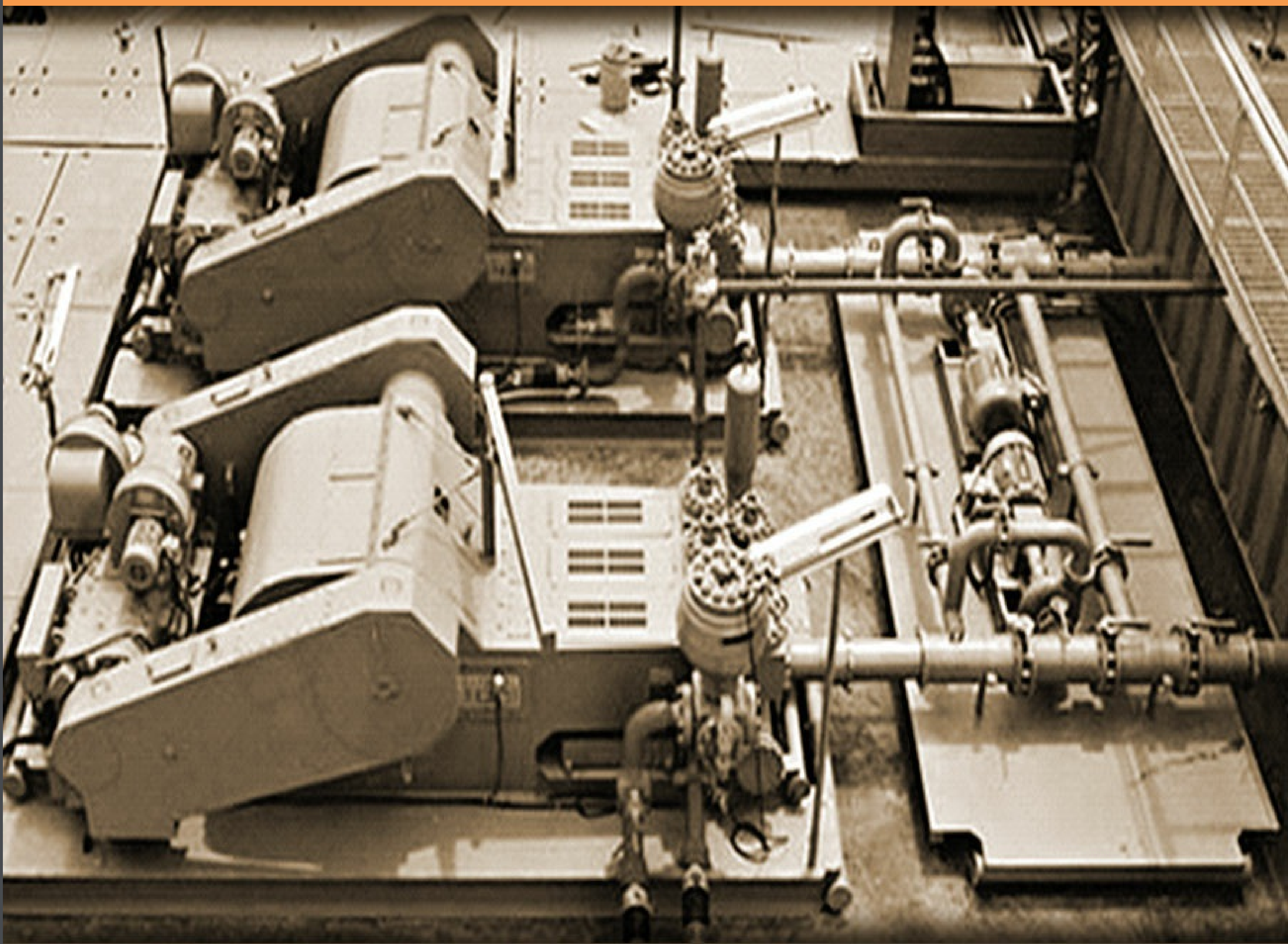
Exercises in Drilling Fluid Engineering

Pål Skalle



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Contents

Preface

These exercises have been developed to fit the content of the text book Drilling Fluid Engineering at www.bookboone.dk. The understanding of the physics and mathematics of the processes has been in focus of both the textbook and the exercises book. Many practical applications have also been created and entered into the collection of exercises. Most of the exercises have been solved and corrected by students in the corresponding course at the Department of Petroleum Engineering and Applied Geophysics at NTNU in Trondheim. If the readers have any comments that could improve the exercises, please contact me at pal.skalle@ntnu.no. Any such comments will be worked into the next year's issue of this book.

Pål Skalle
Trondheim, oktober 2015

1 Fluid Properties

We moved exercises 1.1–1.4 to other chapters.

1.5 Rheology control

a) Will YP, PV and μ_{eff} be influenced by the addition of barite?
b) Why is lye (NaOH) added to the drilling fluid?
c) Define polymers and the purpose of adding them to the drilling fluid.
d) Define pseudo plastic, thixotropic and rheopectic fluid behaviour.
e) Why does viscosity of water increase when Bentonite is added?

1.6 Rheology control

a) Explain how dispersed Bentonite is able to contain up to 18 times its own volume of distilled water. Why is it that the water-holding effect will be reduced when salt is added to the water?
b) Explain the reason behind the non-Newtonian behavior of Bentonite suspensions.

Out on a drilling rig the questions asked are of practical nature: In the upper wellbore section seawater is often used as drilling fluid. If the viscosifying effect drilled-through clay does not produce the proper viscosity, the addition of Bentonite to the water has to be considered. Use Figure 1-6 and assuming that the quality of the drilled-out clay corresponds to Premium Drilling Clay. Assume that ROP is 35 m/hr while using a 26" bit. The pump rate is 3 000 l/min. The required mud viscosity must at least be 15 cP.

c) Will the formation provide the required viscosity?
d) What is the yield (m^3 of mud/ton solids) of Wyoming Bentonite? ($\rho_{Bentonite}$ = 2.4 kg /l)
e) What is the water density (originally 1.0 kg/l) after addition of Bentonite, when an effective viscosity of 50 cP is the upper boundary?

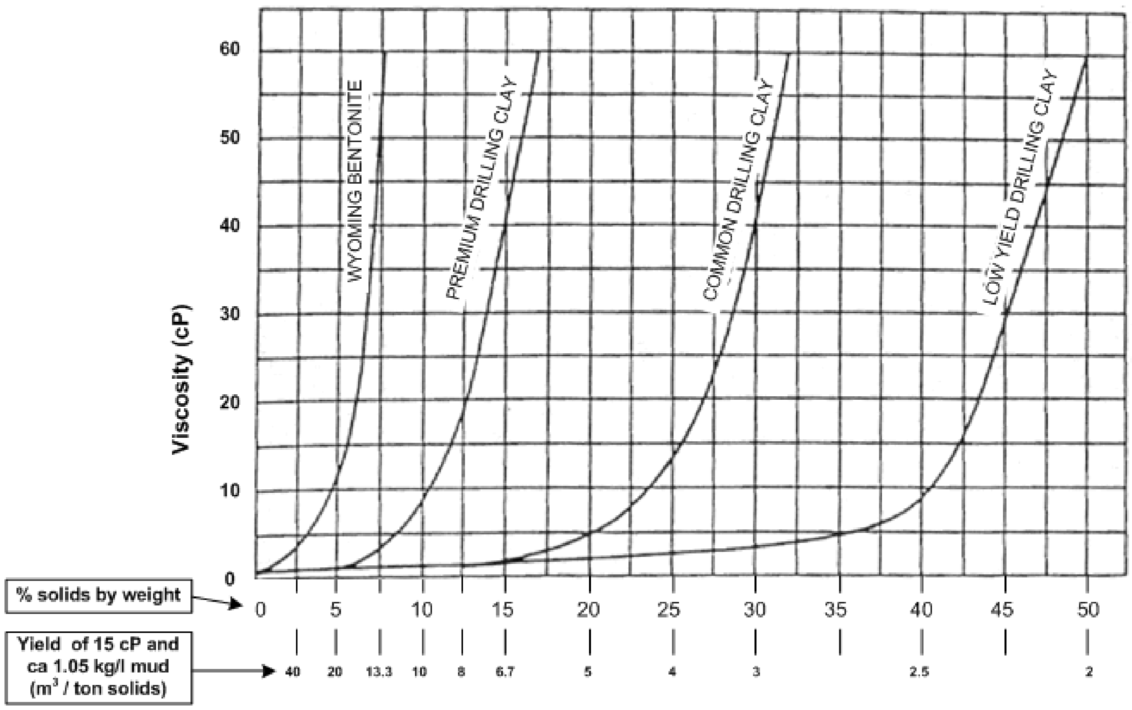


Figure 1-6: The ability of different solids to produce viscosity.

1.7 Flocculation

Bentonite and polymers are the dominating viscosity agents. Bentonite is still widely used and we have to understand its behavior properly. Bentonite behaves different from other additives; it swells and flocculates.

a) Why does Bentonite flocculate (weak flocculation)?
b) What will happen if untreated water based mud is used while drilling through the cement in the casing shoe area? Explain what happens to the mud (strong flocculation) and the respective operational consequences. Sketch the flow curve of the mud before and after having drilled through the cement.
c) Name 3 factors which enhance flocculation of Bentonite.
d) Why does WBM behave shear thinning?
e) Why does WBM behave thixotropic?
f) Explain the difference in flocculation tendency of pure edge-to-face and of cross-linking caused by external agents.

1.8 Mud contamination

a) What is a contaminated mud, and how are the drilling fluid parameters restored?

b) The geologist expected that layers of silty anhydrite would be penetrated at a vertical depth of 1600 m. The mud engineer was therefore told to make measurements every 15 min. of the returning mud as shown in the table below. After penetrating the anhydrite he observed that the viscosity of the mud, a dispersed WBM system, started to rise and became abnormally thick. Drilling continued and after a few hours of drilling/pumping, the viscosity fell back to a lower level than the original viscosity.

Parameter		Shear stress at RPM of 600	300	100	ρ	$V_{filtrate}$
Unit		($lb/100ft^2$)			kg/l	ml / 30 min
Time	0900	42	28	16	1.31	7
	0915	41	27	17	1.31	6
	0930	68	54	37	1.31	18
	1145	31	17	5	1.30	7

Together with the mud engineer you are responsible for the maintenance of the drilling fluid program. Explain changes in the recorded parameters observed at 0930 and at 1145. Suggest countermeasures against these changes.

c) Assume two clay suspensions are flocculating for two different reasons during drilling operations; 1. Edge-to-face. 2. Calcium attack. How do the two clay suspensions behave rheologically? Make a sketch of the in-situ shear stress vs. time; during drilling into the Ca^{++} containing layer. Assume you drill in the contaminated zone for 10 min. Then turn off the pump for 10 min. Continue drilling for 10 min till you are out of the contaminated zone.

1.9 Flocculation

a) Find necessary YP to keep a spherical particle suspended in a mud of ρ_{mud} = 1.1 kg/1. The particle has these characteristics:

d_p = 5 mm
ρ_p = 2.3 kg/l

Similarly, find out what is the maximum size the particles can be kept suspended at when YP = 15 Pa.

b) At 12:00 the ROP became very low and it was decided to change the bit. A 15 min. stop in the operation was made before tripping-out from 2 100 mMD was initiated. When the first pipe was broken mud spilled out on the drill floor. Could this spill have been prevented? How high up can the string be hoisted before gravity pulls the mud down in the following situation:

τ_{gel} 15 min. = 30 lb/100 ft^2 (14.4 Pa)

Inner diameter of the drill pipe = 4.127″ (104.85 mm)

1.10 Fluid additives

a) Define the different concepts and explain their relevance for drilling fluids (e.g.; viscosifyer).

Anhydrite	Lignosulphonate
Caustic soda	Lignite
CEC	MBT
Chalk	PAC
CMC	Pre hydrated
Colloid	PHPA
Dispergator	SAPP
Deflocculators	Sodium Sulphate
Gypsum	Starch
HEC	Xanthan
NaOH	

b) How does the additive called drag reducer reduce turbulent pressure so dramatically?
c) What significance does the K^+ concentration have for the shale?
d) When drilling into swelling clay and swelling shale, problems like sloughing shale and stuck pipe may occur. Explain what happens to the mud. How and why do you convert Bentonite mud into gyp mud?

1.11 Fluid additives

a) How many moles/liter of hydroxyl (OH^-) concentration is required to change the pH of a drilling fluid from 7.5 to 11?
b) How much caustic soda (weight per liter) will be required to increase the pH in question a)?
c) Why potassium hydroxide (KOH) is often preferred to sodium hydroxide (NaOH) in controlling the pH of mud?

How does drilling fluid achieves the following functions:

d) Lift cuttings from the bottom to the surface
e) Releases cuttings at the surface
f) Cools and lubricates the drill bit and the drill stem
g) Prevent blowouts

2 Rheological models

To simplify the evaluation of drilling fluids out in the field, the simplified Bingham field method was developed. In this collection of exercises we distinguish between the simplified field and the standard method of determining rheological model constants.

The simplified field method is applicable in conjunction with the Fann viscometer.

For the standard method the Fann readings are converted to SI units and multiplied with the factor 1.067 (see SPE's Applied Drilling Engineering textbook Appendix A, eqn. A-6b).

Note also that conversion factors are presented in Chapter 6 in present book.

2.1 Bingham/Power law.

In the laboratory the data in Table 2-1 were obtained (θ is the dial reading in the viscometer):

Parameter	RPM	γ	$\dot{\theta}$	τ	τ
Unit		(s^{-1})	(-)	$(lb/100ft^2)$	(Pa)
Data	600	1022	106.4	112.8	54.0
	300	511	75.0	79.5	38.1
	100	170	42.4	44.9	21.5
	6	10	10.0	10.6	5.1
	3	5	6	6.3	3.0

Table 2-1: Rheological data.

The same data are presented as a flow curve in Figure 2-2.

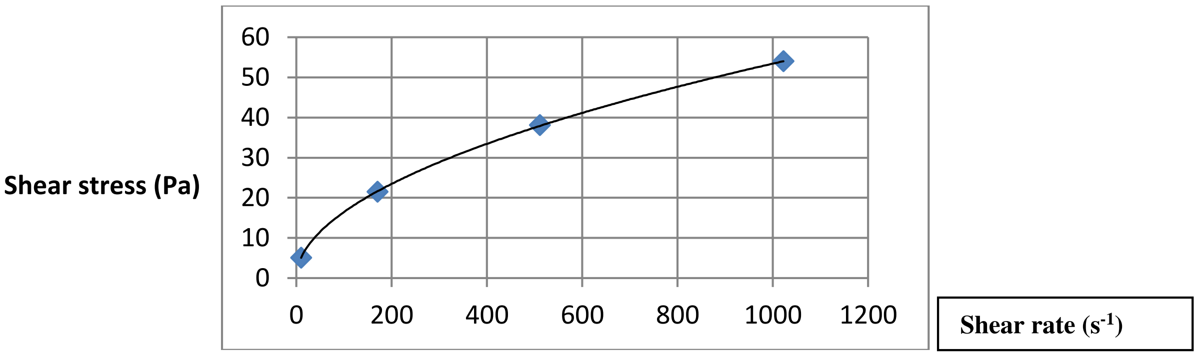


Figure 2-1: Flow curve of the rheological data from the table 2-1.

a) Find rheological constants for the two rheological models Bingham and Power-law. For Bingham model use both field and standard method.
b) Which of the two models in question a) fit the shear stress best at 100 RPM.
c) Which of the three models, Newtonian, Bingham or Power-law, would give the best answer on basis of the given rheology while pumping 1000 l/min through a pipe of 10 cm ID. Hint: Check theoretical vs. measured shear stress.

2.2 Bingham/Power-law

Rheological data are tabulated and presented graphically in Figure 2-2. Note that shear stress, τ, has been multiplied by 1.06 before converting readings, ϴ, to SI-units.

Parameter	RPM	$\dot{\gamma}$	θ	τ
Unit	-	s^{-1}	-	Pa
Data	600	1022	140	71.52
	300	511	98	50.06
	200	340		39.60
	100	170		27.39
	6			8.14
	3			6.14

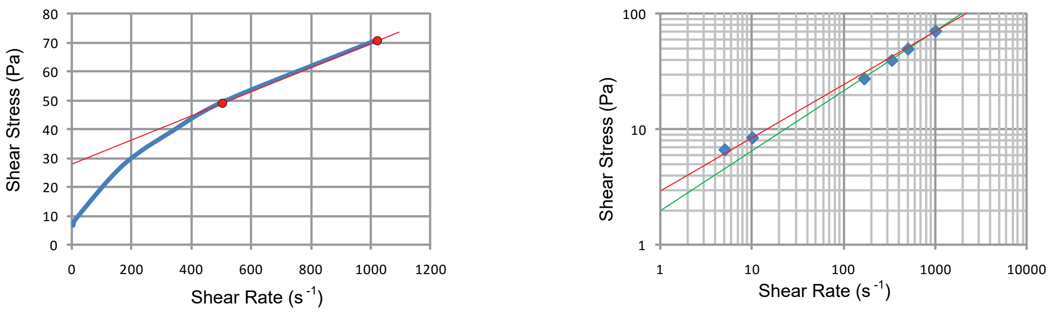


Figure 2-2: Graphical representation of rheological data (flow curve).

a) Select the best 2-data-point-rheology model, either Bingham or Power-law (at an viscometer speed of 100 RPM). Verify selection.

b) Observe the log-log plot. It represents a typical clay-dispersed system.
Why do the two data points of the lowest shear rate deviates from the straight line made on basis of the upper four data points?

2.3 Bingham/Power-law. Regression

After measuring the rheology of the fluid it is always useful to plot its flow curve. The following data are obtained:

Text	Speed		Reading	Shear stress	
Symbol		$\dot{\gamma}$	θ	τ	
Unit	(rpm)	(s^{-1})	(-)	(lbf/100 ft^2)	(Pa)
Data	600	1022	60.4	64	30.6
	300	511	39.6	42	20.2
	200	340	32.1	34	16.3
	100	170	24.5	26	12.5
	6	10	14.2	15	7.2
	3	5	9.4	10	4.8

a) Plot τ vs. γ for three rheological models Newtonian, Bingham, Power-law according to

Field procedure (2 data points) (only for Bingham)
Standard procedure (2 data points) (for all models)
Regression (6 data points). Use Excel linear regression.

Find the constants of the three models. Determine which of the models are best fitted to the readings at high shear rates, i.e. for determining pressure loss in nozzles and inside the drill string (300 rpm and higher), and what model fits best to the lower shear rates, valid for annulus or flow (100 rpm and lower).

b) An exercise without calculations: Will plug flow occur in this mud system?

2.4 Effective viscosity

a) Rheology data:

Rotational speed	Dial Reading
600 rpm	43
300 rpm	30
200 rpm	23
100 rpm	16
6 rpm	8
3 rpm	7

A useful exercise is to plot the effective viscosity (apparent Newtonian viscosity) as a function of shear rate. The non-Newtonian and the shear thinning effect will then appear clearly.

b) The mud is flowing in a 1000 m long pipe with inner diameter of 4 in, flow rate is 6 000 l/min (two pumps) and density is 1.1 kg/l. Use the Power-Law to estimate μ_{eff} at this flow rate.
c) Determine the pressure loss in the pipe (Power-law).
d) Why are drilling fluids often so well suited to the Bingham model?

2.5 All models

The flow rate is 2 500 lpm in a 1000 m long pipe with an inner diameter of 10 cm. Rheological data points are given below. The mud density is 1.1 kg/l.

Shear rate (s^{-1})	Shear stress (Pa)
1022	55
511	40
340	35
5	19

a) Find the rheological constant just for Bingham and Power-law (use only upper 2 data points).
b) For Herschel-Bulkley model, discuss three different ways of obtaining the constants without calculation.
c) Show that the effective viscosity for Bingham fluids is:

$$\mu_{eff} = \mu_{pl} + \frac{\tau_0 d}{6v}$$

d) Explain why the Bingham field model is so useful for evaluating mud behavior.

2.6 All models. Regression

Symbol	$\dot{\gamma}$	RPM	Θ	τ	τ
Unit	Hz	rpm	-	lb/100 ft^2	Pa
Data	1022	600	50	53	25.4
	511	300	34	36	17.2
	340	200	26	27.6	13.2
	170	100	17	18	8.6
	102	60	13	13.8	6.6
	51	30	9	9.5	4.5
	10	6	4	4.2	2.0
	5	3	3	3.2	1.5

a) Use the Fann-viscometer data above to determine model-constants for the first 2 models listed below through standard (2 data points) procedure.

b) Perform linear regression procedure (use Excel spread sheet). Make a plot of the two first models for 2, 4, 6 and 8 data points. Linear regression is in fact possible only for those two models. The lower three models are presented just to give an overview of rheological models.

Bingham	$\tau = \tau_y + \mu_{pl} \cdot \dot{\gamma}$
Power law	$\tau = K \cdot \dot{\gamma}^n$
Herschel Bulkley (H-B)	$\tau = \tau_y + K \dot{\gamma}^n$
Collin-Graves (C-G)	$\tau = (\tau_o + K \dot{\gamma}^n)(1-e^{-\beta\gamma})$
Robertson-Stiff (R-S)	$\tau = K(\gamma_o + \dot{\gamma})^n$
Casson	$\tau = \left[\sqrt{\tau_0} + \sqrt{\mu \cdot \dot{\gamma}}\right]$

c) To solve H-B, use two methods:
c1) Elimination and iteration.
c2) Non-linear regression.

The latter procedure is presented in Chapter 10.6 in the Drilling Fluid Engineering Text book.

2.7 All models

Parameter	Speed	Shear Stress	
Unit	Hz	lb / 100 fl^2	Pa
Data	1022	52.2	25
	511	35.5	17
	340	25.1	12
	170	13.6	6.5
	10	6.3	3.0
	5	4.2	2.0

Which of the following three models, Power-law, Bingham and Herschel-Bulkley would you select for estimating τ at $\dot{\gamma}$ = 10 Hz. Apply the two and three upper data points. Use Field and standard procedure for the Bingham 2-data point model. For the Herschel-Bulkley model, use the τ_5 reading as τ_0. K and n are found from Power-law.

$$\tau = \tau_o + \mu \cdot \dot{\gamma}$$

$$\tau = K \cdot \dot{\gamma}^n$$

$$\tau = \tau_o + K \cdot \dot{\gamma}^n$$

3 Drilling fluid dynamics

3.1 Velocity profile. Continuity equation

The incompressible steady state flow between two parallel plates with breath b in Figure 3.1 is initially uniform at the entrance; $v = \bar{v} = 8$ cm/s. Downstream the flow develops into the parabolic laminar profile $v(z) = az(z_0 - z)$, where a is constant and z_0 the plate distance. If $z_0 = 4$ cm, what is the value of v_{max}?

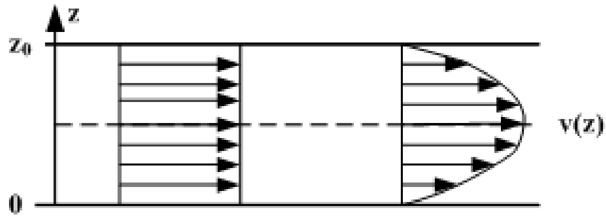


Figure 3-1: Flow data.

3.2 Velocity profile. Momentum flux

a) The fully developed laminar pipe-flow velocity profile is expressed as: $v_z(r) = v_{max}(1 - r^2/R^2)$, $v_\theta = 0$, $v_r = 0$. z indicates here the axial direction: This is an exact solution to the cylindrical Navier-Stoke equation. Neglect gravity and compute the pressure distribution in the pipe; $p(r,z)$, and the shear-stress distribution; $\tau(r,z)$, using R, v_{max} and μ as parameters. Why does the maximum shear occur at the wall?

b) For flow between parallel plates, compute b1) wall shear stress and b2) the average velocity. From the Text book, Chapter 4, we find that $v_z(y) = -dp/dz \cdot h^2/2\ \mu\ (1 - y^2/h^2)$. The most general definition of shear stress is given by: $\tau = \tau_z = \mu\left(\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right)$

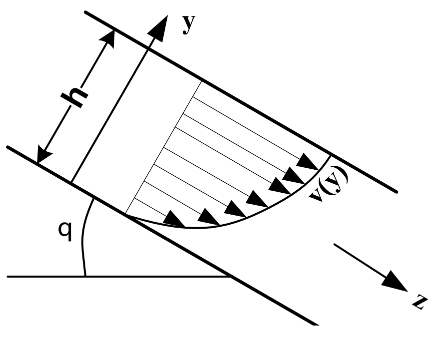


Figure 3-2: The geometry of pipe flow in the z-direction. $\theta = 0$ in this exercise.

3.3 Velocity profile

a) Discuss the meaning of this expression, its assumptions etc.

$$\frac{dp}{dx} = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\mu\,\frac{\partial v_x}{\partial r}\right)$$

b) For stationary non rotational and laminar flow of Newtonian fluids in circular horizontal pipes (z = x), show that $v(r) = \frac{dp/dx}{4\mu}\left(R^2 - r^2\right)$

c) Determine average velocity. The absolute velocity is largest in the center of the pipe. Max velocity compared to the average velocity is forming an expression of the axial dispersion when one fluid is displaced by another. Find this expression.

d) Find wall shear stress and average pipe velocity when the pressure loss is recorded to be 0.9 bar along a 1 000 m long pipe. Radius is 5 cm and viscosity 53.8 cP.

3.4a Pressure loss vs. rheology

Water, assumed incompressible, flows steadily through a pipe of constant diameter 2R. The entrance velocity is constant, $u = u_o$, and the exit velocity approximates turbulent flow, $u = u_{max}(1 - r/R)^{1/4}$. Fluid viscosity is μ. Determine the average velocity and the shear stress at the wall during turbulent flow.

