to the surface in a semi-infinite medium ($z >> D$)

$S = 4D$

($S = 2D$ when $z = 0$)




(13) The edge of two adjoining walls of equal thickness
S = 0.54 w
T2
L
T1
L
T2
w
(14) Corner of three walls of equal thickness
S = 0.15 L
L
T1 (inside)
L
L
T2 (outside)
(15) Isothermal sphere buried in a semi-infinite medium
S = 2πD / (1 − 0.25D/z)
T2
z
T1
D
(16) Isothermal sphere buried in a semi-infinite medium at T2 whose surface is insulated
S = 2πD / (1 + 0.25D/z)
Insulated
T2 (medium)
z
T1
D

==

Prob. 1H.1. A small cubical furnace 500 × 500 × 500 mm on the inside, is constructed of fireclay bricks (k = 1.04 W/m.K) with a wall thickness of 100 mm. The inside of the furnace is maintained at 500 C, and the outside is maintained at 50 C. Calculate the heat lost through the walls. Take shape factor for walls S = A/L, edges: S = 0.54 D, and corners: S = 0.15 L. [VTU – M. Tech. – Dec. 2010]

Mathcad Solution:

Recognise that this problem can be solved by use of 'Shape factors'.

Also, recall that when the interior dimensions of the furnace are greater than one-fifth of the wall thickness, we have, for Shape factors:

$$S_{wall} = \frac{A}{L} \quad , \quad S_{edge} = 0.54 \cdot D \quad \text{and} \quad S_{corner} = 0.15 \cdot L$$

where, A = inside area of the wall, L = wall thickness, and D = length of edge

See Fig. below:

Note that for a cubical structure, there are 6 wall sections, 12 edges and 8 corners. Calculate the Shape factors and compute the total Shape factor by adding all of them.

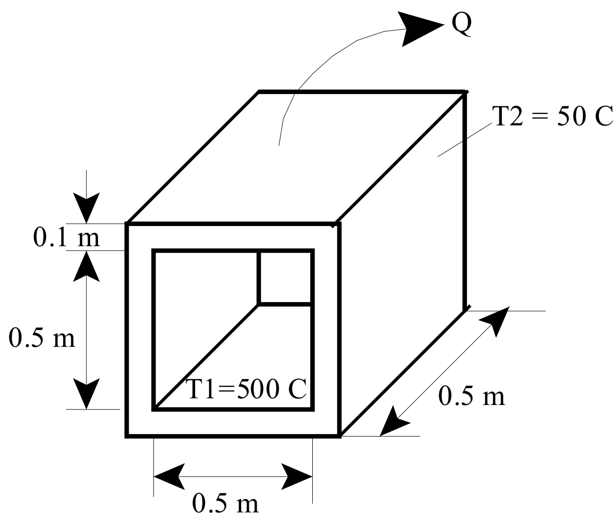


Fig.Prob.1H.1

Data:

Size of furnace: 0.5 m x 0.5 m x 0.5 m i.e. dimension of each wall, D = 0.5 m

$D := 0.5$ m....dimension of one side of cubical furnace

$L := 0.1$ m...thickness of each wall

$A := D \cdot D$ m^2....area of each wall

i.e. $A = 0.25$ m^2...area of each wall

$k := 1.04$ W/(m.C)....thermal cond. of fireclay brick

$T1 := 500$ C...temp. of inside surface of furnace

$T2 := 50$ C...temp. of outside surface of furnace

Calculations:

S for Walls:

S for a single wall section is given by:

$S := \frac{A}{L}$ m...define S for single wall section

i.e. $S = 2.5$ m... Shape factor for single wall section.

Therefore for 6 wall sections:

$S_{walls} := S \cdot 6$ m...S for 6 wall sections

i.e. $S_{walls} = 15$ m...S for 6 wall sections

S for Edges:

S for a single edge is given by:

$S := 0.54 \cdot D$ m...define S for single edge

i.e. $S = 0.27$ m... Shape factor for single edge.

