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Cryogenic Engineering: Software Solutions Vol II B

Liquefaction systems – Problems (Mathcad)

Dr. M. Thirumaleshwar

DR. M. THIRUMALESHWAR

CRYOGENIC ENGINEERING: SOFTWARE SOLUTIONS

VOL II B

LIQUEFACTION SYSTEMS – PROBLEMS (MATHCAD)

Cryogenic Engineering: Software Solutions Vol II B
Liquefaction systems – Problems (Mathcad)
1st edition

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Shri Madhwa Vadiraja Institute of Technology & Management, Karnataka, India.

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To see Part II A, download: Cryogenic Engineering:
Software Solutions Part-II
A Liquefaction systems – Theory+ Problems (EES)

DEDICATION

This work is lovingly dedicated at the lotus feet of

Bhagavan Sri Sathya Sai Baba

**"Strive for the happiness, the joy of all others,
as earnestly as you strive for your own."**

"Help Ever, Hurt Never!"

– Bhagavan Sri Sathya Sai Baba

PREFACE

This book, viz. **Cryogenic Engineering: Software solutions – Part-II, is the second in the series on Cryogenic Engineering: Software Solutions.**

This part is being published in two volumes, viz. Cryogenic Engineering: Software solutions – Part-II A and Part-II B. Part-II A contains the background theory, definitions and formulas and the problems solved with Engineering Equation Solver (EES). Part-II B contains problems solved with Mathcad. So, it is advisable that one refers to both the parts II A and II B.

As with the Part-I of the series, which dealt with: Introduction and properties of cryogenic fluids and properties of materials at low temperatures, the focus is on the solutions of problems in cryogenic engineering using software such as Mathcad and Engineering Equation Solver (EES). Only the essential theory and summary of equations required for calculations are given at the beginning of the chapter.

Advantages of using computer software to solve problems are reiterated:

i) It helps in solving the problems fast and accurately
ii) Parametric analysis (what-if analysis) and graphical visualization is done very easily. This helps in an in-depth analysis of the problem.
iii) Once a particular type of problem is solved, it can be used as a *template* and solving similar problems later becomes extremely easy.
iv) In addition, one can plot the data, curve fit, write functions for various properties or calculations and re-use them.
v) These possibilities create interest, curiosity and wonder in the minds of students and enthuse them to know more and work more.

This book, viz. **Cryogenic Engineering: Software solutions – Part-II** deals with the liquefaction of gases.

In this book: first, a thermodynamic background for cryogenic liquefaction is given; isenthalpic and isentropic expansion, pay-off functions for a liquefier and thermodynamically ideal liquefaction system are explained.

Next, various cycles for liquefaction of gases are explained and the calculation formulas for the pay-off functions are given. The cycles dealt with are: simple Linde-Hampson (L-H) system, pre-cooled L-H system, Linde dual pressure system, Claude system, Kapitza system, Heylandt system, cycles for liquefaction of Neon, Hydrogen and Helium, Collin's Helium liquefier etc.

Finally, various types of heat exchangers, hear transfer correlations, calculation formulas and graphs for analysis by LMTD and NTU methods are given. And, Second Law analysis of the cryogenic systems is explained briefly.

Some catalogue data on commercial liquefiers for Air, Hydrogen and Helium are also given.

Many numerical problems are solved to illustrate the ease of computer calculations using Engineering Equation Solver (EES) and Mathcad software.

Useful data for Nitrogen, Hydrogen, Helium, Argon, Methane, Fluorine and Oxygen are generated from NIST website, i.e. http://webbook.nist.gov/chemistry/fluid/.

EES has built-in functions for properties of several gases, refrigerants and fluids (including cryogenic fluids). Therefore, it is very convenient to use EES in these calculations.

However, Mathcad does not have built-in functions for properties of gases, refrigerants and fluids.

So, Mathcad Functions were written for both saturation properties and single phase gas properties of above mentioned important cryogenic fluids, viz. Nitrogen, Hydrogen, Helium, Argon, Methane, Fluorine and Oxygen. These Functions were used in solving problems, illustrating the ease of using Mathcad in calculations and graphing.

Several Procedures/Functions are written in EES and Mathcad to simplify the standard and most required calculations, which, the students, teachers and researchers may find very useful.

S.I. Units are used throughout this book. Wide variety of worked examples presented in the book should be useful for those appearing for University, AMIE and Engineering Services examinations.

Acknowledgements: Firstly, I would like to **thank all my students**, who have been an inspiration to me in all my academic efforts.

Sincere thanks are due to **Rev. Fr. Joseph Lobo**, Director, St. Joseph Engineering College, Mangalore, for his kindness, regard and words of encouragement.

I am also thankful to **Dr. Thirumaleshwara Bhat**, Principal, Sri Madhwa Vadiraja Institute of Technology and Management, Bantakal, Udupi, for giving me support in my academic activities.

I gratefully remember my former colleagues at the Cryogenics section of Technical Physics Division, Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology, Indore for their sincere cooperation in a true spirit of team-work in all the projects that we undertook.

I particularly salute and admire the vision and foresight of former Heads of Technical Physics Division, BARC viz. late Mr. C. Ambasankaran, Mr. R.Y. Deshpande, Dr. S.R. Gowariker and late Mr. S.S. Ramamurthy in initiating and guiding many of the 'first of its kind' projects for Indian Space Research Organization (ISRO), designed and executed by the Cryogenics section.

I am especially grateful to Prof. R.G.Scurlock, former Director of Institute of Cryogenics, University of Southampton (U.K.), for writing a message for the Part-I of this series.

It was indeed gracious of my former Professor, under whom I studied for M.Sc. in Cryogenics at the University of Southampton, U.K. during 1970–72, and worked as a Visiting Research Fellow during 1993–94, to honor me by writing this message.

My special thanks to **Bookboon.com** for publishing this free ebook. **Ms Karin Jakobsen and Ms Sophie Tergeist** and their editorial staff have been most patient and helpful.

Finally, I would like to express my sincere thanks and appreciation to my **wife, Kala**, who, as usual, has given me continuous support, help and encouragement in all my academic activities, making many silent sacrifices.

M. Thirumaleshwar
March 2016

ABOUT THE AUTHOR

Dr. M. Thirumaleshwar graduated in Mechanical Engineering from Karnataka Regional Engineering College, Surathkal, Karnataka, India, in the year 1965. He obtained M.Sc (cryogenics) from University of Southampton, U.K. and Ph.D. (cryogenics) from Indian Institute of Science, Bangalore, India.

He is a Fellow of Institution of Engineers (India), Life Member, Indian Society for Technical Education, and a Foundation Fellow of Indian Cryogenics Council.

He has worked in India and abroad on large projects in the areas involving heat transfer, fluid flow, vacuum system design, cryo-pumping etc.

He worked as Head of Cryogenics Dept. in Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology (CAT), Indore, from 1966 to 1992.

He worked as Guest Collaborator with Superconducting Super Collider Laboratory of Universities Research Association, in Dallas, USA from 1990 to 1993.

He also worked at the Institute of Cryogenics, Southampton, U.K. as a Visiting Research Fellow from 1993 to 1994.

He was Head of the Dept. of Mechanical Engineering, Fr. Conceicao Rodrigues Institute of Technology, Vashi, Navi Mumbai, India for eight years.

He also worked as Head of Dept. of Mechanical Engineering and Civil Engineering, and then as Principal, Vivekananda College of Engineering and Technology, Puttur (D.K.), India.

He was Professor and coordinator of Post-graduate program in the Dept. of Mechanical Engineering in St. Joseph Engineering College, Vamanjoor, Mangalore, India.

A book entitled **“Fundamentals of Heat and Mass Transfer”** authored by him and published by M/s Pearson Education, India (2006) **has been adopted as a Text book** for third year engineering students by the Visweswaraya Technological University (V.T.U.), Belgaum, India.

He has authored a free e-book entitled **"Software Solutions to Problems on Heat Transfer"** wherein problems are solved using 4 software viz. Mathcad, EES, FEHT and EXCEL. This book, containing about 2750 pages, is presented in 9 parts and all the 9 parts can be downloaded for free from www.bookboon.com.

He has also authored free e-books o**n Thermodynamics** entitled **"Basic Thermodynamics: Software Solutions" and "Applied Thermodynamics: Software Solutions"** wherein problems are solved using 3 software viz. Mathcad, EES, and TEST. Each of these titles is presented in 5 parts and all the books can be downloaded for free from www.bookboon.com.

His earlier free ebook, viz. Cryog**enic Engineering: Software Solutions – Part-I** was published by Bookboon about an year ago.

He has also authored **three motivational, free ebooks**, published by www.bookboon.com, entitled as follows:

1. Towards Excellence… How to Study (A Guide book to Students)
2. Towards Excellence… How to teach (A guide book to Teachers)
3. Towards Excellence… Seminars, GD's and Personal Interviews

Dr. M. Thirumaleshwar has attended several National and International conferences and has more than 50 publications to his credit.

ABOUT THE SOFTWARE USED

Following three software are used while solving problems in this book series:

1. Mathcad 7 and Mathcad 15 (Ref: www.ptc.com)
2. Engineering Equation Solver (EES) (Ref: www.fchart.com), and
3. EXCEL

For a brief introduction to Mathcad, EES and EXCEL see the chapter 1 of the following free ebook by the author:

"Software Solutions to Problems on Heat Transfer – CONDUCTION Part-I":
http://bookboon.com/en/software-solutions-to-problems-on-heat-transfer-ebook

3.4 PROBLEMS SOLVED WITH MATHCAD:

Unlike Engineering Equation Solver (EES), Mathcad does not have built-in functions for properties of cryogenic fluids.

So, we have written Mathcad Functions for properties that are essential for problem solving, for important cryogenic fluids, viz. Helium, Hydrogen,Nitrogen, Argon, Methane, Fluorine and Oxygen.

Cryogenic properties for fluids are taken from NIST [28], i.e.

http://webbook.nist.gov/chemistry/fluid/

Procedure to use the above NIST website to get both the saturation properties and superheat gas properties for different fluids is explained in detail in the earlier free ebook by this author (See pp: 40 to 48 and pp: 60 to 66 of that book), published by Bookboon. Following is the link:

Cryogenic Engineering: Software Solutions: Part-I

http://bookboon.com/en/cryogenic-engineering-software-solutions-part-i-ebook

Note: Cryogenic Engineering: Software solutions Part-II A contains the theory of gas liquefaction and problems solved with Engineering Equation Solver (EES). So, it is suggested that while using this book (i.e. Part-II B) which has problems solved with Mathcad, the student may find it useful to have the Part-II A too readily available, so that he/she may refer to the theoretical aspects if required.

==

Prob. 3.3.1 Write Mathcad Functions to determine properties of Helium.

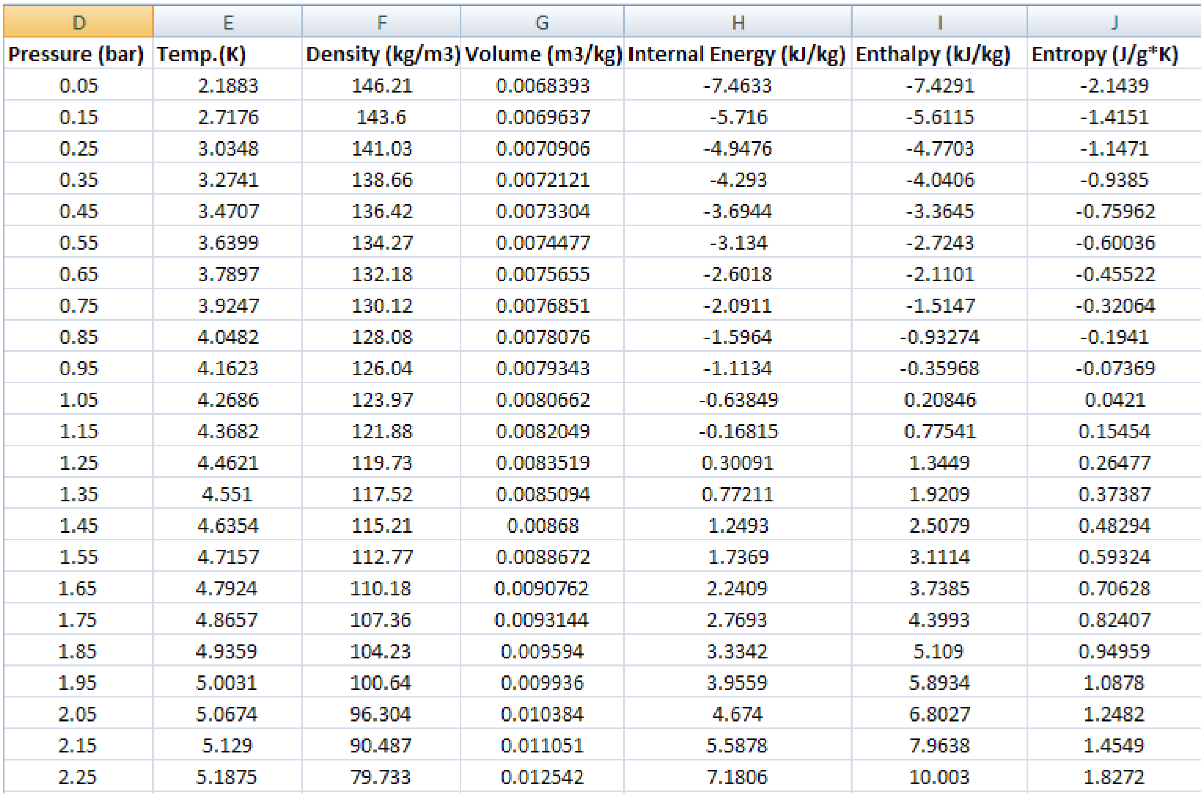
From NIST website, we get properties of Helium and then write the Mathcad Functions.

We have:

For Helium:

Saturation Properties:

Sat. Liquid:



D	E	F	G	H	I	J
Pressure (bar)	Temp.(K)	Density (kg/m3)	Volume (m3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (J/g*K)
0.05	2.1883	146.21	0.0068393	-7.4633	-7.4291	-2.1439
0.15	2.7176	143.6	0.0069637	-5.716	-5.6115	-1.4151
0.25	3.0348	141.03	0.0070906	-4.9476	-4.7703	-1.1471
0.35	3.2741	138.66	0.0072121	-4.293	-4.0406	-0.9385
0.45	3.4707	136.42	0.0073304	-3.6944	-3.3645	-0.75962
0.55	3.6399	134.27	0.0074477	-3.134	-2.7243	-0.60036
0.65	3.7897	132.18	0.0075655	-2.6018	-2.1101	-0.45522
0.75	3.9247	130.12	0.0076851	-2.0911	-1.5147	-0.32064
0.85	4.0482	128.08	0.0078076	-1.5964	-0.93274	-0.1941
0.95	4.1623	126.04	0.0079343	-1.1134	-0.35968	-0.07369
1.05	4.2686	123.97	0.0080662	-0.63849	0.20846	0.0421
1.15	4.3682	121.88	0.0082049	-0.16815	0.77541	0.15454
1.25	4.4621	119.73	0.0083519	0.30091	1.3449	0.26477
1.35	4.551	117.52	0.0085094	0.77211	1.9209	0.37387
1.45	4.6354	115.21	0.00868	1.2493	2.5079	0.48294
1.55	4.7157	112.77	0.0088672	1.7369	3.1114	0.59324
1.65	4.7924	110.18	0.0090762	2.2409	3.7385	0.70628
1.75	4.8657	107.36	0.0093144	2.7693	4.3993	0.82407
1.85	4.9359	104.23	0.009594	3.3342	5.109	0.94959
1.95	5.0031	100.64	0.009936	3.9559	5.8934	1.0878
2.05	5.0674	96.304	0.010384	4.674	6.8027	1.2482
2.15	5.129	90.487	0.011051	5.5878	7.9638	1.4549
2.25	5.1875	79.733	0.012542	7.1806	10.003	1.8272

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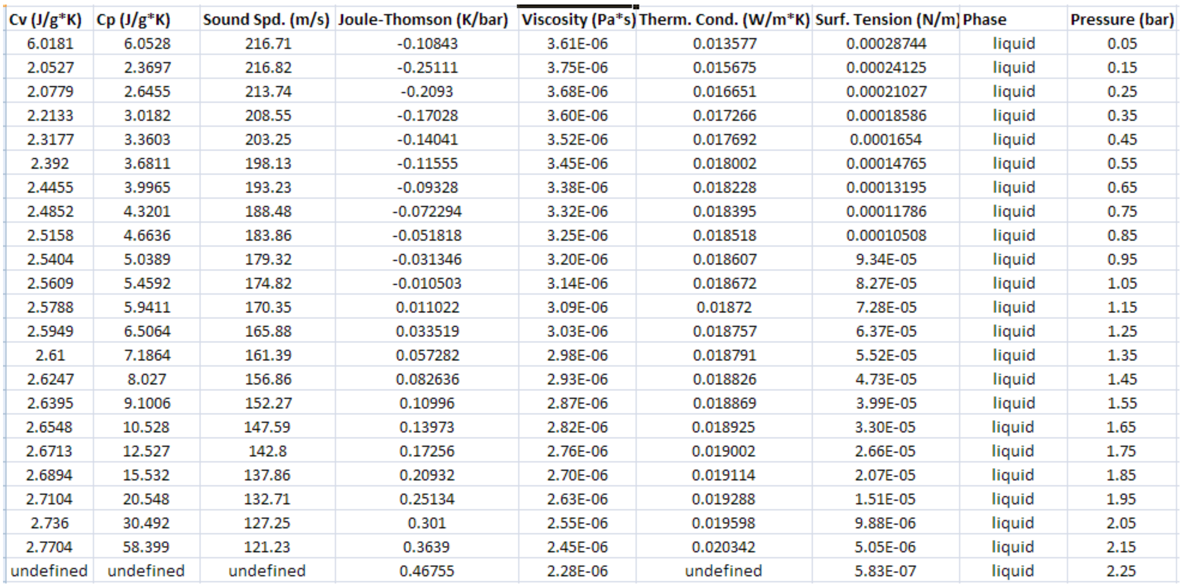
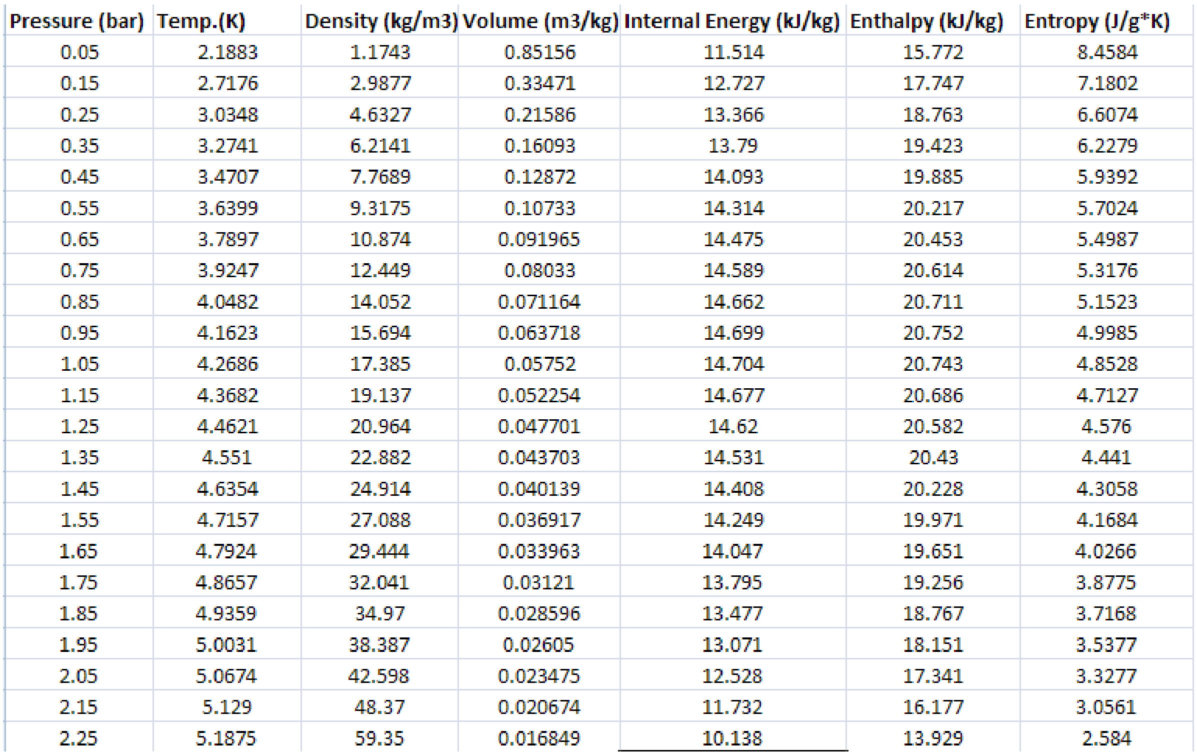