3.4b Pressure loss vs. rheology

Use the viscometer readings from exercise 2.6.

a) Determine the pressure loss pr. 1000 m in a 10 cm ID pipe at a flow rate of 1000 l/min. and a fluid density of 1000 kg/m³. Use three rheological models. The observed pressure loss at these circumstances were 4.5 bar.

b) Calculate shear rates in the pipe for Bingham, Power-law and Newtonian model. Read shear stress from the flow curve, and determine pressure drop through the universal pressure loss model.

$$\Delta p = \tau_w \cdot \frac{L}{d} \cdot 4$$

c) Show that wall shear stress can be expressed as $\tau_w = \frac{1}{2} R \cdot \frac{\Delta p}{\Delta L}$

for laminar, annular pipe flow, and that $\mu_{eff} = \mu_{pl} + \frac{\tau \cdot d_{hydr}}{8 \bar{v}}$

d) What effect has entrance length of a uniform pipe on estimated pressure loss?

3.5 Pressure loss vs. rheology

Mud is pumped at a rate of 800 l/min with these rheological data:

RPM	θ	τ (lb/100 ft²)
600	66.0	70
300	47.2	50
100	25.5	27
6	9.4	10

a) Find Δp_{pipe} in a 1000 m long pipe of d = 0.109 m for a field-Bingham fluid. Fluid density is 1100 kg/m³.

b) Which rheological model, Newtonian or field-Bingham, is better suited for pressure loss estimation when the actual Δp_{pipe} was recorded to be 0.7 MPa. Apply the universal pressure loss model; $\Delta p = 4 \tau L / d$.

3.6 Pressure loss. Power-law

a) Derive the laminar pressure loss expression in pipes for Power Law fluids from the force balance.
b) Show that:

$$\frac{\rho v\, d}{\mu_{eff}} = N_{Re_{generalized}}$$

when μ_{eff} for a Power-law fluid is applied.
c) Show that the Reynolds number for a Power-law fluid increases as the inner pipe diameter of the annulus decreases, while for a Newtonian it decreases. Apply annular diameter in terms of $d_{hydr} = d_o - d_i$, and let flow rate be constant.

3.7 Pressure loss. Turbulent. Energy equation

Oil of the density ρ = 900 kg/m³ and a kinematic viscosity ν = 0.00001 m²/s, flows at a rate of 0.2 m³/s through a 500 m new cast-iron pipe with a diameter of 200 mm and a roughness of 0.26 mm. Determine the head loss.

3.8 Pressure loss vs. flow rate

Make a graph of pressure loss vs. flow rate in a 1000 m long pipe with inner diameter of 10 cm. Rheological data points are given in Exercise 2.6, and the mud density is 1.1 kg/l. Select the Power law model.

3.9 Pressure loss. Use field data to evaluate model

Prior to a pre flush/cementing operation the driller performed pressure tests to verify theoretical pressure estimations. Previously comparisons between the actual pressure readings during drilling with theoretically estimated pressure loss resulted in large derivations. He was convinced that the derivations could be back-tracked to the pressure drop across the mud motor and the bit. These two losses can only be estimated by means of empirical models and are thus highly uncertain. Now he had the chance to record pressure loss without these two disturbing pieces of equipment installed. After lowering the 5″ * 4.127″ DP (without the bit) down to the casing shoe he circulated for 45 min. to neutralize temperature effects. The casing, a 13⅜″, 68 lb/ft, (ID=12.40″) had its casing shoe at 4 500 mMD. The pump was a relatively new (volumetric efficiency = 0.96) Garden-Denver PZ-11-1600 HP triplex mud pump, 6″ liner. One complete pump stroke delivered 15.29 l. At the following pump speeds, with no drill string rotation, he read the average stand pipe pressures (SPP) which was the average of 3 tests:

5 spm – 13 (+/-2) bars
20 spm – 70 (+/-3) bars
50 spm – 100 (+/-5) bars
100 spm – 200 (+/-10) bars

Rheology of drilling fluid at this temperature is identified with the one in Task 2-1. Apply the Power -aw model. Mud density was 1.21 kg/l. The recorded pressure losses are presented in Figure 3-10. Estimate pressure losses and compare them with the recorded ones.

3.10 Pressure loss. Effect of rotation

In this task you need to use your imagination. The driller made now an additional test: At the lowest and the highest pump speeds [5 and 100 SPM from Task 3.9 above] he rotated the drill string at 100 RPM for a short time and saw that the average reading changed to 8 and 248 bars respectively at the two selected pump speeds. Determine the effect of rotation on the mud's rheology for the given flow rate (the rheology is obviously dictating the annular pressure loss). Assume that the rotational movement is additive to the axial flow with respect to shearing effect on the fluid. When determining the shear effect, simply use the average rotational velocity across the annular gap. Assume also that the drilling fluid is the same as in task 3.9, a Power-law model with n = 0.5, K = 1.63.

Change or remove; too many assumptions

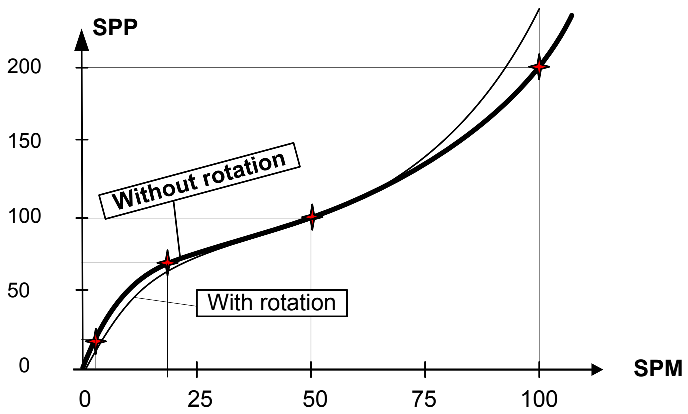


Figure 3-10: Recorded SPP (thick line), with rotation (thin line).

3.11 Pressure loss. Nozzles. OFU

After having drilled a 17.5″ hole the 13⅜″ casing was set and cemented at 2 300 mMD. Finally a 12¼″ hole was drilled down to the reservoir at 2 500 m.

The drill string consisted of a drill pipe (54.276″) and 100 m of drill collars (6.25 · 3.1″). The circulating rate was 800 GPM during drilling and the mud density was 12.9 PPG. A Fann-VG viscometer gave the following readings:

$\theta_{300} = 50$

$\theta_{600} = 85$

Assume the mud rheology is best described through the Bingham model. OFU-equations are copied from Applied Drilling Engineering SPE-text book Table 4.6:

$$N_{\mathrm{Re}} = 928\,\rho \bar{v} d_h / \mu_{eff} \qquad \bar{v} = q / (2.448\, d_h^2)$$

$$\Delta p_{lam} / \Delta l = \frac{\mu_{pl} \bar{v}}{1500 d^2} + \frac{\tau_y}{225 d}$$

$$\Delta p_{turb} / \Delta l = \frac{\rho^{0.75} \mu_{pl}^{0.25} \bar{v}^{1.75}}{1800 d^{1.25}}$$

$$\Delta p_{\mathrm{bit}} = \frac{8.311 \cdot 10^{-5} \cdot \rho \cdot q^2}{C_d^2 \cdot A_t^2} [\mathrm{psi}]$$

The OFU units are:

ρ	PPG
q	GPM
A	in^2
p	psi
C_d	0.95

a) The pressure drop through the annulus above the BHA was equal to100 psi, but must be calculated along the BHA. What is the equivalent circulating density at a depth of 2 500 m?
b) The bit had 5 nozzles, each of a diameter og 14/32 inches. Pressure loss through the surface pipes was 200 psi. What pressure is required from the pump when drilling at a depth of 2 500 m?. Compare the results with bit pressure loss estimated in SI-units

$$\Delta p_{bit} = 1.11 \cdot \frac{1}{2} \rho v_{Av}^2$$

c) Determine the pressure loss in the complete circulation system

3.12 Swab pressure. Cling factor

Due to the no-slip conditions on all surfaces, the mud will also cling to the drill string. The cling factor is used during estimation of surge & swab pressure. How would you, in a stepwise fashion, go about to define and estimate the cling factor?

3.13 Swab pressure model

Assume you are tripping out while simultaneously pumping. Your task is to start the process of derivation, which later, will lead to an expression of surge pressure during laminar flow. When making a drawing of the process, use parameters like v_p (pipe), q_p (pump), R_w (wellbore), R_p (pipe), R_0 (the point where the flow velocity is zero), etc, as required for your explanation.

4 Hydraulic program

The exercises which are related to the hydraulic program distinguish between two different approaches of preparing the hydraulic program:

a) The standard method: Each liner represents the pump's capability. The hydraulic program is planned for one well section at a time.
b) The extended method: All the piston sizes are treated as part of one process, and they are divided into two operating ranges. The hydraulic program is planned for all sections in one common operation.

Both the methods are based on maximizing the ROP, which (in this book) is expressed through the following equation:

$$ROP = A^* (q / d_{nozzle})^{a8}$$

4.1 Mud pump issues

a) Characterize a mud pump as detailed as possible with respect to
  - Effect
  - Efficiency
b) Why are several mud pumps sometimes arranged in parallel or in series?
c) Compare centrifugal with piston pumps
d) Explain the term hydraulic knocking in pumps
e) A tri-cone bit has 3 nozzles; each nozzle is 15/32nd inch in diameter; ρ_{mud} = 1.3 kg/l; drilling at a depth of 2 500 mMD. At two pump rates (which are close to the actual operating flow rates), the following pressures (stand-pipe pressures) were recorded:

q_{pump} (lpm)	p_p (bar)
2 000	230
670	33

Determine the value of K_1 and m in the expression of the parasitic pressure in an oil well.

4.2 Optimal nozzles? Section wise

A tricone bit is equipped with 3 · 15/32″ nozzles. The following data are given:

$p_{p\,1}$ = 186 bar at 2 000 l/min (p is measured at the standpipe)
$p_{p\,2}$ = 120 bar at 1 750 l/min
p_{bit} = 1.11 · 1/2 ρv^2_{nozzle}
$p_{pumpe,max}$ = 270 bar for piston in use
$q_{pumpe,max}$ = 2 700 l/min for piston in use
$q_{min.ann}$ = 1 700 l/min (below this value cuttings will accumulate)
$q_{max.ann}$ = 2 600 l/min (above this value the wellbore adjacent the BHA will start to erode)
D = 3 000 m
ρ_{mud} = 1 400 kg/m^3
$q_{opt} = \left[\frac{2p_{pump.\max}}{K_1 D(m+2)}\right]^{1/m}$

a) Determine the two constants in the parasitic pressure loss equation.
b) Determine optimal flow rate at 3 000 m depth.
c) What are the optimal nozzle size at 3 000 m? (in terms of x/32 inch).

4.3 Liner selection. Section wise

The operating data of a National 12-P-160 (this number indicates a1600 HP pump) triplex pump are presented in this book's Chapter 6 – Supportive Information.

D = 2 500 m
ρ = 1 200 kg/m^3
$K_1 = 1.6 \cdot 10^6$
$m = 1.7$
$q_r = 0.018\ m^3/s \left(d_{bit} = 12\frac{1}{4}''\right)$
$q_{\max}(turb) = 0.04\,m^3/s$

a) Derive an expression of q_{opt} and determine numerically the optimal liner at this depth?
b) Select the most optimum liner at this depth
c) Determine the optimal bits pressure when the 6″ liner is used
d) At what depth would you change from 6″ to 5¾″ liners?

4.4 Hydraulic program. Section wise (i.e. all liners are treated as if in range I)

a) Assume the rate of penetration (ROP) is a function of bottom hole cleaning:

$$ROP = A \cdot \left(\frac{q}{d_e}\right)^{a_8}$$

Show that ROP will decrease with depth when the pump pressure is expressed through the equations below:

$$p_{pump} = p_{loss} + p_{bit}$$

$$p_{loss} = K_1 D q^m$$

$$p_{bit} = 1.11 \frac{1}{2} \rho \bar{v}^{\,2}$$

b) The 12¼″ section starts at the 13⅜″ casing shoe at 1950 m MD, and is planned to reach a depth of 4000 mMD before setting the next casing. The following is known:

K_1	=	$2 \cdot 10^6$
m	=	1.65
$q_{r,\ vertical\ \ 1}$	=	0.025 m^3/s
$q_{r,\ horiz.}$	=	0.040 m^3/s
$q_{max,\ vertical}$	=	0.035 m^3/s
$q_{max,\ horiz.}$	=	0.045 m^3/s

What flow rate and liners size would you recommend through this depth interval when drilling either a vertical well or a horizontal well? Use the 1600 HP pump as defined in Supportive Information.

4.5 Optimal parameters for BHHP. OFU. Section wise

This exercise includes Oil Field Units, just to indicate for you how much simpler it is to work with SI units.

The bit has 3 · 12/32″ nozzles, and ρ_{mud} = 10 PPG while drilling at 8 200 ftMD. The following pump rates and pump pressures (which are within the operating flow rates q_r = 240 GPM) were recorded while the bit was close to the bottom of the well:

q_{pump} (GPM)	p_p (psi)
500	3000
250	800

The pump is characterized through:

p_{max} = 3620 psi, $E_{p,max}$ = 1000 Hp

Determine optimal pump rate and nozzle size when applying Bit Hydraulic HP (BHHP) as the optimization criteria. Assume that the rate of penetration is linearly related to it. The pump volume efficiency is 0.9.

$$BHHP = \frac{\Delta p_{bit} \cdot q}{1714}\ (H_p)$$

Pressure drop in OFU are ($\rho(PPG),\ q\ (GPM), A(in^2), C_d = 0.952$)

$$\Delta p_{bit} = 8.311 \cdot 10^{-5} \cdot \rho\ [q/(C_d \cdot A_{nozzle})]^2$$

4.6 Liner selection. Complete well

The characteristics of a 1600 HP piston pump are found in Supportive Information, while the optimal operational functions are presented graphically in Figure 4-6. The optimal pump pressure values are in general the maximum ones, given for each liner size. Maximum values are the recommended values in the liner table, which in fact are around 85% of the absolute maximum. In Figure 4-6 the flow rates are in correct scale, while R_p-values are only qualitative. Hydraulic parameters, the bit program and minimum and maximum flow rates are presented below and in Table 4-6:

$K_{1 \text{ in range II}} = 1.80 \cdot 10^6$, $m_{II} = 1.6$
$K_{1 \text{ in range I}} = 2.20 \cdot 10^6$, $m_I = 1.5$

Bit diameter	Start depth	Permissible annular flow rate (m3/s)	
In	m	min	max
36	0	0.035	0.050
26	100	0.030	0.045
17 ½	500	0.027	0.035
12 ¼	1500	0.015	0.030
8 ½	3000	0.010	0.020

Table 4-6: Bit program and flow rate ranges

a) Operating range I is defined as the pump operating range of the smallest liner, range II is defined by the remaining liners. Derive optimal flow rate, $q_{opt\,II}$, in pump area II by maximizing the ROP:

$$q_{optII} = \left[\frac{(E_p)_{\max}}{K_1 D(m+2)}\right]^{\frac{1}{m+1}}$$

b) Determine at which depth Range II stops (while drilling downwards) and at what depth the maximum flow rate turns into the theoretical optimum flow rate of Range I.

c) Draw also into the graph the optimal hydraulic program (the graph is a principal drawing of ROP vs. q and thus not a quantitatively correct drawing).

d) What pump rate is optimum at 2 000 m.

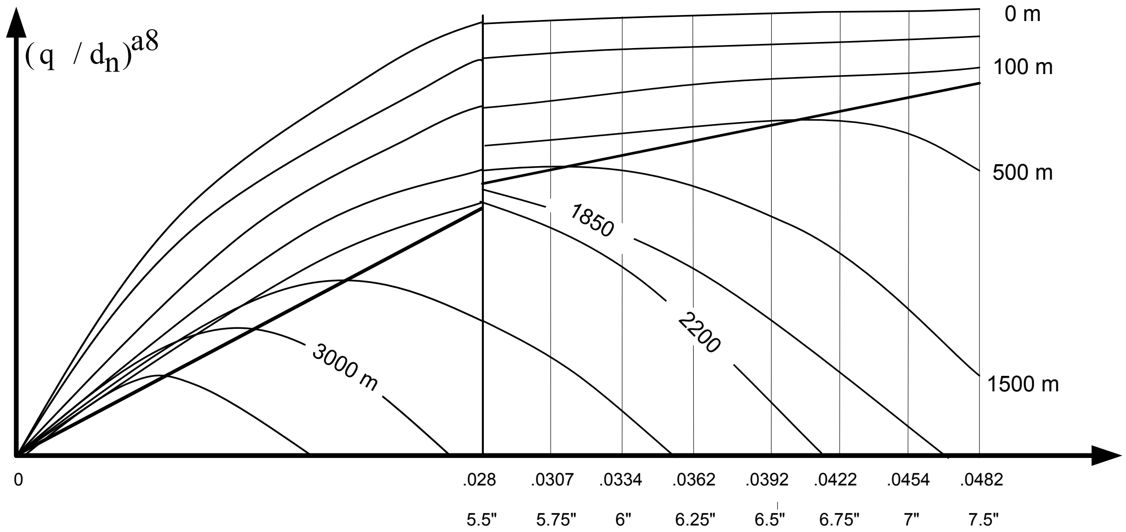


Figure 4-6: Hydraulic Data.

4.7 Liner selection. Complete well

A 1660 HP pump is used (see supportive info).

a) Find at what depths the transition from operating area II to I occur. A 1660 HP pump is used (see supportive info. K_1 and m are the same for both ranges.)

$K_1 = 1.72 \cdot 10^6$

$m = 1.51$

b) Make a flow chart of a computer program of how to determine when to change from working area II to I during drilling.

5 Well challenges

5.1 Filtration control

a) How can we plan the mud composition in WBM to minimize the fluid loss through the filter cake?
b) If the fluid loss shows an increasing tendency during drilling, how is it detected and how is the problem treated?
c) Two sand formations of nearly equal pore pressure are encountered. Will filtrate invasion be greater in the sand of high permeability and high porosity, than in one with porosity? The final filter cake permeability in both sands is assumed to end up at around 10^{-3} mD.
d) A reduction of water flow into shale is beneficial because this will reduce unwanted reaction between the drilling fluid's water phase and shale further away from the wall, where the water activity of the pore water may be different. How can the water-flow into clay be controlled?

5.2 Filtration control

We want to obtain a physical picture of how far the filtrate and the particles penetrate a porous formation and gradually stops due to a tight filter cake. A 15″ hole with open hole length of 4 000 ft is being drilled. The bottom 10% of the borehole length is of porous formation (defining the filter area) with a porosity of 15%. Assume that this porosity corresponds to the porosity of the filter paper. The filter area A of the filter press is 45 cm^2 (r = 3.9 cm.).

A laboratory test of the mud showed an API water loss of 25 ml/30 min. The cumulative loss is proportional to the square root of time. Assume therefore that the accumulative fluid loss V_f is expresses as:

$$V_f = A \cdot \sqrt{\frac{k \cdot \Delta p}{\mu}} \cdot \sqrt{t} = A \cdot C \cdot \sqrt{t}$$

We assume the parameters defined by C are constants.

a) To simplify the fluid loss estimation imagine that the time of drilling the well is negligible. Construct a plot of filtration loss vs. time. Estimate the fluid loss after 24 hours.
b) Calculate the radius of the invaded zone after 24 hours, assuming 100% displacement of the pore fluid.
c) We want to minimize the fluid loss to porous formation during overbalanced drilling, both with OBM and WBM.
   How would you specify the drilling fluid (focus on the part related to filtrate loss)?
   How would you follow up filtration control during the drilling phase?
   Why is filtration control important?
   The lab filtration showed an increasing tendency during drilling. Why?

5.3 Cuttings concentration

A horizontal section has been drilled at more or less constant ROP and flow rate. The cuttings concentration generated at the bit during drilling is c_1 = 0.02. Discuss what could be the concentration at these positions:

a) At the end of the horizontal section (= c_2)
b) At the surface, when the mud is entering the return flow line (= c_3)
c) What determines the cuttings bed height in the horizontal section?
d) Why is cuttings accumulation in wellbore expansions (washouts) a problem during tripping?
e) Mention 5 downhole problems related to poor solids control during drilling, and explain why or how poor solids control is the cause behind the problems.

5.4 Cuttings concentration

a) What forces and mechanisms are involved when cuttings are transported in horizontal wellbores?

b) Slip velocity of perfect spheres is

$$V_{settling} = \frac{d^2{}_{cutting} . g\left(\rho_{cutting} - \rho_{mud}\right)}{6\pi\mu} f(c)$$

What is the meaning of f(c) in the given equation? Present a graphical representation of f(c) vs. particle concentration.

c) Figure out with level and mentioning the forces are involved in cutting transportation in high deviation wellbore.

d) Cleaning of horizontal wells is a challenge. Your task is to:

- Explain the principles of how cleaning works and which processes and parameters are involved.
- Why does the drill string RPM needs to be > 120 RPM before the cleaning process become really efficient?

5.5 Density control

a) Define a weighted mud system (as opposed to an un-weighted)? Explain how to clean weighted muds.
b) Derive a simple formula of necessary volume increase, ΔV_{add}, involving weight material with density ρ_{add} (4.3 kg/l) to increase the density from ρ_1 to ρ_2. Use this formula to estimate how much mass of barite must be added to increase mud density from 1.3 to 1.4 kg/l. Original mud volume was 60 m^3.
c) 1 m^3 of mud has a density of 1.5 kg/l. Adjust the mud density to 1.72 by adding 100 l mud of density 1.8, 40 kg Bentonite (to adjust rheology) of density 2.3 kg/l and barite of density 4.2 kg/l. Find how much Barite of density 4.3 kg/l is needed to obtain a mud density of 1.72 kg/l, all ingrediences added together simultaneously.
d) Different water based muds defined below are stored in three different mud pits. All 3 pits should be mixed into one tank and water added until the density becomes 1.55. How much volume of water must be added?

V_1 = 10 m^3, ρ_1 = 1.5 kg/l
V_2 = 20 m^3, ρ_2 = 1.6 kg/l
V_3 = 3 m^3, ρ_3 = 1.9 kg/l

5.6 Density control

Sometimes the mud viscosity increases unintentionally due to accumulation of fines in the mud. These fines are referred to as Low Gravity Solids Content (LGSC). The fines are too fine to be removed by the cleaning equipment. Typical density of LGS is 2.4 kg/l. They are inert, but builds viscosity since particle size is small (< 5 μ). Their unwanted effect can be reduced by diluting the mud with water. Here follows 3 examples:

a) The mud volume is 100 m^3 with a density 1.8 kg/l. The fraction of low gravity solids is too high, 5 weight %, and has to be decreased to 3% by water addition. Calculate the mud volume to be discarded and the amounts of fresh water and Barite that should be added. The original volume and density has to be unchanged.
b) A tank containing 90 m^3 of mud has a density of 1.6 kg/l and should be increased to 1.7 kg/l. The volume fraction of low-gravity solids must first be reduced from 0.055 to 0.030 by water dilution. It is required that you first discard a part of the original mud volume, so that after adding of water the volume of the mud is 90 m^3 before the barite is added. How much barite must be added, and what exactly is the new LGSC?
c) Calculate the volumes of old mud (< 10 m^3) and barite that has to be mixed in order to fill a 10 m^3 large pit with mud which must balance a pore pressure of 410 bar in a depth of 3000 m. Barite has a density of 4.3 kg/l. The density of the old mud is 1.2 kg/l.

5.7 ECD. Barite

Explain the reasons behind and suggest potential solution to the following problem:

This well was drilled with WBM, weighted by Barite to 15 PPG. While POOH to change the 12¼″ bit, the driller experienced no problems. When GIH with the new bit some weight reductions (took weight) were experienced in the build-up zone. It took around 5 h from the bit left the bottom of the well till it returned. After the bit reached the bottom, the well was circulated for some time, and the returning mud behaved strangely. The mud weight, which originally was 15 PPG displayed an initial sharp decrease, then increased again as shown in Figure 5-3, before finally stabilizing at 15 PPG.

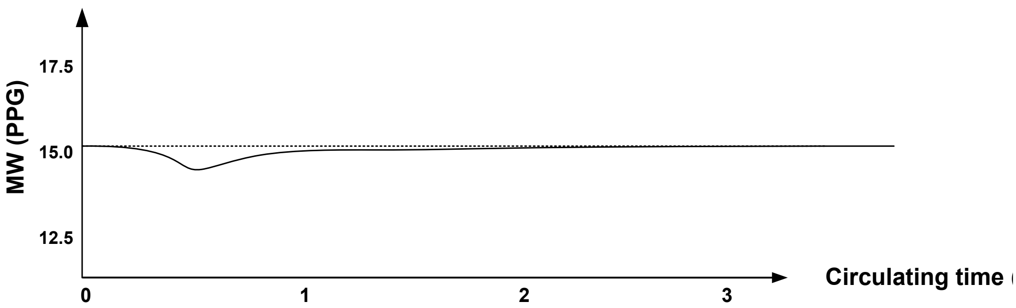


Figure 5-3: Density of returning mud after tripping.

5.8 ECD. Fluid and flow

A 17½″ hole was drilled from the 20″ casing shoe at 1100 mTVD to 2 100 mTVD. The bottom hole assembly consisted of 120 m of 9½″ Drill Collars (DC). A 5½″ drillpipe (DP) was used.