Therefore for 12 edges:

$S_{edges} := S \cdot 12$ m...S for 12 edges

i.e. $S_{edges} = 3.24$ m...S for 12 edges

S for Corners:

S for a single corner is given by:

$S := 0.15 \cdot L$ m...define S for single corner

i.e. $S = 0.015$ m... Shape factor for single corner.

Therefore for 8 corners:

$S_{corners} := S \cdot 8$ m...S for 8 corners

i.e. $S_{corners} = 0.12$ m...S for 8 corners

Total Shape factor:

Therefore, total shape factor is obtained by summing up the shape factors for all the walls, edges and corners:

$S_{total} := S_{walls} + S_{edges} + S_{corners}$ m...total shape factor

i.e. $S_{total} = 18.36$ m...total shape factor

Heat transfer rate, Q:

Therefore, total heat loss from the furnace is given by:

$Q := k \cdot S_{total} \cdot (T1 - T2)$ W....define Q

i.e. $Q = 8.592 \cdot 10^3$ W = 8.592 kW...**total heat loss rate from furnace...Ans.**

==

Prob. 1H.2. A horizontal pipe 15 cm in dia and 4 m long carrying hot water is buried in the earth at a depth of 20 cm. The pipe wall temp is 75 C and the earth surface temp is 5 C. Assuming that k = 0.8 W/m.C for earth, calculate the heat lost by the pipe. [VTU – M. Tech. – May–June. 2010]

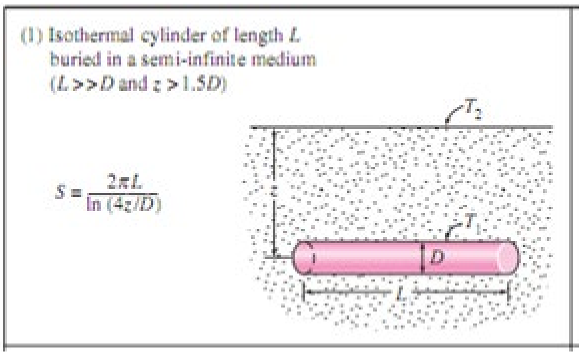


Fig.Prob.1H.2

Mathcad Solution:

Data:

$D := 0.15$ m .... dia of pipe

$L := 4$ m ... length of pipe

$T1 := 75$ C ... temp of pipe surface

$T2 := 5$ C ... temp of earth surface

$z := 0.2$ m ...depth at which pipe is buried.

$k := 0.8$ W/m.C ... thermal cond.

See Table 1H.1, case (a).

Calculations:

We see from Table that:

$$S := \frac{2 \cdot \pi \cdot L}{\ln\left(\frac{4 \cdot z}{D}\right)}$$ m ... Shape factor for the configuration

i.e $S = 15.014$ m

And, heat transfer:

$Q := k \cdot S \cdot (T1 - T2)$ W ...

i.e. $Q = 840.773$ **W ... heat transfer.....Ans.**

===

Prob. 1H.3. Radioactive wastes are temporarily stored in a spherical container, the centre of which is buried a distance of 10 m below the earth's surface. The outside dia of the container is 2 m and 500 W of heat are released as a result of the radioactive decay process. If the soil surface temp is 20 C, what is the outside surface temp of the container under steady state conditions? [Ref. 3]

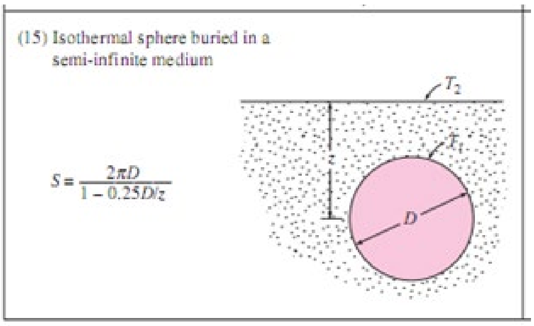


Fig.Prob.1H.3

Mathcad Solution:

This case refers to case (15) of Table 1H.1.

Data:

$D := 2$ m ... dia of sphere

$z := 10$ m ... depth below the surface

$Q := 500$ W ... heat released

$T2 := 20$ C .. soil temp

$k := 0.9$ W/m.C .... for soil, assumed.