Cv (J/g*K)	Cp (J/g*K)	Sound Spd. (m/s)	Joule-Thomson (K/bar)	Viscosity (Pa*s)	Therm. Cond. (W/m*K)	Surf. Tension (N/m	Phase	Pressure (bar)
6.0181	6.0528	216.71	-0.10843	3.61E-06	0.013577	0.00028744	liquid	0.05
2.0527	2.3697	216.82	-0.25111	3.75E-06	0.015675	0.00024125	liquid	0.15
2.0779	2.6455	213.74	-0.2093	3.68E-06	0.016651	0.00021027	liquid	0.25
2.2133	3.0182	208.55	-0.17028	3.60E-06	0.017266	0.00018586	liquid	0.35
2.3177	3.3603	203.25	-0.14041	3.52E-06	0.017692	0.0001654	liquid	0.45
2.392	3.6811	198.13	-0.11555	3.45E-06	0.018002	0.00014765	liquid	0.55
2.4455	3.9965	193.23	-0.09328	3.38E-06	0.018228	0.00013195	liquid	0.65
2.4852	4.3201	188.48	-0.072294	3.32E-06	0.018395	0.00011786	liquid	0.75
2.5158	4.6636	183.86	-0.051818	3.25E-06	0.018518	0.00010508	liquid	0.85
2.5404	5.0389	179.32	-0.031346	3.20E-06	0.018607	9.34E-05	liquid	0.95
2.5609	5.4592	174.82	-0.010503	3.14E-06	0.018672	8.27E-05	liquid	1.05
2.5788	5.9411	170.35	0.011022	3.09E-06	0.01872	7.28E-05	liquid	1.15
2.5949	6.5064	165.88	0.033519	3.03E-06	0.018757	6.37E-05	liquid	1.25
2.61	7.1864	161.39	0.057282	2.98E-06	0.018791	5.52E-05	liquid	1.35
2.6247	8.027	156.86	0.082636	2.93E-06	0.018826	4.73E-05	liquid	1.45
2.6395	9.1006	152.27	0.10996	2.87E-06	0.018869	3.99E-05	liquid	1.55
2.6548	10.528	147.59	0.13973	2.82E-06	0.018925	3.30E-05	liquid	1.65
2.6713	12.527	142.8	0.17256	2.76E-06	0.019002	2.66E-05	liquid	1.75
2.6894	15.532	137.86	0.20932	2.70E-06	0.019114	2.07E-05	liquid	1.85
2.7104	20.548	132.71	0.25134	2.63E-06	0.019288	1.51E-05	liquid	1.95
2.736	30.492	127.25	0.301	2.55E-06	0.019598	9.88E-06	liquid	2.05
2.7704	58.399	121.23	0.3639	2.45E-06	0.020342	5.05E-06	liquid	2.15
undefined	undefined	undefined	0.46755	2.28E-06	undefined	5.83E-07	liquid	2.25

Sat. vapor:



Pressure (bar)	Temp.(K)	Density (kg/m3)	Volume (m3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (J/g*K)
0.05	2.1883	1.1743	0.85156	11.514	15.772	8.4584
0.15	2.7176	2.9877	0.33471	12.727	17.747	7.1802
0.25	3.0348	4.6327	0.21586	13.366	18.763	6.6074
0.35	3.2741	6.2141	0.16093	13.79	19.423	6.2279
0.45	3.4707	7.7689	0.12872	14.093	19.885	5.9392
0.55	3.6399	9.3175	0.10733	14.314	20.217	5.7024
0.65	3.7897	10.874	0.091965	14.475	20.453	5.4987
0.75	3.9247	12.449	0.08033	14.589	20.614	5.3176
0.85	4.0482	14.052	0.071164	14.662	20.711	5.1523
0.95	4.1623	15.694	0.063718	14.699	20.752	4.9985
1.05	4.2686	17.385	0.05752	14.704	20.743	4.8528
1.15	4.3682	19.137	0.052254	14.677	20.686	4.7127
1.25	4.4621	20.964	0.047701	14.62	20.582	4.576
1.35	4.551	22.882	0.043703	14.531	20.43	4.441
1.45	4.6354	24.914	0.040139	14.408	20.228	4.3058
1.55	4.7157	27.088	0.036917	14.249	19.971	4.1684
1.65	4.7924	29.444	0.033963	14.047	19.651	4.0266
1.75	4.8657	32.041	0.03121	13.795	19.256	3.8775
1.85	4.9359	34.97	0.028596	13.477	18.767	3.7168
1.95	5.0031	38.387	0.02605	13.071	18.151	3.5377
2.05	5.0674	42.598	0.023475	12.528	17.341	3.3277
2.15	5.129	48.37	0.020674	11.732	16.177	3.0561
2.25	5.1875	59.35	0.016849	10.138	13.929	2.584

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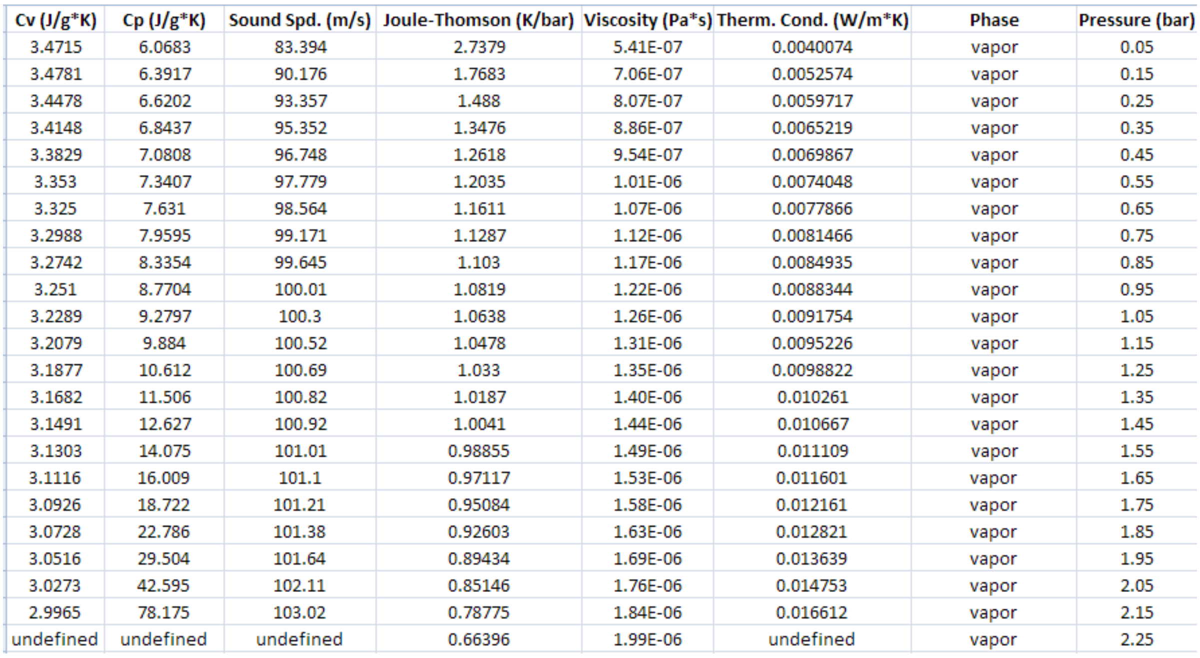
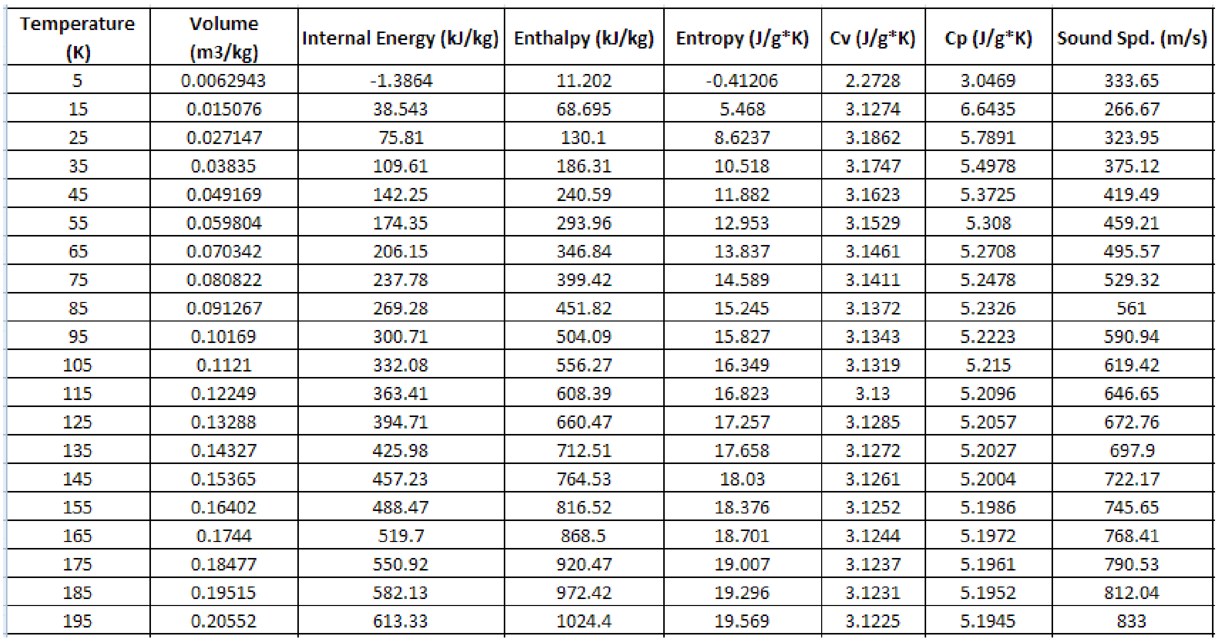
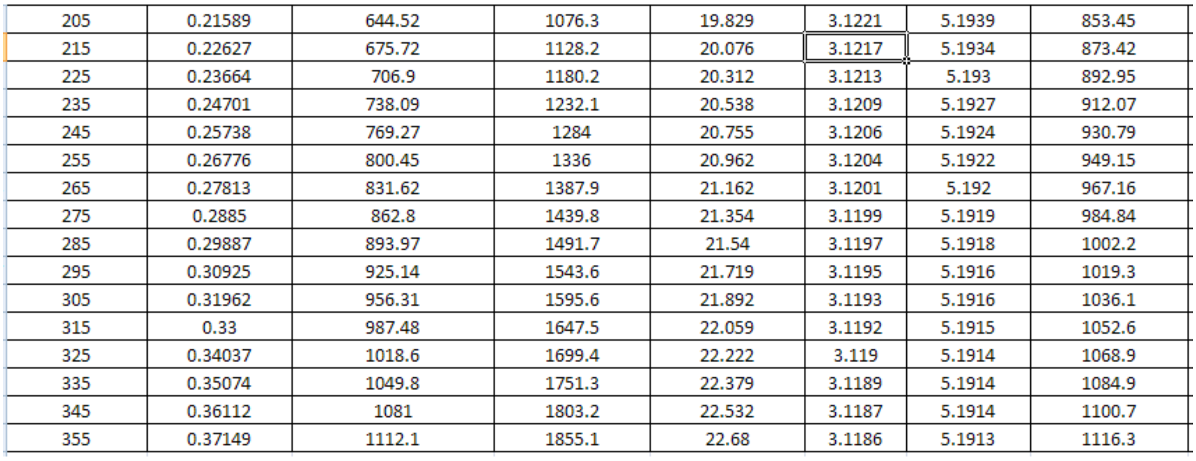


Cv (J/g*K)	Cp (J/g*K)	Sound Spd. (m/s)	Joule-Thomson (K/bar)	Viscosity (Pa*s)	Therm. Cond. (W/m*K)	Phase	Pressure (bar)
3.4715	6.0683	83.394	2.7379	5.41E-07	0.0040074	vapor	0.05
3.4781	6.3917	90.176	1.7683	7.06E-07	0.0052574	vapor	0.15
3.4478	6.6202	93.357	1.488	8.07E-07	0.0059717	vapor	0.25
3.4148	6.8437	95.352	1.3476	8.86E-07	0.0065219	vapor	0.35
3.3829	7.0808	96.748	1.2618	9.54E-07	0.0069867	vapor	0.45
3.353	7.3407	97.779	1.2035	1.01E-06	0.0074048	vapor	0.55
3.325	7.631	98.564	1.1611	1.07E-06	0.0077866	vapor	0.65
3.2988	7.9595	99.171	1.1287	1.12E-06	0.0081466	vapor	0.75
3.2742	8.3354	99.645	1.103	1.17E-06	0.0084935	vapor	0.85
3.251	8.7704	100.01	1.0819	1.22E-06	0.0088344	vapor	0.95
3.2289	9.2797	100.3	1.0638	1.26E-06	0.0091754	vapor	1.05
3.2079	9.884	100.52	1.0478	1.31E-06	0.0095226	vapor	1.15
3.1877	10.612	100.69	1.033	1.35E-06	0.0098822	vapor	1.25
3.1682	11.506	100.82	1.0187	1.40E-06	0.010261	vapor	1.35
3.1491	12.627	100.92	1.0041	1.44E-06	0.010667	vapor	1.45
3.1303	14.075	101.01	0.98855	1.49E-06	0.011109	vapor	1.55
3.1116	16.009	101.1	0.97117	1.53E-06	0.011601	vapor	1.65
3.0926	18.722	101.21	0.95084	1.58E-06	0.012161	vapor	1.75
3.0728	22.786	101.38	0.92603	1.63E-06	0.012821	vapor	1.85
3.0516	29.504	101.64	0.89434	1.69E-06	0.013639	vapor	1.95
3.0273	42.595	102.11	0.85146	1.76E-06	0.014753	vapor	2.05
2.9965	78.175	103.02	0.78775	1.84E-06	0.016612	vapor	2.15
undefined	undefined	undefined	0.66396	1.99E-06	undefined	vapor	2.25

Isobaric data is collected for pressures, P = 1, 5…220 bar.

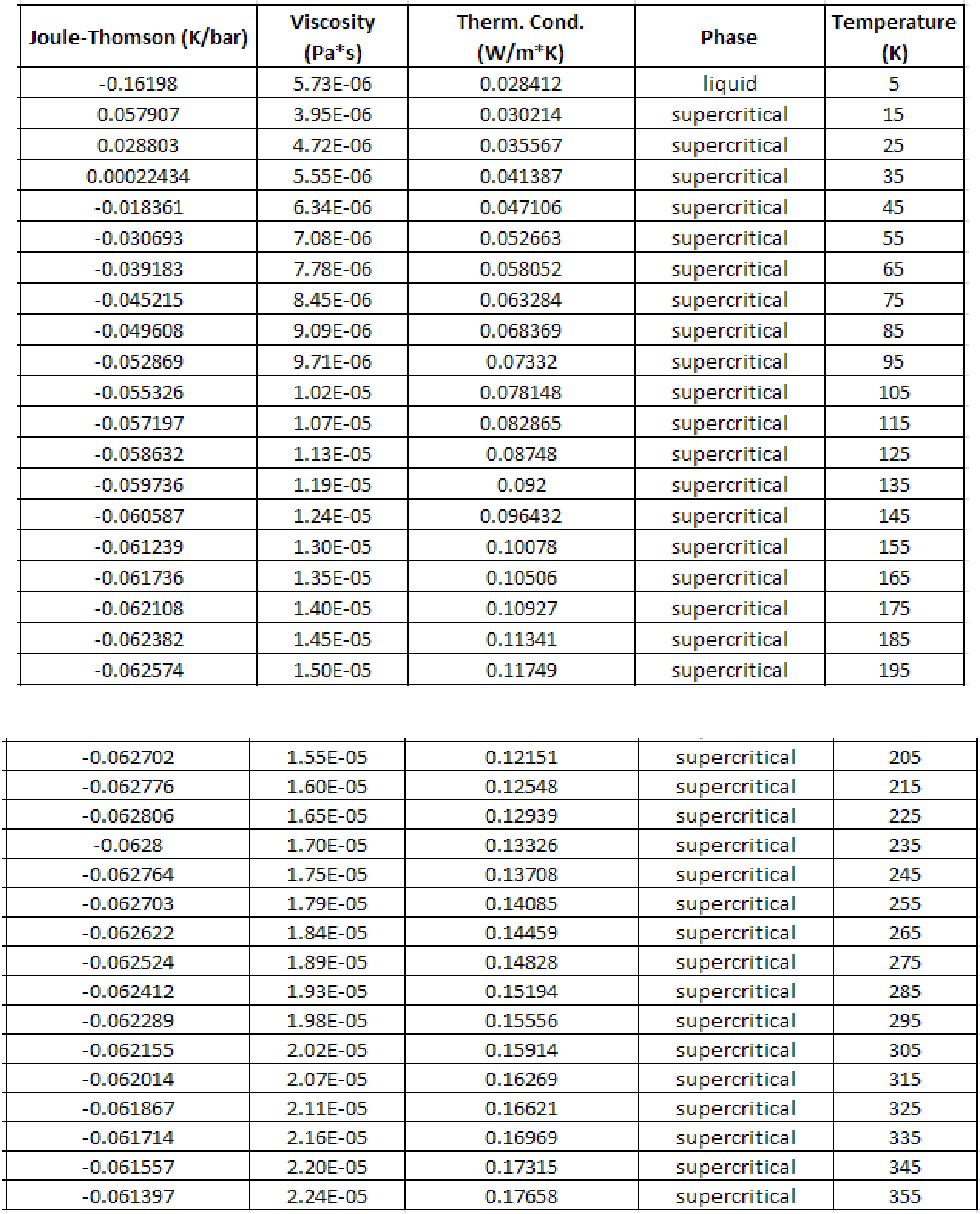
Sample data for P = 20 bar for temps from 5 K to 366 K is shown below:





Temperature (K)	Volume (m3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (J/g*K)	Cv (J/g*K)	Cp (J/g*K)	Sound Spd. (m/s)
5	0.0062943	-1.3864	11.202	-0.41206	2.2728	3.0469	333.65
15	0.015076	38.543	68.695	5.468	3.1274	6.6435	266.67
25	0.027147	75.81	130.1	8.6237	3.1862	5.7891	323.95
35	0.03835	109.61	186.31	10.518	3.1747	5.4978	375.12
45	0.049169	142.25	240.59	11.882	3.1623	5.3725	419.49
55	0.059804	174.35	293.96	12.953	3.1529	5.308	459.21
65	0.070342	206.15	346.84	13.837	3.1461	5.2708	495.57
75	0.080822	237.78	399.42	14.589	3.1411	5.2478	529.32
85	0.091267	269.28	451.82	15.245	3.1372	5.2326	561
95	0.10169	300.71	504.09	15.827	3.1343	5.2223	590.94
105	0.1121	332.08	556.27	16.349	3.1319	5.215	619.42
115	0.12249	363.41	608.39	16.823	3.13	5.2096	646.65
125	0.13288	394.71	660.47	17.257	3.1285	5.2057	672.76
135	0.14327	425.98	712.51	17.658	3.1272	5.2027	697.9
145	0.15365	457.23	764.53	18.03	3.1261	5.2004	722.17
155	0.16402	488.47	816.52	18.376	3.1252	5.1986	745.65
165	0.1744	519.7	868.5	18.701	3.1244	5.1972	768.41
175	0.18477	550.92	920.47	19.007	3.1237	5.1961	790.53
185	0.19515	582.13	972.42	19.296	3.1231	5.1952	812.04
195	0.20552	613.33	1024.4	19.569	3.1225	5.1945	833
205	0.21589	644.52	1076.3	19.829	3.1221	5.1939	853.45
215	0.22627	675.72	1128.2	20.076	3.1217	5.1934	873.42
225	0.23664	706.9	1180.2	20.312	3.1213	5.193	892.95
235	0.24701	738.09	1232.1	20.538	3.1209	5.1927	912.07
245	0.25738	769.27	1284	20.755	3.1206	5.1924	930.79
255	0.26776	800.45	1336	20.962	3.1204	5.1922	949.15
265	0.27813	831.62	1387.9	21.162	3.1201	5.192	967.16
275	0.2885	862.8	1439.8	21.354	3.1199	5.1919	984.84
285	0.29887	893.97	1491.7	21.54	3.1197	5.1918	1002.2
295	0.30925	925.14	1543.6	21.719	3.1195	5.1916	1019.3
305	0.31962	956.31	1595.6	21.892	3.1193	5.1916	1036.1
315	0.33	987.48	1647.5	22.059	3.1192	5.1915	1052.6
325	0.34037	1018.6	1699.4	22.222	3.119	5.1914	1068.9
335	0.35074	1049.8	1751.3	22.379	3.1189	5.1914	1084.9
345	0.36112	1081	1803.2	22.532	3.1187	5.1914	1100.7
355	0.37149	1112.1	1855.1	22.68	3.1186	5.1913	1116.3