Capacities:

- 17½″ open hole capacity: 155.2 l/m
- DC / Open hole capacity: 109.4 l/m
- DP Open hole capacity: 139.2 l/m
- DP / Casing capacity: 161.8 l/m
- 9½″ DC / capacity: 4.56 l/m
- 5½″ DP capacity 10.77 l/m

Mud Parameters:

• Mud density:		1.25 kg/l
• Rheology:	600 / 300 rpm:	51.7 / 30.6 Pa
	200 / 100 rpm:	22 / 12 Pa
	6 / 3 rpm:	3 / 4 Pa
	Gel: 10s / 10 min:	5 / 13 Pa

a) Prior to drilling, the hole was circulated at 3 500 l/min. What is the annular pressure loss, and what is the corresponding ECD?

b) The drilling commenced from 2 100 m. The same rheology and flow rate was applied. At 2 300 m the average drilling rate was 50 m/hr. Formation bulk density was 2.4 kg/liter. What is the ECD in this situation? Transport ratio is 0.75. At the casing shoe at 1 800 m TVD the formation fracture pressure was 228 bar. Check if everything is OK.

c) A new mud was being prepared for the 8½″ section. The 13⅜″ csg shoe was located at 15 000 ft vertical depth. While drilling at 16 500 ft the well started losing mud and it was decided to lower the MW from 15 to 14 PPG. The well had very narrow pressure window, and the equivalent pore pressure gradient at this depth was 13.5 PPG. The mud was mixed to 14 PPG with an effective viscosity of 40 and 30 cP at 600 and 300 RPM respectively, at an average surface mud temperature of 40°C. After the new mud was circulated, the pump was shut off, and a flow-check indicated that the well was dead (no influx). While repairing the power swivel the well started to flow by itself, and soon afterwards the kicking well had to be shut in to avoid a complete unloading.

How would you go about to estimate the pressure profile of an initially cold, static fluid column as a function of time. You are asked to present the governing equations of heat transfer in a well on differential form, and work out a flow sheet of how to numerically solve this task.

5.9 Water activity

a) Define water activity, A_w and how to determine A_w in a a) salt water solutions and in b) pore water in shale?
b) Explain why water activity is a function of water salinity?
c) Explain why high water activity causes clay swelling problems?
d) How do you prevent clay swelling problem?
e) How does water activity of the water phase in OBM influence wellbore stability?

5.10 Shale stability

In order to avoid that water enters and causes the shale to swell, the activity of the water in the mud (including the water phase in oil based mud) and in the shale must be equal.

The activity of the water in the water phase in shale cuttings is measured in the field using an electro hygrometer. The probe of the electro hygrometer is placed in the vapor above the sample being tested. The electrical resistance of the probe is sensitive to the amount of water vapor present. Since the test always is conducted at atmospheric pressure, the water vapor pressure is directly proportional to the volume fraction of water in the air/water vapor mixture. The instrument is normally calibrated with saturated solutions of known activity shown in Table 5-6.

Salt	Activity
$ZnCl_2$	0.10
$CaCl_2$	0.30
$MgCl_2$	0.33
$Ca(NO_3)_2$	0.51
NaCl	0.75
$(NH_4)_2SO_4$	0.80
Pure water	1.00

Table 5-6: Saturated solutions of different salts and its vapor's water activity

Sodium chloride and calcium chloride are the salts mostly used to alter the activity of the water in the mud. Calcium chloride is quite soluble, allowing the activity to be varied over a wide range. In addition, it is relatively inexpensive. The resulting water activity for various concentrations of NaCl and $CaCl_2$ are shown in Fig. 5-6.

a) The activity of a sample of shale cuttings drilled with OBM (no foreign fluid invasion) is determined to be 0.69 by an electro hygrometer. Determine the concentration of calcium chloride needed in the water phase of the mud in order to have the activity of the mud equal to the activity of the shale.

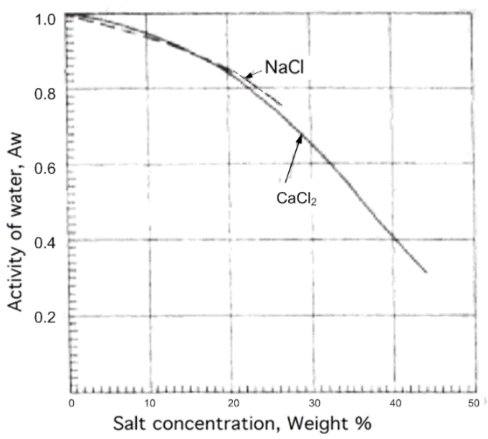


Figure 5-6: Water activity in calcium chloride and sodium chloride at room temperature.

b) A core is taken from a swelling formation. Can you retrieve any useful information from its specific weight, useful with respect to avoid swelling while drilling through it?
c) Explain why wellbores and cuttings stability is so much better when applying OBM instead of WBM. As part of the answer, please explain the principal function of the two different surface active additives that are always added to Oil based mud.

5.11 Shale stability

a) When drilling into swelling clay, problems like sloughing (soft) shale and stuck pipe may occur. Explain how/why this can be avoided by means of the proper oil based drilling fluid and specify the ingredients in the drilling fluid.
b) Why is KOH preferred over NaOH?
c) What significance does the K^+ concentration have for the shale?
d) Can wate flow through shale be controlled?
e) Explain the principal function of the two different surface active additives that are always added to Oil based mud.
f) How is the salt concentration in the water phase, which is added to Oil Based Mud, determined?

While drilling in the 8½″ section, at a depth of 1 500 mTVD / 6 000 mMD, in overbalance, the ECD will fluctuate and at times be high in this long well. Previous experience from that area indicates that instable, swellable shale will be penetrated. Your task now is the following:

g) Define what wellbore stability-related processes may take place in the shale while drilling through it with WBM.
h) Which type of inhibitive mud will you suggest in order to maximize wellbore stability? Explain how this mud type will affect the wellbore.
i) Does fluctuating ECD have any implications for the stability of the wellbore?

5.12 Wellbore problem

a) What are the dominating mechanisms or factors leading to mechanically stuck pipe. Explain the mechanisms of differential sticking in porous/permeable formation.
b) What are the consequences of stuck and how do you suggest combating the problem?
c) What is the most likely stuck pipe mechanism while drilling in salt formations? In the case of presence of halite type salts, what type of mud would you select for safe drilling in such salt section.
d) Wellbore breathing (ballooning) and Seepage losses. Include the headings; Definition, Explanation, Detection; Repair activity. Discuss the two phenomena.

6 Supportive Information

6.1 Pump (National 12-P-160) and hydraulic program data

Line size	in	5 ½	5 ¾	6	6 ¼	6 ½	6 ¾	7	7 ½
Discharge pressure	Psi 10^5·Pa	5555 383.0	5085 350.6	4670 322.0	4305 296.8	3980 274.4	3690 254.4	3430 236.5	3200 220.2
Pump rated at 120 spm	GPM m^3/s	444 0.0280	486 0.0307	529 .0334	574 .0362	621 .0392	669 .0422	720 .0454	772 .0482
HP		1439.0	1441.8	1441.3	1441.7	1442.0	1440.3	1440.8	1441

Power of Efficiency = 1441.7 · 745.7 = $1.0748 \cdot 10^6$ or $322 \cdot 10^5 \cdot 0.0334 = 1.0755 \cdot 10^6$ (watt)

$$ROP_{q/d_n} = C \cdot (q/d_n)^{a_8}$$

$$ROP_{BHHP} = C \cdot (BHHP)^{a_8} \qquad BHHP = \Delta p_{bit} \cdot q$$

$$P_p = q \cdot p_p \text{ (Power)}$$

$$p_p = \Delta p_{bit} + \Delta p_d$$

$$\Delta p_d = K_1 \cdot D \cdot q^m$$

$$p_{bit} = 1.11 \frac{1}{2} \rho v^2$$

$$q_{opt_I - q/d_n} = \left(\frac{2 p_p}{(m+2) K_1 D} \right)^{\frac{1}{m}}$$

$$q_{opt_{II} - q/d_n} = \left[\frac{(E_p)_{\max}}{K_1 \cdot D (m+2)} \right]^{\frac{1}{m+1}}$$

6.2 Pressure loss equations

	Newtonian fluid	Bingham model	Power law model
Lam/pipe	$\Delta p_p = \frac{32\,\bar{v}\,\mu\,L}{d^2}$	$\Delta p_p = \frac{32\,\mu_{pl}\cdot L\cdot\bar{v}}{d^2} + \frac{16\,L\tau_o}{3d}$	$\Delta p_p = 4K\left(\frac{8\bar{v}}{d}\cdot\frac{3n+1}{4n}\right)^n\cdot\frac{L}{d}$
Lam/annulus	$\Delta p_a = \frac{48\,\bar{v}\,\mu\,L}{(d_o - d_i)^2}$	$\Delta p_a = \frac{48\,\mu_{pl}\cdot L\cdot\bar{v}}{(d_o - d_i)^2} + \frac{6\,L\tau_o}{d_o - d_i}$	$\Delta p_a = 4K\left(\frac{12\bar{v}}{d_o - d_i}\cdot\frac{2n+1}{3n}\right)^n\cdot\frac{L}{d_o - d_i}$
Turb/pipe/ann	$\Delta p = \frac{0.092\rho_m^{0.8}\;\bar{v}^{1.8}\;\mu^{0.2}\;L}{d_h^{1.2}}$	$\Delta p = \frac{0.073\,\rho_m^{0.8}\cdot\bar{v}^{1.8}\cdot\mu_{pl}^{0.2}\cdot L}{d_h^{1.2}}$	$\Delta p = a\cdot N_{\mathrm{Re}}^{-b}\cdot\frac{4L}{d_h}\cdot\frac{1}{2}\rho\bar{v}^2$ $a = (\log n + 3.93)/50$ $b = (1.75 - \log n)/7$
Eff. visc. pipe	$\mu_{eff} = \tau/\gamma$	$\mu_{eff} = \mu_{pl} + \frac{\tau_o d}{6\bar{v}}$	$\mu_{eff} = \left(\frac{8\bar{v}}{d}\cdot\frac{3n+1}{4n}\right)^n\cdot\frac{Kd}{8\bar{v}}$
Eff. visc. ann	$\mu_{eff} = \tau/\gamma$	$\mu_{eff} = \mu_{pl} + \frac{\tau_o(d_o - d_i)}{8\bar{v}}$	$\mu_{eff} = \left(\frac{12\bar{v}}{d_h}\cdot\frac{2n+1}{3n}\right)^n\cdot\frac{Kd_h}{12\bar{v}}$
Shear-r. pipe	$\dot{\gamma} = \frac{8\bar{v}}{d}$	$\dot{\gamma} = \frac{8\bar{v}}{d} + \frac{\tau_o}{3\,\mu_{pl}}$	$\dot{\gamma} = \left(\frac{8\bar{v}}{d}\cdot\frac{3n+1}{4n}\right)$
Shear-r. ann.	$\dot{\gamma} = \frac{12\bar{v}}{d_y - d_i}$	$\dot{\gamma} = \frac{12\bar{v}}{d_o - d_i} + \frac{\tau_o}{2\mu_{pl}}$	$\dot{\gamma} = \left(\frac{12\bar{v}}{d_o - d_i}\cdot\frac{2n+1}{3n}\right)$
General $N_{re,pipe}$	$N_{\mathrm{Re}} = \frac{d^n\cdot\bar{v}^{2-n}\cdot\rho}{K_p\cdot(8^{n-1})}$	$K_p = K\cdot\left(\frac{3n+1}{4n}\right)^n$	*Fanning* $f_{lam} = 16/N_{re}$
General $N_{re,ann}$	$N_{\mathrm{Re}} = \frac{d^n\cdot\bar{v}^{2-n}\cdot\rho}{K_a\cdot(12^{n-1})}$	$K_a = K\cdot\left(\frac{2n+1}{3n}\right)^n$	*Fanning* $f_{lam} = 24/N_{re}$

Continuity equation: $-\frac{\partial \rho}{\partial t} = \nabla \cdot (\rho v)$

Microscopic Cylindrical coordinates $-\frac{\partial \rho}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho w)$

Macroscopic $\frac{d\int v\rho dV}{dt} = -\Delta \rho \bar{v} A = \rho_1 \bar{v}_1 A_1 - \rho_2 \bar{v}_2 A_2$

Momentum equation $\rho \frac{Dv}{Dt} = \rho g - \nabla p - \nabla \cdot \tau$

Microscopic Cylindrical coordinates (only the r-component)

$$\rho\left(\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_z)\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Macroscopic $\frac{d}{dt}\int \rho dV = \rho_1 \bar{v}_1^2 A_1 - \rho_2 \bar{v}_2^2 A_2 + p_1 A_1 - p_2 A_2 - F + Mg$

The steady state, one dimensional pipe flow form is: $p_1 A_1 - p_2 A_2 - F = Mg \sin\theta$.

Energy equation

Macroscopic $\left(\frac{p}{\gamma} + \frac{\bar{v}^2}{2g} + z\right)_{in} + h_{pump} = \left(\frac{p}{\gamma} + \frac{\bar{v}^2}{2g} + z\right)_{out} + h_{friction}$

6.3 Conversion factors and formulas:

1 Hp:	745.7 W
Shear stress:	$\tau_{OFU} = \Theta \cdot 1.06$ (Fann VG readings = Θ)
Shear stress:	1 lb/100ft^2 (OFU) = 0.4788 Pa (SI)
τ_{SI} :	$\tau_{OFU} \cdot 0.4788)$
Shear rate:	$\dot{\gamma}\,(s^{-1}) = RPM \cdot 1.703$
1 inch:	0.0254 m
1 bar:	10^5 Pa
1cP:	10^{-3} Pas
P (effect):	q · p (Watt)
A_{sphere}:	$4 \cdot \pi r^2$
V_{sphere}:	$4/3 \cdot \pi r^3$

1 Solutions to exercises in drilling fluid engineering

Content:

1. Fluid Properties
2. Rheological models
3. Drilling fluid dynamics
4. Bit hydraulics
5. Wellbore challenges

1.5 Rheology control

a) The mean size of Barite is typically 20 mm. They will be uniformly dispersed in the drilling fluid and they will lead to increased viscosity. PV and μ_{eff} will increase. YP will be unaffected or decrease, since the smallest Barite particles will behave as physical dispersants.

b) In order to:

Suppress Ca^{++} from dissolving.

Keep anionic colloidal particles dispersed.

Suppress corrosion, H_2S- and CO_2 – attack.

c) Polymers are polymerized monomers. They can be of organic origin or be manufactured synthetically. Polymers have high molecular weight and come mostly as charged particles → they will bind water molecules → increase the hydrodynamic volume → influence viscosity and filter behaviour. Some types of polymers can attract or bind charged particles (clay), and therefore influence the swelling process and contribute to selective flocculation (clear water drilling). The purpose of polymers are mainly to:
- increase fluid viscosity
- reduce fluid viscosity in turbulent flow (drag reducer)
- control flocculation
- improve both the filter cake itself and the fluid loss through the filter cake

d) The description of the three expressions are:
- Pseudo plastic fluids exhibit altered apparent viscosity whenever the shear rate is changed.
- The fluid displays a reduction in viscosity over time at constant shear rate as indicated in Figure 1-5.
- Rheopectic fluids are rare. They exhibit increased shear stress at increasing shear time (at constant shear rate).

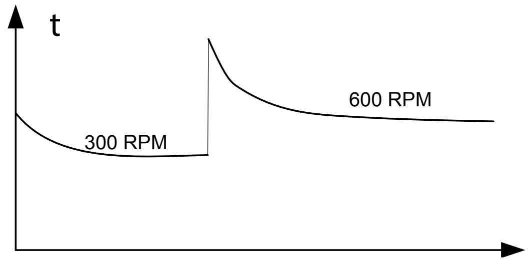


Figure 1-5: Behavior of thixotropic fluids after stillstand (left) and at constant shear rates.

e) Clay particles have a static, negative surface charge localized on the particle edges, but with weakly positively sites on the surface of the platelets. This triggers water! When studying the repulsive – attractive forces, it is experienced that at low salt concentration or high colloidal concentration, a very slow flocculation will take place. When edges come sufficiently close to the surfaces of other Montmorilonite particles, they join.

1.2 Rheology control

a) Bentonite particles attract water molecules (dipoles) in hundreds of layers onto each of its charged surface. Swelled Na-Bentonite sheets separate readily when exposed to shear forces. Salt will reduce the charges on the Bentonite surfaces. In addition the salt (ions) will bind much of the water (reduce the water activity).

b) At still-stand many layers of water molecules are attached to the Bentonite surface, and particles flocculate slowly → viscosity increases. When the dispersion is stirred or pumped, the resulting shear stress will remove attached water layers and break up the flocks. Dispersed Bentonite with many free water molecules, torn off due to high shear stress, has lower viscosity than at still stand.

c) Drilling in sediments of high yield (premium) clay produces the following clay concentration: Fraction of solids by weight =*f*

$$f = \frac{q_{clay} \cdot \rho_{clay}}{q_{pump} \cdot \rho_{mud}} = \frac{ROP \cdot A_{bit} \cdot \rho_{clay}}{q_{pump} \cdot \rho_{mud}} = \frac{0.0097 \cdot 0.34 \cdot 2400}{0.05 \cdot 1\,025} = 0.154$$

$ROP = 35\ m/t = 35 / (60 \cdot 60) = 0.0097$ m/s
$q_{pump} = 3\,000$ l/min $= 3\,000 / (1\,000 \cdot 60) = 0.05\ m^3/s$
$A_{bit} = \pi/4 \cdot (d_{bit}^2) = \pi/4 \cdot 0.662^2 = 0.34\ m^2$
$d_{bit} = 26'' = 26 \cdot 0.0254 = 0.662$ m

Drilled clay contributes to a weight increase of 15.4% and produces a viscosity of 42 cP, which is above the required viscosity of 15cP.

d) The yield of Bentonite is how many m^3 of mud of 15 cP which the amount of one ton Bentonite is able to produce. From Figure 1-6 one ton of Bentonite will produce 6 weigth% = 16.666 m^3 of 15 cP mud.

e) From Figure 1-6 we see that a viscosity of max. 50 cP will correspond to 7.5 w% of Bentonite.

$$\rho_{bentonite} = 2400\ kg/m^3$$

To find the density of the mixture of 100 kg mud $\Rightarrow$ 7.5 kg Bentonite + 92.5 kg water, we need to find the volume:

$$Volume = V_{bentonite} + V_{water} = \frac{7.5\ kg}{2400\ kg/m^3} + \frac{92.5\ kg}{1000\ kg/m^3} = 0.0955\ m^3$$

$$\rho_{mud} = \frac{m}{V} = \frac{100kg}{0.0955\ m^3} = 1\,047\ kg/m^3$$

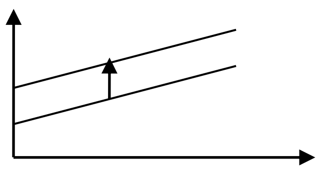
This shows that mud density cannot be increased much higher than 1 047 kg/m^3 by the addition of Bentonite.

1.3 Flocculation

a) When two Bentonite flakes are sufficiently near each other they are electrostatically attracted and will join edge to surface.

b) The cement is not by far hardened and contains a high concentration of lime (CaOH). Lime dissociates and one Ca^{++} ion can crosslink two charged clay platelets. This binding cannot be broken by hydraulic means (shear stress), and the flocculated mud has to be dumped after having been circulated to the surface.

c) The factors which enhance flocculation:
   - High concentration of charged colloids (Bentonite or anionic polymer)
   - High concentration of Ca^{++}ions (from dissolved salt and carbonates)
   - High Temperature (Brownian movements of the water molecules contribute to bringing the colloidal particles more frequently in contact with each other)



d) WBM behaves shear thinning:
   - Amount of water dipoles attached to each charged colloidal is a function of the shear force. High flow rate→ highe shear rate → water molecules are sheared off and become free → lower viscosity of the suspension
   - Weak flocculation occurs, especially at low sehar. At high shear the floccs break up → lower yeld point

   This behavior was presented during lectures.

e) The shearing action explained in e) above also is **time dependent**. Layer by layer are sheared off in concedutive order (presented in lectures).

f) The surface of caly colloids are positively charged by the loosely bonded Na-ions and represents therefore a weak bond compared to the electrostatic bond of double valence cations, mainly Ca^{++}, to the strolngly negative charged colloids at their edges.

1.4 Mud contamination

a) Contaminations are dissolved salt and chalk; drilled through cement etc. all produce Ca^{++} ions which lead to flocculation. Also fines are pollutant. At the surface the mud can be treated with thinners. Low gravity (2.4 kg/l) solids content (LGSC) is reduced by running the mud through centrifuges.

b) Rheology monitoring. First we plot the evolution of the rheology vs. time, as shown in Figure 1-8a.

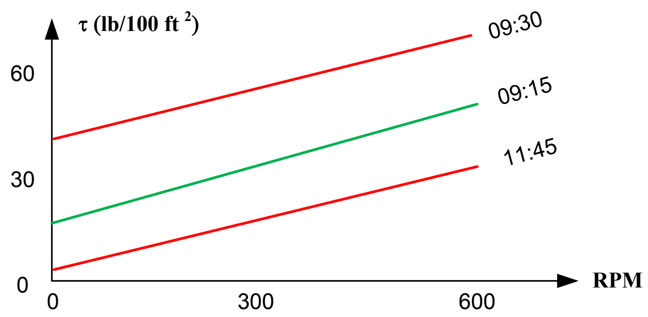


Figure 1-8a: The rheology of the mud at 3 time points

Then to evaluate the mentioned changes of the Bingham field rheology-model is perfect. We estimate the Bingham constants:

μ_{pl}	$= 42 - 28$	$= 14$ cP
	$= 41 - 27$	$= 14$ cP
	$= 68 - 54$	$= 14$ cP
	$= 31 - 17$	$= 14$ cP

We observe that the mud is dispersed at first. Salt will dissolve and Ca^{++} contaminates the mud. We now see that the viscosity is constant, while the yield point (YP) has changed like this:

YP	$= 28 - 14$	$= 14$
	$= 27 - 14$	$= 13$
	$= 54 - 14$	$= 40$
	$= 17 - 14$	$= 3$

The PV is constant (constant inner friction means solids content is constant). Δ YP is caused by higher attractive forces, caused by drilling into a salty formation. Initially, low concentration of Ca^{++} ions leads to strong cross binding (flocculation). Later, when the Ca^{++} concentration has been high for a while the Ca^{++} ions have exchanged the Na^{+}-ions in the Bentonite plates, leading to aggregation of clay platelets. A high concentration of Ca ions will over time lead to lower shear stress than originally due to cation exchange. The development is seen in Figure 1-8b.

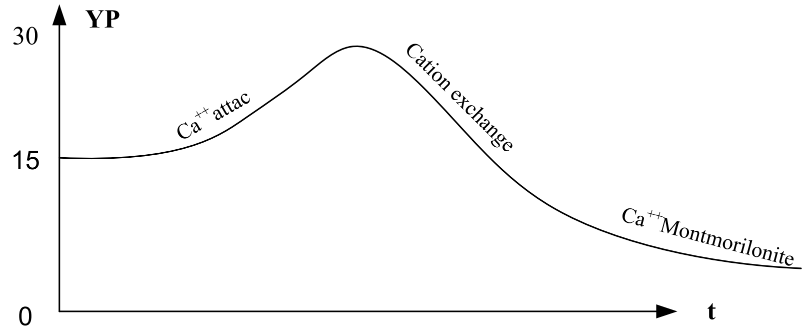


Figure 1-8b: Shear stress vs. time after encountering high concentration of contaminants.

Countermeasures: Add dispersants or use Gyp mud or use a high pH level in the mud when contaminations are expected. This is how to avoid flocculation and aggregation.

c) The edge-to-face flocculation is weak electrostatic forces and is easily broken when sheared. The Ca^{++} ion binds two clay platelets in a much stronger grip, and is not easily broken as indicated in Figure 1-8c.

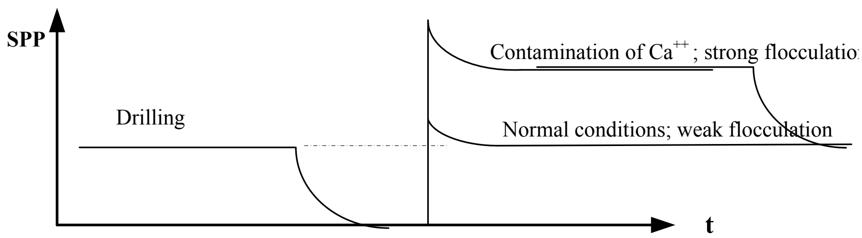


Figure 1-8c: Rheological response at the surface when drilling into contaminants.