Let the surface temp of sphere be T1.

Then,

$$S := \frac{2\cdot\pi\cdot D}{1 - \frac{D}{4\cdot z}}$$ m....Shape factor ...see Table 1H.1

i.e. $S = 13.228$ m

and, $Q = k\cdot S\cdot(T1 - T2)$ W ... heat transferred

Therefore: $T1 := \frac{Q}{k\cdot S} + T2$

i.e. $T1 = 61.999$ **C ... temp of outer surface of Sphere .... Ans.**

In addition: keeping T1 and T2 constant, how does Q vary with D? Plot the results:

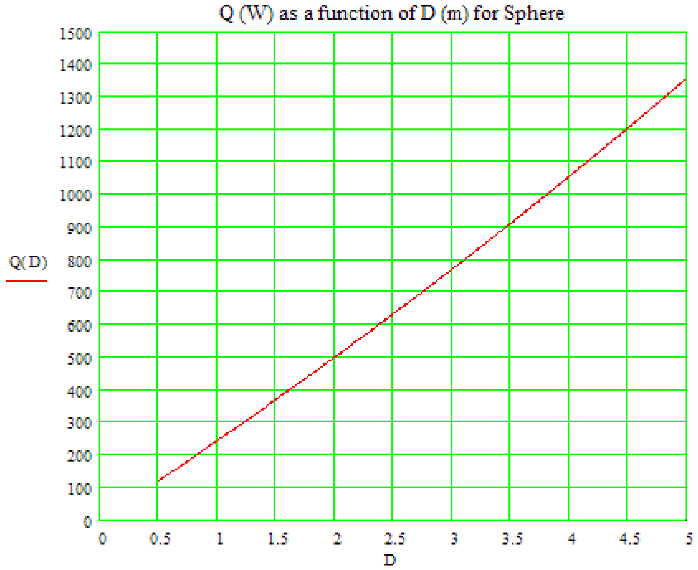
Write Q as a function of D:

$$S(D) := \frac{2\cdot\pi\cdot D}{1 - \frac{D}{4\cdot z}}$$ ...S defined as a function of D

$Q(D) := k \cdot S(D) \cdot (T1 - T2)$ ....Q defined as a function of D

Let D vary from 0.5 to 5 m:

$D := 0.5, 0.51 .. 5$ m .. define a range variable D



Verify from the graph at for D = 2 m, Q is 500 W.

==

"**Prob. 1H.4.** Two parallel pipelines spaced 0.5 m apart are buried in soil (k = 0.5 W/m.K). The pipes have outer diameters of 100 and 75 mm with surface temperatures of 175 and 5 C, respectively. Estimate the heat transfer rate per unit length between the two pipelines. [Ref. 3]"

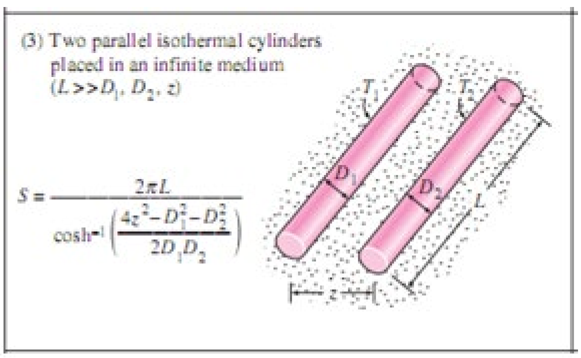


Fig.Prob.1H.4

"EES Solution:"

"Data:"

"See Table 1H.1, case(3):"

D_1 = 0.1 [m]
D_2 = 0.075 [m]
T_1 = 175 [C]
T_2 = 5 [C]
k = 0.5 [W/m-C]
z = 0.5 [m]"…spacing between pipelines"
L = 1 [m]

"Calculations:"

"S is given by:

$$S = \frac{2 \cdot \pi \cdot L}{\operatorname{arccosh}\left[\dfrac{4 \cdot z^2 - D_1^2 - D_2^2}{2 \cdot D_1 \cdot D_2}\right]} \quad \text{[m] ... Shape factor}$$