Contd…



Joule-Thomson (K/bar)	Viscosity (Pa*s)	Therm. Cond. (W/m*K)	Phase	Temperature (K)
-0.16198	5.73E-06	0.028412	liquid	5
0.057907	3.95E-06	0.030214	supercritical	15
0.028803	4.72E-06	0.035567	supercritical	25
0.00022434	5.55E-06	0.041387	supercritical	35
-0.018361	6.34E-06	0.047106	supercritical	45
-0.030693	7.08E-06	0.052663	supercritical	55
-0.039183	7.78E-06	0.058052	supercritical	65
-0.045215	8.45E-06	0.063284	supercritical	75
-0.049608	9.09E-06	0.068369	supercritical	85
-0.052869	9.71E-06	0.07332	supercritical	95
-0.055326	1.02E-05	0.078148	supercritical	105
-0.057197	1.07E-05	0.082865	supercritical	115
-0.058632	1.13E-05	0.08748	supercritical	125
-0.059736	1.19E-05	0.092	supercritical	135
-0.060587	1.24E-05	0.096432	supercritical	145
-0.061239	1.30E-05	0.10078	supercritical	155
-0.061736	1.35E-05	0.10506	supercritical	165
-0.062108	1.40E-05	0.10927	supercritical	175
-0.062382	1.45E-05	0.11341	supercritical	185
-0.062574	1.50E-05	0.11749	supercritical	195
-0.062702	1.55E-05	0.12151	supercritical	205
-0.062776	1.60E-05	0.12548	supercritical	215
-0.062806	1.65E-05	0.12939	supercritical	225
-0.0628	1.70E-05	0.13326	supercritical	235
-0.062764	1.75E-05	0.13708	supercritical	245
-0.062703	1.79E-05	0.14085	supercritical	255
-0.062622	1.84E-05	0.14459	supercritical	265
-0.062524	1.89E-05	0.14828	supercritical	275
-0.062412	1.93E-05	0.15194	supercritical	285
-0.062289	1.98E-05	0.15556	supercritical	295
-0.062155	2.02E-05	0.15914	supercritical	305
-0.062014	2.07E-05	0.16269	supercritical	315
-0.061867	2.11E-05	0.16621	supercritical	325
-0.061714	2.16E-05	0.16969	supercritical	335
-0.061557	2.20E-05	0.17315	supercritical	345
-0.061397	2.24E-05	0.17658	supercritical	355

Now, to write Mathcad Functions, first write the data as Mathcad Matrix/vectors:

Properties of Helium:

Sat. properties: (Ref: NIST)

Units: psat (bar), tsat(K), vf, vg (m3/kg), hf, hg (kJ/kg), sf,sg (kJ/kg.K)

$$M\_sat\_He := \begin{array}{cccccccc} \text{Psat} & \text{Tsat} & \text{vf} & \text{vg} & \text{hf} & \text{hg} & \text{sf} & \text{sg} \\ \left(\begin{array}{cccccccc}
0.05 & 2.1883 & 0.0068393 & 0.85156 & -7.4291 & 15.772 & -2.1439 & 8.4584 \\
0.15 & 2.7176 & 0.0069637 & 0.33471 & -5.6115 & 17.747 & -1.4151 & 7.1802 \\
0.25 & 3.0348 & 0.0070906 & 0.21586 & -4.7703 & 18.763 & -1.1471 & 6.6074 \\
0.35 & 3.2741 & 0.0072121 & 0.16093 & -4.0406 & 19.423 & -0.9385 & 6.2279 \\
0.45 & 3.4707 & 0.0073304 & 0.12872 & -3.3645 & 19.885 & -0.75962 & 5.9392 \\
0.55 & 3.6399 & 0.0074477 & 0.10733 & -2.7243 & 20.217 & -0.60036 & 5.7024 \\
0.65 & 3.7897 & 0.0075655 & 0.091965 & -2.1101 & 20.453 & -0.45522 & 5.4987 \\
0.75 & 3.9247 & 0.0076851 & 0.08033 & -1.5147 & 20.614 & -0.32064 & 5.3176 \\
0.85 & 4.0482 & 0.0078076 & 0.071164 & -0.93274 & 20.711 & -0.1941 & 5.1523 \\
0.95 & 4.1623 & 0.0079343 & 0.063718 & -0.35968 & 20.752 & -0.07369 & 4.9985 \\
1.05 & 4.2686 & 0.0080662 & 0.05752 & 0.20846 & 20.743 & 0.0421 & 4.8528 \\
1.15 & 4.3682 & 0.0082049 & 0.052254 & 0.77541 & 20.686 & 0.15454 & 4.7127 \\
1.25 & 4.4621 & 0.0083519 & 0.047701 & 1.3449 & 20.582 & 0.26477 & 4.576 \\
1.35 & 4.551 & 0.0085094 & 0.043703 & 1.9209 & 20.43 & 0.37387 & 4.441 \\
1.45 & 4.6354 & 0.00868 & 0.040139 & 2.5079 & 20.228 & 0.48294 & 4.3058 \\
1.55 & 4.7157 & 0.0088672 & 0.036917 & 3.1114 & 19.971 & 0.59324 & 4.1684 \\
1.65 & 4.7924 & 0.0090762 & 0.033963 & 3.7385 & 19.651 & 0.70628 & 4.0266 \\
1.75 & 4.8657 & 0.0093144 & 0.03121 & 4.3993 & 19.256 & 0.82407 & 3.8775 \\
1.85 & 4.9359 & 0.009594 & 0.028596 & 5.109 & 18.767 & 0.94959 & 3.7168 \\
1.95 & 5.0031 & 0.009936 & 0.02605 & 5.8934 & 18.151 & 1.0878 & 3.5377 \\
2.05 & 5.0674 & 0.010384 & 0.023475 & 6.8027 & 17.341 & 1.2482 & 3.3277 \\
2.15 & 5.129 & 0.011051 & 0.020674 & 7.9638 & 16.177 & 1.4549 & 3.0561 \\
2.25 & 5.1875 & 0.012542 & 0.016849 & 10.003 & 13.929 & 1.8272 & 2.584
\end{array}\right) \end{array}$$

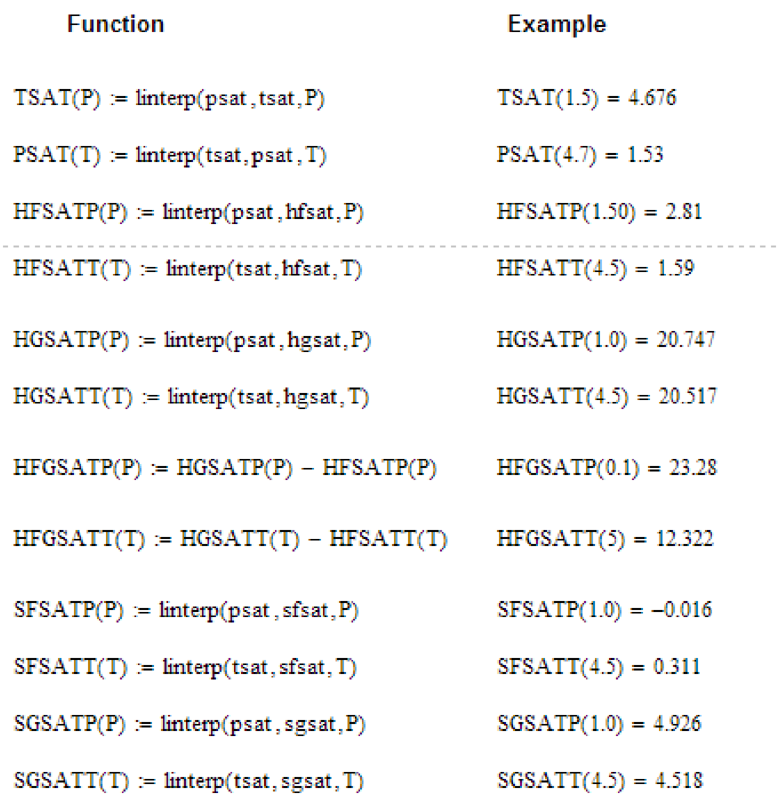
Now, separate the data vectors:

$psat := M\_sat\_He^{\langle 0 \rangle}$ $tsat := M\_sat\_He^{\langle 1 \rangle}$ $vfsat := M\_sat\_He^{\langle 2 \rangle}$

$vgsat := M\_sat\_He^{\langle 3 \rangle}$ $hfsat := M\_sat\_He^{\langle 4 \rangle}$ $hgsat := M\_sat\_He^{\langle 5 \rangle}$

$sfsat := M\_sat\_He^{\langle 6 \rangle}$ $sgsat := M\_sat\_He^{\langle 7 \rangle}$

And, now write the Functions for sat. properties of Helium:



Function	Example
TSAT(P) := linterp(psat, tsat, P)	TSAT(1.5) = 4.676
PSAT(T) := linterp(tsat, psat, T)	PSAT(4.7) = 1.53
HFSATP(P) := linterp(psat, hfsat, P)	HFSATP(1.50) = 2.81
HFSATT(T) := linterp(tsat, hfsat, T)	HFSATT(4.5) = 1.59
HGSATP(P) := linterp(psat, hgsat, P)	HGSATP(1.0) = 20.747
HGSATT(T) := linterp(tsat, hgsat, T)	HGSATT(4.5) = 20.517
HFGSATP(P) := HGSATP(P) − HFSATP(P)	HFGSATP(0.1) = 23.28
HFGSATT(T) := HGSATT(T) − HFSATT(T)	HFGSATT(5) = 12.322
SFSATP(P) := linterp(psat, sfsat, P)	SFSATP(1.0) = −0.016
SFSATT(T) := linterp(tsat, sfsat, T)	SFSATT(4.5) = 0.311
SGSATP(P) := linterp(psat, sgsat, P)	SGSATP(1.0) = 4.926
SGSATT(T) := linterp(tsat, sgsat, T)	SGSATT(4.5) = 4.518

$\mathrm{SFGSATP(P)} := \mathrm{SGSATP(P)} - \mathrm{SFSATP(P)}$ $\quad$ $\mathrm{SFGSATP(1.0)} = 4.941$

$\mathrm{SFGSATT(T)} := \mathrm{SGSATT(T)} - \mathrm{SFSATT(T)}$ $\quad$ $\mathrm{SFGSATT(4.5)} = 4.207$

$\mathrm{VGSATP(P)} := \mathrm{linterp(psat, vgsat, P)}$ $\quad$ $\mathrm{VGSATP(1.0)} = 0.061$ m^3/kg

$\mathrm{VGSATT(T)} := \mathrm{linterp(tsat, vgsat, T)}$ $\quad$ $\mathrm{VGSATT(4.5)} = 0.046$ m^3/kg

$\mathrm{VFSATP(P)} := \mathrm{linterp(psat, vfsat, P)}$ $\quad$ $\mathrm{VFSATP(1.0)} = 8.00025 \times 10^{-3}$ m^3/kg

$\mathrm{VFSATT(T)} := \mathrm{linterp(tsat, vfsat, T)}$ $\quad$ $\mathrm{VFSATT(4.5)} = 8.419046 \times 10^{-3}$ m^3/kg

$\mathrm{VFGSATP(P)} := \mathrm{VGSATP(P)} - \mathrm{VFSATP(P)}$ $\quad$ $\mathrm{VFGSATP(1.0)} = 0.052619$ m^3/kg

$\mathrm{VFGSATT(T)} := \mathrm{VGSATT(T)} - \mathrm{VFSATT(T)}$ $\quad$ $\mathrm{VFGSATT(4.5)} = 0.037578$ m^3/kg

$\mathrm{UGSATP(P)} := \mathrm{HGSATP(P)} - P \cdot 10^{2} \cdot \mathrm{VGSATP(P)}$ $\quad$ $\mathrm{UGSATP(1)} = 14.686$

$\mathrm{UFSATP(P)} := \mathrm{HFSATP(P)} - P \cdot \mathrm{VFSATP(P)} \cdot 10^{2}$

$\mathrm{UFGSATP(P)} := \mathrm{UGSATP(P)} - \mathrm{UFSATP(P)}$

$\mathrm{UGSATT(T)} := \mathrm{HGSATT(T)} - \mathrm{PSAT(T)} \cdot 10^{2} \cdot \mathrm{VGSATT(T)}$ $\quad$ $\mathrm{UGSATT(4.5)} = 14.572$

$\mathrm{UFSATT(T)} := \mathrm{HFSATT(T)} - \mathrm{PSAT(T)} \cdot 10^{2} \cdot \mathrm{VFSATT(T)}$

$\mathrm{UFGSATT(T)} := \mathrm{UGSATT(T)} - \mathrm{UFSATT(T)}$

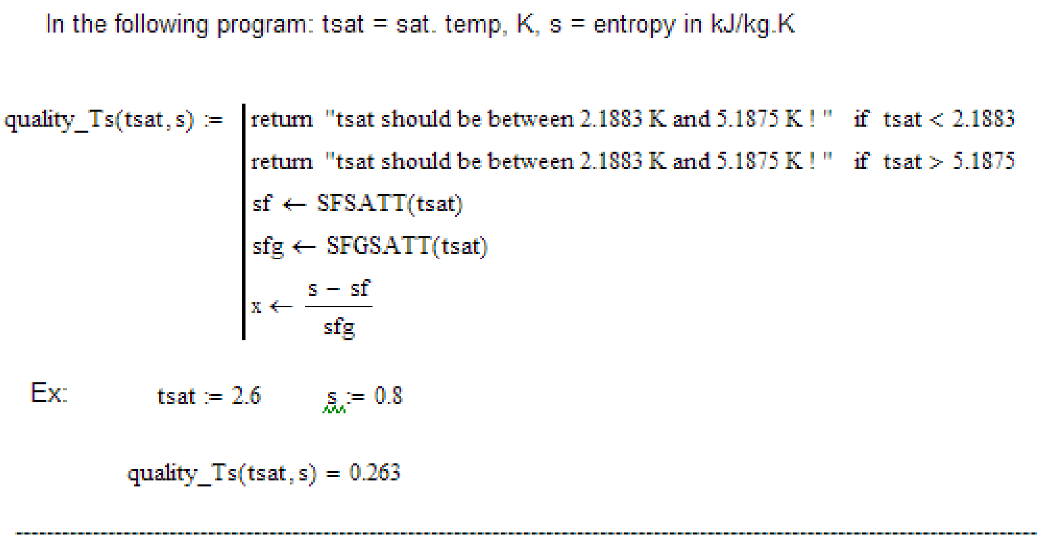
Some more Functions for two-phase properties:

In the following program: psat = sat. pr. in bar, s = entropy in kJ/kg.K

```
quality_Ps(psat, s) :=  | return "psat should be between 0.05 bar and 2.25 bar ! "  if  psat < 0.05
                        | return "psat should be between 0.05 bar and 2.25 bar ! "  if  psat > 2.25
                        | sf ← SFSATP(psat)
                        | sfg ← SFGSATP(psat)
                        | x ← $\frac{s - sf}{sfg}$
```

Ex: $\quad$ $\mathrm{psat} := 2.25$ $\quad$ $s := 2.5$

$\mathrm{quality\_Ps(psat, s)} = 0.889$

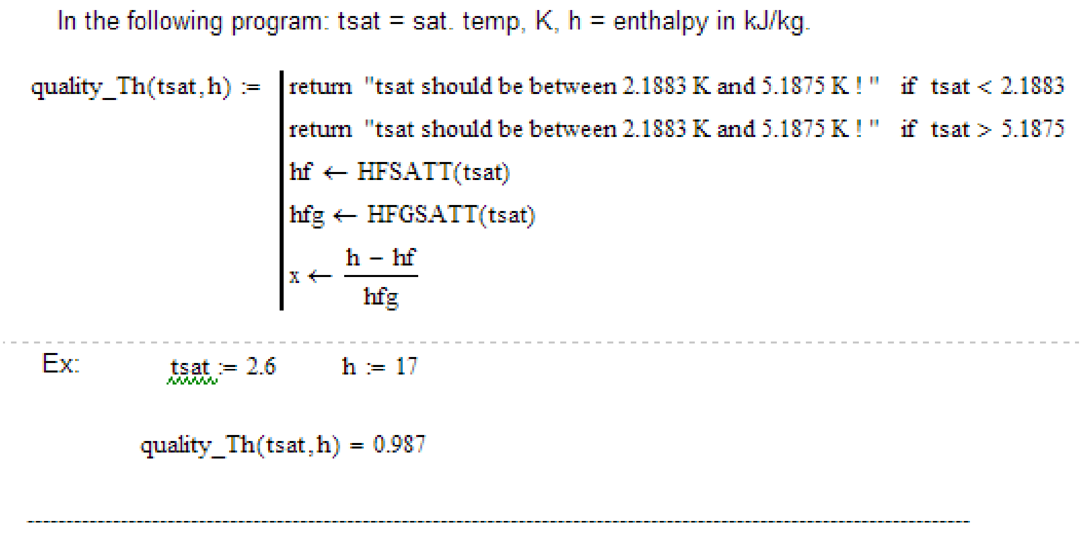


In the following program: tsat = sat. temp, K, s = entropy in kJ/kg.K

```
quality_Ts(tsat, s) :=
    return "tsat should be between 2.1883 K and 5.1875 K ! "  if  tsat < 2.1883
    return "tsat should be between 2.1883 K and 5.1875 K ! "  if  tsat > 5.1875
    sf ← SFSATT(tsat)
    sfg ← SFGSATT(tsat)
    x ← $\frac{s - sf}{sfg}$
```

Ex: tsat := 2.6   s := 0.8

quality_Ts(tsat, s) = 0.263

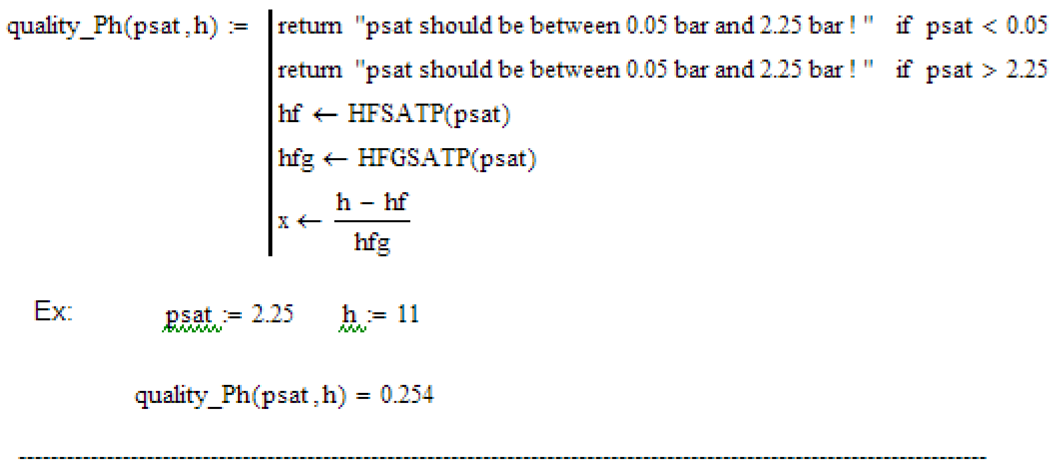


In the following program: tsat = sat. temp, K, h = enthalpy in kJ/kg.

```
quality_Th(tsat, h) :=
    return "tsat should be between 2.1883 K and 5.1875 K ! "  if  tsat < 2.1883
    return "tsat should be between 2.1883 K and 5.1875 K ! "  if  tsat > 5.1875
    hf ← HFSATT(tsat)
    hfg ← HFGSATT(tsat)
    x ← $\frac{h - hf}{hfg}$
```

Ex: tsat := 2.6   h := 17

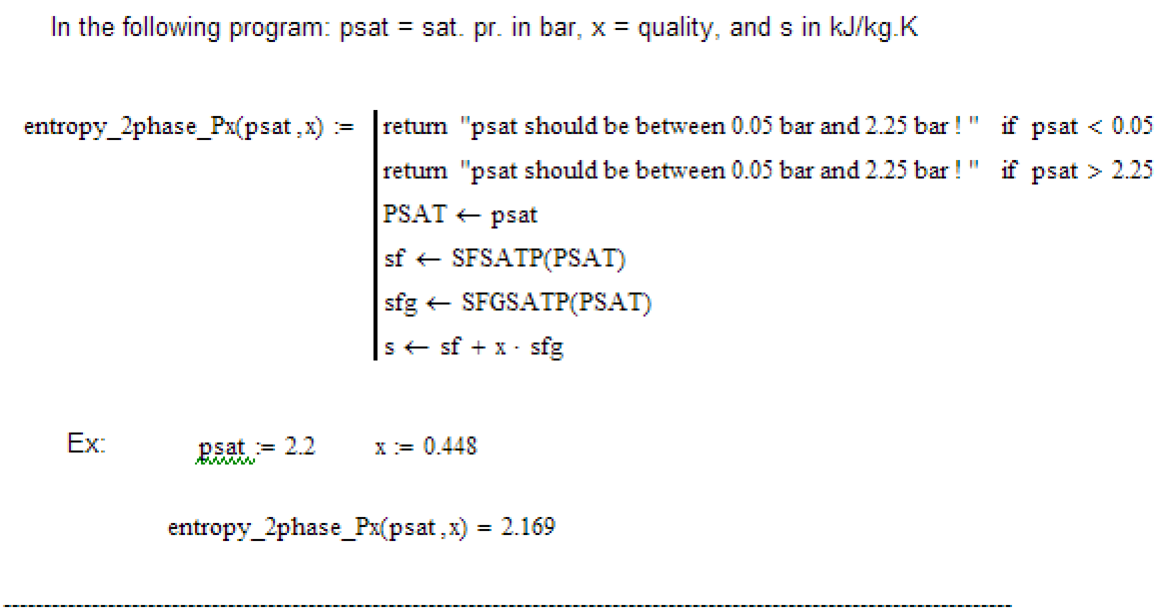
quality_Th(tsat, h) = 0.987



```
quality_Ph(psat, h) :=
    return "psat should be between 0.05 bar and 2.25 bar ! "  if  psat < 0.05
    return "psat should be between 0.05 bar and 2.25 bar ! "  if  psat > 2.25
    hf ← HFSATP(psat)
    hfg ← HFGSATP(psat)
    x ← $\frac{h - hf}{hfg}$
```