1.5 Flocculation

a) Consider the force balance between shear and gravity acting on a particle:

$$A_p \cdot \tau_y = m_p \cdot g = V_p \cdot (\rho_p - \rho_{fl})\, g$$

$$4\,\pi\, d_p^2 \cdot \tau_y = \frac{1}{6}\pi\, d_p^3\, (\rho_p - \rho_{fl})\, g$$

Solve for yield point

$$\tau_y = \frac{d_p}{24}\left(\rho_p - \rho_{fl}\right) g = \frac{0.005}{24}(2\,300 - 1\,100)\,9.81 = 2.45\ Pa$$

Now solve the same balance of force while particles are settling, with respect to particle diameter:

$$d_p = \tau_y \cdot 24 / [(\rho_p - \rho_{fl})\ g] = 15\ \mathrm{Pa} \cdot 24 / [(2300 - 1100) \cdot g] = 0.0304 = 30.4\ \mathrm{mm}$$

b) The gel structure inside the drill string could have broken either by just pumping, by hitting the pipe with a hammer, or by inserting a heavy pill in the upper part of the drill string (U-tube the level in the drill string downwards). Consider the force balance between shear force and gravity acting on the fluid column inside the pipe:

$$A_{fluid} \cdot \tau_y = \rho g \cdot \Delta h$$
$$\pi d_{pipe} \cdot L \cdot \tau_y = \rho g \cdot \Delta h$$

Solve for Δh, the additional hoisting height before gravity breaks the gel

$$\Delta h = \pi \cdot d_{pipe} \cdot L \cdot \frac{\tau_y}{\rho \cdot g} = \pi \cdot 4.127 \cdot 0.0254 \cdot 2100 \cdot 14.4 / (1100 \cdot 9.81) = 0.3\ m$$

1.6 Fluid additives

a)

Material	Definition	Relevance
Anhydrite	$CaSO_4$ = sedimentary salt	dissolves in water → Ca^{++}
caustic soda	NaOH, Sodium hydroxide = lye	adjust pH (dispersant)
Ca SO_4	anhydrite	contaminator, leads to flocculation
Gypsum	Ca $SO_4 \cdot H_2O$ Sedimentary salt	Restructure from Na- to Ca-Bentonite; shale control
CEC	Cation Exchange Capacity	ability of reactive clay to exchange cations
CMC	natural polymer	filter loss/viscosifyers
Chalk	$CaCO_3$ = sedimentary salt	dissolves in water → Ca^{++}
Colloid	particles of size < 2 μ, clay, silt	build viscosity
Deflocculator	dispersion of colloids	reduce/neutralize electrostatic attraction
Dispergator	spreading of colloids	thinner
HEC	natural polymer	viscosifyer, filtrate reducer, deflocculant.
Lignite	anionic polymer	Thinner, filtrate reducer
Lignosulphonate	anionic polymer	thinner (dispergator)
Dispergator	thinner	hinder particles to coalesce
MBT	Methylene Blue Test	determines amount of reactive clay in mud. Clays present in the mud will adsorb MB

PAC	natural polymer	filter reducer, shale control
PHPA	synthetic polymer	shale control, Bentonite extender
Prehydrated	Bentonite hydrated in water	
Salt	Ionic compound, NaCl	dissolves in water →Na^+
SAPP	anionic polymer	thinner (dispergator)
Starch	------- " ---------	----- " -----
Xanthan	natural polymer	viscosifyer

b) A drag reducer consists of long-chained, neutral polymers added in small amounts to the viscous fluid. In a turbulent flow regime, the fluid molecules move in a random manner, causing much of the energy applied to them to be wasted as eddy currents (and corresponding high pressure loss). Due to shear stress the polymers are stretched out where the shear is high (close to the wall). In outstretched state the diffusion is higher due to lower effective surface area and thus lower resistance to movements. The polymers diffuse more in the direction of the pipe centre since the wall hinders diffusion in the opposite direction. The concentration will therefore become low after some flowing distance. The small concentration of stretched out long polymers along the wall will tend to turn the turbulent flow into laminar or streamlined.

c) The higher concentration, the more K^+ is exchanged with Na^+ shale, the less shale swelling is seen. K+ is geometrically suitable and leads to high platelet attraction.

d) Figure 5-10 shows both how a Ca attacks is changing the dispersed mud, but also how Gyp mud is made. Bentonite is composed mainly of the mineral called Montmorilonite. It is an aluminium silicate in which the silicon-oxygen sandwich is a layer of aluminium hydroxide. Some of the aluminium in the lattice structure is, during creation, replaced with ions of a lower charge (electron valence) such as magnesium (from 3 to 2). The *substitution*, without any other change of structure, creates surface negative charges in the lattice.

The negative charge sites created in the clay sheet are partly balanced by close association of positively charged ions (cation), normally Na^+. The creation of these charged ions and charged clay surfaces creates very strong attractive forces for polar water molecules that readily force themselves in between the unit layers.

Gypsum is added to prevent shale swelling. When lime or gypsum is added to Bentonite-treated mud, sodium Montmorilonite will convert to calcium Montmorilonite, which first produces flocculation and eventually aggregation of the Montmorilonite (see Figure 5-10). Caustic soda is added for pH control to suppress further dissolution of Ca^{++}. Lignosulphate is added for deflocculating. CMC may be added for fluid control. Then we get gyp mud.

If a lot of swellable clays have to be drilled through, apply gypsum mud; Pre-hydrate Na^+-Bentonite Add gypsum to the mud. Gypsum (Ca SO_4) dissolves and Ca^{++} ions are formed. First reaction causes flocculation. Over time Ca^{++} replaces Na^+ in Bentonite platelets, to make Ca^{++} – Bentonite and they aggregate and viscosity goes down. But from now on, there is an access of Ca^{++} in the mud (and in filtrate). No new flocculation will occur, and swellable clay does not swell much.

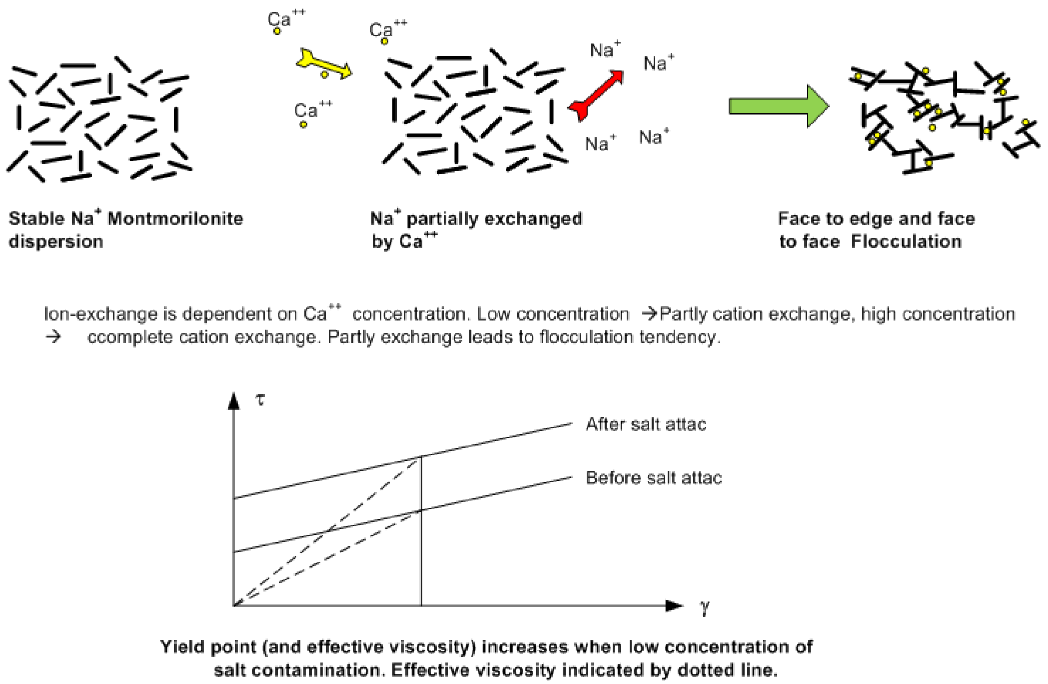


Figure 5-10. Clay behavior.

1.7 Fluid additives

The pH value of WBM are held at 9–11 to increase the solubility of anionic compounds.

a) The change in OH^- concentration required to increase the pH from a particular value to another is given by:

$\Delta [OH^-] = \Delta [OH^-]_{final} - \Delta [OH^-]_{initial}$

$\Delta [OH^-] = 10^{(11-14)} - 10^{(7.5-14)} = 0.001 - 0.000000316 = 0.00099$ mol/l

b) Mass of NaOH in g/l = concentration molecular weight (of Caustic Soda)

= 0.00099 moles/l (23 + 16+1) g/moles = 0.0198 g /l

c) K^+ is geometrically suiting in between the Na-Montmorilonite platelets, and leads to higher platelet attraction (low swelling) than for Na-Montmorilonite. Recall in your chemistry or periodic table (Group 1) that potassium (atom # 20) is more reactive than sodium (atom # 11). Why? The atomic radius of potassium is larger than that of sodium. Therefore, the single valence electron that exists for all alkali metal is located further away from the nucleus for potassium than sodium. This results in less energy required to remove that valence electron from potassium than from sodium, leading to increased reactivity. This trend continues as you move down Group 1 on the periodic table; i.e. Rubidium is more reactive than K.

d) The mud must have sufficient viscosity & velocity to exceed the settling velocity of the cuttings.

e) The mud must be thixotropic, i.e. must have the ability to gel when stationary or at low laminar flow, but becomes less viscous during circulation or when rigorously shaken by the shale shaker.

f) A large proportion of the mechanical energy (in the form of WOB and rotation) and hydraulic energy will generate friction and is dissipated as heat. Drilling fluid, which has a high heat capacity, is voluminous (typically 100 m^3) and absorbs this heat and allows the drill bit and the rest of the drill string not to be intensely heated.

g) The drilling fluid prevents blowouts by providing a hydrostatic pressure at least greater than the formation pressure.

2 Rheological models

2.1 Bingham / Power-law

a) Field method:

$\mu_{pl} = PV = \theta_{600} - \theta_{300} = 106.4 - 75.0 = 31.4$ cP

$YP = \theta_{300} - PV = 75 - 31.4 = 43.6$ lb/100 ft^2 $= 43.6 \cdot 1.06 \cdot 0.4788 = 22.1$ Pa

Standard method:

$\mu_{pl} = \frac{\Delta\tau}{\Delta\rho} = \frac{54.0-38.1}{511} = 0.0311$ Pas $= 31.1$ cP

$YP = \tau_{300} - \mu_{pl} \cdot \dot{\gamma} = 38.1 - 0.0311 \cdot 511 = 22.2$ Pa

$\mu_{Newton} = \frac{\tau_{600}}{\dot{\gamma}_{600}} = \frac{54.0}{1022} = 0.053\ Pas$

$n = 3.32 \log (54 / 38.1) = 0.50$

$K = 38.1 / 511^{0.5} = 1.69$

b) 100 PPM = 100 · 1.703 = 170 s^{-1}

At a shear rate of 170 s^{-1} the shear stress is seen in Table 2-1 to be 21.5 Pa. To answer the question please estimate the shear stress at this shear rate and compare.

$\tau_{B\text{-}H,170} = 27.4$ Pa

$\tau_{P\text{-}L,170} = 21.5$ Pa

P-L is obviously best.

c) To answer the question we need to know at which shear rate to compare the two models. Given data gives us $\bar{v}$ = q/A = [1 000 / (1 000· 60)] $/ \frac{\pi}{4} \cdot 0.1^2 = 2.12$ m/s.

This results in shear rates for the three models, taken from Chapter 6.2 in this exercise book. Pressure loss equations, equal to:

$$\dot{\gamma}_{newt} = 8\frac{v}{d} = 8 \cdot \frac{2.12}{0.1} = 170\ s^{-1}$$

$$\dot{\gamma}_{bingh=} 8\frac{v}{d} + \frac{\tau_0}{3\mu_{pl}} = 170 + \frac{22.1}{(3 \cdot 0.0311)} = 407\ s^{-1}$$

$$\dot{\gamma}_{PL} = 8\frac{v}{d} \cdot \frac{3n+1}{4n} = \frac{8 \cdot 2.12}{0.1} \cdot \frac{3 \cdot 0.5+1}{4 \cdot 0.5} = 212\ s^{-1}.$$

Checking the three models theoretically, at the estimated shear rates:

$$\tau_{newt} = \mu\,\dot{\gamma} = 0.0601 \cdot 170 = 10.2 Pa$$

$$\tau_{Bingh} = \tau_0 + \mu_{pl} \cdot \dot{\gamma} = 22.1 + 0.0311 \cdot 407 = 34.7 \text{ Pa}$$

$$\tau_{PL} = K \cdot \gamma^n = 1.63 \cdot 212^{0,5} = 24.4 \text{ Pa}$$

Then the recorded; From the graph we read the following shear stress

Model	Shear rate	Read	Theoretically	Δ
Newton	170	21.5	10.2	-11.3
P-L	212	23.4	24.4	+1.0
Bingham	407	33.2	34.9	+1.7

From this investigation we may conclude that P-L and Bingham model predict the rheology rather well at this flow rate.

2.2 Bingham/Power-law

a) The flow curve (τ vs. γ) data show obviously closer nearness to the Power-low model, especially for low shear rates. The Bingham model reads about 12 Pa higher than the measured data. To verify we need model constants:

$$n = \frac{\log(71.05\ /\ 49.75)}{\log(1022\ /\ 511)} = \frac{\log\ 1.428}{\log\ 2} = 0.51 \qquad K = \frac{\tau}{\dot{\gamma}^n} = \frac{71.05}{1022^{0.51}} = 2.07\ Pas^{-n}$$

$$\mu_{pl} = \theta_{600} - \theta_{300} = 140 - 98 = 42cP, \qquad \tau_0 = \theta_{300} - \mu_{pl} = 98 - 42 = 56$$
$$= 56 \cdot lb/100ft^2$$
$$= 56 \cdot 1.06 \cdot 0.478\ = 28.42\ Pa$$

Verification at 100 RPM (170 s^{-1}), where we read shear stress to be 27.39

$$\tau_{100,PL} = K \cdot \gamma^n = 2.07 \cdot 170^{0,51} = 28.41\ Pa$$
$$\tau_{100,BH} = \tau_0 + \mu_{pl} \cdot \dot{\gamma}\ = 35.6\ Pa = 28.42 + 0.042\ \cdot 170 = 35.56\ Pa$$

We observe that the Bingham over predict as expected, while P-L fits well.

b) This is a time-effect which occurs while performing rheological readings in the lab. The time being spent while performing the test at low speed gives the colloidal clay particles the possibility to flocculate (edge to face). The flocculation process is much more pronounced at low shear rates.

2.3 Bingham/Power-law. Regression

a) All model constants are necessary for plotting of the three models:
Newtonian model: We initially select the 300 rpm reading

$$\mu = \frac{\tau}{\dot{\gamma}} = \frac{42\ \cdot\ 0.4788}{300\ \cdot\ 1.703} = 0.039\ \ Pas$$

Bingham model (Field procedure):

$$\mu_{pl} = \theta_{600} - \theta_{300} = 60.4 - 39.6 = 20.8\ cP =\ 0.021\ Pas$$

$$\tau_y\ = \tau_{300} - \mu_{pl} = 39.6 - 20.8 = 18.8\ \frac{lb}{100}ft^2 = 18.8\ \cdot 1.06 \cdot 0.4788 = 9.6\ Pa$$

Bingham standard procedure:

$$\mu_{pl} = \frac{\Delta\tau}{\Delta\gamma} = (30.6 - 20.2)\ /\ 511 = 0.021\ Pas$$
$$\tau_y\ = \tau_{300} - \mu_{pl}\ \cdot \dot{\gamma} = 20.2 - 0.021 \cdot 511 = 9.5\ Pa$$

Both procedures gave the same answers. This assumes that the factor 1.06 is taken into consideration in standard procedure.

Exponent model:

$$n = \frac{\log\tau_{600} - \log\tau_{300}}{\log\dot\gamma_{600} - \log\dot\gamma_{300}} = \frac{\log\dfrac{\tau_{600}}{\tau_{300}}}{\log\dfrac{\dot\gamma_{600}}{\dot\gamma_{300}}} = \frac{\log\dfrac{64}{42}}{\log\dfrac{600\cdot 1.703}{300\cdot 1.703}} = \frac{\log\dfrac{64}{42}}{\log 2} = 0.60$$

$$K = \frac{\tau}{\dot\gamma^{n}} = \frac{64\cdot 0.4788}{(600\cdot 1.703)^{0.608}} = 0.454\ Pas^{n}$$

To plot the three curves (see Figure 2-3.1), τ is calculated at 50, 100 and 800 rpm:

Bingham:	τ	=	$9.5 + 0.021\cdot 50\cdot 1.703 = 11.3$ Pa
	τ	=	$9.5 + 0.021\cdot 800\cdot 1.703 = 37.8$ Pa
Power-law:	τ	=	$0.454\cdot(50\cdot 1.703)^{0.61} = 6.8$ Pa
	τ	=	$0.454\ (100\cdot 1.703)^{0.61} = 10.4$ Pa
	τ	=	$0.455\ (800\cdot 1.703)^{0.61} = 36.5$ Pa

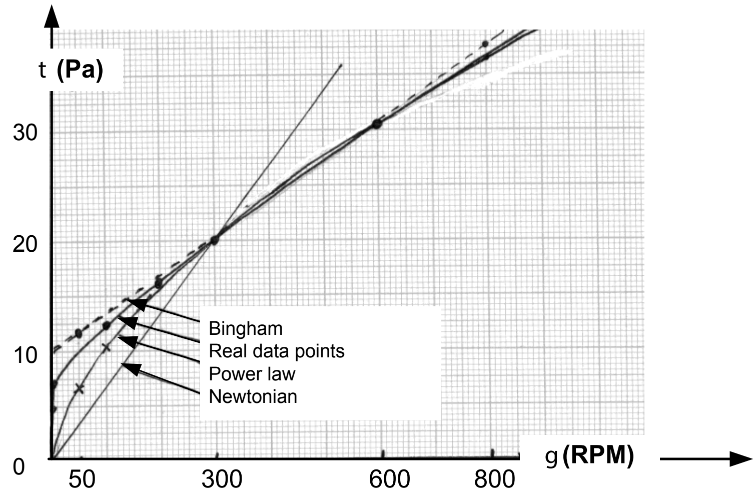


Figure 2-3.1: Rheogram (flow curve) interpreted through 3 rheological models

Readings on flow curve: 50 rpm = 10.0 Pa
100 rpm (already measured)
800 rpm = 36.5 Pa (extrapolated)

From this we see that the Bingham model suits the mud is flow curve best at 50 s^{-1}, since $\tau_{50B\text{-}M}$ is much closer to the real data than $\tau_{50\ P\text{-}L}$, while at higher shear rates Power law is the better one.

b) Plug flow will occur at any laminar flow since shear stress is low in the middle of the pipe. The Bingham model predicts an YP = 9.1 Pa. This is a theoretical value, but given time at steady-state laminar flow, gel strength will develop, in fact approaching the level of the YP. Shear stresses below this level will lead to gel (flocculation) as shown in Figure 2-3.2

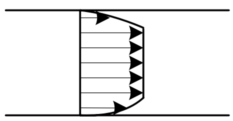


Figure 2-3.2: Plug flow of a Binghamian fluid.

2.4 Effective viscosity

a) The effective viscosity (Newtonian model):

For 600 rpm: $$\mu = \frac{\tau}{\dot{\gamma}} = \frac{43 \cdot 0.4788 \cdot 1.06}{600 \cdot 1.703} = 0.021\ Pas = 21.2\ cP$$

For 3 rpm: $$\mu = \frac{\tau}{\dot{\gamma}} = \frac{3.2 \cdot 0.4788 \cdot 1.06}{3 \cdot 1.703} = 0.318\ Pas = 318\ cP$$

These and more data points result in Figure 2-4:

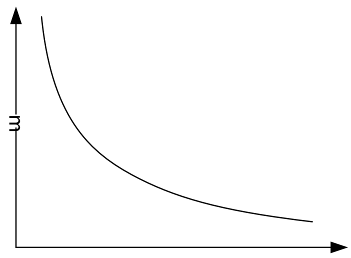


Figure 2-4: Viscosity for a non-Newtonian fluid.

b) When determining the effective viscosity we assume the fluid is closest to a Power law fluid:

2 data points-procedure

$$n = \frac{\log \tau_2/\tau_1}{\log \gamma_2/\gamma_1} = \frac{\log 43/30}{\log 2} = 0.52, \quad K = \frac{\tau}{\dot{\gamma}^n} = \frac{43 \cdot 0.4788 \cdot 1.06}{(600 \cdot 1.703)^{0.52}} = \underline{0.59\ Pa\ s^{-n}}$$

$$\bar{v} = q / A = \frac{4 \cdot 6000/(1000 \cdot 60)}{\pi \cdot 0.102^2} = 12.33\ m/s$$

c) Always check the Reynolds number:

$$N_{\text{Re}} = \frac{\rho \bar{v}\ d}{\mu_{eff}} = \frac{1100 \cdot 12.3 \cdot 0.102}{0.024} = 57\ 500 \rightarrow turbulent$$

$$\mu_{eff} = \left(8\frac{\bar{v}}{d} \cdot \frac{3n+1}{4n}\right)^n \cdot \frac{Kd}{8v} = \left(\frac{8 \cdot 12.33}{0.012} \cdot \frac{3 \cdot 0.52 + 1}{4 \cdot 0.52}\right)^{0,52} \cdot \frac{0.59 \cdot 0.102}{8 \cdot 12.33} = 0.024\ Pas$$

Use therefore a turbulent pressure loss model:

$$\Delta p = aN_{\text{Re}}^{-b} \cdot 4\frac{L}{d} \cdot \frac{1}{2}\rho v^2$$

d) High YP is caused by slow flocculation/gelling. At long still stand (10 min or more) the gel strength will approach the YP.

2.5 All models

α) μ_{pl} = (55-40) / 511 = 0.0294 Pas = 29 cP

$\tau_y = \tau - \dot{\gamma} \cdot \mu_{pl}$ = 55 – 1022 · 0.0294 = 25.4 Pa

n = 3.32 · log 55/40 = 0.459

K = τ/γ = 55 / 1022 $^{0.459}$ = 2.28 Pas^n

b) H & B:
1. Field approach: Take τ_3 *as* τ_y, and n and K from Power law
2. Standard: 3 points, iteration, select the 3 data points closest to actual shear rate, normally the three upper points
3. Nonlinear regression: 6 data points

c) Compare the two laminar friction expressions $\Delta p_{Newton} = \Delta p_{Bingham}$, and solve for $\mu_{Newton} (= \mu_{eff})$.

d) Increased PV is an indicator of more suspended, inert fine particles (more mechanical friction). Increased YP is an indicator of more surface interaction between surface active colloidal particles.

2.6 All models. Regression

a) Standard 2 data points:

Bingham: $\mu_{pl} = (25.4-17.2) / 511 = 0.016 \text{ Pas} = 16 \text{ cP}$

$\tau_0 = \tau_{300} - \gamma_{300} \cdot \mu_{pl} = 17.2 - 0.016 \cdot 511 = 9 \text{ Pa}$

Power-law: $n = \log (\tau_2/\tau_1)/ \log \gamma_2/\gamma_1 = 3.32 \cdot \log 25 / 17 = 0.56$

$K = \tau_{600}/\gamma_{600}^{\,n} = 53 \cdot 0.4788 / 1022^{\,0.56} = 0.52 \text{ Pas}^n$

b) Regression results for Bingham (straight line) and Power law (curved line) models are presented in Figure 2-6:

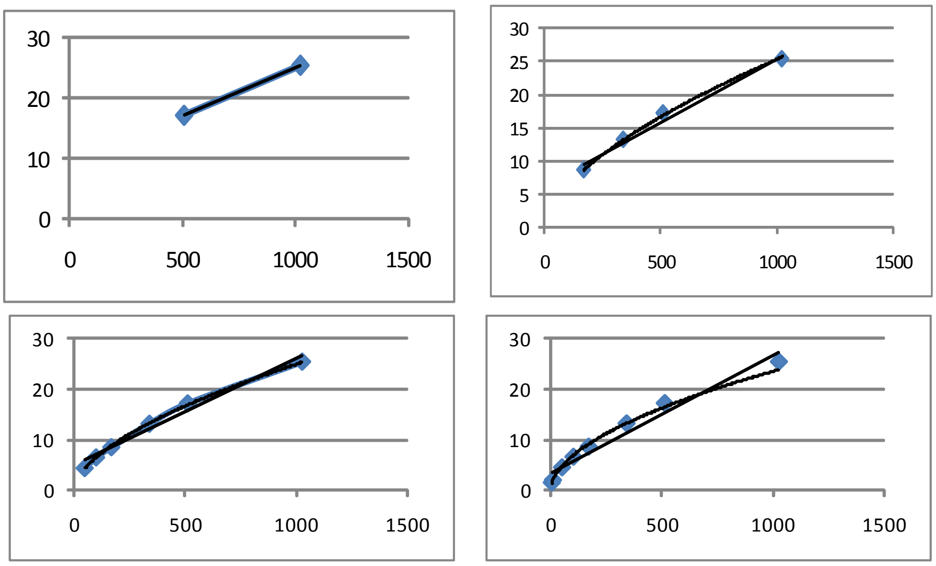


Figure 2-6: Regression result as a function of # of viscometer data points. (2, 4, 6, and 8 data points).

A summary of the results of # of data points:

Data points	Bingham model			Power Law model		
	PV	YP	R^2	n	K	R^2
2	0.0160	9.0	-	0.56	0.52	-
4	0.0192	6.3	0.983	0.61	0.39	0.999
6	0.0212	4.8	0.977	0.58	0.45	0.999
8	0.0232	3.4	0.960	0.53	0.59	0.997

The 4 upper points are the most reliable data points. The two lower ones (3 and 6 RPM) are influenced by the time delay inflicted by the test procedure (see Task 2.2).

c1) In the H-B model the constants are found through elimination;

$$\tau_2 = \tau_y + K \cdot \dot{\gamma}_1^n \qquad 25.4 = \tau_y + K \cdot 1\,022^n \qquad (1)$$

$$\tau_2 = \tau_y + K \cdot \dot{\gamma}_2^n \qquad 17.3 = \tau_y + K \cdot 511^n \qquad (2)$$

$$\tau_2 = \tau_y + K \cdot \dot{\gamma}_3^n \qquad 13.2 = \tau_y + K \cdot 340^n \qquad (3)$$

Use (1) and (2): $\tau_y = K \cdot 1022^n - 25.4 = K \cdot 511^n - 17.2$ (4)

to find K: $K = \frac{25.4-17.2}{1022^n-511^n}$ (5)

Enter (5) in (2): $17.2 = \tau_y + \frac{8.1}{1022^n-511^n} \cdot 511^n$ (6)

and in (3): $13.2 = \tau_y + \frac{8.1}{1022-511} \cdot 340^n$ (7)

Set (6) = (7): $(-\tau_y =)\frac{8,1}{1022^n-511^n} \cdot 511^n - 17.2 = \frac{8.1}{1022-511} \cdot 340^n - 13.2$ (8)

τ_y and K are now eliminated: $\frac{8.1}{1022^n-511^n} \cdot (511^n - 340^n) = 17.3 - 13.2 = 4.1$ (9)

$$\frac{8.1}{4.1} \cdot (511^n - 340^n) = (1022^n - 511^n) \qquad (10)$$

To solve equation (10) we suggest iteration. Start with n = 0,5, and we get:

1.976 (22.6 – 18.4)	= 32.0 – 22.6	
8.3	= 9.4	-1.1

Now try n = 0.52:

1.976 (25.6 – 20,7)	= 36.7 – 2536	
9.6	= 11,1 =	-1.5

We are moving away. Try n = 0.48:

1.976 (20.0 -16,4)	=27.8 – 20	
7.01	= -7.8 =	-0.7

Further iteration with decreasing n to 0.35 gives a deviation cloe to 0 → n = 0.35. When n is found, the other constants can be found easily. Now K = 3.4, resulting in τ_y = – 13 På.

c2) Non-linear regression. Please refer to textbook Chapter 10.6 of how to step-wise go about in Excel to determine the constants.