And, it is entered as follows in EES:"

S = (2 * pi * L) / arccosh((4 * z^2 – D_1^2 – D_2^2) / (2 * D_1 * D_2)) " [m] … Shape factor"

"Then:"

Q = S * k * (T_1 – T_2) "[W] … heat transfer"

Results:

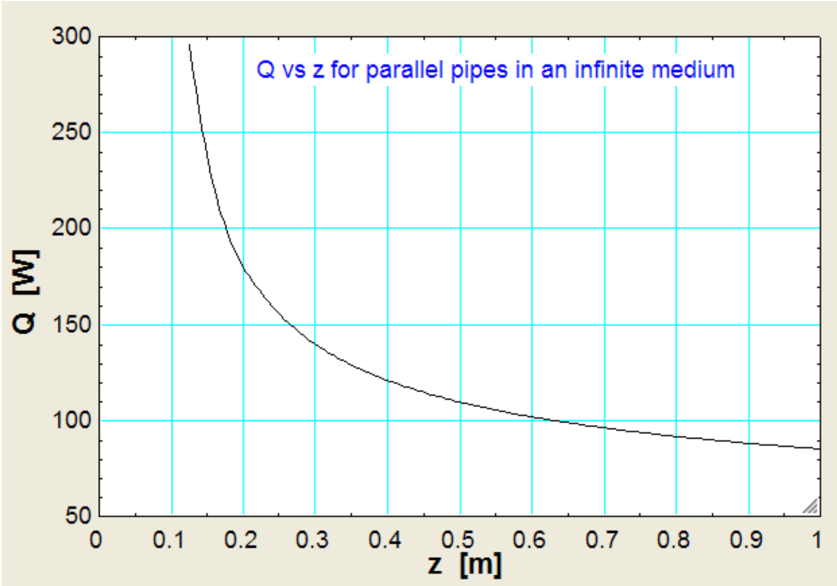
Unit Settings: SI C kPa kJ mass deg

D_1 = 0.1 [m]	D_2 = 0.075 [m]	k = 0.5 [W/m-C]	L = 1 [m]
Q = 109.5 [W]	S = 1.288 [m]	T_1 = 175 [C]	T_2 = 5 [C]
z = 0.5 [m]			

Thus:

Q = 109.5 W ….. heat transfer between two pipelines …. Ans.

Plot the variation of Q with space between the pipes, z:



It may be noted that as the spacing between the pipes increases, initially the reduction in Q is rapid up to about z = 0.5 m, and then the rate of reduction decreases.

==

"**Prob. 1H.5.** A long power transmission cable is buried at a depth (ground to cable centre-line distance) of 2 m. The cable is encased in a thin-walled pipe of 0.1 m dia, and to render the cable superconducting, the space between the cable and pipe is filled with liquid nitrogen at 77 K. If the pipe is covered with a super insulator (k_ins = 0.005 W/m.K) of 0.05 m thickness and the surface of the earth (k_g = 1.2 W/m.K) is at 300 K, what is the cooling load in W/m that must be maintained by the cryogenic refrigerator per unit pipe length? [Ref. 3]"

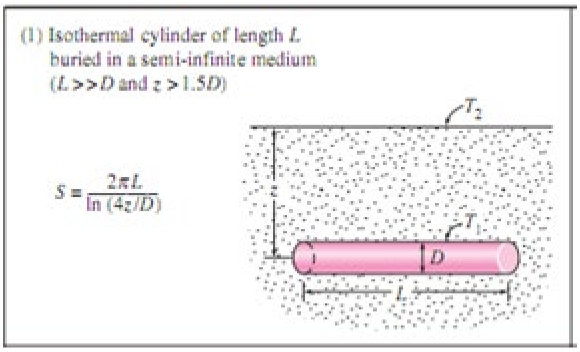


Fig.Prob.1H.5

"EES Solution:"

"Data:"

"See Table 1H.1, case(1):"