Ex: psat := 2.25   h := 11

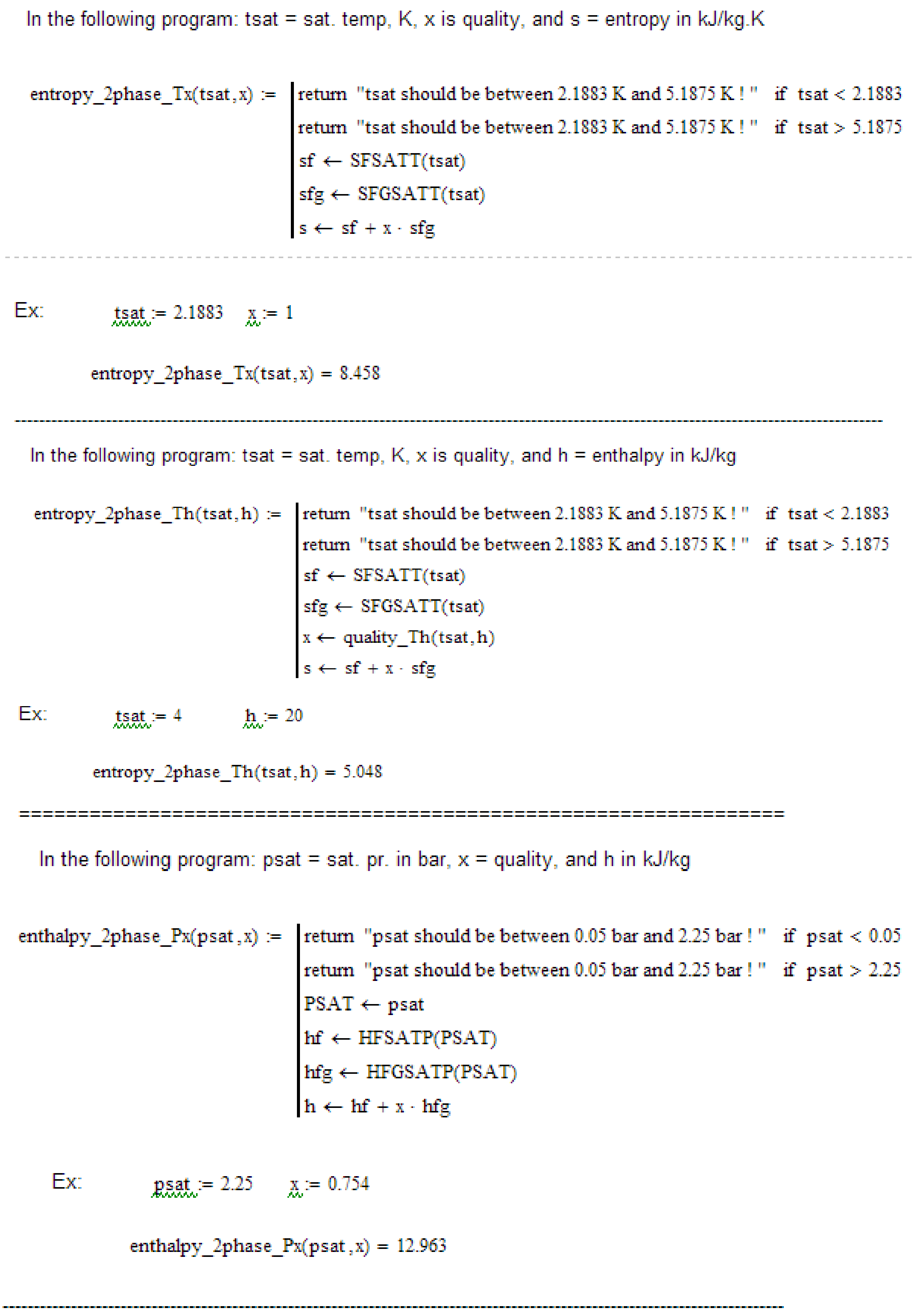
quality_Ph(psat, h) = 0.254



In the following program: psat = sat. pr. in bar, x = quality, and s in kJ/kg.K

```
entropy_2phase_Px(psat,x) := | return "psat should be between 0.05 bar and 2.25 bar ! "  if  psat < 0.05
                             | return "psat should be between 0.05 bar and 2.25 bar ! "  if  psat > 2.25
                             | PSAT ← psat
                             | sf ← SFSATP(PSAT)
                             | sfg ← SFGSATP(PSAT)
                             | s ← sf + x · sfg
```

```
Ex:        psat := 2.2        x := 0.448

        entropy_2phase_Px(psat,x) = 2.169
```



In the following program: tsat = sat. temp, K, x is quality, and s = entropy in kJ/kg.K

```
entropy_2phase_Tx(tsat,x) := | return "tsat should be between 2.1883 K and 5.1875 K ! "  if tsat < 2.1883
                             | return "tsat should be between 2.1883 K and 5.1875 K ! "  if tsat > 5.1875
                             | sf ← SFSATT(tsat)
                             | sfg ← SFGSATT(tsat)
                             | s ← sf + x · sfg
```

Ex: tsat := 2.1883 x := 1

entropy_2phase_Tx(tsat,x) = 8.458

In the following program: tsat = sat. temp, K, x is quality, and h = enthalpy in kJ/kg

```
entropy_2phase_Th(tsat,h) := | return "tsat should be between 2.1883 K and 5.1875 K ! "  if tsat < 2.1883
                             | return "tsat should be between 2.1883 K and 5.1875 K ! "  if tsat > 5.1875
                             | sf ← SFSATT(tsat)
                             | sfg ← SFGSATT(tsat)
                             | x ← quality_Th(tsat,h)
                             | s ← sf + x · sfg
```

Ex: tsat := 4 h := 20

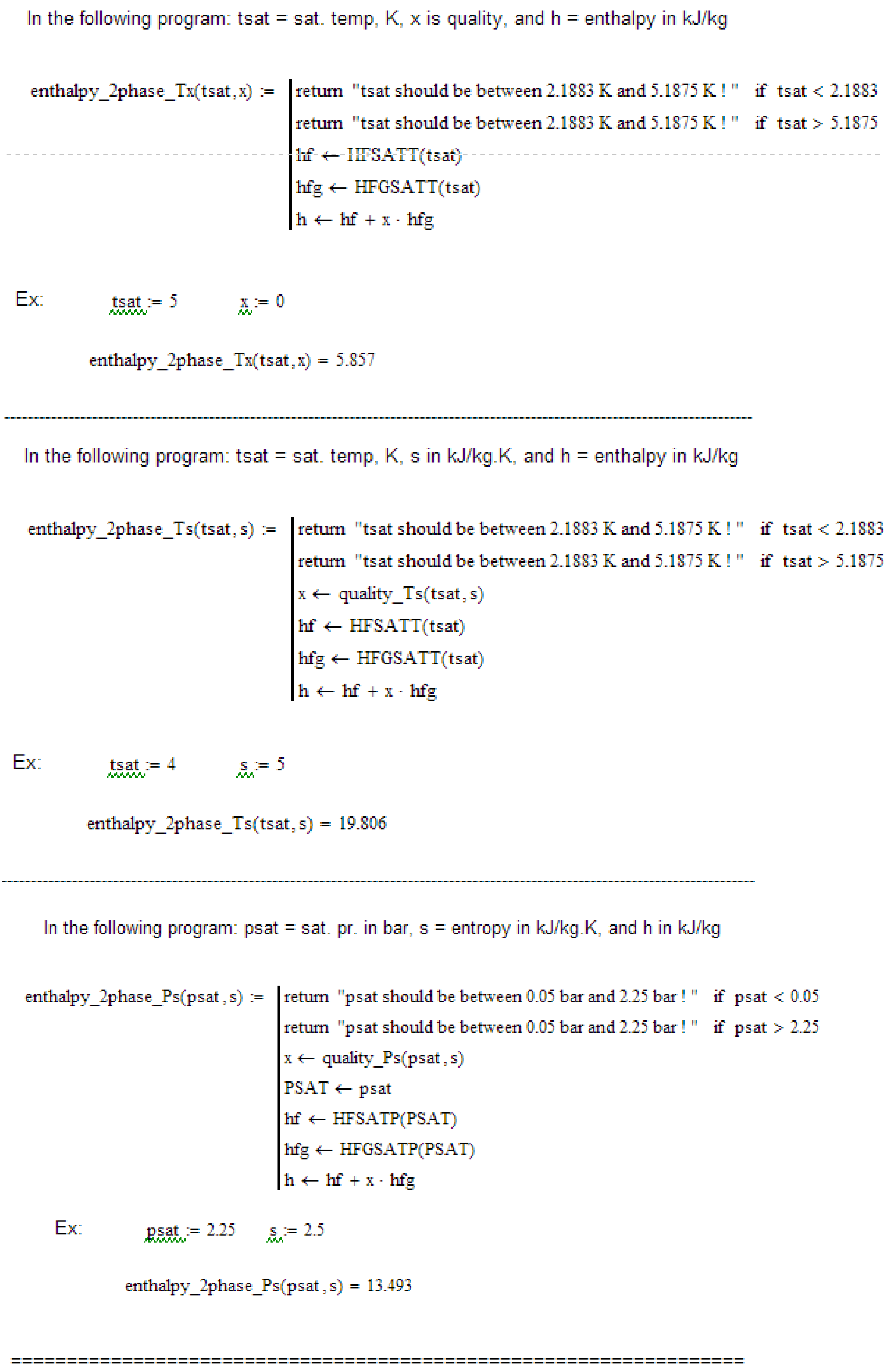
entropy_2phase_Th(tsat,h) = 5.048

In the following program: psat = sat. pr. in bar, x = quality, and h in kJ/kg

```
enthalpy_2phase_Px(psat,x) := | return "psat should be between 0.05 bar and 2.25 bar ! "  if psat < 0.05
                              | return "psat should be between 0.05 bar and 2.25 bar ! "  if psat > 2.25
                              | PSAT ← psat
                              | hf ← HFSATP(PSAT)
                              | hfg ← HFGSATP(PSAT)
                              | h ← hf + x · hfg
```

Ex: psat := 2.25 x := 0.754

enthalpy_2phase_Px(psat,x) = 12.963



In the following program: tsat = sat. temp, K, x is quality, and h = enthalpy in kJ/kg

```
enthalpy_2phase_Tx(tsat,x) :=  | return "tsat should be between 2.1883 K and 5.1875 K ! "  if tsat < 2.1883
                               | return "tsat should be between 2.1883 K and 5.1875 K ! "  if tsat > 5.1875
                               | hf ← HFSATT(tsat)
                               | hfg ← HFGSATT(tsat)
                               | h ← hf + x · hfg
```

Ex: tsat := 5   x := 0

enthalpy_2phase_Tx(tsat,x) = 5.857

In the following program: tsat = sat. temp, K, s in kJ/kg.K, and h = enthalpy in kJ/kg

```
enthalpy_2phase_Ts(tsat, s) :=  | return "tsat should be between 2.1883 K and 5.1875 K ! "  if tsat < 2.1883
                                | return "tsat should be between 2.1883 K and 5.1875 K ! "  if tsat > 5.1875
                                | x ← quality_Ts(tsat, s)
                                | hf ← HFSATT(tsat)
                                | hfg ← HFGSATT(tsat)
                                | h ← hf + x · hfg
```

Ex: tsat := 4   s := 5

enthalpy_2phase_Ts(tsat, s) = 19.806

In the following program: psat = sat. pr. in bar, s = entropy in kJ/kg.K, and h in kJ/kg

```
enthalpy_2phase_Ps(psat , s) :=  | return "psat should be between 0.05 bar and 2.25 bar ! "  if psat < 0.05
                                 | return "psat should be between 0.05 bar and 2.25 bar ! "  if psat > 2.25
                                 | x ← quality_Ps(psat , s)
                                 | PSAT ← psat
                                 | hf ← HFSATP(PSAT)
                                 | hfg ← HFGSATP(PSAT)
                                 | h ← hf + x · hfg
```

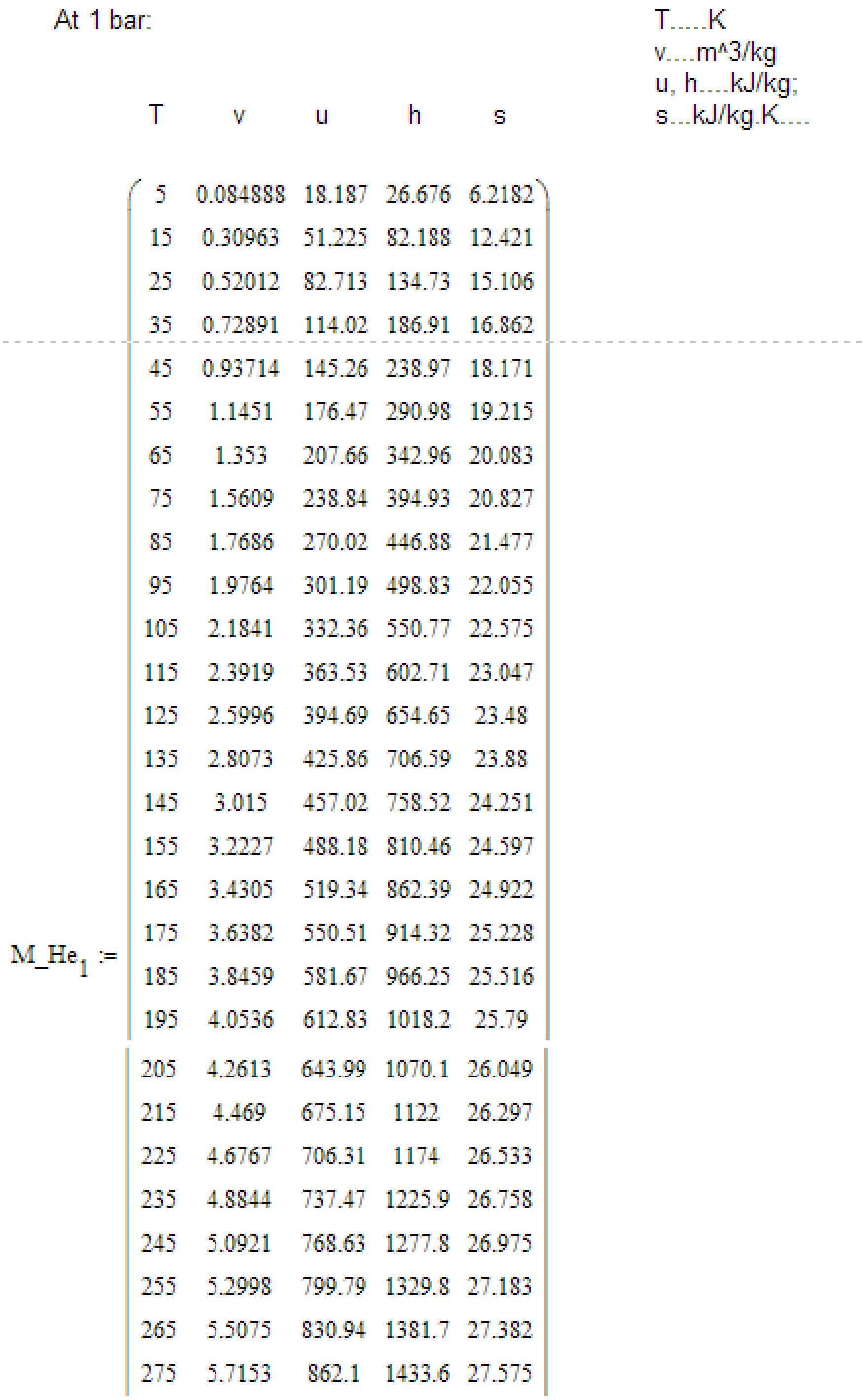
Ex: psat := 2.25   s := 2.5

enthalpy_2phase_Ps(psat , s) = 13.493

===

Now, properties at 1, 5, 10…220 bar for single phase helium gas are written as Mathcad matrices.

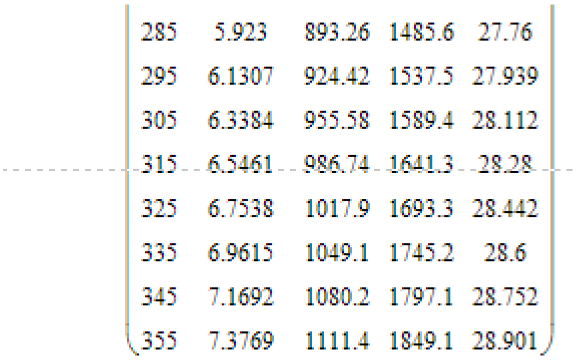
Example for P = 1 bar and 220 bar only are shown, to conserve space:



At 1 bar:

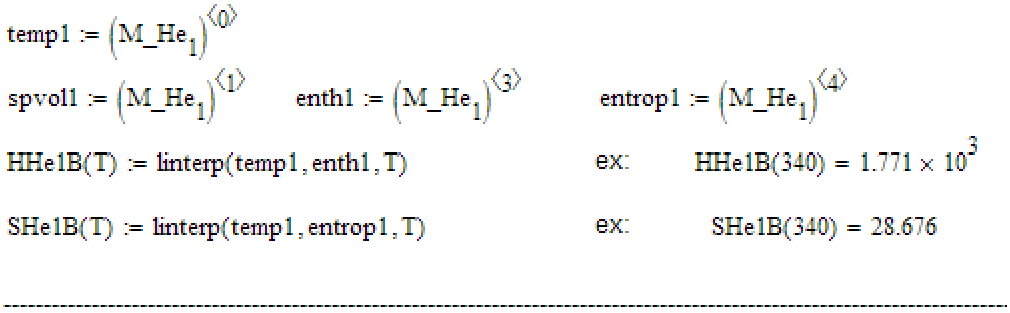
T.....K
v....m^3/kg
u, h....kJ/kg;
s...kJ/kg.K....

$$M\_He_1 := \begin{array}{ccccc} T & v & u & h & s \\ \left(\begin{array}{c} 5 \\ 15 \\ 25 \\ 35 \\ 45 \\ 55 \\ 65 \\ 75 \\ 85 \\ 95 \\ 105 \\ 115 \\ 125 \\ 135 \\ 145 \\ 155 \\ 165 \\ 175 \\ 185 \\ 195 \\ 205 \\ 215 \\ 225 \\ 235 \\ 245 \\ 255 \\ 265 \\ 275 \end{array}\right. & \begin{array}{c} 0.084888 \\ 0.30963 \\ 0.52012 \\ 0.72891 \\ 0.93714 \\ 1.1451 \\ 1.353 \\ 1.5609 \\ 1.7686 \\ 1.9764 \\ 2.1841 \\ 2.3919 \\ 2.5996 \\ 2.8073 \\ 3.015 \\ 3.2227 \\ 3.4305 \\ 3.6382 \\ 3.8459 \\ 4.0536 \\ 4.2613 \\ 4.469 \\ 4.6767 \\ 4.8844 \\ 5.0921 \\ 5.2998 \\ 5.5075 \\ 5.7153 \end{array} & \begin{array}{c} 18.187 \\ 51.225 \\ 82.713 \\ 114.02 \\ 145.26 \\ 176.47 \\ 207.66 \\ 238.84 \\ 270.02 \\ 301.19 \\ 332.36 \\ 363.53 \\ 394.69 \\ 425.86 \\ 457.02 \\ 488.18 \\ 519.34 \\ 550.51 \\ 581.67 \\ 612.83 \\ 643.99 \\ 675.15 \\ 706.31 \\ 737.47 \\ 768.63 \\ 799.79 \\ 830.94 \\ 862.1 \end{array} & \begin{array}{c} 26.676 \\ 82.188 \\ 134.73 \\ 186.91 \\ 238.97 \\ 290.98 \\ 342.96 \\ 394.93 \\ 446.88 \\ 498.83 \\ 550.77 \\ 602.71 \\ 654.65 \\ 706.59 \\ 758.52 \\ 810.46 \\ 862.39 \\ 914.32 \\ 966.25 \\ 1018.2 \\ 1070.1 \\ 1122 \\ 1174 \\ 1225.9 \\ 1277.8 \\ 1329.8 \\ 1381.7 \\ 1433.6 \end{array} & \left.\begin{array}{c} 6.2182 \\ 12.421 \\ 15.106 \\ 16.862 \\ 18.171 \\ 19.215 \\ 20.083 \\ 20.827 \\ 21.477 \\ 22.055 \\ 22.575 \\ 23.047 \\ 23.48 \\ 23.88 \\ 24.251 \\ 24.597 \\ 24.922 \\ 25.228 \\ 25.516 \\ 25.79 \\ 26.049 \\ 26.297 \\ 26.533 \\ 26.758 \\ 26.975 \\ 27.183 \\ 27.382 \\ 27.575 \end{array}\right) \end{array}$$



$$\begin{array}{ccccc}
285 & 5.923 & 893.26 & 1485.6 & 27.76 \\
295 & 6.1307 & 924.42 & 1537.5 & 27.939 \\
305 & 6.3384 & 955.58 & 1589.4 & 28.112 \\
315 & 6.5461 & 986.74 & 1641.3 & 28.28 \\
325 & 6.7538 & 1017.9 & 1693.3 & 28.442 \\
335 & 6.9615 & 1049.1 & 1745.2 & 28.6 \\
345 & 7.1692 & 1080.2 & 1797.1 & 28.752 \\
355 & 7.3769 & 1111.4 & 1849.1 & 28.901
\end{array}\Bigg)$$