2.7 All models.

The rheological data are plotted in Fig. 2-7.

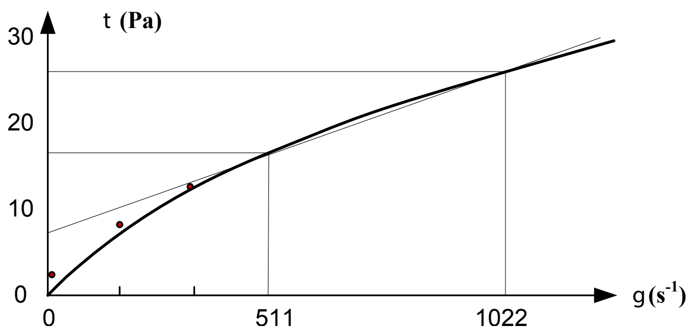


Figure 2-7: The Bingham and the power-law model together with raw-data.

We see that the Bingham model overpredicts and P-L underpredicts. This observation we test below at $10\ s^{-1}$.

Bingham Field procedure: From the two upper viscometer data points, we obtain:

$$\mu_{pl} = \theta_{600} - \theta_{300} = (52.2 - 35.5) \cdot \frac{1}{1{,}06} = 15.8\ cP$$

$$\tau_o = \theta_{300} - \mu_{pl} = \frac{35.5}{1.06} - 15.8 = 17.7\ \text{lb/100ft}^2 = 17.7 \cdot 1.06 \cdot 0.4788 = 9\ Pa$$

Bingham standard SI-procedure:

$$\mu = \frac{25-17}{511} = 0.0157\ Pas$$

$$\tau_o = 25 - 0.0157 \cdot 1022 = 8.9\ \text{Pa}$$

Power-law, two upper data points:

$$n = 3.32 \cdot \log 25/17 = 0.556$$

$$K = \frac{25}{1022^{0.556}} = 0.52$$

Herschel-Bulkley is determined through the field method: iteration and elimination, based on the three upper readings;

$$\tau = \tau_0 + K \cdot \dot{\gamma}^n = 2 + 0.52 \cdot \gamma^{0.556}$$

Now we test the models at $\gamma = 10\ s^{-1}$

$$\tau_{PL} = 0.52 \cdot 10^{0.556} = 1.9\ Pa$$
$$\tau_{BH} = 8.9 + 0.0157 \cdot 10 = 9.1\ Pa$$
$$\tau_{HB} = 2 + 0.52 \cdot \gamma^{0.556} = 3.1\ Pa$$

Since the solution is 3.0 Pa the H-B is closest. The Bingham model is relatively far away, as expected, since theoretical model is based only on the two upper readings, which is far from the tested shear rate. Therefore it is, as stated in the text book, recommended to apply the data points closest to the actual shear rate of the model. Since these specifics are not always known ahead of time, the next best is to use all the data points (regressi9on), not only the two upper ones or a few selected ones.

3 Drilling fluid dynamics

3.1 Velocity profile. Continuity equation

Define cross sectional flow area and discretize in the z-direction and then integrate velocity over the A.

$$A = z \cdot b \quad \rightarrow \quad dA = b \cdot dz$$

Average velocity:

$$\bar{v} = \frac{1}{A}\int az(z_0 - z)b \cdot dz$$

$$\bar{v} = \frac{ab}{z_0 b}\int\left(z_0 z - z^2\right)dz$$

$$\bar{v} = \frac{a}{z_0} \cdot \left[\frac{z_0}{2}z^2 - \frac{1}{3}z^3\right]$$

$$\bar{v} = \frac{a}{z_0}\left(\frac{z_0^3}{2} - \frac{z_0^3}{3}\right) = az_0^2\left(\frac{1}{2} - \frac{1}{3}\right) = az_0^2 \cdot \frac{1}{6}$$

υ_{max} is found when the derivative of υ (z)' = 0

$$\frac{dv}{dz} = \frac{d\left(azz_0 - az^2\right)}{dz} = az_0 - 2az = 0$$

$$2az = az_0 \rightarrow \text{max at } z = \frac{z_0}{2}$$

$$v_{max} = a\frac{z_0}{2}\left(z_0 - \frac{z_0}{2}\right) = \frac{az_0^2}{4}$$

Find v_{max} when: $\bar{v} = 8cm/s \text{ and } z_0 = 4\ cm$

$$\bar{v} = az_0^2 \cdot \frac{1}{6}$$

$$8 = a \cdot 16 \cdot \frac{1}{6}$$

$$a = \frac{8 \cdot 6}{16} = \frac{12}{4} = 3\ m/s$$

Finally: $v_{max} = \frac{3 \cdot 4^2}{4} = 12\ cm/s$

3.2 Velocity profile. Momentum flux

a) To find the pressure distribution: We start with Navier-Stoke in the z-direction, cylindrical coordinates:

$$v\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v}{\partial\theta^2}+\frac{\partial^2 v}{\partial z^2}\right]+g_z$$

Under stationary laminar flow the equation reduces to:

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)\right]$$

$$\frac{\partial p}{\partial z}=\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)$$

To replace the latter part of the equation with manageable parameters we go through 3 steps:

Step 1: Define $\partial v/\partial r$ *from* $v(r)$: $v=v_{max}\left(1-\frac{r^2}{R^2}\right), v_\Theta=v_r=0$

Step 2: $\frac{\partial v}{\partial r}=v_{max}\left(-\frac{2r}{R^2}\right)$

Step 3: Now we derive the part $r\cdot\frac{\partial v}{\partial r}$ with respect to r: $\frac{\partial}{\partial r}r\cdot v_{max}\left(-\frac{2r}{R^2}\right)=-\frac{v_{max}\cdot 2 \quad 2r}{R^2}$

The pressure distribution or pressure gradient now becomes:

$$\frac{\partial p}{\partial z} = \mu \cdot \frac{1}{r} \cdot \left(-\frac{v_{\max} \cdot 4r}{R^2} \right)$$

Total pressure loss along z:

$$\int_{p(o)}^{p(z)} \partial p = -\int_{o}^{z} \frac{4\mu v_{\max}}{R^2} \partial z$$

$$\Delta p = -\left(0 - \frac{4\mu v_{\max} z}{R^2} \right) = 4\mu v_{\max} z / R^2$$

The shear stress is found as: $\tau = -\mu \cdot \frac{\partial v}{\partial r} = \mu v_{\max} \left(\frac{2r}{R^2} \right)$.

Maximum of τ is found when r = R, i.e. at the wall as always. Navier Stoke deals with incompressible fluids; density does not change.

$$\tau_{\max} = \mu \cdot v_{\max} \cdot \frac{2}{R}$$

b1) Wall shear stress: The wall shear follows from the definition of a Newtonian fluids: The x and r direction must be replaced by y and z respectively.

$$\tau_w = \tau_{wxywall} = \mu \left(\frac{\partial v_z}{\partial y} + \frac{\partial v_{xy}}{\partial z} \right) |_{y=\pm h} = \mu \frac{\partial}{\partial y} \left[\left(-\frac{dp}{dz} \right) \left(\frac{h^2}{2\mu} \right) \left(1 - \frac{y^2}{h^2} \right) \right] |_{y=\pm h}$$

$$\tau_w = -\mu \frac{\partial v_z}{\partial y} \quad (since\ v_{y=0})$$

$$\tau_w = \pm\mu \left(-\frac{dp}{dz} \right) \cdot \frac{h^2}{2\mu} \left(-\frac{2y}{h^2} \right) = \frac{dp}{dx} h = \frac{dp}{dx} (h - -h) = 2h \frac{dp}{dx}$$

v_{max} is reached when y = 0:

$$v_{max} = \frac{dp}{dx} \cdot \frac{h^2}{2\mu} \rightarrow \frac{dp}{dx} = v_{max} \cdot \frac{2\mu}{h^2}$$

b2) Average velocity:

It is defined as $\bar{v}$ = q/A, where q = $\int v$ dA over the cross section.

For our particular distribution: A = b · y → dA = b · dy,

$$\bar{v} = \frac{1}{A} \int vdA = \frac{1}{b(2h)} \int_{-h}^{+h} \frac{dp}{dz} \cdot \frac{h^2}{2\mu} \left(1 - \frac{y^2}{h^2} \right) bdy$$

$$= \frac{1}{2h} \cdot v_{max} \int \left(1 - \frac{y^2}{h^2} \right) dy = \frac{v_{max}}{2h} \left[h - \frac{h^3}{3h^2} - \left(-h - \frac{-h^3}{3h^2} \right) \right] = \frac{2}{3} v_{max}$$

In plane Poiseulle flow between parallel plates, the average velocity is two-thirds of the maximum value.

3.3 Flow profile

a) The equation represents the conservation of the momentum for stationary, incompressible, curl free, laminar pipe flow, also called the Navier Stoke equation.

b) From momentum equation and stated condition;

$$0 = -\frac{\partial p}{\partial x} + \mu\left[1/r \cdot \partial/\partial r(\frac{\partial(rv)}{\partial r})\right]$$

From boundary condition, rearranging/integrating twice, $v(r)$ is found.

Alternatively, start with the shear stress vs. friction pressure for laminar pipe flow; or, one step later, the general shear stress equation:

$$\tau = \frac{r}{2}\frac{dp}{dx} = -\mu\frac{dv(r)}{dr}$$

$$dv(r) = -\frac{1}{2\mu} \cdot r\frac{dp}{dx} \cdot dr$$

$$v(r) = -\frac{1}{2\mu} \cdot \frac{dp}{dx}\frac{r^2}{2} + C_1$$

$$v(R) = 0, C_1 = \frac{1}{4\mu} \cdot R^2\frac{dp}{dx}$$

$$v(r) = \frac{1}{4\mu}\frac{dp}{dx}\left(R^2 - r^2\right)$$

Dp/dx is the local pressure gradient. Since it is constant, the integrated pressure gradient is equal to the total pressure loss over the total length; Δp/L.

c) Average velocity is found by taking th areal average of the integrated velocity v(r):

$$\bar{v} = \frac{1}{A}\int v\,dA = \frac{1}{\pi R^2}\frac{dp/dx}{4\mu}\int_o^R\left(R^2 - r^2\right)2\pi\, r\, dr = \frac{R^2}{8} \cdot \frac{dp/dx}{\mu}$$

To compare v_{max} and $\bar{v}$ we need fist to express v_{max};

$$v_{max} = v(r = 0) = \frac{1}{4\mu} \cdot \frac{dp}{dx}R^2$$

By comparing the two we find an expression of axial dispersion:

$$\frac{v_{max}}{\bar{v}} = \frac{1 \cdot dp/dx \cdot R^2 \cdot \mu}{4\mu \cdot R^2/8 \cdot dp/dx} = 2$$

From this evaluation we can conclude that axial dispersion is independent of the factors shown above and that it is always positive; the max velocity is twice the average velocity.

d) The wall shear stress is found in two steps:

Step1: dp/dx = 0.09 MPa/1 000 m = 90 Pa/m

Step 2: $\tau_w = \frac{dp}{dx} \cdot \frac{R}{2} = \frac{0.05}{2} \cdot 90 = 2.25\ Pa$

$$\bar{v} = \frac{R^2}{8} \cdot \frac{dp/dx}{\mu} = \frac{0.05^2}{8} \cdot \frac{90}{0.0538} = 0.52\ m/s$$

3.4a Pressure loss vs. rheology

Average velocity is u_o

$$\bar{u} = \frac{1}{A} \int u(r) dA = u_o \qquad A = \pi r^2 \rightarrow dA = 2prdr$$

$$= \frac{1}{\pi R^2} u_{max} \int \left(1 - \frac{r}{R}\right)^{1/4} \cdot 2\pi dr = 2 \frac{u_{max}}{R^2} \int (\ ) rdr$$

Substitution is the solution:

$$\int_0^R \left(1 - \frac{r}{R}\right)^m rdx =$$

$$y = \left(1 - \frac{r}{R}\right) \rightarrow r = (1 - y)R \qquad r/R \text{ vs. } Y: 0 \text{ vs. } 1,\ 1 \text{ vs. } 0$$

$$dr. = - dy \cdot R$$

$$\bar{y}^m = \int y^m \cdot (1-y)R\,(-dyR) = -R^2 \int y^m(1-y)\,dy$$

$$-R^2\int y^m dy + R^2\int y^{m+1}dy$$

$$= R^2\left(-\frac{y^{m+1}}{m+1} + \frac{y^{m+2}}{m+2}\right)\Big|_0^1$$

$$= R^2\left(-\frac{\left(1-\frac{r}{R}\right)^{m+1}}{m+1} + \frac{\left(1-\frac{r}{R}\right)^{m+2}}{m+2}\right)\Big|_0^R$$

$$= R^2\left(-\frac{0}{m+1} + \frac{0}{m+2} - \left(-\frac{1}{m+1} + \frac{1}{m+2}\right)\right)$$

$$= R^2\left(\frac{1}{m+1} - \frac{1}{m+2}\right) = \frac{R^2}{(m+2)(m+1)} = \frac{R^2}{\frac{9}{4}\cdot\frac{5}{4}} =$$

$$\bar{y}^{1/4} = 0.355\,R^2$$

$$\bar{u} = \frac{2U_m \cdot 0.355\,R^2}{R^2} = 0.71\,U_m$$

$$\tau_w = \mu \cdot \frac{du}{dr} \rightarrow \frac{du}{dr} = \frac{-U_m}{R}\,\frac{\left(1-\frac{1}{R}\right)}{\left(1-\frac{r}{R}\right)^{3/4}} \qquad r \rightarrow R, \quad \tau \rightarrow \infty$$

3.4b Pressure loss vs. rheology

To find pressure loss we first need to check the Reynolds number:

(μ_{pl} = 0.016, τ_o = 9 Pa, n = 0.56, K = 0.52)

a) $N_{Re} = \rho v d/\mu_{eff}$

$$\bar{v} = \frac{q}{A} = \frac{\frac{1}{60}}{\frac{\pi}{4}0.1^2} = 2.12\,m^3/s$$

$N_{Re} = \frac{d^n \cdot \bar{v}^{2-n} \cdot \rho}{K_p\,[(3n+1)/4n]^n \cdot 8^{n-1}} = \frac{0.1^{0.56} \cdot 2.12^{1{,}44} \cdot 1000}{0.52 \cdot \left(\frac{3\cdot 0.56+1}{4\cdot 0.56}\right)^{0{,}56} \cdot 8^{-0{,}44}} =$ 3 500 => Turbulent, but in this exercise we need to assume laminar flow

For the Newtonian model we determine the viscosity at the estimated shear rate γ = 8v/d = 8 **.** 2.12 /0.1 = 169.6 s^{-1}. Here the viscosity is:

μ = τ / γ = 8.6 (from Task 2.6) / 169.6 = 0.0507 Pas

Laminar pressure loss equations give these results:

Newton: $\Delta p_N = 32\ \mu\ L\ v\ /\ d^2 = 32\ .\ 0.0507\ .1000\ .\ 2.12\ /\ 0.10 =$ $3.4\ .10^5$ Pa

Power-law: $\Delta p_{PL} = 4K(8v/d\ .\ (3n+1)/4n)^n\ .\ 1/d\ .\ L$
$= 4.\ 0.52\ (8.\ 2.12\ /0.1\ .\ 2.68/2.24)^{0.56}\ .\ 1/0.1\ .\ 1\ 000 =$ $4.1.\ 10^5$ Pa

Bingham: $\Delta p_{BH} = 32\ .\ 0.016\ .\ 1\ 000\ .\ 2.12\ /0.1^2 + 16\ .\ 1\ 000\ .\ 9\ /\ 3\ .\ 0.1 = 5.9\ .10^5$ Pa

We observe that Bingham "aims" high at low shear rates, and will always produce higher results than the Power-law model at low shear rates.

b) Calculated shear rate for all models:

$\gamma_{Newt} = 8v/d = 8\ .\ 2.12\ /\ 0.1 = 170\ s^{-1}$

$\gamma_{PL} = 8v/d\ .(3n + 1)/\ 4n = 170\ .\ (3\ .\ 0.56 + 1\ )/\ 4\ .\ 0.56 = 203\ s^{-1}$

$\gamma_{BH} = 8v/d + \tau_0\ /\ (3\mu_{pl}) = 8\cdot 2.12\ /\ 0.1 + 9\ /\ (3\cdot 0.016) = 170 + 188 = 358\ s^{-1}$

We read from Figure 3-4 for all the models;

$\tau_N = 8.5$ Pa at $170\ s^{-1}$:
$\tau_{PL} = 10$ Pa at $203\ s^{-1}$:
$\tau_{BH} = 14$ Pa at $358\ s^{-1}$:

And the resulting pressure loss becomes:

$\Delta p_N = 8.5 \cdot 1000\ /\ 0.1 \cdot 4 =$ $3.4 \cdot 10^5$ Pa
$\Delta p_{P\text{-}L} = 10 \cdot 1000\ /\ 0.1 \cdot 4 =$ $4.0 \cdot 10^5$ Pa
$\Delta p_{BH} = 14 \cdot 1000\ /\ 0.1 \cdot 4 =$ $5.6 \cdot 10^5$ Pa

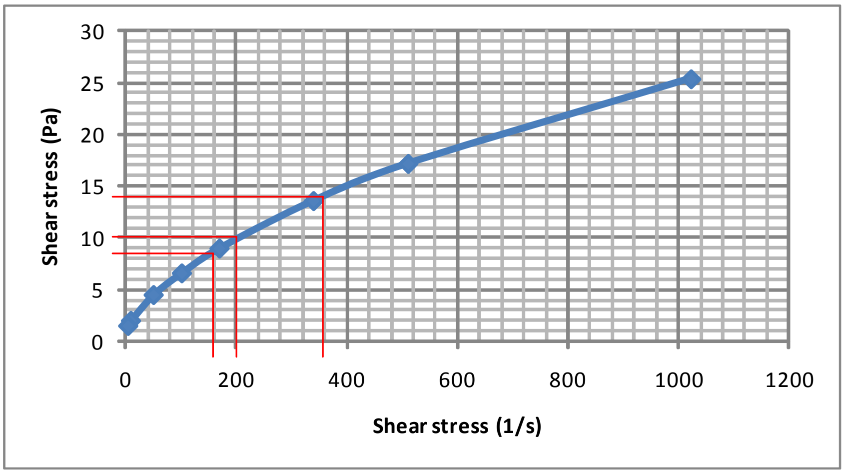


Figure 3-4: Graphical presentation of viscometer data and reading of shear stress for individual shear rates.

Model	$\dot{\gamma}$ (s^{-1})	Read from Fig. 3-4 (Pa)	Estimated from universal. eqn (bar)	Estimated from ordinary model (bar)
Newton	170	8.5	3.4	3.4
Power-law	203	10	4.0	4.3
Bingham	358	14	5.2	5.9

One conclusion is that the two non-Newtonian ordinary and universal models give similar results. But the different models give different results. The best fit is as expected the P-L.

c) Force Balance:

$$\tau \cdot 2\pi \cdot \Delta L = \Delta p \cdot \pi r \quad \rightarrow \quad \tau_w = \frac{1}{2} R \cdot \frac{\Delta p}{\Delta L}$$

And solving $\Delta p_{Newton} = \Delta p_{Bingham}$ for μ_{eff} gives the answer.

d) A uniform pipe has an entrance length. Increasing length with increasing N_{Re} Undeveloped flow at the entrance is turbulent and has higher shear until boundary layer expands and meets in the middle.

3.5 Pressure loss vs. Rheology

RPM	$\dot{\gamma}$	θ		τ	τ
rpm	s^{-1}	-		lb/100 ft^2	Pa
600	1022	66.0		70	33.5
300	511	47.2		50	23.9
100	170	25.5		27	12.9
6	10	9.4		10	4.8

$$\mu_{pl} = \theta_{600} - \theta_{300} = 66 - 47.2 = 18.8\ cP = 0.019\ Pas$$

$$\tau_0 = \theta_{300} - \mu_{pl} = 47.2 - 18.8 = 28.4 \frac{lb}{100} ft^2 = 28.4 \cdot 1.06 \cdot 0.4788 = 14.4\ Pa$$

$$\mu_{pl} = \frac{\Delta\tau}{\Delta\gamma} = \frac{33.5 - 23.9}{511} = 0.0188\ Pas$$

$$\tau_0 = \tau_{300} - \mu_{pl} \cdot \dot{\gamma} = 33.5 - 0.0188 \cdot 1022 = 14.3\ Pa$$

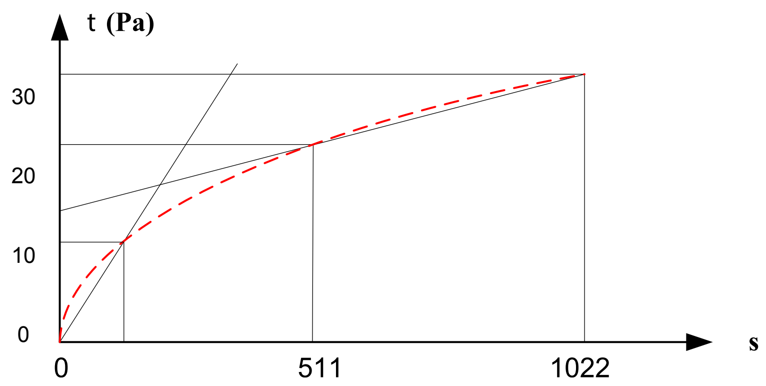


Figure 3-5: Rheogram for relevant rheology.

The Newton model can also be based on the two upper readings like the non-Newtonian. Use then the average value.

a) At this low flow rate we assume laminar flow, but still have to check;

$$\bar{v} = q/A = \frac{4}{\pi}\cdot\frac{q}{d^2} = \frac{4}{\pi}\cdot\frac{800}{0.109^2\cdot 1000\cdot 60} = 1.425\ m/s$$

$$N_{re} = \frac{\rho v d}{\mu_{eff}} = 1100\cdot 1.425\cdot 0.109/\ 0.2\ = 855$$

$$Above\ we\ need: \mu_{eff} = \mu_{pl} + \frac{\tau_0\cdot d}{6\bar{v}} = 0.0188 + \frac{14.3\cdot 0.109}{6\cdot 1.425} = 0.2\ \text{Pas}$$

$$\Delta p_{pipe} = \frac{32\mu_{pl}\cdot L\cdot\bar{v}}{d^2} + \frac{16L\tau_0}{3d} = \frac{32\cdot 0.0188\cdot 1.425\cdot 1000}{0.109^2} + \frac{16\cdot 1000\cdot 14.3}{3\cdot 0.109} = 7.8\cdot 10^5\ Pa$$

b) To compare the two we need to compute Newtonian pressure loss at the correct shear rate.

$$\dot{\gamma}_{Newt} = \frac{8\bar{v}}{d} = \frac{8\cdot 1.425}{0.109} = 105\ s^{-1}$$

$$\dot{\gamma}_{BH} = \frac{8\bar{v}}{d} + \frac{\tau_0}{3\mu_{pl}} = 105 + \frac{14.3}{3\cdot 0.019} = 356\ s^{-1}$$

Read from Figure 3-5 that τ = ca10 and 25 Pa at the two shear rates 105 and 356 s^{-1} respectively.

$$\Delta p_{univ,N} = 4\tau_w\cdot\frac{L}{d} = 4\cdot 10\cdot\frac{1000}{0.109} = 3.7\cdot 10^5\ Pa$$

$$\Delta p_{univ,BH} = 4\cdot 25\cdot\frac{1000}{0.109} = 9.2\cdot 10^5\ Pa$$

Bingham model gives the best answers. This is natural since the mud has a typical Binghamian characteristic at laminar flow. We could also have compared the two by using the normal pressure loss models. We must then first determine the specific viscosity, just like for the universal. See also Exercise 3.4 in this regard.