D_pipe = 0.1 [m] "..dia of pipe"
D_ins = 0.2 [m] "…outside dia after putting insulation of 0.05 m thickness"
T_1 = 77 [K]
T_2 = 300 [K]
k_ins = 0.005 [W/m-K] "…thermal cond. of insulation"
k_g = 1.2 [W/m-K] "… thermal cond. of earth"
z = 2 [m] "…depth at which the pipe is buried"
L = 1 [m]
"Let T_3 be the temp on insulation surface"

"Calculations:"

"In steady state, Q flowing through the earth,and that flowing through insulation to the pipe is same. This is the refrigeration capacity required by the refrigerator:"

"Shape factor, S for the Insulated pipe: See Table 1H.1. case (1):"

S = (2 * pi * L) / ln ((4 * z) / D_ins) " [m] … Shape factor"

Q = S * k_g * (T_2 – T_3) "[W] … heat transfer between the earth's surface and insulation surface"

Q = (T_3 – T_1) / (ln (D_ins / D_pipe) / (2 * pi * k_ins * L)) "[W] … heat flow through insulation"

Results:

Unit Settings: SI C kPa kJ mass deg

D_{ins} = 0.2 [m]	D_{pipe} = 0.1 [m]	k_g = 1.2 [W/m-K]
k_{ins} = 0.005 [W/m-K]	L = 1 [m]	Q = 9.888 [W]
S = 1.703 [m]	T_1 = 77 [K]	T_2 = 300 [K]
T_3 = 295.2 [K]	z = 2 [m]	

Thus:

Q = 9.888 W …. Refrigeration capacity required of the refrigerator …. Ans.

Also, S = 1.703 m …. Shape factor for the geometry, and

T_3 = 295.2 K … temp on the surface of super-insulation.

==

Prob.1H.6. Hot water at an average temp of 80 C and an average velocity of 1.5 m/s is flowing through a 25 m section of a pipe that has an outer dia of 5 cm. The pipe extends 2 m in the ambient air above the ground, dips in to the ground (k = 1.5 W/m.C) vertically for 3 m, and continues horizontally at this depth for 20 m more before it enters the next building. The first portion of the pipe is exposed to the ambient air at 80 C, with h = 22 W/m^2.C. If the surface of the ground is covered with snow at 0C, determine:

(a) the total rate of heat loss from the hot water, and (b) the temperature drop of the hot water.

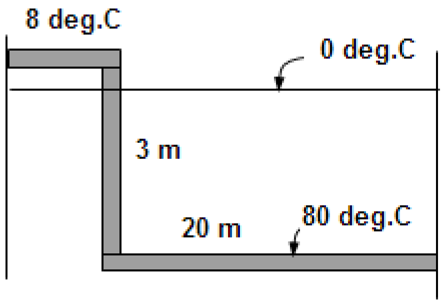


Fig. 1H.6

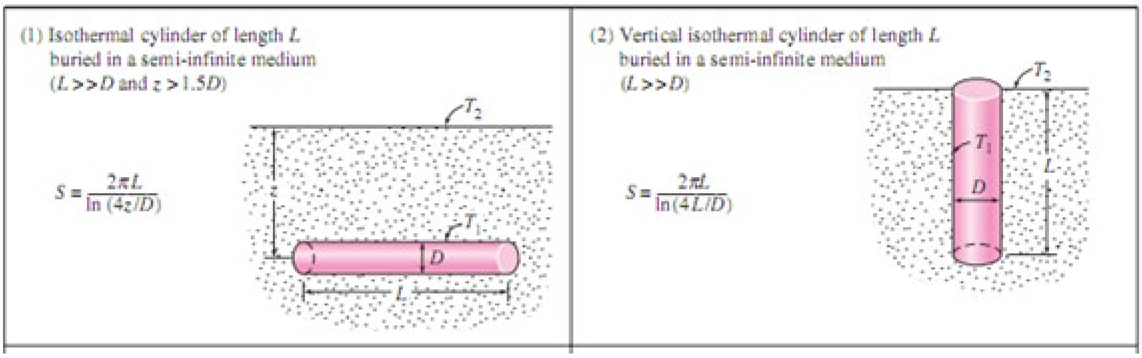
EXCEL Solution:

Here, there are three sections of piping for which we have to calculate the heat losses:

1. Section of 2 m length, above the ground…. It loses heat by convection = Q_{conv}
2. Section of 3 m length, dipping vertically into the ground…loses heat by conduction = Q_{cond1}
3) Section of 20 m length going horizontally in the ground…. Loses heat by conduction = Q_{cond2}

Q_{conv} is calculated by Newton's Law, and Q_{cond1} and Q_{cond2} are calculated using the 'Shape factor' for the respective configurations. Qcond = S. k. ΔT

See from Table 1H.1 that eqns for Shape factors for cases 2 and 3 above are given as follows:



Following are the steps in EXCEL Solution:

1. Set up the EXCEL worksheet, enter data and name the cells:

T_surface | f_x | 0

	A	B	C	D	E
1					
2		**Data:**			
3			**T_pipe**	**80**	C
4			**U_avg**	**1.5**	m/s
5			**dia**	**0.05**	m/s
6			**L_1**	**2**	m
7			**L_2**	**3**	m
8			**L_3**	**20**	m
9			**k_soil**	**1.5**	W/m.C
10			**T_amb**	**8**	C
11			**h**	**22**	W/m^2.C
12			**T_surface**	**0**	C

2. Perform the calculations, as per the explanations and eqns given above:

S_3 | f_x | =(2* PI() * L_3)/LN(4 * L_2/dia)

	A	B	C	D	E
8			L_3	20	m
9			k_soil	1.5	W/m.C
10			T_amb	8	C
11			h	22	W/m^2.C
12			T_surface	0	C
13					
14		Calculations:			
15		Conv. heat tr.	Q_conv	497.6283	W .. Ans.
16		To find Qcond1:			
17		Shape factor	S_2	3.439299	m
18		Cond. heat tr	Qcond1	412.7159	W .... Ans.
19					
20		To find Qcond2:			
21		Shape factor	S_3	22.92866	m
22		Cond. heat tr	Qcond2	2751.439	W ... Ans
23					
24		Total heat tr	Qtot =	3661.783	W ... Ans.

$$Qconv = h\cdot(\pi\cdot dia\cdot L\_1)\cdot(T\_pipe - T\_amb)$$

$$S\_2 = \frac{2\cdot\pi\cdot L\_2}{\ln\left(\frac{4\cdot L\_2}{dia}\right)}$$

$$Qcond1 = S\_2\cdot k\_soil\cdot(T\_pipe - T\_surface)$$

$$S\_3 = \frac{2\cdot\pi\cdot L\_3}{\ln\left(\frac{4\cdot L\_2}{dia}\right)}$$

$$Qcond2 = S\_3\cdot k\_soil\cdot(T\_pipe - T\_surface)$$

$$Qtot = Qconv + Qcond1 + Qcond2$$

Thus, total heat transfer = 3661.78 W … Ans.

In the above worksheet, the formulas used are shown separately for clarity.

In the Formula bar, eqn for S_3, the Shape factor for section-3 of pipe, can be seen.

3. Next, calculate the temp drop in water as it flows through the 25 m section. Heat lost is equal to (m_dot * cp * ΔT_{drop}), where m_dot is the mass flow rate = (rho * A_c * U_avg), cp is the sp.heat of water. This calculation is shown in the following section of worksheet:

D32 | f_x | =Qtot/(m_dot*cp)

	A	B	C	D	E
26		To find temp drop in water:			
27					
28		Density of water	rho	1000	kg/m^3
29		sp. heat of water	cp	4180	J/kg.C
30		Area of crosssection of pipe	A_c	0.001963	m^2
31		Mass flow	m_dot	2.945243	kg/s
32		Temp drop	delta_T	0.297437	deg.C....Ans.

Thus, temp drop = 0.297 deg.C.

==

To see Part III download
Software Solutions to Problems on Heat Transfer
Conduction – Part III