Data vectors are created from the above matrix, and Functions are written for enthalpy and entropy:



$\text{temp1} := \left(\text{M\_He}_1\right)^{\langle 0\rangle}$

$\text{spvol1} := \left(\text{M\_He}_1\right)^{\langle 1\rangle}$ $\text{enth1} := \left(\text{M\_He}_1\right)^{\langle 3\rangle}$ $\text{entrop1} := \left(\text{M\_He}_1\right)^{\langle 4\rangle}$

$\text{HHe1B}(T) := \text{linterp}(\text{temp1}, \text{enth1}, T)$ ex: $\text{HHe1B}(340) = 1.771 \times 10^{3}$

$\text{SHe1B}(T) := \text{linterp}(\text{temp1}, \text{entrop1}, T)$ ex: $\text{SHe1B}(340) = 28.676$

At 220 bar:

T.....K
v....m^3/kg
u, h....kJ/kg;
s...kJ/kg.K....

$$\begin{array}{rl} & \begin{array}{ccccc} T & v & u & h & s \end{array} \\ M\_He_{220} := & \left(\begin{array}{ccccc} 15 & 0.0047112 & 27.364 & 131.01 & 1.1655 \\ 25 & 0.0054133 & 58.786 & 177.88 & 3.5391 \\ 35 & 0.0062499 & 93.235 & 230.73 & 5.3142 \\ 45 & 0.0071602 & 128.2 & 285.73 & 6.6957 \\ 55 & 0.008106 & 162.89 & 341.23 & 7.8095 \\ 65 & 0.0090657 & 197.12 & 396.56 & 8.734 \\ 75 & 0.010029 & 230.87 & 451.5 & 9.5202 \\ 85 & 0.010991 & 264.22 & 506.01 & 10.202 \\ 95 & 0.011949 & 297.22 & 560.1 & 10.804 \\ 105 & 0.012904 & 329.94 & 613.84 & 11.342 \\ 115 & 0.013856 & 362.44 & 667.27 & 11.828 \\ 125 & 0.014805 & 394.74 & 720.44 & 12.272 \\ 135 & 0.01575 & 426.89 & 773.4 & 12.679 \\ 145 & 0.016694 & 458.9 & 826.17 & 13.056 \\ 155 & 0.017636 & 490.81 & 878.8 & 13.407 \\ 165 & 0.018576 & 522.63 & 931.29 & 13.735 \\ 175 & 0.019514 & 554.36 & 983.68 & 14.044 \\ 185 & 0.020452 & 586.03 & 1036 & 14.334 \\ 195 & 0.021389 & 617.64 & 1088.2 & 14.609 \\ 205 & 0.022325 & 649.2 & 1140.3 & 14.87 \\ 215 & 0.02326 & 680.71 & 1192.4 & 15.118 \\ 225 & 0.024195 & 712.19 & 1244.5 & 15.355 \\ 235 & 0.02513 & 743.63 & 1296.5 & 15.581 \\ 245 & 0.026064 & 775.05 & 1348.5 & 15.797 \\ 255 & 0.026998 & 806.44 & 1400.4 & 16.005 \\ 265 & 0.027932 & 837.8 & 1452.3 & 16.205 \\ 275 & 0.028866 & 869.15 & 1504.2 & 16.397 \\ 285 & 0.0298 & 900.48 & 1556.1 & 16.582 \\ 295 & 0.030734 & 931.79 & 1607.9 & 16.761 \\ 305 & 0.031668 & 963.08 & 1659.8 & 16.934 \\ 315 & 0.032602 & 994.37 & 1711.6 & 17.101 \\ 325 & 0.033536 & 1025.6 & 1763.4 & 17.263 \\ 335 & 0.03447 & 1056.9 & 1815.2 & 17.42 \\ 345 & 0.035404 & 1088.1 & 1867 & 17.573 \\ 355 & 0.036338 & 1119.4 & 1918.8 & 17.721 \end{array}\right) \end{array}$$

$$\mathrm{temp220} := \mathrm{S220}^{\langle 0 \rangle}$$

$$\mathrm{spvol220} := \mathrm{S220}^{\langle 1 \rangle} \quad \mathrm{enth220} := \mathrm{S220}^{\langle 3 \rangle} \quad \mathrm{entrop220} := \mathrm{S220}^{\langle 4 \rangle}$$

$$\mathrm{HHe220B}(T) := \mathrm{linterp}(\mathrm{temp220}, \mathrm{enth220}, T) \qquad \text{ex:} \qquad \mathrm{HHe220B}(340) = 1.841 \times 10^{3}$$

$$\mathrm{SHe220B}(T) := \mathrm{linterp}(\mathrm{temp220}, \mathrm{entrop220}, T) \qquad \text{ex:} \qquad \mathrm{SHe220B}(340) = 17.497$$

Now, using the above Data Matrices, write a Function to find enthalpy and entropy of superheated helium as a function of P and T:

First, create a vector of all pressure values for which we have collected data Matrices:

$$\mathrm{Pressures\_He} := \begin{pmatrix} 1 & 5 & 10 & 15 & 20 & 30 & 40 & 60 & 80 & 100 & 120 & 140 & 160 & 180 & 200 & 220 \end{pmatrix}$$

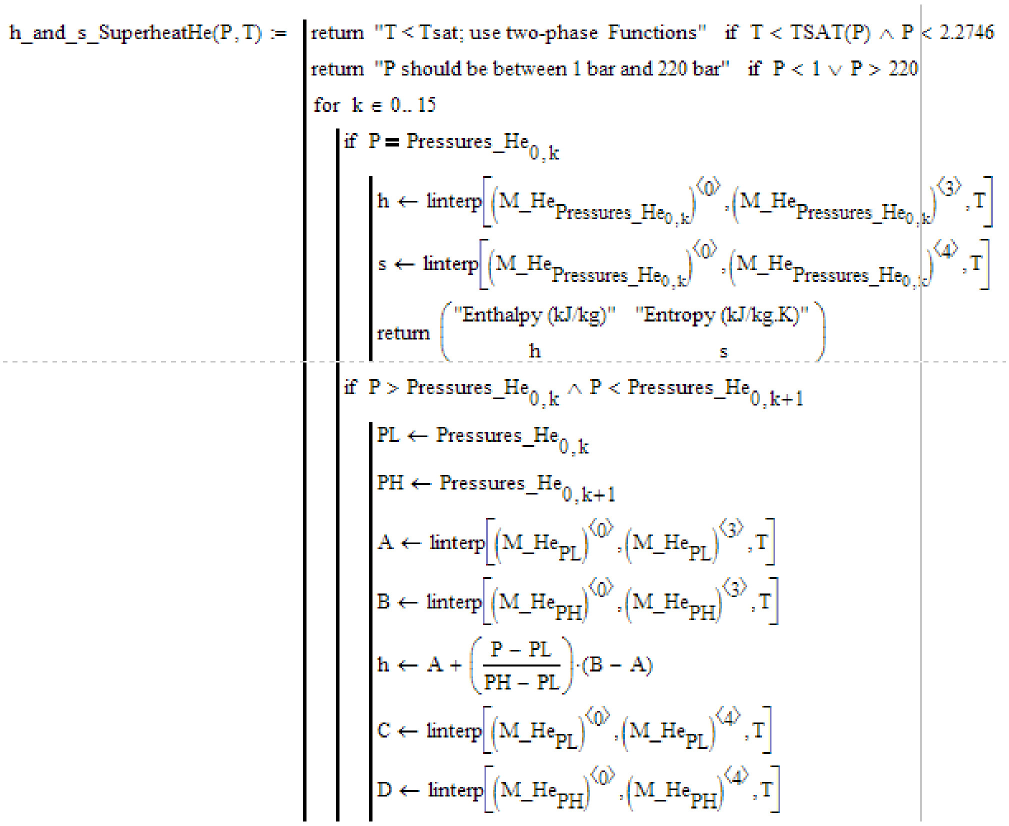
Explanation of the Function:

In the Function below, P and T are the Inputs, and h and s are the outputs.

If the input P value exactly matches with one of the values in the Pressures_He data vector, that particular Matrix is accessed and the h and s are found out by linear interpolation.

However, if the input P does not exactly match with any of the values in the Pressures_He data vector, then the pressures PL and PH which bracket P are found out, and the h and s are first calculated for pressures corresponding to PL and PH, and then the h and s values at the required P are found out by linear interpolation:

Now, write the function:



```
h_and_s_SuperheatHe(P, T) :=
    return "T < Tsat; use two-phase  Functions"  if  $T < TSAT(P) \wedge P < 2.2746$
    return "P should be between 1 bar and 220 bar"  if  $P < 1 \vee P > 220$
    for  $k \in 0..15$
        if  $P = Pressures\_He_{0,k}$
            $h \leftarrow linterp\left[\left(M\_He_{Pressures\_He_{0,k}}\right)^{\langle 0\rangle}, \left(M\_He_{Pressures\_He_{0,k}}\right)^{\langle 3\rangle}, T\right]$
            $s \leftarrow linterp\left[\left(M\_He_{Pressures\_He_{0,k}}\right)^{\langle 0\rangle}, \left(M\_He_{Pressures\_He_{0,k}}\right)^{\langle 4\rangle}, T\right]$
            return  $\begin{pmatrix} \text{"Enthalpy (kJ/kg)"} & \text{"Entropy (kJ/kg.K)"} \\ h & s \end{pmatrix}$
        if  $P > Pressures\_He_{0,k} \wedge P < Pressures\_He_{0,k+1}$
            $PL \leftarrow Pressures\_He_{0,k}$
            $PH \leftarrow Pressures\_He_{0,k+1}$
            $A \leftarrow linterp\left[\left(M\_He_{PL}\right)^{\langle 0\rangle}, \left(M\_He_{PL}\right)^{\langle 3\rangle}, T\right]$
            $B \leftarrow linterp\left[\left(M\_He_{PH}\right)^{\langle 0\rangle}, \left(M\_He_{PH}\right)^{\langle 3\rangle}, T\right]$
            $h \leftarrow A + \left(\frac{P - PL}{PH - PL}\right) \cdot (B - A)$
            $C \leftarrow linterp\left[\left(M\_He_{PL}\right)^{\langle 0\rangle}, \left(M\_He_{PL}\right)^{\langle 4\rangle}, T\right]$
            $D \leftarrow linterp\left[\left(M\_He_{PH}\right)^{\langle 0\rangle}, \left(M\_He_{PH}\right)^{\langle 4\rangle}, T\right]$
```

```
            s ← $C + \left(\frac{P - PL}{PH - PL}\right)\cdot(D - C)$
            return $\begin{pmatrix}\text{"Enthalpy (kJ/kg)"} & \text{"Entropy (kJ/kg.K)"}\\ h & s\end{pmatrix}$
```

Ex: $P := 16 \quad T := 355$

$$h\_and\_s\_SuperheatHe(P,T) = \begin{pmatrix}\text{"Enthalpy (kJ/kg)"} & \text{"Entropy (kJ/kg.K)"}\\ 1.854\times 10^{3} & 23.158\end{pmatrix}$$

Now, write Functions for enthalpy and entropy in terms of P and T:

Remember: For He: Pc =2.2746 bar, Tc = 5.1953 K, NBP = 4.23 K

```
enthalpy_He(P,T) :=
    return "P should be between 1 bar and 220 bar"  if $P < 1 \vee P > 220$
    return "T should be between 5 K and 355 K"  if $T < 5 \vee T > 355$
    tsat ← TSAT(P)  if $P \le 2.2746$
    h ← $h\_and\_s\_SuperheatHe(P,T)_{1,0}$  if $(T \ge tsat \wedge P \le 2.2746)$
    h ← $h\_and\_s\_SuperheatHe(P,T)_{1,0}$  if $(P > 2.2746)$
    (return "State point in two phase region--- use 2 phase Functions")  if $(T < tsat \wedge P < 2.2746)$
```

Ex:

$P := 200$ bar $\quad T := 355$ K

$$enthalpy\_He(P,T) = 1.913\times 10^{3}$$

```
entropy_He(P,T) :=
    return "P should be between 1 bar and 220 bar"  if $P < 1 \vee P > 220$
    return "T should be between 5 K and 355 K"  if $T < 5 \vee T > 355$
    tsat ← TSAT(P)  if $P \le 2.2746$
    s ← $h\_and\_s\_SuperheatHe(P,T)_{1,1}$  if $(T \ge tsat \wedge P \le 2.2746)$
    s ← $h\_and\_s\_SuperheatHe(P,T)_{1,1}$  if $(P > 2.2746)$
    (return "State point in two phase region--- use 2 phase Functions")  if $(T < tsat \wedge P < 2.2746)$
```

Ex:

$P := 200$ bar $\quad T := 350$ K $\qquad TSAT(16) = 13.231$

$$entropy\_He(P,T) = 17.843$$

Function to find h when P and s are known:

As a first step, get T when P and s are known:

P := 200 bar

s := 5.727 kJ/kg.C

T := 100 K....guess value

Given

entropy_He(P, T) = s

Temp_He(P, s) := Find(T)

Temp_He(P, s) = 36.628 K

Now, write the Function to get h:

```
enthalpy_He_Ps(P, s) := | return "P should be between 1 bar and 220 bar"  if  P < 1 ∨ P > 220
                        | T ← Temp_He(P, s)
                        | h ← enthalpy_He(P, T)
```

Ex: P := 80 bar s := 15.87 kJ/kg.C

enthalpy_He_Ps(P, s) = 894.915 kJ/kg

Function to find T when P and h are known:

P := 100 bar h := 870 kJ/kg

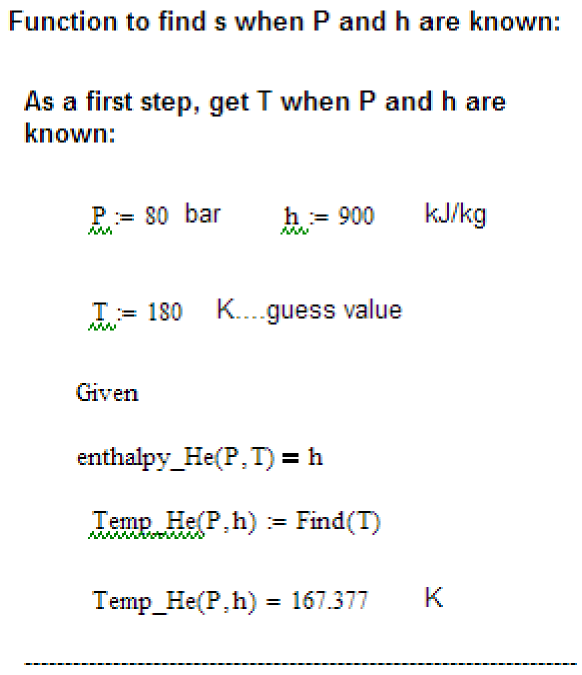
T := 150 K....guess value

Given

enthalpy_He(P, T) = h

Temp_He(P, h) := Find(T)

Temp_He(P, h) = 160.415 K



Function to find s when P and h are known:

As a first step, get T when P and h are known:

$P := 80$ bar $\quad h := 900$ kJ/kg

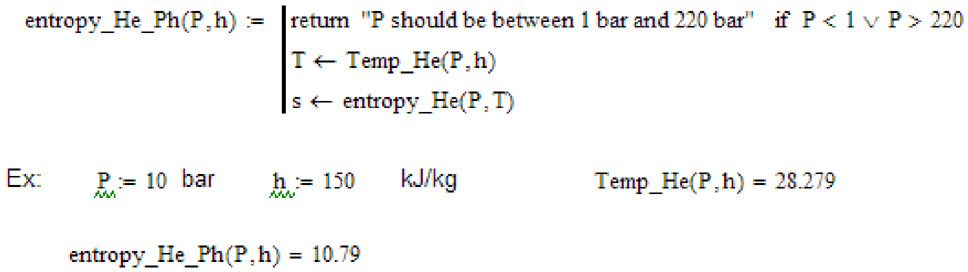
$T := 180$ K....guess value

Given

$\text{enthalpy\_He}(P,T) = h$

$\text{Temp\_He}(P,h) := \text{Find}(T)$

$\text{Temp\_He}(P,h) = 167.377$ K



```
entropy_He_Ph(P,h) := | return "P should be between 1 bar and 220 bar"  if  P < 1 ∨ P > 220
                      | T ← Temp_He(P,h)
                      | s ← entropy_He(P,T)

Ex:     P := 10  bar        h := 150     kJ/kg            Temp_He(P,h) = 28.279

        entropy_He_Ph(P,h) = 10.79
```

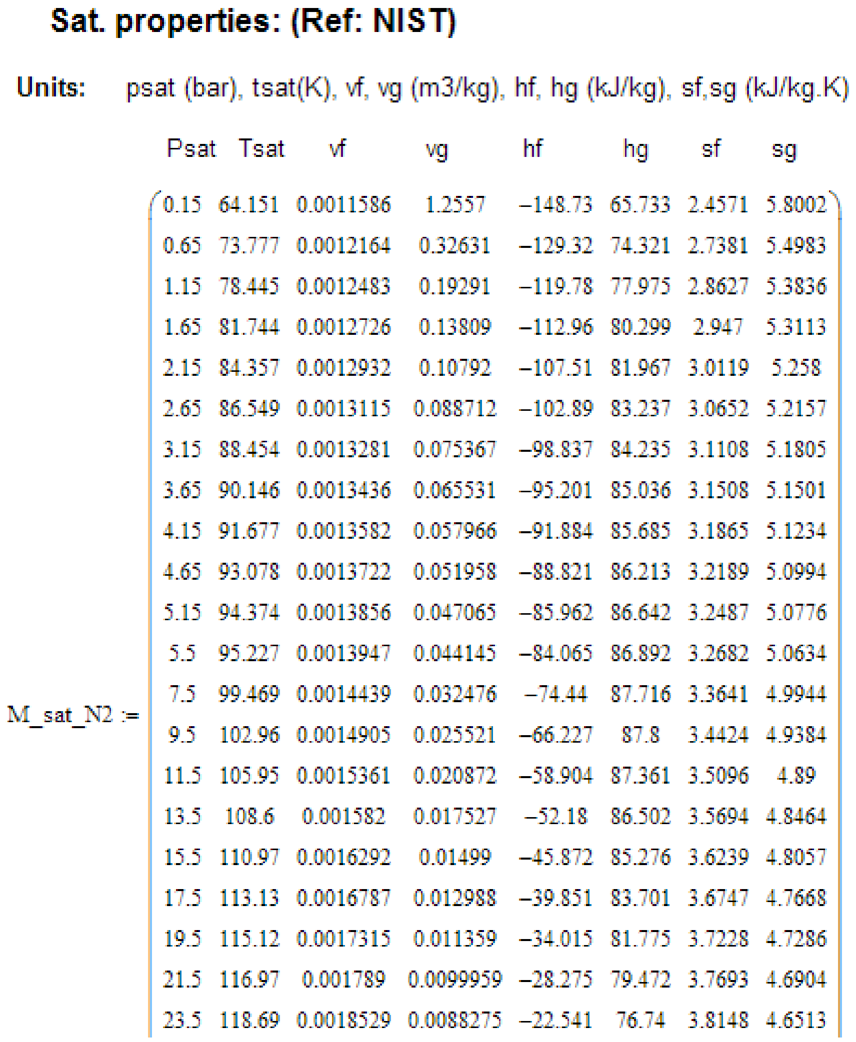
==

Prob. 3.3.2 Write Mathcad Functions to determine properties of Nitrogen.

From NIST website, we get properties of Nitrogen and then write the Mathcad Functions.

First, saturation properties are generated from the NIST website, just as we did for Helium.