3.6 Pressure loss. Power law

a) Power law: $\tau = k\cdot\gamma^n$, $\gamma = -\dfrac{dv}{dr}$

From force balance we derive:

$$\tau\cdot 2\pi r L = (p_1 - p_2)\pi r^2 \quad\rightarrow\quad \tau = \frac{\Delta p\cdot r}{2L}$$

Using Power law model, we obtain:

$$K\cdot\gamma^n = \frac{\Delta p\cdot r}{2L} \quad\rightarrow\quad \gamma = \left(\frac{\Delta p}{2LK}\right)^{\frac{1}{n}}\cdot r^{\frac{1}{n}}$$

Knowing that $\gamma = dv/dr$ we can find v(r) from:

$$\int_{v(r)}^{0} - dv = \int_{r}^{R} \left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} r^{\frac{1}{n}} \, dr$$

$$v(r) = \left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} \cdot \frac{n}{n+1}\left[R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right]$$

We need to integrate over the cross sectional area:

$$q = \bar{v} \; \pi r^2 \quad \rightarrow \quad dq = v(r)\, \pi 2r \, dr$$

$$q = \int_0^R \left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} \cdot \frac{n}{n+1}\left[R^{\frac{n+1}{n}} - r^{\frac{2n+1}{n}}\right]\pi \cdot 2 \cdot r dr = \pi \cdot 2\left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} \frac{n}{n+1}\left[R^{\frac{n+1}{n}} \cdot \frac{R^2}{2} - \frac{n}{3n+1} \cdot R^{\frac{3n+1}{n}}\right]$$

$$= \pi \cdot 2\left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} \frac{n}{n+1}\left[\frac{1}{2} R^{\frac{3n+1}{n}} - \frac{n}{3n+1} \cdot R^{\frac{3n+1}{n}}\right] = \pi \cdot 2\left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} \frac{n}{n+1}\left[\frac{n+1}{2(3n+1)} - \frac{2n}{2(3n+1)}\right] R^{\frac{3n+1}{n}}$$

With this expression of q_{tot} we set up this relationship:

$$q = \pi R^2 \cdot \bar{v} \Rightarrow \quad \pi R^2 \cdot \bar{v} = \pi\left[\frac{\Delta p}{2LK}\right]^{\frac{1}{n}} \cdot n \cdot \left[\frac{n}{3n+1} \cdot R^{\frac{3n+1}{n}}\right]$$

and solved finally for Δp:

$$\Delta p = 4K\left[\frac{8\bar{v}}{d}\cdot\frac{3n+1}{4n}\right]\cdot\frac{L}{d}$$

b) Enter effective (Newtonian) viscosity for Power law fluids into the N_{Re} expression:

$$\mu_{eff,PL} = \left(\frac{8\bar{v}}{d}\right)^{n-1}\cdot K_p \; where \; K_p = K\cdot\left(\frac{3\cdot n+1}{4\cdot n}\right)^n$$

$$N_{Re} = \frac{\rho\cdot\overline{vd}}{\mu_{eff}} = \frac{d^{n-1+1}}{v^{n-1-1}}\cdot\frac{\rho}{K_p\cdot 8^{n-1}} = \frac{d^n\cdot\bar{v}^{2-n}\cdot\rho}{K_p\cdot 8^{n-1}} = N_{Re,pipe}$$

c) Power law fluids: $\frac{d_h^n\cdot v^{2-n}\cdot\rho}{K_a\cdot 12^{n-1}} = \frac{\rho}{K_a(12^{n-1})}\cdot (d_o-d_i)^n\cdot\left(\frac{q}{\frac{\pi}{4}d_o^2-d_i^2}\right)^{2-n} = C\frac{(d_o-d_i)^n}{(d_o^2-d_i^2)^{2-n}} =$

$$\frac{(d_o-d_i)^n}{\left(d_o-d_i\right)^{2-n}(d_o+d_i)^{2-n}} = \frac{(d_o-d_i)^{2(n-1)}}{(d_o+d_i)^{2-n}}$$

Newtonian fluids:

$$N_{Re,Newt} = \frac{\rho_1 v\cdot d_h}{\mu} = \frac{c_1 4q\,(d_0-d_i)}{C_2\cdot\pi\cdot d_h(d-d_i)^2} = \frac{\rho_1\cdot 4\cdot q}{\mu}\cdot\frac{d_0-d_i}{(d_0-d_i)(d\cdot d_i)} = \frac{c}{d_0\cdot d_i}$$

N_{Re} increases for Newtonian fluids when d_i increases, while for Power law fluids it is reversed.

The physical explanation for this behavior is the forced streamlining of the flow in narrow gaps. Out in the ocean, on the other hand, ocean water turns turbulent at very low shear rates.

3.7 Pressure loss. Turbulent flow. Energy equation

Average velocity:

$$\bar{v} = \frac{q}{\pi R^2} = \frac{0.2\,m^3/s}{\pi(0.1\,m)^2} = 6.4\,m/s$$

$$N_{Re} = \frac{\rho\bar{v}d}{\vartheta} = \frac{\bar{v}d}{\mu} = \frac{6.4\cdot 0.2}{0.00001} = 128\,000 \rightarrow turbulent\;(Kinematic\;viscosity;\;\vartheta = \mu/\rho)$$

From Chapter 4.3 in textbook we find ε = 0.26 mm for cast-iron pipe. Then

$$\frac{\varepsilon}{d} = \frac{0.26\,mm}{200\,mm} = 0.0013$$

Enter the Moody chart (Figure 4-12 in the Textbook) on the right hand side at ε/d = 0.0013 (need to interpolate), and move to the left to intersect with N_{Re} = 128 000. Read: f = 0.0225. The head loss becomes:

$$h_f = f\,\frac{L}{d}\cdot\frac{\bar{v}^2}{2g} = (0.0225)\,\frac{500\,m}{0.2\,m}\cdot\frac{(6.4\,m/s)^2}{2\cdot(9.81\,m/s^2)} = 117\,m$$

3.8 Pressure loss vs. flow rate

From exercise 2-7 we copy these data and present the flow curve:

n = 0.56, K = 0.52 Pa $s^{0.56}$

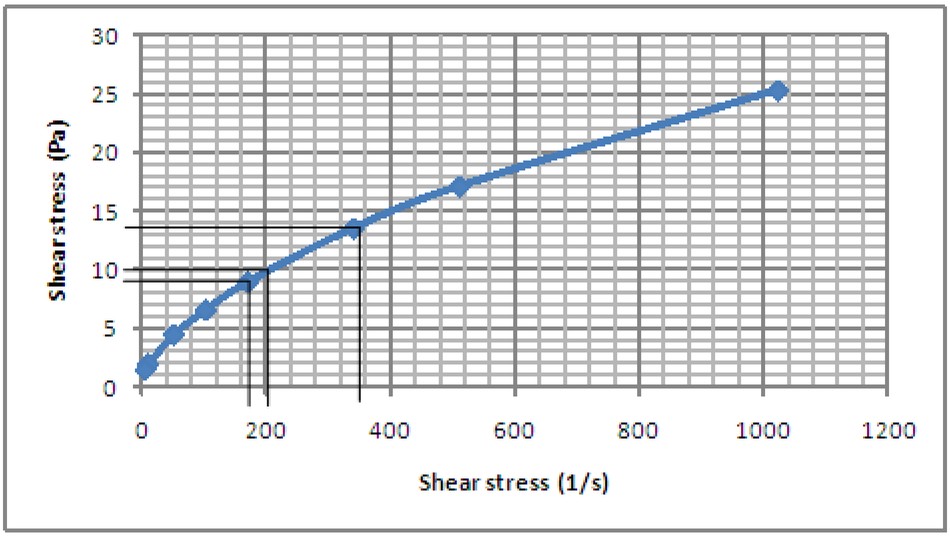


Figure 3-8: Flow curve.

The pressure loss equations:

$$\Delta p_{lam} = 4\,K\left(\frac{8\bar{v}}{d}\cdot\frac{3n+1}{4\,K}\right)^{n}\frac{1}{d}\cdot\Delta L = 4\cdot 0.52\left(\frac{8\cdot v}{0.1}\cdot\frac{2.68}{2.24}\right)^{0.56}\frac{1}{0.1}\cdot 1000 = 267\,553\cdot v^{0.56}$$

$$\Delta p_{turb} = a\cdot N_{\mathrm{Re}}^{-b}\cdot\frac{4L}{d_h}\cdot\frac{1}{2}\rho\bar{v}^2 = 1.619\cdot 10^6\cdot N_{\mathrm{Re}}{}^{-0.214}\cdot \mathrm{v}^2$$

a = (log n + 3.93) / 50 = (log 0.56 + 3.93)/ 50 = 0.0736, b = 0.286
b = (1.75 + log n) / 7 = (1.75 + log 0.56)/ 7 = 0.286

$$\bar{v} = \frac{q}{A} = \frac{q}{\pi/4\cdot 0.1^2} = 127.4\,q$$

$$N_{Re} = \frac{\rho\cdot\bar{v}d}{\mu_{eff}} = \frac{d^n\cdot\bar{v}^{2-n}\cdot\rho}{K_p\cdot 8^{n-1}} = \frac{0.1^{0.56}\cdot v^{1.44}\cdot 1100}{0.56\cdot\left(\frac{3\cdot 0.56+1}{4\cdot 0.56}\right)^{0.56}\cdot 8^{-0.44}} = 1\,206\cdot v^{1.44}$$

Select three data points in the laminar and three in higher flow/turbulent regime and plot the result (Figure 3-8):

Symbol	q		v	N_{Re}	Δp_{lam}	Δp_{turb}
Unit	l/min	m^3/s	m/s	-	MPa	MPa
Data	100	0.00170	0.22	136	0.11	
	250	0.00417	0.53	1083	0.18	
	500	0.00833	1.06	2170	0.28	0.20
	1000	0.017	2.12	4340	0.41	0.66
	1500	0.025	3.18	6380		1.34
	2000	0.033	4.25	8687		2.19

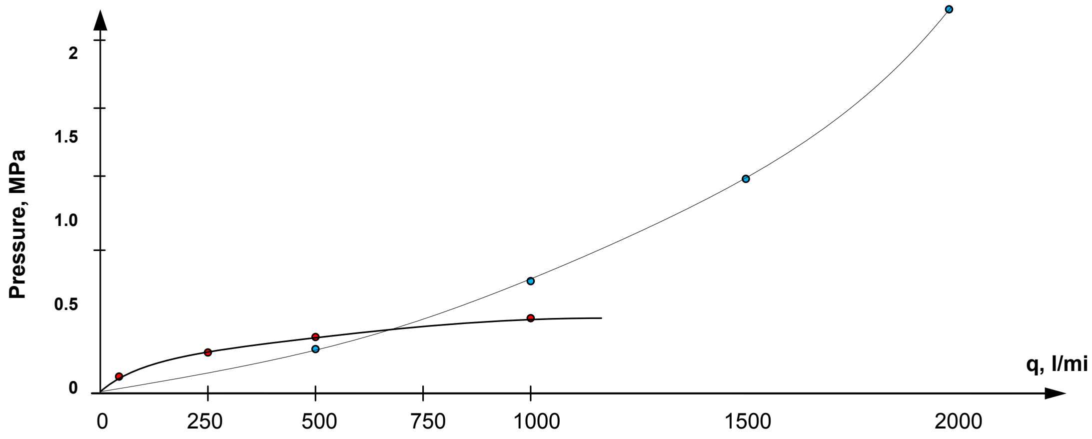


Figure 3-8: Laminar and turbulent pressure loss.

3.9 Pressure loss. Field data

Check first the solution for 5 spm. Then repeat for the remaining speeds. Here a check is made only for the 5 spm case. Here we think the flow is laminar. Pressure reading at 5 spm = 13 bars. To check theoretical Δp at 5 spm we go through these steps:

Step 1: Rheology: n = 0.50, K= 1.63 Pas^{-n}

Step 2: Check flow regime: $N_{Re} = \frac{\rho v d}{\mu_{eff}}$

$$\mu_{eff,DP} = \left(\frac{8v}{d}\cdot\frac{3n+1}{4n}\right)^n \cdot \frac{Kd}{8v} = \left(\frac{8\cdot 0.15}{0.105}\cdot\frac{3\cdot 0.5+1}{4\cdot 0.5}\right)^{0.5}\cdot\frac{1.63\cdot 0.105}{8\cdot 0.15} = 0.54\ Pas$$

$$\mu_{eff,Ann} = \left(\frac{12v}{d_0-d_i}\cdot\frac{2n+1}{3n}\right)^n \cdot \frac{Kd}{12v} = \left(\frac{12\cdot 0.02}{0.188}\cdot\frac{2\cdot 0.5+1}{3\cdot 0.5}\right)^{0.5}\cdot\frac{1.63\cdot 0.188}{12\cdot 0.02} = 1.47\ Pas$$

$$v = \frac{q}{A}, \qquad d = 4.127\cdot 0.0254 = 0.105\ m$$

$$q = 15.29\cdot 10^{-3}\cdot 5 = 0.0013\ m^2/s$$

$$A_{0p} = {}^{\pi}/_{4}\cdot 0.105^2 = 0.0087\ m/s$$

$$A_{Ann} = (12.40^2 - 5^2)\cdot 0.0254^2 = 0.065\ m^2$$

$$v_{dp} = {}^{q}/_{A_{0p}} = 0.0013/0.0087 = 0.15\ m/s$$

$$v_{Ann} = {}^{q}/_{A_{nn}} = 0.0013/0.065 = 0.02\ m/s$$

$$d_o - d_i = (12.4 - 5)\ 0.0254 = 0.188\ m$$

$$N_{Re,DP} = \frac{\rho\cdot d\cdot\bar{v}}{\mu_{eff}} = \frac{1120\cdot 0.15\cdot 0.105}{0.54}$$

$$= 32\ (\textit{since it is so low it is assumed laminar also in the annulus})$$

Step 3: Find total pressure drop in flow system: $\Delta p = \Delta p_{dp} + \Delta p_{ann}$

$$\Delta p_{dp} = 32 v \mu_{eff} \cdot \frac{L}{d^2} = 32 \cdot 0.15 \cdot 0.54 \cdot \frac{4500}{0.105^2} = 10.6 \cdot 10^5 Pa$$

$$\Delta p_{ann} = 48 v \mu_{eff} \cdot \frac{L}{d_u^2} = 32 \cdot 0.02 \cdot 0.147 \cdot \frac{4500}{0.188^2} = 1.2 \cdot 10^5 Pa$$

$$\Delta p_{tot} = \Delta p_{dp} + \Delta p_{ann} = (10.6 + 1.2) 10^5 = 11.8 \cdot 10^5 Pa$$

The task was to evaluate and compare the estimated pressure with the recorded pump pressure. As exemplified at 5 spm flow, the theory compared to reality, 11.8 vs. 13, differed by -9%. One possible explanation for this difference is that the theoretical Power law underestimates the real shear rate. Other error sources exist; selecting the correct rheology model; fluid is very Binghamian. When does flow turn from laminar to turbulence? In real wells it turns turbulent earlier than theoretically predicted, due to roughness and uneven flow path. Turbulent pressure loss is highly empirical.

3.10 Pressure loss. Effects of rotation

During rotation of the pipe, external energy is added to the fluid (from the top drive). If the pipe is exposed to strict, controlled rotation the mud will be stirred (like in a viscometer). The mud is non-Newtonian and shear-thinning ($n < 1.0$) during laminar flow. For laminar flow at SPM = 5, the friction loss will be reduced from 13 at pure flow to 8 bars at added rotation.

To check this fact we will try to estimate the reduction in effective viscosity. Assume the fluid behaves according to the Power-Law.

Step 1: Find $\bar{v}_{rot}$ = peripherical velocity of pipe. One rotation corresponds to the length $2\pi r$. The outer wall velocity is zero. Divide by 2 to obtain the average of the two extreme velocities:

$$v = \left(2\pi r \cdot RPM \cdot 1 \frac{min}{60} s + 0\right)/2 \quad = 2\pi \cdot 5/2 \cdot 0.0254 \cdot \frac{100}{60} \cdot \frac{1}{2} = 0.33\ m/s$$

Step 2: Translate the rotational velocity variation to shear rate:

$$\dot{\gamma} = \frac{12v}{d_0 - d_i} \cdot \frac{2n+1}{3n} = \left(\frac{12 \cdot 0.33}{13.4 - 5) \cdot 0.0254} \cdot \frac{2 \cdot 0.5 + 1}{3 \cdot 0.5}\right) = 30\ s^{-1}$$

Step 3: Calculate the shear rate caused by pure axial flow at 5 SPM: v_{ann} at 5 spm was found to be 0.02 m/s in the solution of Task 3.9 above.

$$\gamma = \frac{12v}{d_0 - d_i} \cdot \frac{2n+1}{3n} = \left(\frac{12 \cdot 0.02}{13.4 - 5) \cdot 0.0254} \cdot \frac{2 \cdot 0.5 + 1}{3 \cdot 0.5}\right) = 1.5\ s^{-1}$$

Step 4: Effective viscosity: In Figure 3-10 the shear rate has been increased from 1.5 to additionally 30 s^{-1} (the two were assumed to be additive). We compare the effect of rotation in terms of changes in the Newtonian viscosity: Use the flow curve in Figure 3-10. Here we are left to assume the shear stress at these low shear rate values. From the rheological data in Task 2-2 we select the smallest recordable one, 3 Pa.

$$\mu_{5\ spm} = \tau / \gamma = 1 / 1.5 = 0.67\ Pas$$
$$\mu_{5\ spm+rotation\ at\ 100\ rpm} = \tau / \gamma = 8 / (1.5 + 30) = 0.25\ Pas$$

We see that the effective viscosity has reduced to less than half of its original at that flow rate. Note also that the shear stress seen from the pump (SPP) is not increased, it is “for free”, only the effective viscosity is decreased, as shown in Figure 3-10.

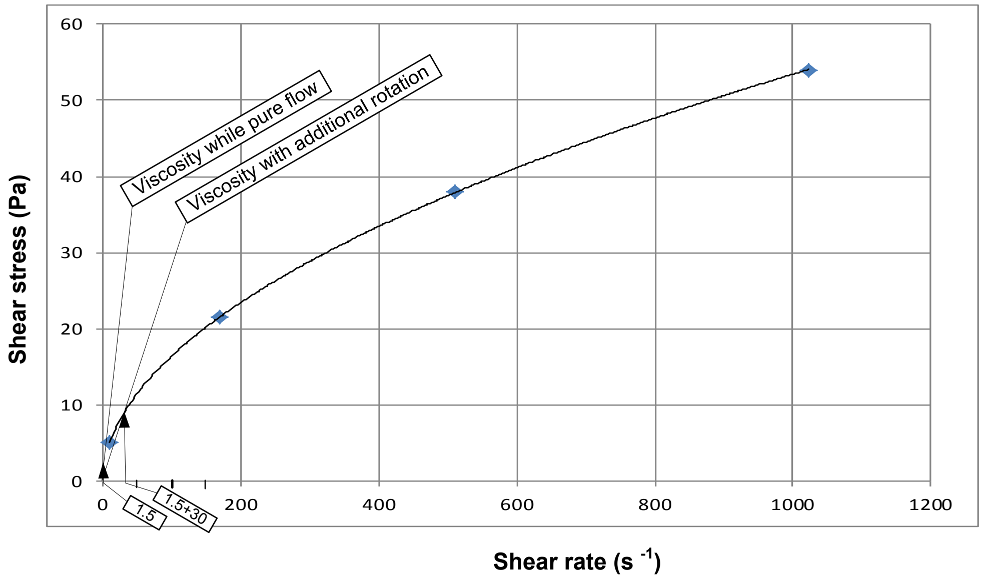
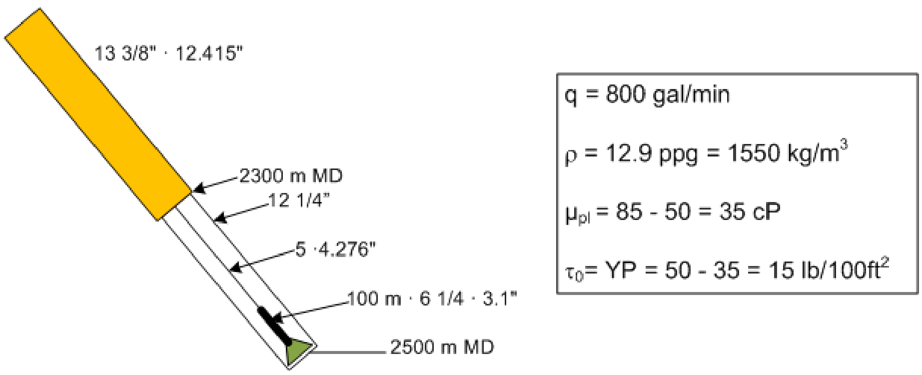


Figure 3-10: Effect of rotations on the rheology results in new ECD for instance at the low shear stress of 10 Pa.

If the pipe is exposed to vigorous rotation and possibly vibrations, which often is the case in a real well, then additional turbulence will introduce **higher** pressure loss, not a lower one. However, at the highest pump speed (100 spm) flow is already turbulent and the complex motion of the drill string will only negligibly influence the pressure.

3.11 Pressure loss. Bit nozzle. OFU

The exercise information is summarized below:



a) In order to estimate the annular pressure drop, the mud velocity, the effective viscosity and the Reynolds number are needed, valid for the BHA-annulus, while drilling at a depth of 2500 m:

$$\bar{v} = \frac{q}{A} = \frac{q\,\dfrac{Gal}{\min}\cdot\dfrac{0.1377\,ft^3/gal}{60\ s/\min}}{\dfrac{\pi}{4}d\left(in\cdot\dfrac{1}{12\,in/ft}\right)^2} = 0.40856\frac{q}{d_h^2} = \frac{q}{2.448\,d_h^2} = \frac{800}{2.448\,\left(12.25^2-6.25^2\right)} \rightarrow 2.94\ \ ft/s$$

First check the flow regime before estimating Δp:

$$N_{\text{Re}} = \frac{757\rho\bar{v}(d_2-d_1)}{\mu_{eff}} = \frac{757\cdot 12.9\cdot 2.94\,(12.25-6.25)}{\mu_{eff}} = 945 \Rightarrow \textit{not turbulent}$$

$$\mu_{eff} = \mu_{PL} + \frac{5\,\tau_y\cdot d_b}{\bar{v}} = 35 + \frac{5\cdot 15\cdot(12.25-6.25)}{2.94} = \underline{179}\;cP$$

$$\frac{\Delta p}{\Delta l_{dc}} = \frac{\mu_{pl}\cdot\bar{v}}{1000(d_2-d_1)^2} + \frac{\tau_y}{200(d_2-d_1)} = 0.0154\,psi/ft \quad\Rightarrow\quad 0.0154\cdot\frac{100}{0.3048} = 5.05\;psi$$

Eqv. ρ in 2500 MD:

$$\rho_{ekv} = \rho + \frac{\Delta p_{ann}}{0.052\cdot L} = 12.9 + \frac{5.05\;+\;100}{0.052\cdot\frac{2500}{0.3048}} = \underline{\underline{13.15\;\;PPG}}$$

b) Bit pressure drop in OFU and in SI-units:

$$\Delta p_{bit-OFU} = \frac{8.311\cdot 10^{-5}\cdot 12.9\cdot 800^2}{0.95^2\cdot\left(5\cdot\frac{\pi}{4}\cdot\left(\frac{14}{32}\right)^2\right)^2} = \underline{\underline{1\,346\;\;psi}}$$

$$\Delta p_{bit} = 1.11\cdot\frac{1}{2}\cdot 1550\left[\frac{800\;\cdot\;\frac{3.78533}{60\;\cdot 1000}}{5\;\frac{\pi}{4}\left(\frac{14}{32}\;\cdot\;0.0254\right)^2}\right]^2 = 9.318\;\;MPa\; = \underline{1369\;\;psi}$$

Conclusion: Choice of units does not make any difference for this formula.

c) Total pressure loss in the system:

DP: $\bar{v} = \frac{800}{2.448\cdot 4.276^2} = \underline{17.87\;\;ft/s}$

DC: $\bar{v} = \frac{800}{2.448\cdot 3.1^2} = \underline{34.0\;\;\;ft/s}$

Turbulent or laminar in largest area?

$$N_{\text{Re}} = \frac{928\rho\bar{v}d}{\mu_{eff}} = \frac{928\cdot 12.9\cdot 17.87\cdot 4.276}{35+} = \underline{26\,136} \rightarrow \text{turbulent by far}$$

Pressure losses, turbulent:

DP: $$\Delta p_{turb}\;/\Delta l = \frac{\rho^{0.75}\mu_{pl}^{\;0.25}\bar{v}^{1.75}}{1800d^{1.25}} = \frac{12.9^{0.75}\cdot 17.87^{1.75}\cdot 85^{0.25}}{1800\cdot 4.276^{1.25}} = 0.29\;\;psi/ft$$

Total pressure loss in the circulating system (not all details shown):

$\Delta p_{tot} = \Delta p_{DP} + \Delta p_{dc} + \Delta p_{bit} + \Delta p_{ann1} + \Delta p_{ann2}$

$\Delta p_{tot} = 2\,320 + 300 + 1\,346 + 5 + 100 = 4\,071$ psi

3.12 Swab pressure. Clinging factor

a) Make a detailed drawing. The clinging fluid is defined by R_{cling} in Figure 3-12, causing a certain volume to accompany the string upwards.

b) Identify an expression of the velocity profile with the correct boundary condition;

$$v(r)_{r=R0} = 0$$
$$v(r)_{r=Rcling} = 0$$
$$v(r)_{r=Ri} = v_p$$

From the velocity profile we can obtain the cling volume pr. unit time:

$$q_{cling} = \int_{R_{cling}}^{R_i} v(r) 2\pi r dr$$

c) Mass balance: $q_{up} = v_{pipe} \cdot A_{pipe} + q_{cling}$. An equivalent fluid mass has to move down

d) Cling factor = q_{cling} / q_{down}

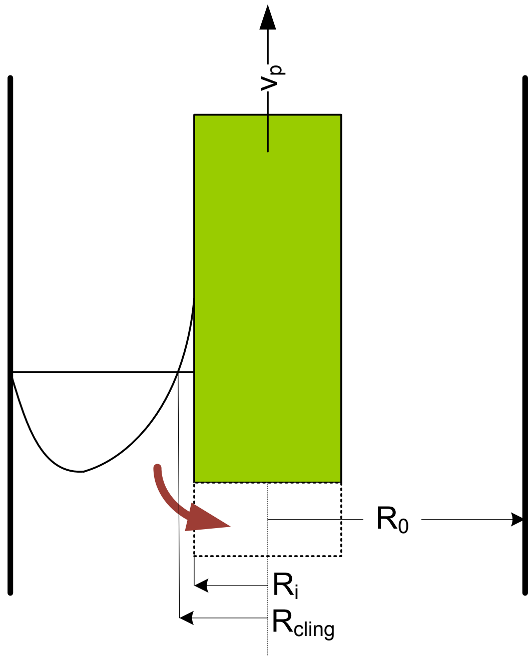


Figure 3-12: Flow streaming down the annulus when pulling a closed-end drill string. The frictional pressure loss causes a pressure deficit below the string's bottom end.