Relevant properties are shown in the following Mathcad Matrix:



Sat. properties: (Ref: NIST)

Units: psat (bar), tsat(K), vf, vg (m3/kg), hf, hg (kJ/kg), sf,sg (kJ/kg.K)

$$
\mathrm{M\_sat\_N2} := \begin{array}{cccccccc}
\text{Psat} & \text{Tsat} & \text{vf} & \text{vg} & \text{hf} & \text{hg} & \text{sf} & \text{sg} \\
0.15 & 64.151 & 0.0011586 & 1.2557 & -148.73 & 65.733 & 2.4571 & 5.8002 \\
0.65 & 73.777 & 0.0012164 & 0.32631 & -129.32 & 74.321 & 2.7381 & 5.4983 \\
1.15 & 78.445 & 0.0012483 & 0.19291 & -119.78 & 77.975 & 2.8627 & 5.3836 \\
1.65 & 81.744 & 0.0012726 & 0.13809 & -112.96 & 80.299 & 2.947 & 5.3113 \\
2.15 & 84.357 & 0.0012932 & 0.10792 & -107.51 & 81.967 & 3.0119 & 5.258 \\
2.65 & 86.549 & 0.0013115 & 0.088712 & -102.89 & 83.237 & 3.0652 & 5.2157 \\
3.15 & 88.454 & 0.0013281 & 0.075367 & -98.837 & 84.235 & 3.1108 & 5.1805 \\
3.65 & 90.146 & 0.0013436 & 0.065531 & -95.201 & 85.036 & 3.1508 & 5.1501 \\
4.15 & 91.677 & 0.0013582 & 0.057966 & -91.884 & 85.685 & 3.1865 & 5.1234 \\
4.65 & 93.078 & 0.0013722 & 0.051958 & -88.821 & 86.213 & 3.2189 & 5.0994 \\
5.15 & 94.374 & 0.0013856 & 0.047065 & -85.962 & 86.642 & 3.2487 & 5.0776 \\
5.5 & 95.227 & 0.0013947 & 0.044145 & -84.065 & 86.892 & 3.2682 & 5.0634 \\
7.5 & 99.469 & 0.0014439 & 0.032476 & -74.44 & 87.716 & 3.3641 & 4.9944 \\
9.5 & 102.96 & 0.0014905 & 0.025521 & -66.227 & 87.8 & 3.4424 & 4.9384 \\
11.5 & 105.95 & 0.0015361 & 0.020872 & -58.904 & 87.361 & 3.5096 & 4.89 \\
13.5 & 108.6 & 0.001582 & 0.017527 & -52.18 & 86.502 & 3.5694 & 4.8464 \\
15.5 & 110.97 & 0.0016292 & 0.01499 & -45.872 & 85.276 & 3.6239 & 4.8057 \\
17.5 & 113.13 & 0.0016787 & 0.012988 & -39.851 & 83.701 & 3.6747 & 4.7668 \\
19.5 & 115.12 & 0.0017315 & 0.011359 & -34.015 & 81.775 & 3.7228 & 4.7286 \\
21.5 & 116.97 & 0.001789 & 0.0099959 & -28.275 & 79.472 & 3.7693 & 4.6904 \\
23.5 & 118.69 & 0.0018529 & 0.0088275 & -22.541 & 76.74 & 3.8148 & 4.6513
\end{array}
$$

$$\begin{pmatrix} 25.5 & 120.31 & 0.0019261 & 0.0078012 & -16.706 & 73.482 & 3.8605 & 4.6101 \\ 27.5 & 121.83 & 0.0020133 & 0.0068748 & -10.619 & 69.527 & 3.9075 & 4.5653 \\ 29.5 & 123.27 & 0.0021241 & 0.0060068 & -4.0125 & 64.53 & 3.958 & 4.5141 \\ 31.5 & 124.63 & 0.0022833 & 0.005138 & 3.7511 & 57.646 & 4.0171 & 4.4496 \\ 33.5 & 125.91 & 0.0026279 & 0.0040462 & 15.897 & 44.636 & 4.1101 & 4.3384 \end{pmatrix}$$

Now, write Mathcad Functions for saturated properties:

First, for Sat. properties:

Separate the data vectors from the above matrix:

$psat := M\_sat\_N2^{\langle 0 \rangle}$ $tsat := M\_sat\_N2^{\langle 1 \rangle}$ $vfsat := M\_sat\_N2^{\langle 2 \rangle}$

$vgsat := M\_sat\_N2^{\langle 3 \rangle}$ $hfsat := M\_sat\_N2^{\langle 4 \rangle}$ $hgsat := M\_sat\_N2^{\langle 5 \rangle}$

$sfsat := M\_sat\_N2^{\langle 6 \rangle}$ $sgsat := M\_sat\_N2^{\langle 7 \rangle}$

Now, write the Functions for sat. properties of N2:

$TSAT(P) := linterp(psat, tsat, P)$ $TSAT(1.5) = 80.754$

$PSAT(T) := linterp(tsat, psat, T)$ $PSAT(90) = 3.607$

$HFSATP(P) := linterp(psat, hfsat, P)$ $HFSATP(1.150) = -119.78$

$HFSATT(T) := linterp(tsat, hfsat, T)$ $HFSATT(90) = -95.515$

$HGSATP(P) := linterp(psat, hgsat, P)$ $HGSATP(1.50) = 79.602$

$HGSATT(T) := linterp(tsat, hgsat, T)$ $HGSATT(90) = 84.967$

$HFGSATP(P) := HGSATP(P) - HFSATP(P)$ $HFGSATP(1.5) = 194.608$

$HFGSATT(T) := HGSATT(T) - HFSATT(T)$ $HFGSATT(90) = 180.482$

$SFSATP(P) := linterp(psat, sfsat, P)$ $SFSATP(1.0) = 2.825$

$SFSATT(T) := linterp(tsat, sfsat, T)$ $SFSATT(90) = 3.147$

$SGSATP(P) := linterp(psat, sgsat, P)$ $SGSATP(1.5) = 5.333$

$SGSATT(T) := linterp(tsat, sgsat, T)$ $SGSATT(90) = 5.153$

$SFGSATP(P) := SGSATP(P) - SFSATP(P)$ $SFGSATP(1.5) = 2.411$

$SFGSATT(T) := SGSATT(T) - SFSATT(T)$ $SFGSATT(90) = 2.005$

$VGSATP(P) := \mathrm{linterp}(psat, vgsat, P)$ $VGSATP(1.5) = 0.155$ m^3/kg

$VGSATT(T) := \mathrm{linterp}(tsat, vgsat, T)$ $VGSATT(90) = 0.066$ m^3/kg

$VFSATP(P) := \mathrm{linterp}(psat, vfsat, P)$ $VFSATP(1.5) = 1.26531 \times 10^{-3}$ m^3/kg

$VFSATT(T) := \mathrm{linterp}(tsat, vfsat, T)$ $VFSATT(90) = 1.342263 \times 10^{-3}$ m^3/kg

$VFGSATP(P) := VGSATP(P) - VFSATP(P)$ $VFGSATP(1.5) = 0.153271$ m^3/kg

$VFGSATT(T) := VGSATT(T) - VFSATT(T)$ $VFGSATT(90) = 0.065037$ m^3/kg

$UGSATP(P) := HGSATP(P) - P \cdot 10^{2} \cdot VGSATP(P)$ $UGSATP(1.5) = 56.421$

$UFSATP(P) := HFSATP(P) - P \cdot VFSATP(P) \cdot 10^{2}$

$UFGSATP(P) := UGSATP(P) - UFSATP(P)$

$UGSATT(T) := HGSATT(T) - PSAT(T) \cdot 10^{2} \cdot VGSATT(T)$ $UGSATT(90) = 61.025$

$UFSATT(T) := HFSATT(T) - PSAT(T) \cdot 10^{2} \cdot VFSATT(T)$

$UFGSATT(T) := UGSATT(T) - UFSATT(T)$

In the following program: psat = sat. pr. in bar, s = entropy in kJ/kg.K

```
quality_Ps(psat, s) := | return "psat should be between 0.15 bar and 33.5 bar ! "   if psat < 0.15
                       | return "psat should be between 0.15 bar and 33.5 bar ! "   if psat > 33.5
                       | sf ← SFSATP(psat)
                       | sfg ← SFGSATP(psat)
                       | x ← $\frac{s - sf}{sfg}$
```

Ex: $psat := 2.25$ $s := 4.5$

$quality\_Ps(psat, s) = 0.663$

In the following program: tsat = sat. temp, K, s = entropy in kJ/kg.K

```
quality_Ts(tsat, s) := | return "tsat should be between 64.151 K and 125.91 K ! "   if tsat < 64.151
                       | return "tsat should be between 64.151 K and 125.91 K ! "   if tsat > 125.91
                       | sf ← SFSATT(tsat)
                       | sfg ← SFGSATT(tsat)
                       | x ← $\frac{s - sf}{sfg}$
```

Ex: $tsat := 70$ $s := 3$

$quality\_Ts(tsat, s) = 0.125$

In the following program: tsat = sat. temp, K, h = enthalpy in kJ/kg.

```
quality_Th(tsat, h) := | return "tsat should be between 64.151 K and 125.91 K ! "   if tsat < 64.151
                       | return "tsat should be between 64.151 K and 125.91 K ! "   if tsat > 125.91
                       | hf ← HFSATT(tsat)
                       | hfg ← HFGSATT(tsat)
                       | x ← $\frac{h - hf}{hfg}$
```

Ex: $tsat := 70$ $h := 17$

$quality\_Th(tsat, h) = 0.74$

```
quality_Ph(psat, h) :=  | return "psat should be between 0.15 bar and 33.5 bar ! "  if psat < 0.15
                        | return "psat should be between 0.15 bar and 33.5 bar ! "  if psat > 33.5
                        | hf ← HFSATP(psat)
                        | hfg ← HFGSATP(psat)
                        | $x \leftarrow \frac{h - hf}{hfg}$
```

Ex: psat := 2.25 h := 11

quality_Ph(psat, h) = 0.623

In the following program: psat = sat. pr. in bar, x = quality, and s in kJ/kg.K

```
entropy_2phase_Px(psat, x) :=  | return "psat should be between 0.15 bar and 33.5 bar ! "  if psat < 0.15
                               | return "psat should be between 0.15 bar and 33.5 bar ! "  if psat > 33.5
                               | PSAT ← psat
                               | sf ← SFSATP(PSAT)
                               | sfg ← SFGSATP(PSAT)
                               | s ← sf + x · sfg
```

Ex: psat := 2.2 x := 0.448

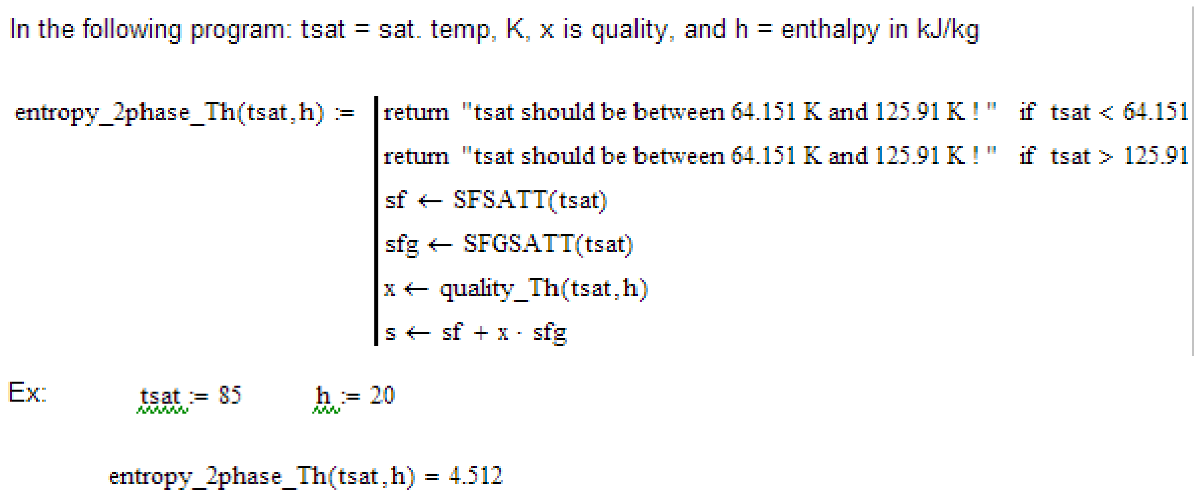
entropy_2phase_Px(psat, x) = 4.019

In the following program: tsat = sat. temp, K, x is quality, and s = entropy in kJ/kg.K

```
entropy_2phase_Tx(tsat, x) :=  | return "tsat should be between 64.151 K and 125.91 K ! "  if tsat < 64.151
                               | return "tsat should be between 64.151 K and 125.91 K ! "  if tsat > 125.91
                               | sf ← SFSATT(tsat)
                               | sfg ← SFGSATT(tsat)
                               | s ← sf + x · sfg
```

Ex: tsat := 85 x := 1

entropy_2phase_Tx(tsat, x) = 5.246



In the following program: tsat = sat. temp, K, x is quality, and h = enthalpy in kJ/kg

```
entropy_2phase_Th(tsat,h) := | return "tsat should be between 64.151 K and 125.91 K ! "  if  tsat < 64.151
                             | return "tsat should be between 64.151 K and 125.91 K ! "  if  tsat > 125.91
                             | sf ← SFSATT(tsat)
                             | sfg ← SFGSATT(tsat)
                             | x ← quality_Th(tsat,h)
                             | s ← sf + x · sfg
```

Ex: tsat := 85   h := 20

      entropy_2phase_Th(tsat,h) = 4.512

In the following program: psat = sat. pr. in bar, x = quality, and h in kJ/kg

```
enthalpy_2phase_Px(psat,x) := | return "psat should be between 0.15 bar and 33.5 bar ! "  if psat < 0.15
                              | return "psat should be between 0.15 bar and 33.5 bar ! "  if psat > 33.5
                              | PSAT ← psat
                              | hf ← HFSATP(PSAT)
                              | hfg ← HFGSATP(PSAT)
                              | h ← hf + x · hfg
```

Ex: psat := 2.25 x := 0.754

enthalpy_2phase_Px(psat,x) = 35.774

In the following program: tsat = sat. temp, K, x is quality, and h = enthalpy in kJ/kg

```
enthalpy_2phase_Tx(tsat,x) := | return "tsat should be between 64.151 K and 125.91 K ! "  if tsat < 64.151
                              | return "tsat should be between 64.151 K and 125.91 K ! "  if tsat > 125.91
                              | hf ← HFSATT(tsat)
                              | hfg ← HFGSATT(tsat)
                              | h ← hf + x · hfg
```

Ex: tsat := 85 x := 0

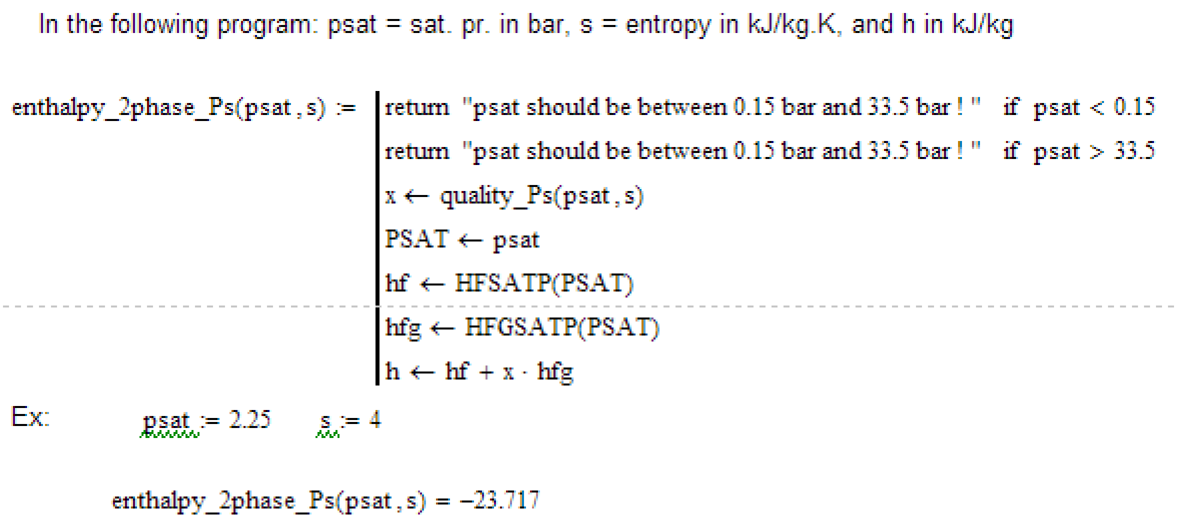
enthalpy_2phase_Tx(tsat,x) = −106.155

In the following program: tsat = sat. temp, K, s in kJ/kg.K, and h = enthalpy in kJ/kg

```
enthalpy_2phase_Ts(tsat,s) := | return "tsat should be between 64.151 K and 125.91 K ! "  if tsat < 64.151
                              | return "tsat should be between 64.151 K and 125.91 K ! "  if tsat > 125.91
                              | x ← quality_Ts(tsat,s)
                              | hf ← HFSATT(tsat)
                              | hfg ← HFGSATT(tsat)
                              | h ← hf + x · hfg
```

Ex: tsat := 85 s := 5

enthalpy_2phase_Ts(tsat,s) = 61.469



In the following program: psat = sat. pr. in bar, s = entropy in kJ/kg.K, and h in kJ/kg

```
enthalpy_2phase_Ps(psat, s) := | return "psat should be between 0.15 bar and 33.5 bar ! "  if  psat < 0.15
                               | return "psat should be between 0.15 bar and 33.5 bar ! "  if  psat > 33.5
                               | x ← quality_Ps(psat, s)
                               | PSAT ← psat
                               | hf ← HFSATP(PSAT)
                               | hfg ← HFGSATP(PSAT)
                               | h ← hf + x · hfg
```

Ex: psat := 2.25   s := 4

enthalpy_2phase_Ps(psat, s) = −23.717

Superheated N2 properties:

Isobaric data is collected for pressures, P = 1, 5…240 bar.

And, properties at these pressures are written as Mathcad matrices.