3.13 Swab pressure model

The V_{cling} will be added to the fluid volume displaced by the drill string.

$$q_{DS} = A_{DS} \cdot v_{DS}, \; v_{displaced} = q_{DS}/A_{flow}$$
$$V_{total} = V_{pump} - V_{displaced} - V_{cling}$$

Step 1. Understand the physics and forces in a Drawing
Step 2. Make a control box and entering exiting forces.

$$\Delta p \cdot \pi r^2 = \tau \cdot 2\pi r \Delta L$$
$$\Delta p/\Delta L \cdot r = 2\tau$$

Step 3. Differentiate: $\Delta p/\Delta L \cdot dr = 2\, d\tau$
Step 4. Integrate over the control volume (along r)

4 Hydraulic program

4.1 Mud pump issues

a) Characteristic diagrams of mud pumps are shown in Figure 4-1:

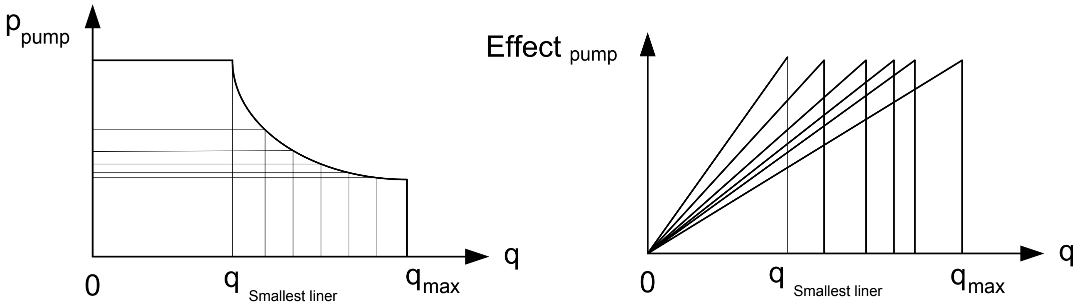


Figure 4-1: Maximum pump pressure for each liner (left) and maximum effect for the same (right)

Power or effect = E = p · q, is expressed in Horse Power (HP) or in watt (W). As shown in Figure 4-1 (to the right) the maximum effect is constant for all liners.

Volumetric efficiency = 0.96 for new pumps. When it has reduced to 0.93 the pump has to be upgraded (replace valves and / or piston / liner).

b) Parallel → higher q, used for upper wellbore sections. Series → higher p (only for very special application, not done for mud pumps).
c) Centrifugal pumps can deliver high flow rate but limited pressure (max 3 bars) since there is an open slot between statot and rotor.
d) Knocking is a result of high acceleration. At the highest acceleration the water is not able to fully follow the high accelleration, and the following takes place: -> vacuum effect -> vapor pressure is surpassed and water starts boiling (vapor bubbles are formed on the steel surface) -> implosion of vapor bubbles during retardation of piston movement → repeated shock vaves weakenes the steel.

e) Find first $\Delta p_d = \Delta p_p - \Delta p_{bit}$

Bit pressure loss can be determined since nozzle sizes are known:

$$p_{bit} = 1.11\frac{1}{2}\rho v^2 = 1.11 \cdot \frac{1}{2} \cdot 1300 \cdot 100^2 = 72.15 \cdot 10^5 Pa$$

$\bar{v} = q/A_{nozzle}$

$A_{nozzles} = (\pi/4)d_e^2 \;\; = \pi/4 \cdot 0.0206^2 = 3.3333 \cdot 10^{-4}$ m^2

$$d_e = \sqrt{d_1^{\,2} + d_2^{\,2} + d_3^{\,2}} = \sqrt{3} \cdot d_1 = \sqrt{3} \cdot \frac{15}{32} 0.0254 = 0.0206\, m$$

$q_1 = 2\,500 /(1\,000 \cdot 60) = 0.04167$ m^3/s $\quad v_1 = q_1/A = 0.04167 / 3.33 \cdot 10^{-4} = 125.2$ m/s
$q_2 = 670 /(1\,000 \cdot 60) = 0.011$ m^3/s $\quad v_2 = q_2/A = 0.017 / 3.33 \cdot 10^{-4} = 33.5$ m/s

Resulting in these values:

Parameter	q	p_p	Δp_{bit}	Δp_d
Unit	lpm	bar	bar	bar
Data	2500	230	72	157.9
	670	33	8.1	24.9

K_1 and m are determined from: $\Delta p_d = K_1 D\, q^m$. Selecting the operator ln or log does not make any diffidence here.

$m = \ln(\Delta pd_1/\Delta pd_2) / \ln(q_1/q_2) = \ln(157.9 / 24.9) / \ln(2\,500 / 670) = 1.6$
$K_1 = \Delta p_d/q^m \cdot D = 157.9 \cdot 10^5 / (0.04167^{1.69} \cdot 2\,500) = 2 \cdot 10^6$

4.2 Nozzle selection. Section wise

a) Existing parasitic pressure loss and its parameters (assume 3 identical nozzles):

$$d_e = \sqrt{n} \cdot d_1 = \sqrt{3}\, \frac{15}{32} \cdot 0.0254 = 0.0206\, m$$

$$v_{nozzle1} = \frac{q}{A} = \frac{2000}{1000 \cdot 60 \cdot \frac{\pi}{4} 0.0206^2} = \frac{0.0333}{0.000333} = 100.2\ m/s$$

$v_{nozzle,2} = 87.7\ m/s$

$p_{bit1} = 1.11 \cdot 1/2 \cdot 1400 \cdot 100^2 = 78 \cdot 10^5 Pa\ /\ 59.8 \cdot 10^5 Pa \quad (represents\ pump\ rate\ 1/rate\ 2)$

$\Delta p_{loss} = 186 - 121.8 = 108.2$ bar

$\Delta p_{loss} = 120 - 59.8 = 60.2$

$$\Delta p_{loss} = K_1 D \cdot q^m$$

$$m = \frac{\log(\Delta p_{loss1} / \Delta p_{loss2})}{\log(q_1 / q_1)} = \frac{\log(108.2 / 60.2)}{\log(2500 / 1750)} = 1.64$$

$$K_1 = \frac{108.2 \cdot 10^5}{2500 \cdot 0.0333^{1.64}} = 1.16 \cdot 10^6$$

b) From existing hydraulic program parameters we obtain:

q_{opt} = $[2p_{pump,m} /(m+2)K_1D]^{1/m}$ = $[2·270·10^5(1.64+2)·1.16·10^6·3\ 000)]^{1/1,64}$ = $0.0043^{0.61}$ = 0.0358 m^3/s

Check the flow rate at the boundary conditions (in SI-units to be able to compare):

$$q_{max,p} = 2\ 700\ lpm = 2\ 700/(1\ 000 \cdot 60) = 0.045\ m^3/s$$

$$q_r = 1\ 700/(1\ 000 \cdot 60) = 0.028\ m^3/s$$

$$q_{\max,ann} = 2\ 600 / 60\ 000 = 0.0433\ m^3 / s$$

Since q_{opt} is in between q_r and $q_{max,ann}$, 0.0358 is the optimal solution at this depth.

c) The optimal bit pressure loss is given by this equation:

$$\Delta p_{bit} = p_{max,p} \cdot m/(m+2) = 0.458 \cdot 270 \cdot 10^5 \cdot 1.64(3.64) = 121.6 \cdot 10^5\ Pa$$

To find the optimal nozzle, solve pressure drop equation vs. optimal bit pressure drop

$$1.11 \cdot \frac{1}{2}\rho\left(\frac{q_{opt}}{\frac{\pi}{4}d_e^2}\right)^2 = 121.6 \cdot 10^5\ Pa$$

$$d_e^4 = \frac{1.11 \cdot 0.5\rho \cdot q^2}{\left(\frac{\pi}{4}\right)^2 \cdot 121.6 \cdot 10^5}$$

$$d_e^4 = \sqrt[4]{\frac{1.11 \cdot 0.5 \cdot 1\,400 \cdot 4^2 \cdot 0.0358^2}{\pi^2 \cdot 121.6 \cdot 10^5}} = 0.019\ m$$

Nozzle diameter is not optimal at 3000 meters depth but close to. It should have been:

$$d_{SI} = \frac{d_e}{\sqrt{3}} = \frac{0.019}{\sqrt{3}} \Rightarrow d_{OFU} = \frac{0.019}{\sqrt{3}} \cdot \frac{32}{0.0254} = 0.011 \cdot \frac{32}{0.0254} = 13.9 \text{ (32nds in)}$$

To stay below maximum pump pressure we select:

14/32″, 14/32″, 14/32″

4.3 Liner selection. Section wise

a) To optimize q_{opt} you need criteria; maximize ROP. First find a suitable expression of ROP:

$$ROP = A \cdot (q/d_{nozzle})^{a8}$$

Further we need to know the boundaries:

$$p_p = \Delta p_{loss} + \Delta p_{bit}$$

$$p_p = K_1 D q^m + K_{bit} \cdot \left(\frac{q}{d_e}\right)^2 \quad \rightarrow$$

$$p_p = K_1 D q^m + K_{bit} \cdot \left(\frac{q}{{d_e}^2}\right)^2 \quad | \, q^2$$

$$p_p \cdot q^2 = K_1 D q^{m+2} + K_{bit} \cdot \left(\frac{q}{d_e}\right)^2$$

$$\frac{q}{d_e} = \left({p_p}^2 - k_1 D q^{m+2}\right)^{0.5} \cdot \frac{1}{K_{bit}^{0.5}}$$

Entering the ezpression of q/d into the ROP expression, differentiate and fid maximum when dROP/dq = 0, resulting in an optimal expression

$$2p_p q - K_1 D(m+2)q^{m+1} = 0$$

b) In order to cover the complete range of possible flow rates, we see from the data and the pump characteristics that the 6¾″ liner has a q_{max} of 0.0422. Only this liner is large enough to supply the q_{max} of 0.04 m^3/s if necessary. Intuitively we would select the smallest liner possible, the 5½″, since it has the highest available pressure. This is what we also recommend. Alternatively we could check what optimal flow rate each liner produced at this depth:

$$q_{opt5.5} = \left[\frac{2(p_p)_m}{K_1 \cdot D(m+2)}\right]^{\frac{1}{m}} = \left[\frac{2 \cdot 383 \cdot 10^5}{1.6 \cdot 10^6 \cdot 2500\,(1.7+2)}\right]^{\frac{1}{1.7}} = 0.0454 m^3/s$$

$$q_{opt6} = \left[\frac{2(p_p)_m}{K_1 \cdot D(m+2)}\right]^{\frac{1}{m}} = \left[\frac{2 \cdot 322 \cdot 10^5}{1.6 \cdot 10^6 \cdot 2500\,(1.7+2)}\right]^{\frac{1}{1.7}} = 0.0405 m^3/s$$

$$q_{opt6.5} = \left[\frac{2(p_p)_m}{K_1 \cdot D(m+2)}\right]^{\frac{1}{m}} = \left[\frac{2 \cdot 274 \cdot 10^5}{1.6 \cdot 10^6 \cdot 2500\,(1.7+2)}\right]^{\frac{1}{1.7}} = 0.0369 m^3/s$$

$$q_{opt7.5} = \left[\frac{2(p_p)_m}{K_1 \cdot D(m+2)}\right]^{\frac{1}{m}} = \left[\frac{2 \cdot 220 \cdot 10^5}{1.6 \cdot 10^6 \cdot 2500\,(1.7+2)}\right]^{\frac{1}{1.7}} = 0.0324 m^3/s$$

We observe that the optimal flow rate is outside the boundaries for 5.5, 5.75 and 6. We could claim that the 6.25″ liner is the smallest liner with its optimm within the rang and therefore select that one.

An expression of optimal bit pressure is found from the same optimal solution as above. Substitute $p_d = K_1 D q^m$:

$$2p_p - (m+2)p_d = 0$$

$$2p_p - (m+2)(p_p - p_{bit}) = 0$$

Optimum bit pressure drop is thus: → $p_{bit} = \frac{m}{m+2} p_p$

From pump data of 6″ liner we read: $p_{pump,\,max} = 322 \cdot 10^5$ Pa

$$p_{bit,opt} = \frac{1.7}{3.7} \cdot 322 \cdot 10^5 Pa = \underline{150 \cdot 10^5\, Pa}$$

c) The depth of altering liner is found when q_{opt_1} of 6″ reaches q_{max} of liner 5¾″ which is 0.0307 m^3/s

$$\left[\frac{2(p_p)_{max}}{K_1 \cdot D(m+2)}\right]^{\frac{1}{m}} = 0.0307 \qquad \Rightarrow \qquad K_1 \cdot D(m+2) = \frac{2(p_p)_{max}}{(0.0307)^m}$$

$$D = \frac{2(p_p)_{max}}{K_1 \cdot (m+2)(0.0307)^m}$$ = (2 * 322) / [1.6 * 10^6 * (3.7 * $(0.0307)^{1.7}$] = 3 841 m

4.4 Hydraulic program. Section wise

a) By including all the variables in the ROP equation we might succeed.

$$ROP = A\ (q/d_e)^{a_8}$$

$$p_p = \Delta p_{loss} + \Delta p_{bit}$$

$$p_p = K_1 D q^m + 1.11 \cdot 0.5 \rho \left(\frac{q}{d_e^{\,2}}\right)^2 \cdot \frac{1}{(\pi/4)^2} \quad \rightarrow \quad k_{bit} = 1.11 \,.\, 0.5 \,.\, \rho \,/\, (\pi/4)^2$$

$$p_p = K_1 D q^m + k_{bit} \cdot \left(\frac{q}{d_e^{\,2}}\right)^2 \quad \rightarrow \quad \frac{q}{d_e} = \left(p_p - k_1 D q^m\right)^{0.5} \cdot \frac{1}{k_{bit}^{0.5}}$$

Substitute into the ROP equation: $ROP = A\left[\left(p_p - k_1 D q^m\right)^{0.5} \cdot \frac{1}{k_{bit}^{0.5}}\right]^{a_8} = \left(\frac{1}{k_{bit}^{0.5}}\right)^{a_8} \cdot A \cdot \left(p_p - k_1 D q^m\right)^{0.5 \cdot a_8}$

From this equation we see that for increasing depth the ROP-function will decrease.

b) Only the case of vertical wells is presented here. The horizontal case is solved identically, only the flow rate has higher importance / higher upper boundaries. Within the narrow flow rate interval for vertical wells, the 6¼″ liner covers the maximum flow rate. First check what is the optimum flow rate for each of the liners, from 6¼″ and downwards at 1 950 mMD. Since the boundary conditions do not change with depth, intuitively we would suggest that the smallest liner possible should be selected, which we do; the 5½″ liner. Just for the interest we also check the optimal flow rate at the two depths:

$$q_{opt,1950,5.5"} = \left[\frac{2 \cdot \Delta p_p}{(m+2)K_1 D}\right]^{1/m} = \left[\frac{2 \cdot 383 \cdot 10^5}{(1.65+2)2 \cdot 10^6 \cdot 1950}\right]^{1/1.65} = 0.0418\ m^3/s$$

$$q_{opt,4\,000,5.5"} = \left[\frac{2 \cdot 383 \cdot 10^5}{(1.65+2)2 \cdot 10^6 \cdot 4\,000}\right]^{1/1.65} = 0.0273\ m^3/s$$

We observe that when drilling at 4 000 m the hydraulic program is inside the theoretical limits, but not while drilling at 1 950 m. Our suggestion is therefore to use the q_{max}, 0.028, until a depth at which q_{opt} is reached:

$$D = \frac{2 \cdot \Delta p_p}{(m+2)\ K_1 q^m} = \frac{2.383 \cdot 1\ 0^5}{(1.65+2)\ 2 \cdot 10^6 \cdot 0.028^{1.65}} = 3\ 829\ m$$

Then use q_{opt} till TD. If change of liner is allowed with inclreasing depth, then a better solution is to select the liner corresponding to optimum flow rate.

4.5 Optimal parameters with BHHP. OFU. Section wise

Optimum parameters for this criterion can be derived as follows:

$$ROP = C \cdot BHHP = \frac{c}{1714} \cdot \Delta p_{bit} \cdot q = c_1\left(p_p - \Delta p_{loss}\right) \cdot q$$

$$ROP = c_1\left(p_p \cdot q - K_1 D q^m \cdot q\right)$$

Only the derivation of the core produces a sensible answer.

$$\frac{d\ ROP}{dq} = p_p - (m+1)K_1 D q^m = 0$$

$$\Delta p_{p,opt} = (m+1)K_1 D q^m \quad \Rightarrow \quad q_{opt} = \left[\frac{p_p}{(m+1) \cdot K_1 D}\right]^{1/m}$$

We observe also that optimal parasitic and bit pressure drop can be derived from the equation above:

$$\Delta p_{loss} = P_p \frac{1}{m+1}, \quad \Delta p_{bit,opt} = \frac{m}{m+1} p_p$$

To estimate q_{opt}, K_1D and m must be determined from: $\Delta p_{loss} = K_1 D q^m$

Input		Output	
q (GPM)	p_p (psi)	Δp_{bit} (psi)	Δp_d (psi)
500	3 000	2 097.0	903.0
250	800	524.3	275.7

$$m = \frac{\ln\,(\Delta p_{loss1}/\Delta p_{loss2})}{\ln(q_1/q_2)} = \frac{\ln(903/275.7)}{\ln(500/250)} = 1.712$$

$$K_1 D = \Delta p_{loss}/q^m = \frac{903}{500^{1.712}} = 0.02168\ (psi/GPM^m)$$

Now q_{opt} can be found (note that the units are correct):

$$q_{opt} = \left[\frac{\Delta p_{p.max}}{(m+1)K_1 D}\right]^{1/m} = \left[\frac{3620}{(1.71+1)\ 0.0217}\right]^{1/1.712} = 627\ GPM$$

Including the volumetric efficiency factor, e_{vol}, the practical q_{opt} is:

$q_{opt} = 627 \cdot 0.9 = 564$ GPM

Now the bit nozzles can be estimated from $\Delta p_{bit,opt} = \Delta p_{bit}$:

$$\frac{m}{m+1} \cdot p_{p,max} = 8.311 \cdot 10^{-5} \cdot \rho \cdot q^2/{C_d}^2 \cdot A_{nozzle}$$

$$A_{nozzle,opt} = \sqrt{\frac{8{,}311 \cdot 10^{-5} \cdot \rho q_{opt}^2}{C_d^2 \cdot \Delta p_{bit}}} = \sqrt{\frac{8.311 \cdot 10^{-5} \cdot 564^2}{0.95^2 \cdot 2284}} = 0.33\ in^2$$

$$p_{bit,opt} = \frac{m}{m+1} \cdot 3620 = \frac{1.71}{1.71+1} \cdot 3620 = 2284\ psi$$

Finally the area can be expressed as:

$$A = \sqrt{3} \cdot \frac{\pi}{4}\left(\frac{d_e}{32}\right)^2 \ \rightarrow\ d_e^2 = \frac{A \cdot 4 \cdot 32^2}{\pi\sqrt{3}} \rightarrow$$

$$d_e = \sqrt{\frac{0.33 \cdot 4 \cdot 32^2}{\sqrt{3} \cdot \pi}} = x\ (32nd\ in)$$

Assume x = 15.8 → 16/32″, 16/32″, 16/32″
Assume x = 16.8 → 16/32″, 17/32″, 17/32″

4.6 Liner selection. Complete well

a) Optimal flow rate in pump range II is derived initially like in Exercise 4.4.

$$ROP = K \cdot \left(\frac{q}{d_{nozzle}} \right)^{a_8} = \frac{K}{K_{bit}} \left(p_p q^2 - K_1 D \cdot q^{m+2} \right)^{\frac{a_8}{4}}$$

In this equation we substitute q·p = E

$$ROP_{II} = \frac{K_a}{K_{bit}} (p_{max} \cdot q^2 - K_1 D \cdot q^{m+2})^{a_8/4} = K'(E_p \cdot q - K_1 D q^{m+2})^{a_8/4}$$

$$\frac{\partial ROP_{II}}{\partial q} = \frac{K^1 \left(E_p - (m+2) K_1 D q^{m+1} \right)^{\frac{a_8}{4}}}{(\ )^{\frac{a_8}{4} 1}} = 0 \qquad q_{opt\ II} = \left[\frac{E_p}{K_1 \cdot D(m+2)} \right]^{\frac{1}{m+1}}$$

b) Range II stops towards lower q, when $q_{opt,II} = q_{5,5",max}$, while range I stops towards higher q when $q_{opt,I} = q_{5,5",max}$.

$$q_{opt,II} = \left(\frac{E_p}{K_1 D(m+2)} \right)^{1/(m+1)} \Rightarrow D_{II} = 1852\ m$$

$$q_{opt,I} = \left(\frac{2p_p}{(m+2)K_1 D} \right)^{1/m} = \left(\frac{2 \cdot 383 \cdot 10^5}{3.5 \cdot 2.20 \cdot 10^6 \cdot D} \right)^{1/1.5} = 0.028 \Rightarrow D_I = 2123\ m$$

c) The graphical solution speaks for itself:

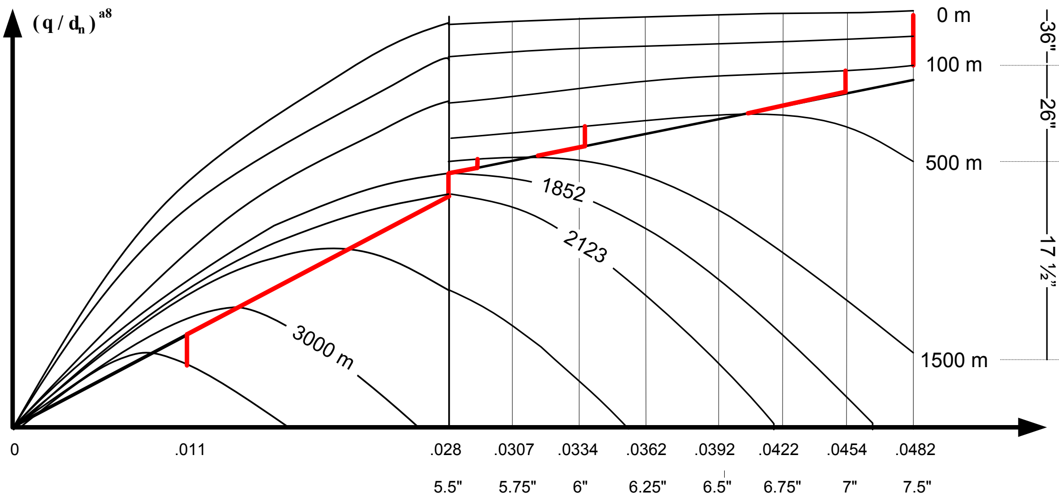


Figure 4-6: Graphical solution.

d) From Table 4-6 in the Task section we can see that between 1 500 and 3 000 m there are two potential options. However, from Figure 4-6 above we already know that at 1 852 m we are inside pump range I, and applying the 5½″ liner. We also see that the maximum flow rate of the 5½″ liner is the optimum one for most of its operational time. This has already been shown in task b) above.

4.7 Liner selection. Complete well

a) Find first what working area prevails in the boundary zone between pump are II and I, which are valid as indicated in Figure 4-7.1. Area II ends with the 5.75″ liner when $q_{max,\,5.5}$ is reached:

$$q_{opt,II} = 0.028 = \left[\frac{E}{K_1 D(m+2)}\right]^{1/(m+1)} = \left[\frac{1.076 \cdot 10^6}{1.72 \cdot 10^6 \cdot D \cdot 3.52}\right]^{1/2.52} = D_{II-I} = 1457\ m$$

At 1 457 m depth the pump area I is entered, coming from II. From the pump chart we note that 5½″ liner has p_{max} = 383 bar. Solving q_{opt1} eqn. for D we find where $q_{opt,I}$ starts.

$$q_{opt,I} = 0.028 = \left[\frac{2p_p}{K_1 D(m+2)}\right]^{1/m} = \left[\frac{2 \cdot 383 \cdot 10^5}{1.72 \cdot 10^6 \cdot D \cdot 3.52}\right]^{1/1.52} = D_{I-opt} = 2\,900\ m$$

b) In Figure 4-7.1 we see the division between the two pump areas. A preliminary solution of how to distinguish between them is given in Figure 4-7.2, its upper part.

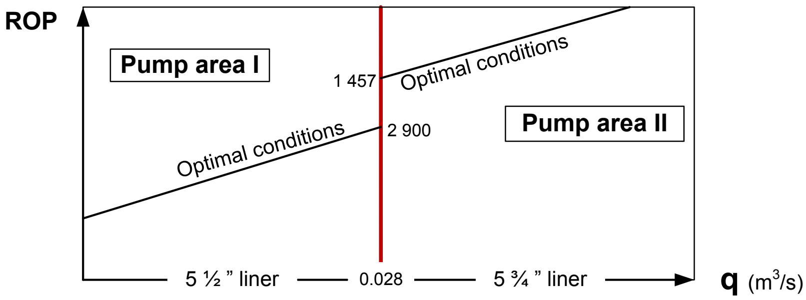


Figure 4-7.1: Boundary area between pump area II and I

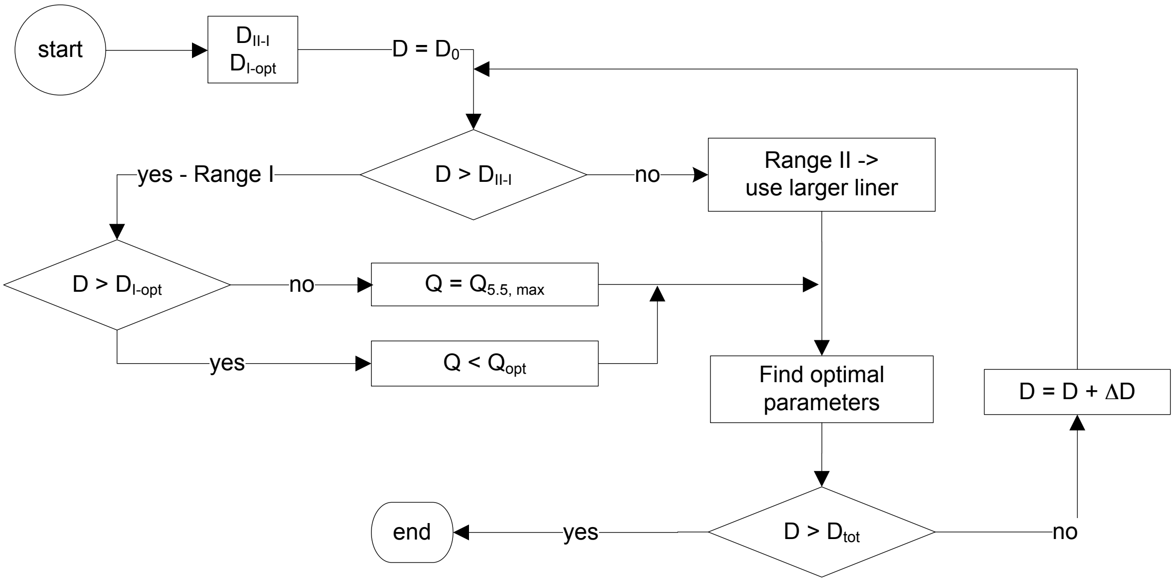


Figure 4-7.2: Flow chart of estimating hydraulic parameters while in the boundary zone between the pump area ranges.