Example for P = 1 bar only is shown below, to conserve space:

At 1 bar:

T.....K
v....m^3/kg
u, h....kJ/kg;
s...kJ/kg.K....

$$M\_N2_1 := \begin{array}{c} \begin{array}{ccccc} T & v & u & h & s \end{array} \\ \begin{pmatrix} 77.244 & 0.21947 & 55.126 & 77.073 & 5.4116 \\ 85 & 0.24425 & 61.248 & 85.672 & 5.5178 \\ 95 & 0.27552 & 68.982 & 96.534 & 5.6386 \\ 105 & 0.30635 & 76.615 & 107.25 & 5.7458 \\ 115 & 0.3369 & 84.188 & 117.88 & 5.8425 \\ 125 & 0.36725 & 91.723 & 128.45 & 5.9307 \\ 135 & 0.39748 & 99.233 & 138.98 & 6.0117 \\ 145 & 0.4276 & 106.73 & 149.49 & 6.0868 \\ 155 & 0.45765 & 114.2 & 159.97 & 6.1567 \\ 165 & 0.48765 & 121.67 & 170.44 & 6.2222 \\ 175 & 0.5176 & 129.14 & 180.9 & 6.2837 \\ 185 & 0.54752 & 136.59 & 191.34 & 6.3418 \\ 195 & 0.5774 & 144.05 & 201.79 & 6.3967 \\ 205 & 0.60726 & 151.49 & 212.22 & 6.4489 \\ 215 & 0.6371 & 158.94 & 222.65 & 6.4986 \\ 225 & 0.66693 & 166.38 & 233.07 & 6.546 \\ 235 & 0.69673 & 173.82 & 243.5 & 6.5913 \\ 245 & 0.72653 & 181.26 & 253.92 & 6.6347 \\ 255 & 0.75632 & 188.7 & 264.33 & 6.6764 \\ 265 & 0.78609 & 196.14 & 274.75 & 6.7165 \\ 275 & 0.81586 & 203.58 & 285.16 & 6.755 \\ 285 & 0.84562 & 211.01 & 295.58 & 6.7922 \\ 295 & 0.87538 & 218.45 & 305.99 & 6.8281 \\ 305 & 0.90512 & 225.89 & 316.4 & 6.8629 \\ 315 & 0.93487 & 233.33 & 326.82 & 6.8965 \\ 325 & 0.96461 & 240.77 & 337.23 & 6.929 \\ 335 & 0.99434 & 248.22 & 347.65 & 6.9606 \\ 345 & 1.0241 & 255.66 & 358.07 & 6.9912 \\ 355 & 1.0538 & 263.11 & 368.49 & 7.021 \\ 365 & 1.0835 & 270.57 & 378.92 & 7.05 \end{pmatrix} \end{array}$$

$temp1 := \left(M\_N2_1\right)^{\langle 0\rangle}$

$spvol1 := \left(M\_N2_1\right)^{\langle 1\rangle}$ $enth1 := \left(M\_N2_1\right)^{\langle 3}$ $entrop1 := \left(M\_N2_1\right)^{\langle 4\rangle}$

$HN21B(T) := \mathrm{linterp}(temp1, enth1, T)$ ex: $HN21B(340) = 352.86$

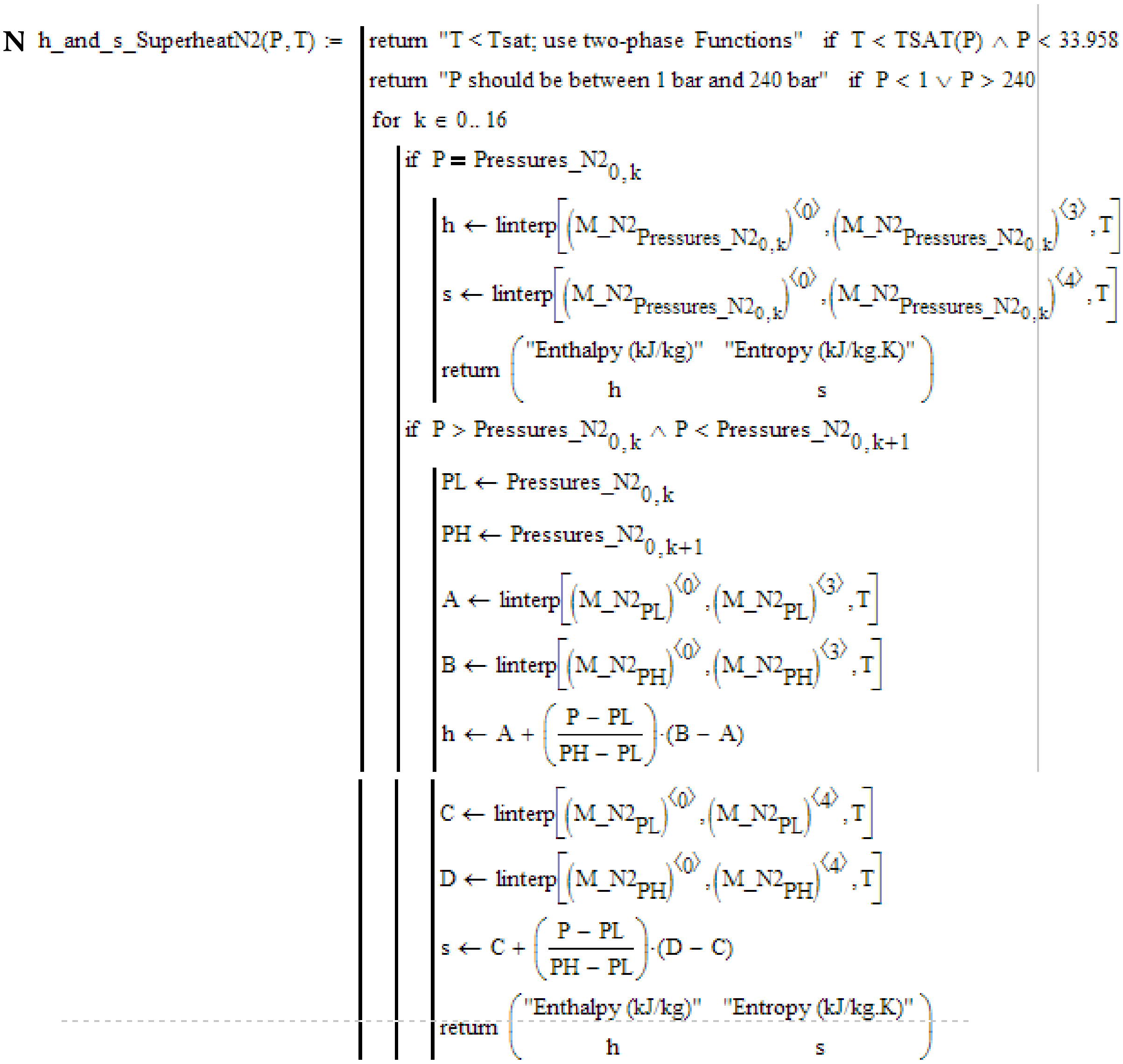
$SN21B(T) := \mathrm{linterp}(temp1, entrop1, T)$ ex: $SN21B(340) = 6.976$

Now, using the above data Matrices, write a Function to find enthalpy and entropy of superheated nitrogen when P and T are given.

Procedure is exactly as we did in the case of Helium, earlier:

First, create a vector of all pressure values for which we have collected data Matrices:

$\text{Pressures\_N2} := (1\ \ 5\ \ 10\ \ 15\ \ 20\ \ 30\ \ 40\ \ 60\ \ 80\ \ 100\ \ 120\ \ 140\ \ 160\ \ 180\ \ 200\ \ 220\ \ 240)$



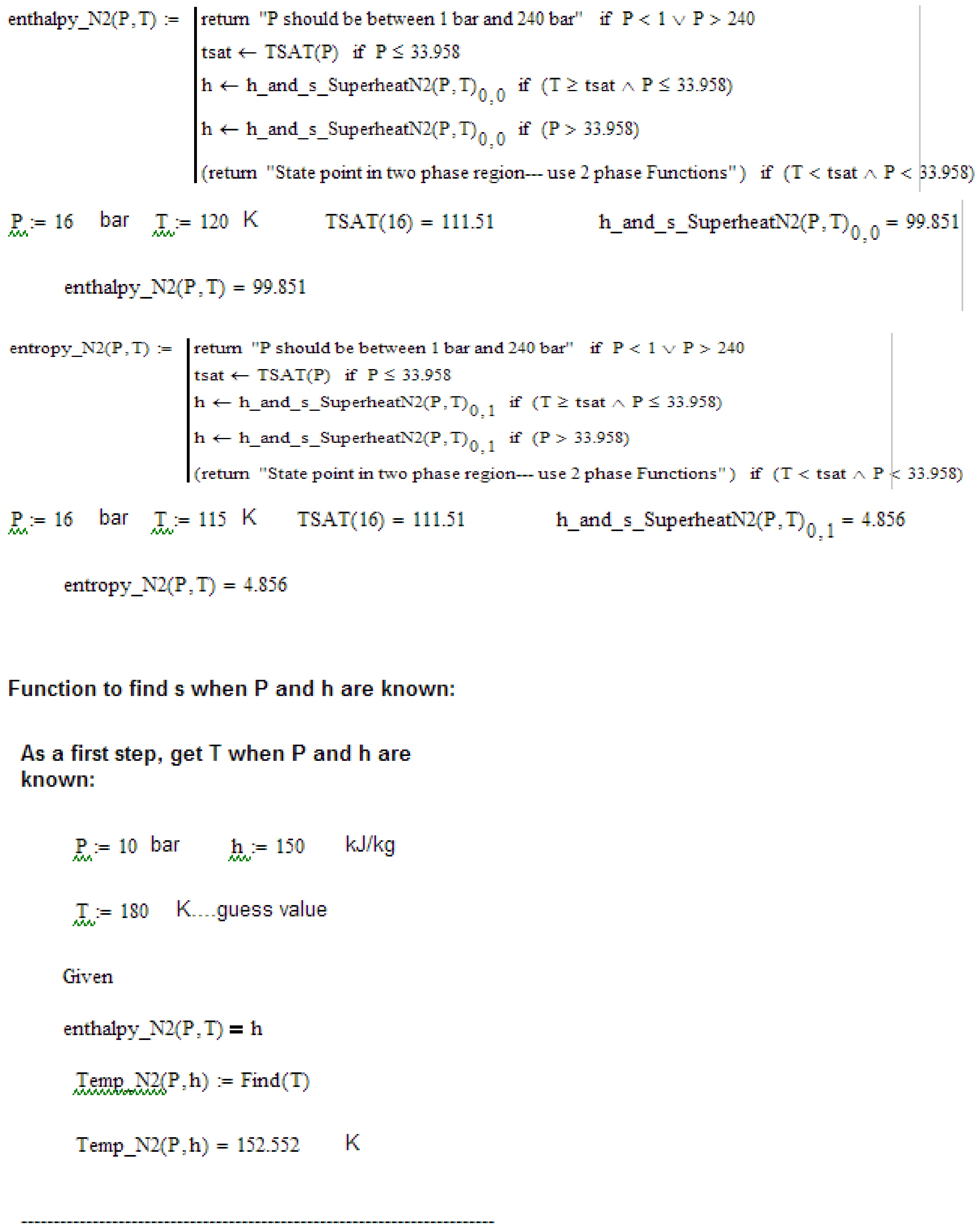
```
N  h_and_s_SuperheatN2(P,T) :=
    return "T < Tsat; use two-phase Functions"  if  T < TSAT(P) ∧ P < 33.958
    return "P should be between 1 bar and 240 bar"  if  P < 1 ∨ P > 240
    for k ∈ 0..16
        if P = Pressures_N2_{0,k}
            h ← linterp[(M_N2_{Pressures_N2_{0,k}})^<0>, (M_N2_{Pressures_N2_{0,k}})^<3>, T]
            s ← linterp[(M_N2_{Pressures_N2_{0,k}})^<0>, (M_N2_{Pressures_N2_{0,k}})^<4>, T]
            return ( "Enthalpy (kJ/kg)"  "Entropy (kJ/kg.K)" )
                   (        h                    s          )
        if P > Pressures_N2_{0,k} ∧ P < Pressures_N2_{0,k+1}
            PL ← Pressures_N2_{0,k}
            PH ← Pressures_N2_{0,k+1}
            A ← linterp[(M_N2_PL)^<0>, (M_N2_PL)^<3>, T]
            B ← linterp[(M_N2_PH)^<0>, (M_N2_PH)^<3>, T]
            h ← A + ((P − PL)/(PH − PL))·(B − A)
            C ← linterp[(M_N2_PL)^<0>, (M_N2_PL)^<4>, T]
            D ← linterp[(M_N2_PH)^<0>, (M_N2_PH)^<4>, T]
            s ← C + ((P − PL)/(PH − PL))·(D − C)
            return ( "Enthalpy (kJ/kg)"  "Entropy (kJ/kg.K)" )
                   (        h                    s          )
```

Ex: $P := 16\quad T := 355\qquad \text{TSAT}(16) = 111.51$

$$\text{h\_and\_s\_SuperheatN2}(P,T) = \begin{pmatrix} \text{"Enthalpy (kJ/kg)"} & \text{"Entropy (kJ/kg.K)"} \\ 366.32 & 6.193 \end{pmatrix}$$

Now, write Functions for enthalpy and entropy in terms of P and T:

Remember: For N2: Pc =33.958 bar, Tc = 126.192 K, NBP = 77.355 K



```
enthalpy_N2(P,T) := | return "P should be between 1 bar and 240 bar"  if  $P < 1 \lor P > 240$
                    | tsat ← TSAT(P)  if  $P \le 33.958$
                    | h ← $\text{h\_and\_s\_SuperheatN2}(P,T)_{0,0}$  if  $(T \ge tsat \land P \le 33.958)$
                    | h ← $\text{h\_and\_s\_SuperheatN2}(P,T)_{0,0}$  if  $(P > 33.958)$
                    | (return "State point in two phase region--- use 2 phase Functions")  if  $(T < tsat \land P < 33.958)$
```

P := 16 bar T := 120 K TSAT(16) = 111.51 $\text{h\_and\_s\_SuperheatN2}(P,T)_{0,0} = 99.851$

enthalpy_N2(P,T) = 99.851

```
entropy_N2(P,T) := | return "P should be between 1 bar and 240 bar"  if  $P < 1 \lor P > 240$
                   | tsat ← TSAT(P)  if  $P \le 33.958$
                   | h ← $\text{h\_and\_s\_SuperheatN2}(P,T)_{0,1}$  if  $(T \ge tsat \land P \le 33.958)$
                   | h ← $\text{h\_and\_s\_SuperheatN2}(P,T)_{0,1}$  if  $(P > 33.958)$
                   | (return "State point in two phase region--- use 2 phase Functions")  if  $(T < tsat \land P < 33.958)$
```

P := 16 bar T := 115 K TSAT(16) = 111.51 $\text{h\_and\_s\_SuperheatN2}(P,T)_{0,1} = 4.856$

entropy_N2(P,T) = 4.856

Function to find s when P and h are known:

As a first step, get T when P and h are known:

P := 10 bar h := 150 kJ/kg

T := 180 K....guess value

Given

enthalpy_N2(P,T) = h

Temp_N2(P,h) := Find(T)

Temp_N2(P,h) = 152.552 K

```
entropy_N2_Ph(P,h) := | return "P should be between 1 bar and 240 bar"  if  P < 1 ∨ P > 240
                      | T ← Temp_N2(P,h)
                      | s ← entropy_N2(P,T)
```

Ex: $P := 10$ bar $h := 150$ kJ/kg $\text{Temp\_N2}(P,h) = 152.552$

$\text{entropy\_N2\_Ph}(P,h) = 5.422$

==

Function to find T when P and h are known:

$P := 20$ bar $h := 170$ kJ/kg

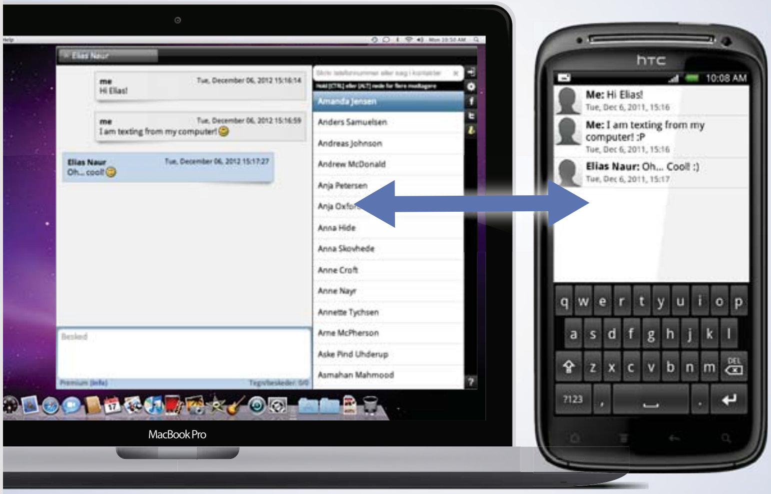
$T := 150$ K....guess value

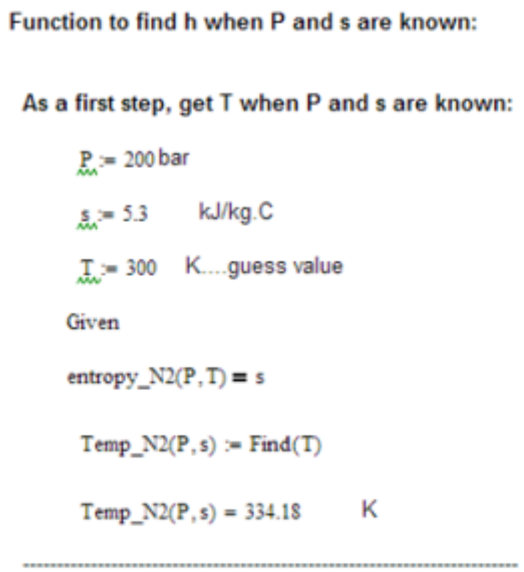
Given

$\text{enthalpy\_N2}(P,T) = h$

$\text{Temp\_N2}(P,h) := \text{Find}(T)$

$\text{Temp\_N2}(P,h) = 176.159$ K





Function to find h when P and s are known:

As a first step, get T when P and s are known:

$P := 200$ bar

$s := 5.3$ kJ/kg.C

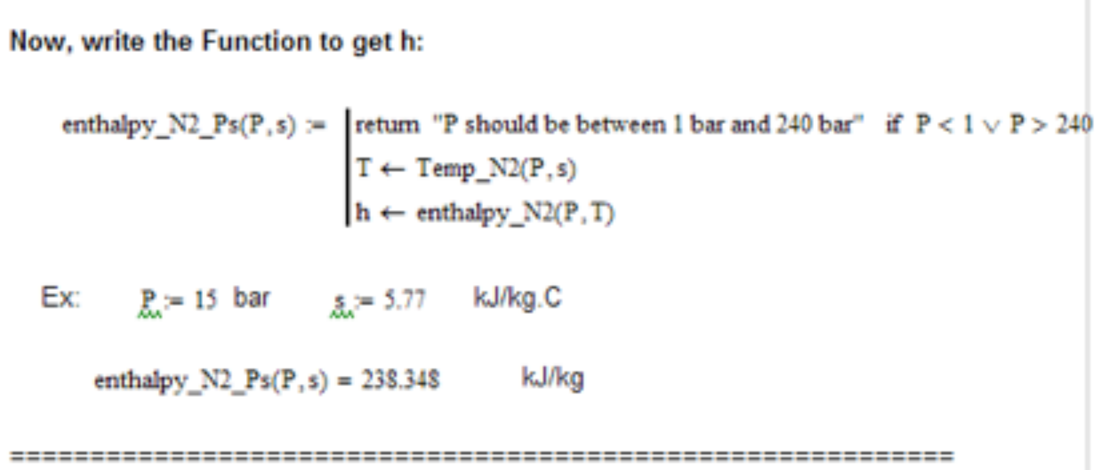
$T := 300$ K....guess value

Given

$\text{entropy\_N2}(P,T) = s$

$\text{Temp\_N2}(P,s) := \text{Find}(T)$

$\text{Temp\_N2}(P,s) = 334.18$ K



Now, write the Function to get h:

$$\text{enthalpy\_N2\_Ps}(P,s) := \left| \begin{array}{l} \text{return "P should be between 1 bar and 240 bar" if } P < 1 \vee P > 240 \\ T \leftarrow \text{Temp\_N2}(P,s) \\ h \leftarrow \text{enthalpy\_N2}(P,T) \end{array} \right.$$

Ex: $P := 15$ bar $s := 5.77$ kJ/kg.C

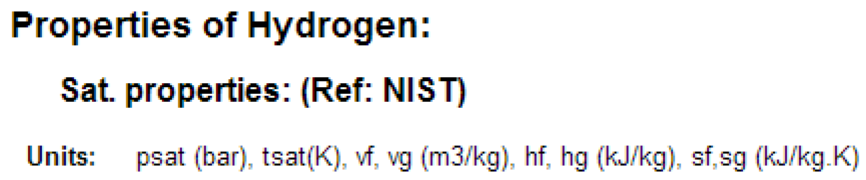
$\text{enthalpy\_N2\_Ps}(P,s) = 238.348$ kJ/kg

===

Prob. 3.3.3 Write Mathcad Functions to determine properties of Hydrogen

From NIST website, we get properties of Hydrogen and then write the Mathcad Functions.