5 Wellbore challenges

5.1 Filtration control

a) Fluid loss can be optimized in three ways:
   1. Ensure a good filter in the wellbore wall by including a wide range of particles in the mud (Bentonite, Barite and cuttings are already there. It may be enough)
   2. Ensure a good filter cake by making sure the mud is dispersed
   3. Losses through the filter cake in WBM are treated by increasing the viscosity of the fluid phase (starch, CMC and sodium polyacrylate). OBM rarely requires such additives

b) Increased fluid loss is detected through the filter press test. Depending on the fluid loss, its rheology and other test results you normally find out which of the 3 qualities in question a) above are the problem. See more details in textbook Chapter 2.

c) Filtrate will lead to approximately the same invasion depth (typically 3 feet). Particles will invade both sands typically 1 ft, assuming the particle size distribution of the added or the existing particles in the mud covers at least the range of 1/7th to 1/3rd of the pore size distribution. That assumption is normally fulfilled through the presence of Bentonite / polymers + Barite + cuttings.

d) Water-flow into the shale can be controlled by
   1. Increasing the viscosity of thefluid face
   2. Establishing a filter; same principles as in c) above

5.2 Filtration control

a) Standard fluid loss = filtrate volume in ml after 30 min

$$V_f = AC \cdot \sqrt{t}$$

API filter press $A_1 = 45\ cm^2 = 45 \cdot 10^{-4}\ m^2$
Flow into porous formation:
Effective formation height (10%) $h = 4000\ ft \cdot 0.3048 \cdot 0.10 = 122\ m$
Wellbore surface area $A_2 = \pi \cdot d \cdot L = 15 \cdot 0.0254 \cdot \pi \cdot 122 = 149\ m^2$

Assuming that an identical filter is built up on the sandstone surface, the loss during the first 30 minutes can be estimated:

$$\frac{V_{f_2}}{V_{f_1}} = \frac{A_2}{A_1}$$

$$V_{f_{2,0,5}} = V_{f_1} \cdot \frac{A_2}{A_1} = 25 \cdot 10^{-6} \cdot \frac{149}{45 \cdot 10^{-4}} = 0.83\ m^3$$

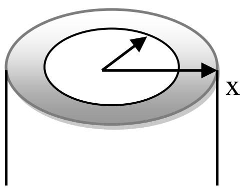
$$V_{f\ 2.24} = V_{f\,2,0.5} \cdot \frac{\sqrt{24}}{\sqrt{0,5}} = 0.83 \cdot 6.93 = 5.74\ m^3$$

The filter will have the same progress and same constants.

b) Intrusion after 24 h with 100% displacement of 5.74 m^3. Since only the pores are invaded, the affected volume to be invaded will be accordingly larger.

Affected volume = $V_{eff} = \frac{V_f}{0.15}$

Hole radius: $r_{hole} = \frac{15}{2} \cdot 0.0254 = 0.19\ m$



Effected or invaded volume can be expressed by means of the radius, $r_{invaded}$

$$V_{eff} / Ø = \left(\pi \cdot r_{invaded} - \pi \cdot r_{hole}^{2}\right) \cdot h$$

$$\pi \cdot r_{invaded}^{2} \cdot h = V_{eff} / Ø + \pi \, r_{hole}^{2} \cdot h$$

$$R_{inv} = \sqrt{(V_{eff} / Ø + \pi \cdot r_{hole}^{2}) \cdot 1/(\pi h)}$$

$$R_{inv} = \sqrt{5.74/0.15 + \pi \cdot 0.59^2 - 122)/(\pi \cdot 122)} = 0.369 \text{ m (which is round 1 ft)}$$

c) Obtain knowledge of the sandstone's pore size distribution. Apply
- Particles in the mud with a mean size = ⅓ of pore size.
- Colloidal particles must be dispersed
- Base fluid viscosity can be increased.

For OBM the particle distribution is manly taken care of by emulsified water droplets, with a wide droplet distribution.

Follow up by making a filter press test every 30 min. In case filter loss increases, every bullet point above must be evaluated through more tests, viscosity especially and specific countermeasures are set in motion.

Filter test is important mainly to obtain a thin and tight filter cake to avoid stuck. Secondly, while drilling in the reservoir, to minimize fm-damage.

Filter loss is increasing because we are drilling trough contaminants like salt, chalk, etc., which are causing flocculation; resulting in thick filter cake and perhaps in stuck pipe.
Solution/explanation: Add thinners/-dispersants/deflocculators.
If massive attack; dump the returning mud.

5.3 Cuttings concentration

a) At the end of the horizontal section the concentration would also be close to 0.02. A bed has been established which is in equilibrium between erosional and depositional forces. The bed height is a result of the steady state process. Settling and lifting of cuttings is a function of flow rate, ROP, etc. The bed height can be estimated theoretically. We select not to do that, but rather assume that a certain % of the generated cuttings are left behind in the wellbore. E.g. 10% of the wellbore cross sectional area is in average filled with cuttings. This means 10% of what is drilled out is left in the well. The initial concentration is accordingly reduced by 10%;

$$c_2 = c_1 \cdot (1 - 0.1) = 0.2 \cdot 0.9 = 0.018$$

b) In the vertical section the concentration will increase due to the slipping of cuttings, expressed through the transport ratio R_t. This could have been estimated through Stokes Law. We select here just to assume a typical value of R_t, which is 0.75. We then have:

$$c_3 = c_2 / R_t = 0.018 / 0.75 = 0.024$$

c) The cuttings removal forces are opposed by gravity and cohesive forces. Gravitational forces are given by Stokes law. Removal forces are given by drag and lift forces. The drag force is a function of particle Reynolds number and spherisity. The drag and lift forces can be developed info the critical lift velocity or rolling velocity. String RPM will induce a drag force onto the cuttings, referred to as the viscous coupling. At a given cuttings feed rate, a stationary bed height will form as a function of the equilibrium state between all involved input variables, some eroding, some building.

d) Here the bed height will be higher due to higher cross sectional area and thus lower fluid velocity, assuming the entrance effects do not ruin this assumption. Settling rate will increase, but first after the entrance effects are passed. When BHA during tripping is shoveling much cuttings into a narrower wellbore it is easy to imagine that the BHA may become jammed.

e) At least five issues are related to poor solids control:
   - Building of solids bed → Mechanical stuck pipe
   - Building of viscosity / high gel → high pressure losses → high ECD → fracture
   - High cuttings concentration in annulus → higher mud density in annulus → high ECD → lower ROP
   - Higher mechanical friction → loss of WOB → reduced ROP
   - Loss of filter control → thicker mud cake → differential sticking

5.4 Cuttings

a) Friction, drag, lifts cohesive, gravity. Make drawing. Rolling when $\sum M \geq 0$, lifting when $\sum F_z \geq 0$.
b) Stoke's law is valid only for infinite dilution (one spheres), for small spheres (r < 0.1mm) and for laminar flow around the particle.
c) High ROP, high inclination, highly viscous mud, rolling, lifting etc.
d) Two drawings are needed: One for Drag, Lift, Cohesion, Gravity, and one for a rotating string with viscous coupling.
   The viscous coupling radius relative to R_0, increase with RPM and climb the TJ at around 120.

5.5 Density control

a) When weight material has to be added to the mud it is called weighted mud. To clean it, normally a 60 + 250 Mesh shakers + a centrifuge would be perfect. Barite is preliminary taken out by the centrifuge and later re-injected.
b) By combining this information:

   $\rho_2 = \frac{\sum m}{\sum V}$ and $m_1 = V_1/\rho_1$, we obtain $\Rightarrow \Delta V_B = \frac{(\rho_2 - \rho_1)}{\rho_B - \rho_2} V_1$

   Using it for the specified task:

$$\Delta V_B = \frac{60\,000\,(1.4 - 1.3)}{4.3 - 1.4} = \underline{\underline{2\,092\, l}}$$

   However, barite is controlled by its mass, not volume:

$$\Delta m_B = \Delta V_B \cdot \rho_B = 2.092\, m^3 \cdot 4.3\, ton/m^3 = 9.1\, ton$$

c) In the next two tasks we simply take the weighted average of all densities:

$$\bar{\rho} = \frac{\sum \rho V}{\sum V} = \frac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_s + \rho_w \cdot V_w}{V_1 + V_2 + V_3 + V_w} = \frac{1\,500 \cdot 1 + 1\,800 \cdot 0.1 + 40 + m_B \cdot (m_B/4300)}{1 + 0.1 + 40/2300 + m_B/4300}$$

   m_B = 143.2 kg

d) $\bar{\rho} = 1.55 = \dfrac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_s + \rho_w \cdot V_w}{V_1 + V_2 + V_3 + V_w}$

$= \dfrac{1\,500 \cdot 10 + 1\,600 \cdot 20 + 1\,900 \cdot 3 + V_w \cdot 1\,000}{10 + 20 + 3 + V_w} = 1\,550 \rightarrow \qquad V_w = 2.8 \text{ m}^3$

5.6 Density control

a) Remaining volume $V_r = \frac{3}{5} V = 60 \text{ m}^3 \rightarrow$ Remove 40 m³ and then add $V_{add} = V_w + V_b = 40 \text{ m}^3$

Density of added volume must also be 1.8 kg/l

$$\rho_{add} = \frac{m_1 + m_w + m_b}{V_1 + V_w + V_b}$$

$m_1 = \rho_1 \cdot V_1 = 1800 \cdot 60 = 108\,000$ kg

$V_w + V_b = 40\ m^3$

$$1800 = \frac{108\,000 + m_w + m_b}{100}\ kg/m^3$$

$m_w + m_b = 72\,000$ kg

$m_w = V_w \cdot 1\,000$

$m_b = V_b \cdot 4\,300 = (40 - V_w)\,4\,300$

$V_w \cdot 1\,000 + (40 - V_w)\,4300 = 72\,000$ kg

$V_w = 30.3$ m³

$V_b = 40 - 30.3 = 9.7$ m³

$m_b = 72\,000 - m_w = 72\,000 - 30.3 \cdot 1\,000 = 41\,700$ kg

Conclusion: Remove 40 m³ mud, add 30.3 m³ of water and 41 700 kg Barite to obtain 100 m³ mud of $\rho = 1800$ kg/m³ and LSGS of 3%.

b) Dump first $V_2 = 90 \cdot (0.055 - 0.035) / 0.055 = 40.9\ m^3$

Remaining volume $V_1 = 49.1$, $\rho_1 = 1\,600$, $m_1 = 78\,560$ kg

$\rho_w = 1\,000$, $\rho_b = 4\,300$ kg

Step 1: Add 40 900 kg of water and find new density:

$\rho_{new} = (78\,560 + 40\,900) / 90 = 1\,327$ kg/m³ $= \rho_{old}$ in step 2.

Step 2: Find out how much Barite must be added to reach the desired density of 1.7 kg/l

$$V_{add} = V_1 \frac{\rho_{new} - \rho_{old}}{\rho_{add} - \rho_{new}} \quad = \quad 90 \cdot (1.6 - 1.327) / (4.3 - 1.6) = 9.1\ m^3$$

The new volume fraction of Low gravity solids, becomes:

$LSGC_2$ = 0.03 * (90 +9.1)/90 = 0.033 → 3.3%

c) Necessary r2: $r_2gh = 410 \cdot 10^5 \rightarrow \quad \rho_2 = \dfrac{410 \cdot 10^5}{9.81 \cdot 3000} = \underline{1393\ kg/m^3}$

Fill a tank of 10 m^3 with new mud of ρ_2:

From data: V_1 = volume old mud $V_2 = 10\ m^3$
$\rho_1 = 1\,200\ kg/m^3$ V_b = volume barite $\rho_b = 4\,300\ kg/m^3$

Increase from ρ1 to ρ2:

$$\rho_2 = \frac{m_1 + m_b}{V_1 + V_b}, \quad V_b = \frac{m_b}{\rho_b}, \quad \rho_1 = \frac{m_1}{V_1}$$

$$V_1 + V_b = 10\,m^3 \quad \rightarrow \quad V_1 = (10 - V_b)$$

$$\rho_2 = \frac{(10 - V_b)\cdot \rho_1 + V_b \rho_b}{(10 - V_b) + V_b} \quad \Rightarrow V_b = 0.6226\ m^3 \quad \Rightarrow V_1 = 9.3774\ m^3$$

It is shown that 0.6226 m^3 of Barite requires 9.3774 m^3 of water to produce 10 m^3 mud of density 1.393 kg/l.

5.7 ECD. Barite

Start by converting to SI units. When ρ_{mud} > ~ 1.8 kg/l, the barite concentration is high. At stillstand, most mud types quickly develop a yield point, and barite is suspended in the gelled mud. During pumping of mud in laminar mode, any existing gel is broken, and barite will not be held in suspension as during stillstand with gelled mud. However, the mud will under these conditions exhibit a high effective viscosity but barite will, nevertheless, slip slowly downwards due to gravity. Agglomeration may also affect the settling rate. Barite has a specific density of 4.2, and an average particle size of approximately 20 μm. In accordance with Stokes's law of settling, barite particles will settle, but as indicated above, very slowly. However, wellbore inclination will shorten the settling distance to only a few centimetres, and a stratified bed of barite and cuttings will form. The stratified layers of solids will, when the angle of repose is surpassed, slide downwards. The critical angle of repose in found mainly in the build-up section of the well, and here sliding takes place. After the first "landslide", the barite concentration in the mud has become lower and accordingly, also the viscosity. Barite settling will now be a fraction faster, and the process continues at accelerated speed. The self-preserving dynamics may lead to separation of almost all the barite in the fluid in the build-up section, especially during still stand after tripping-out of the hole. The pile of Barite at the bottom of the build-up section may cause pipe sticking. Worse is the reduction of mud weight. It may disrupt the safety.

The following summary of guidelines to obtain efficient hole-cleaning is based on field experiences and on many laboratory investigations:

a) Apply micro barite or equivalent. Such material systems have an average particle size of 2 μm
b) Use of top-drive to allow for pipe rotation and redistribution of the settling barite and cuttings while tripping and drilling
c) Maximize fluid velocity during drilling and reaming by increasing pumping output and/or use large diameter drill string. This improves hydraulic and agitation of cuttings. Use in addition downhole flow enhancers
d) Design mud rheology so that it enhances turbulence in the annulus (if ECD allows). Turbulent flow will bring the cuttings and barite back into the flow stream if point 3 above "failed"
e) Perform frequent wiper trips

5.8 ECD. Flow rate & fluid consistency

a) In order to determine the ECD we need to estimate the pressure loss in the annulus. The mud has some gel, so we assume the Bingham model will suit well.

$$\mu_{pl} = \frac{\tau_{600} - \tau_{300}}{\gamma_{600} - \gamma_{300}} = \frac{51.7 - 30.6}{1022 - 511} = 0.0413\ Pas$$

$$\tau_0 = 51.7\ Pa - 1022 \cdot 0.0413\ Pas = 9.5\ Pa$$

Although the flow rate is rather high, we assume laminar flow in the annulus. However, if the purpose were to find the exact Δp we would need to check the Reynolds number to confirm this. Annular pressure loss:

$$\Delta p_a = 48 \cdot \mu_{pl} \cdot \left[\frac{L_{DC/OH} \cdot v_{DC/OH}}{(d_o - d_i)_{DC/OH}^{\ 2}} + \frac{L_{DP/OH} \cdot v_{DP/OH}}{(d_o - d_i)_{DP/OH}^{\ 2}} + \frac{L_{DP/C} \cdot v_{DP/C}}{(d_o - d_i)_{DP/C}^{\ 2}} \right] +$$

$$6 \cdot \tau_0 \cdot \left[\frac{L_{DC/OH}}{(d_o - d_i)_{DC/OH}} + \frac{L_{DP/OH}}{(d_o - d_i)_{DP/OH}} + \frac{L_{DP/C}}{(d_o - d_i)_{DP/C}} \right] = \underline{\Delta p_a = 385\,179\ Pa = 3.85\ bar}$$

Now the ECD:

$$ECD = \frac{P_{hydr} + \Delta p_{ann}}{g \cdot h} = \frac{p \cdot g \cdot h + \Delta p_{ann}}{g \cdot h}$$

$$ECD = 1250 + \frac{3.85 \cdot 10^5}{0.0981 \cdot 2100} = 1270\ kg/m^3$$

b) What is the ECD during drilling? Production of mud and cuttings:

q_{mud} = 3 500 /(1000 · 60) = 0.058 m^3/s
$q_{cuttings}$ = ROP · A_{bit} = 3 500/(1000 · 60) · 3.14/4 · $(17.5 \cdot 0.0254)^2$ = 0.0022 m^3/s

Total flow rate in annulus: $q_{ann} = q_{mud} + q_{cuttings} \cong q_{mud}$

Volumetric concentration of cuttings:

$c_{cuttings} = q_{cuttings} / (q_{mud})$ = 0.0022 / 0.058 = 0.038

We assume slip; R_t = 0.75

$c_{cuttings}$ = 0.038 / 0.75 = 0.051

ECD contribution will be enhanced by the higher density of cuttings:

$$ECD = \rho_{mud} \cdot c_{mud} + \rho_{cuttings} \cdot c_{cuttings} = 1.25(1 - 0.051) + 2.4 \cdot 0.051 = 1.31\ kg/l$$

The effect of longer well is neglected.

The casing shoe equivalent fracture density: ρ_{fr} = p/ gh = 228 . 10^5 / (1 800 . 9.81) = 1 291 kg/m^3

c) Suggested procedure to determine pressure profile, which is a f(T):

1) Find the stable temperature profile in the well in accordance with Chapter 7 of the text book. Default temperature gradient in sedimentary rocks = 10 + 0.03°C/m.
2) Look up the thermal coefficients of materials, a, in a handbook;

$\alpha_{steel} =$

$\alpha_{water} =$

3) $\rho_i = \rho_o - \rho_o \cdot \alpha_{(T_i - T_{i-1})}$ at each depth interval Δh
4) Estimate $\rho_i = f(D(f(T)))$
5) Find p_{bottom} $as \sum \rho_i \, g \, \Delta h$
6) $\rho_{equiv} = \dfrac{p_{bottom}}{g \sum \Delta h}$

5.9 Water activity

a) One definition of water activity is the vapor pressure of its salt water solution and compares it to the water vapor pressure of distilled water.

$$A_w = \frac{p_{saline\ water}}{p_{destiilled\ water}}$$

Water solution: Distilled water thus has a water activity of 1.00. All water molecules are freely moving around, not electrostatically bound to salt ions. A_w will diminish at increasing salt content.

Pore water: Find the A_w of clay through its in situ weight and then by comparing with clays with known A_w. Thus A_w of water phase is determined, and type of salt.

b) Salt ions attract polar water and make it in-active. Distilled water has a water activity 1.0 whereas for saline water it is less than 1.0.
c) Clay swelling leads to increased pressure according to this equation:

$$p_{swelling} = -\frac{RT}{V_w}\ln\left(\frac{p_{w.clay}}{p_{w.mud}}\right)$$

V_w = Molar volume of water vapor;

Most shale formations contain water since shale is (water wet). Water sensitive clay materials, such as smectite, illite and mixed – layer clays, will adsorb water and swell. This leads to an elevated localized pressure around the wellbore. The pressure may surpass the material strength; the material will disintegrate and collapse. Water is transported into the shale through different mechanisms (hydraulic pressure, osmosis, diffusion etc.).

d) Adding K-salt to WBM. Dissolved salt will bind water and hinder it to evaporate; the water vapor pressure will declines and so also the measured A_w. The A_w is inversely proportional to salinity.
e) If $A_{w, OBM} \geq A_{w, pore}$, will be drawn from the OBM and into the formation, making the shale swell brittle.
If $A_{w, OBM} \leq A_{w, pore}$, the shale will become brittle.

Yes, it can be reduced by creating a filter in the shale. This can be done by adding particles to the water phase where the average particle size distribution $\simeq$ 1/3 of the average pore throat size distribution.

5.10 Shale stability

a) Using Figure 5-6 in the task section a calcium chloride concentration of 28.2% by weight is needed to give an activity of 0.69.
b) Take an "in-situ" cut of the core, crush it and place it in a desiccator (completely air tight), expose it to different humilities (vapour pressure) through designed brines. When weight of the exposed sample has been converted to typical weight evolution of this type of shale. Through this comparison the water activity in the core can be determined. Since this is the water activity of the shale's pore water, the water activity on the mud should be identical to avoid swelling or shrinking of the shale.

c) Oil does not penetrate clay because the capillary forces are too large;

$$p_c = \frac{2 \cdot \sigma_f \cdot \cos\theta}{r}$$

This depends on the wettability (which governs the θ). For oil to be the wetting phase against the formation/cuttings/steel a wetting agent must be added to the base oil. If water is emulsified into the oil, the water activity of the water phase must be controlled properly.

Emulsifier is used to reduce the surface tension between oil and water, enabling smaller droplets, enhancing stability. Wetting agents make sure that oil is wetting the shale.

5.11 Shale stability

a) By adding salt to the mud's water phase corresponding to A_w of pore water. No osmotic force between mud and clay, and thus no water driven swelling. There are 5 ingredients in OBM, the A_w and the salt type should be as close to pore water's as possible.

1. Base oil
2. Water
3. Emulsifier (surface active additive I)
4. Surface wetting (surface active additive II)
5. Salt

a) K^+ is geometrically suitable in between Montmorilonite platelets and leads to high platelet attraction (low swelling).
b) The higher concentration, the more Na^{++} is exchanged.
c) Yes, it can be reduced by creating a filter in the shale. This can be done by adding particles to the water phase where the average particle size distribution ≃ 1/3 of the average pore throat size distribution.
d) One emulsifies, the other one oil-wets solid surfaces
e) Through Chenevert's dissicator method
f) Water flow, ion-flux, pressure flux
g) Osmotic, non-invading drilling fluid
h) Cyclic spalling, but also fractures and kick

5.12 Wellbore problems

a) Lots of material (cuttings, cavings, sloughing shale) combined with small and large washouts where material tends to accumulate. During tripping, material is scraped and squeezed.

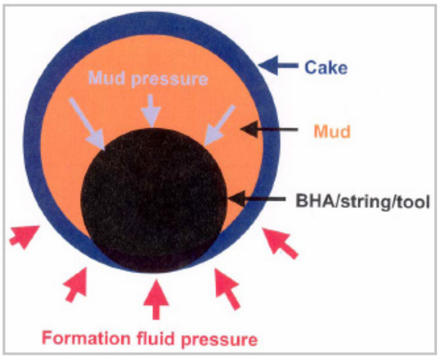


Figure 5-7. Differential sticking mechanism.

Differential sticking: Once the drill string touches the wellbore wall, the mud pressure, which is higher than the formation fluid pressure, holds the string in place. As mud cake builds around the string, a pressure seal is formed and sticking occurs as shown in Figure 5-7.

b) The consequences of a stuck pipe are very costly. They include:

- Lost drilling time spent on freeing the pipe, and on fishing if not ask to jar out string.
- Abandon tools in the hole because fishing was given up.

Rules of Thumb

1. Begin working the string immediately. Jar in the opposite direction to the pipe movement prior to becoming stuck.
2. Work the pipe to the limits.
3. If getting movement down, concentrate on expanding downward movement – and vice versa.
4. In cases of hole bridging/packing off, concentrate on downward working. Increase applied working force in gradual increments up to the maximum.

c) Salt tends to "flow" at high temperature and pressure. Use high mud weight and a mud that tend to dissolve salt (to keep the wellbore diameter as large as possible).

WBM can be used with some additives. Halite has little creep tendency.

d) Dominating mechanisms at lost circulation are:

- low pressure window
- high ECD (for many reasons)

Counter measures are:

- improved hole cleaning
- improved ECD – control

Wellbore breathing (ballooning) (any two sentence is enough for get full marks)

- The onset of wellbore breathing, often referred to as wellbore ballooning, is typically an indicator of imminent lost circulation.
- Wellbore breathing is associated with fractures that open when annular pressure is applied to the Wellbore and close when the pressure is reduced.
- These fractures fill with drilling fluid when open and subsequently return the fluid is observed as a flow out of the Wellbore when the pumps are off.
- One of the more severe consequences of wellbore breathing is the misinterpretation of the observed flow as a kick when the pumps are shut down.
- Implementing well-control procedures and increasing the mud weight is often enough to propagate the existing fractures leading to severe loss of circulation, a situation that is much more difficult to manage.