We have: For Hydrogen, summary of relevant Saturation Properties:



Properties of Hydrogen:

Sat. properties: (Ref: NIST)

Units: psat (bar), tsat(K), vf, vg (m3/kg), hf, hg (kJ/kg), sf,sg (kJ/kg.K)

$$\mathrm{M\_sat\_H2} := \begin{array}{cccccccc} \mathrm{Psat} & \mathrm{Tsat} & \mathrm{vf} & \mathrm{vg} & \mathrm{hf} & \mathrm{hg} & \mathrm{sf} & \mathrm{sg} \\ \hline 0.1 & 14.51 & 0.013063 & 5.8678 & -49.972 & 404.88 & -2.7969 & 28.55 \\ 0.6 & 18.717 & 0.013765 & 1.2005 & -15.821 & 438.59 & -0.78005 & 23.499 \\ 1.1 & 20.65 & 0.014179 & 0.69693 & 2.8538 & 450.2 & 0.13316 & 21.796 \\ 1.6 & 22.021 & 0.01452 & 0.49523 & 17.475 & 456.5 & 0.78479 & 20.721 \\ 2.1 & 23.113 & 0.014828 & 0.38484 & 30.066 & 460.19 & 1.3102 & 19.92 \\ 2.6 & 24.033 & 0.015116 & 0.31459 & 41.416 & 462.29 & 1.7599 & 19.272 \\ 3.1 & 24.836 & 0.015395 & 0.26566 & 51.932 & 463.26 & 2.159 & 18.721 \\ 3.6 & 25.552 & 0.015668 & 0.22947 & 61.855 & 463.39 & 2.522 & 18.236 \\ 4.1 & 26.202 & 0.01594 & 0.20151 & 71.343 & 462.84 & 2.8581 & 17.8 \\ 4.6 & 26.798 & 0.016213 & 0.17919 & 80.507 & 461.7 & 3.1736 & 17.399 \\ 5.1 & 27.35 & 0.01649 & 0.1609 & 89.434 & 460.06 & 3.4731 & 17.024 \\ 5.6 & 27.866 & 0.016773 & 0.1456 & 98.191 & 457.94 & 3.7602 & 16.67 \\ 6.1 & 28.35 & 0.017065 & 0.13258 & 106.83 & 455.37 & 4.0376 & 16.332 \\ 6.6 & 28.806 & 0.017369 & 0.12132 & 115.42 & 452.37 & 4.3078 & 16.005 \\ 7.1 & 29.239 & 0.017686 & 0.11146 & 123.99 & 448.93 & 4.5728 & 15.686 \\ 7.6 & 29.651 & 0.018021 & 0.10272 & 132.59 & 445.03 & 4.8348 & 15.372 \\ 8.1 & 30.044 & 0.018377 & 0.094892 & 141.29 & 440.65 & 5.0956 & 15.06 \\ 8.6 & 30.419 & 0.01876 & 0.087801 & 150.13 & 435.74 & 5.3573 & 14.747 \\ 9.1 & 30.779 & 0.019177 & 0.081312 & 159.19 & 430.24 & 5.6223 & 14.429 \\ 9.6 & 31.125 & 0.019636 & 0.075306 & 168.55 & 424.04 & 5.8936 & 14.102 \\ 10.1 & 31.458 & 0.020153 & 0.06968 & 178.35 & 417 & 6.175 & 13.761 \\ 10.6 & 31.778 & 0.020747 & 0.06433 & 188.77 & 408.9 & 6.472 & 13.399 \\ 11.1 & 32.087 & 0.021455 & 0.059143 & 200.09 & 399.38 & 6.7934 & 13.004 \\ 11.6 & 32.385 & 0.022342 & 0.053967 & 212.83 & 387.79 & 7.1549 & 12.557 \\ 12.1 & 32.673 & 0.023561 & 0.048532 & 228.17 & 372.71 & 7.5912 & 12.015 \\ 12.6 & 32.95 & 0.025629 & 0.042089 & 249.89 & 349.72 & 8.2153 & 11.245 \end{array}$$

Now, write Mathcad Functions for saturation properties:

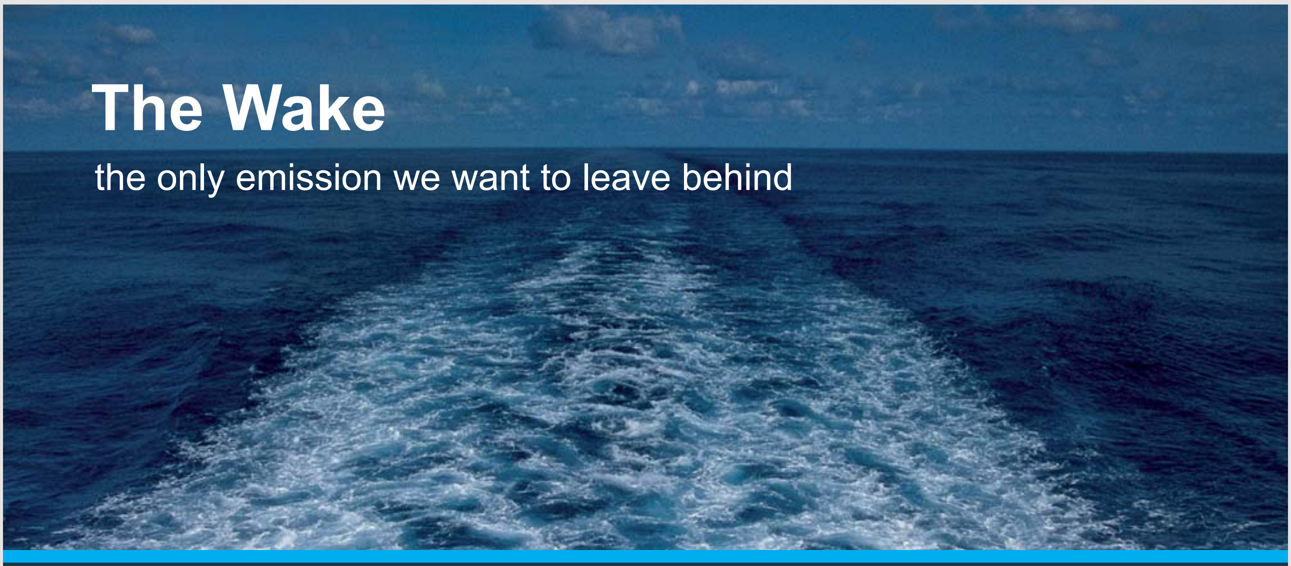
First, separate the data vectors from the Matrix:

$\mathrm{psat} := \mathrm{M\_sat\_H2}^{\langle 0\rangle}$ $\mathrm{tsat} := \mathrm{M\_sat\_H2}^{\langle 1\rangle}$ $\mathrm{vfsat} := \mathrm{M\_sat\_H2}^{\langle 2\rangle}$

$\mathrm{vgsat} := \mathrm{M\_sat\_H2}^{\langle 3\rangle}$ $\mathrm{hfsat} := \mathrm{M\_sat\_H2}^{\langle 4\rangle}$ $\mathrm{hgsat} := \mathrm{M\_sat\_H2}^{\langle 5\rangle}$

$\mathrm{sfsat} := \mathrm{M\_sat\_H2}^{\langle 6\rangle}$ $\mathrm{sgsat} := \mathrm{M\_sat\_H2}^{\langle 7\rangle}$

Function	Example
$TSAT(P) := linterp(psat, tsat, P)$	$TSAT(1.5) = 21.747$
$PSAT(T) := linterp(tsat, psat, T)$	$PSAT(25) = 3.215$
$HFSATP(P) := linterp(psat, hfsat, P)$	$HFSATP(1.50) = 14.551$
$HFSATT(T) := linterp(tsat, hfsat, T)$	$HFSATT(25) = 54.205$
$HGSATP(P) := linterp(psat, hgsat, P)$	$HGSATP(1.5) = 455.24$
$HGSATT(T) := linterp(tsat, hgsat, T)$	$HGSATT(25) = 463.29$
$HFGSATP(P) := HGSATP(P) - HFSATP(P)$	$HFGSATP(1.5) = 440.689$
$HFGSATT(T) := HGSATT(T) - HFSATT(T)$	$HFGSATT(25) = 409.085$



SFSATP(P) := linterp(psat, sfsat, P)   SFSATP(1.5) = 0.654

SFSATT(T) := linterp(tsat, sfsat, T)   SFSATT(25) = 2.242

SGSATP(P) := linterp(psat, sgsat, P)   SGSATP(1.5) = 20.936

SGSATT(T) := linterp(tsat, sgsat, T)   SGSATT(25) = 18.61

SFGSATP(P) := SGSATP(P) − SFSATP(P)   SFGSATP(1.5) = 20.282

SFGSATT(T) := SGSATT(T) − SFSATT(T)   SFGSATT(25) = 16.368

VGSATP(P) := linterp(psat, vgsat, P)   VGSATP(1.50) = 0.536   m^3/kg

VGSATT(T) := linterp(tsat, vgsat, T)   VGSATT(25) = 0.257   m^3/kg

VFSATP(P) := linterp(psat, vfsat, P)   VFSATP(1.0) = 0.014096   m^3/kg

VFSATT(T) := linterp(tsat, vfsat, T)   VFSATT(25) = 0.015458   m^3/kg

VFGSATP(P) := VGSATP(P) − VFSATP(P)   VFGSATP(1.0) = 0.783548   m^3/kg

VFGSATT(T) := VGSATT(T) − VFSATT(T)   VFGSATT(25) = 0.241913   m^3/kg

UGSATP(P) := HGSATP(P) − P · 10^2 · VGSATP(P)   UGSATP(1) = 368.114

UFSATP(P) := HFSATP(P) − P · VFSATP(P) · 10^2

UFGSATP(P) := UGSATP(P) − UFSATP(P)

UGSATT(T) := HGSATT(T) − PSAT(T) · 10^2 · VGSATT(T)   UGSATT(4.5) = 2.174×10^3

UFSATT(T) := HFSATT(T) − PSAT(T) · 10^2 · VFSATT(T)

UFGSATT(T) := UGSATT(T) − UFSATT(T)

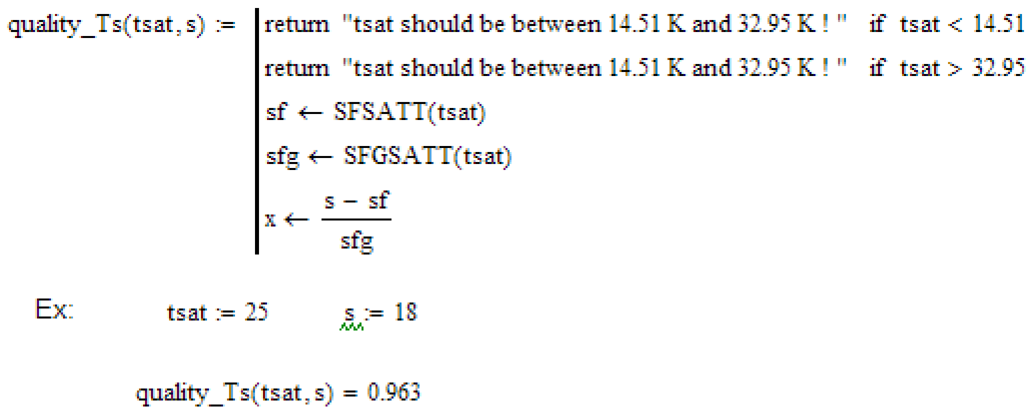
In the following program: psat = sat. pr. in bar, s = entropy in kJ/kg.K

```
quality_Ps(psat, s) := | return "psat should be between 0.1 bar and 12.6 bar ! "  if psat < 0.1
                       | return "psat should be between 0.1 bar and 12.6 bar ! "  if psat > 12.6
                       | sf ← SFSATP(psat)
                       | sfg ← SFGSATP(psat)
                       | x ← $\frac{s - sf}{sfg}$
```

Ex: psat := 2.25   s := 19

quality_Ps(psat, s) = 0.96

In the following program: tsat = sat. temp, K, s = entropy in kJ/kg.K

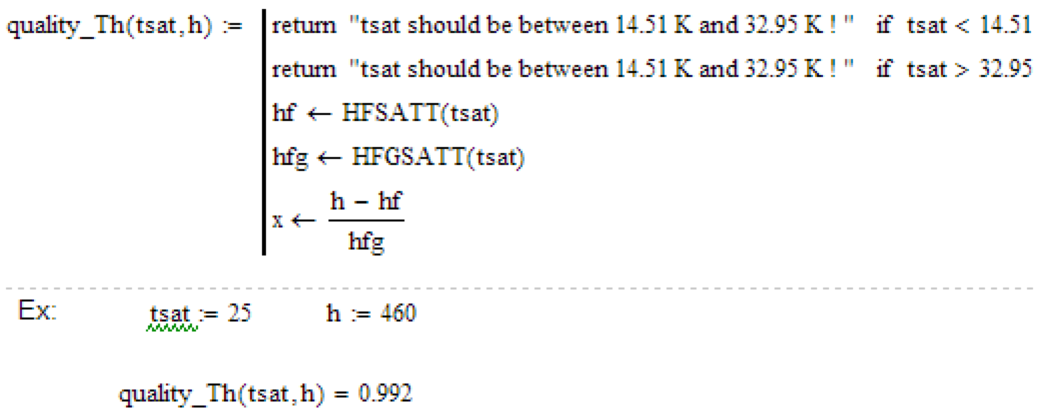


```
quality_Ts(tsat, s) := | return "tsat should be between 14.51 K and 32.95 K ! "  if  tsat < 14.51
                       | return "tsat should be between 14.51 K and 32.95 K ! "  if  tsat > 32.95
                       | sf ← SFSATT(tsat)
                       | sfg ← SFGSATT(tsat)
                       | $x \leftarrow \frac{s - sf}{sfg}$
```

Ex: tsat := 25   s := 18

quality_Ts(tsat, s) = 0.963

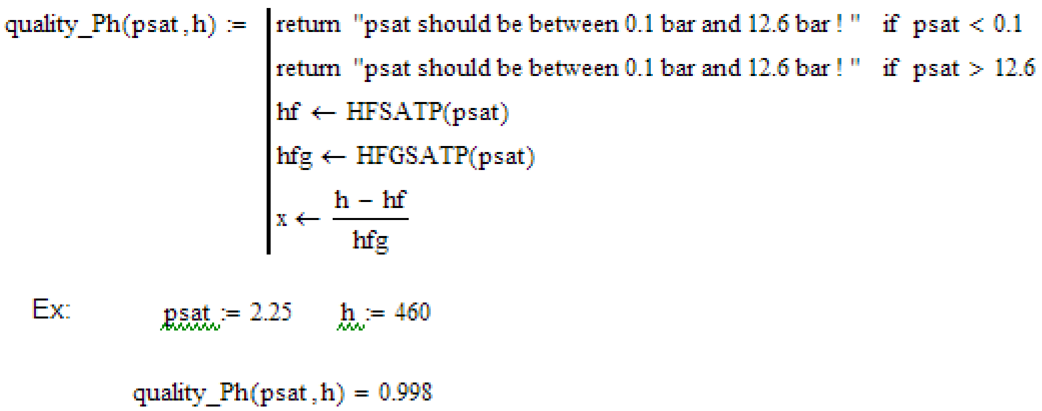
In the following program: tsat = sat. temp, K, h = enthalpy in kJ/kg.



```
quality_Th(tsat, h) := | return "tsat should be between 14.51 K and 32.95 K ! "  if  tsat < 14.51
                       | return "tsat should be between 14.51 K and 32.95 K ! "  if  tsat > 32.95
                       | hf ← HFSATT(tsat)
                       | hfg ← HFGSATT(tsat)
                       | $x \leftarrow \frac{h - hf}{hfg}$
```

Ex: tsat := 25   h := 460

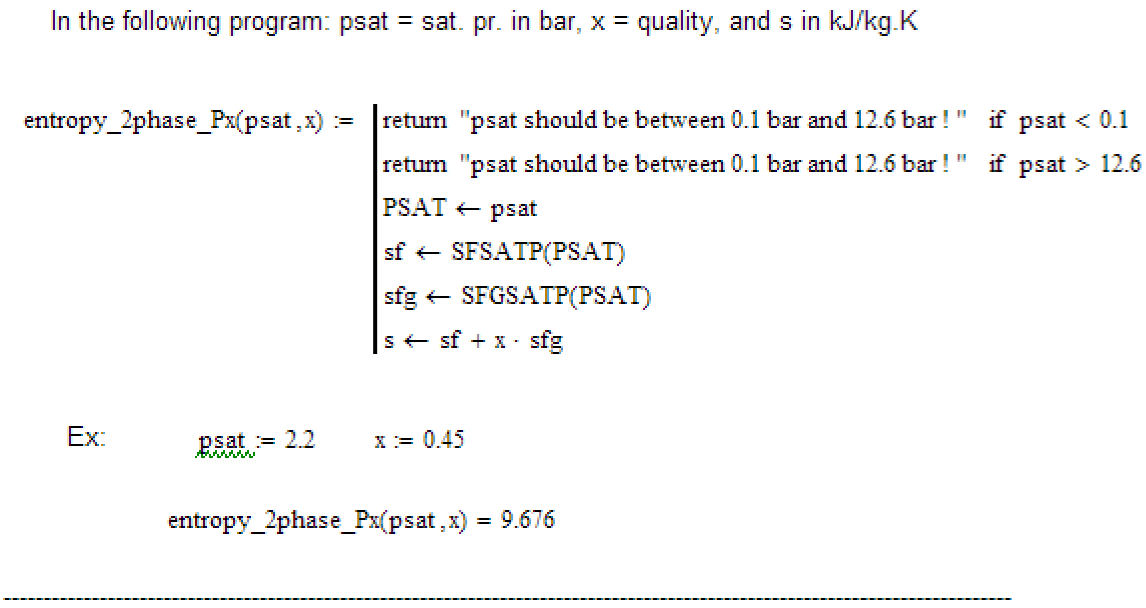
quality_Th(tsat, h) = 0.992



```
quality_Ph(psat, h) := | return "psat should be between 0.1 bar and 12.6 bar ! "  if  psat < 0.1
                       | return "psat should be between 0.1 bar and 12.6 bar ! "  if  psat > 12.6
                       | hf ← HFSATP(psat)
                       | hfg ← HFGSATP(psat)
                       | $x \leftarrow \frac{h - hf}{hfg}$
```

Ex: psat := 2.25   h := 460

quality_Ph(psat, h) = 0.998

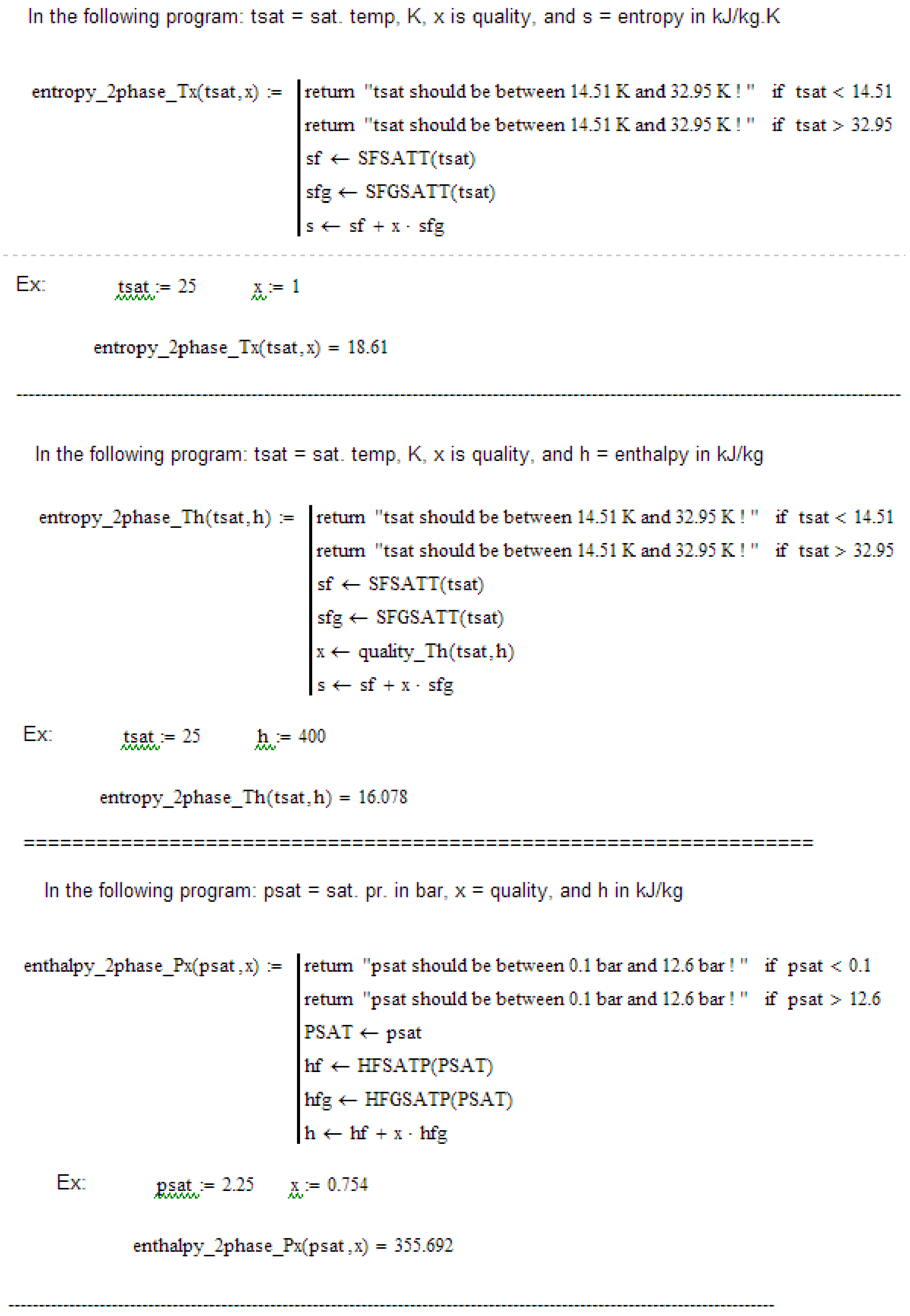


In the following program: psat = sat. pr. in bar, x = quality, and s in kJ/kg.K

```
entropy_2phase_Px(psat,x) := | return "psat should be between 0.1 bar and 12.6 bar ! "  if psat < 0.1
                             | return "psat should be between 0.1 bar and 12.6 bar ! "  if psat > 12.6
                             | PSAT ← psat
                             | sf ← SFSATP(PSAT)
                             | sfg ← SFGSATP(PSAT)
                             | s ← sf + x · sfg
```

Ex: psat := 2.2   x := 0.45

entropy_2phase_Px(psat,x) = 9.676



In the following program: tsat = sat. temp, K, x is quality, and s = entropy in kJ/kg.K

```
entropy_2phase_Tx(tsat,x) := | return "tsat should be between 14.51 K and 32.95 K ! "  if tsat < 14.51
                             | return "tsat should be between 14.51 K and 32.95 K ! "  if tsat > 32.95
                             | sf ← SFSATT(tsat)
                             | sfg ← SFGSATT(tsat)
                             | s ← sf + x · sfg
```

```
Ex:        tsat := 25        x := 1

        entropy_2phase_Tx(tsat,x) = 18.61
```

In the following program: tsat = sat. temp, K, x is quality, and h = enthalpy in kJ/kg

```
entropy_2phase_Th(tsat,h) := | return "tsat should be between 14.51 K and 32.95 K ! "  if tsat < 14.51
                             | return "tsat should be between 14.51 K and 32.95 K ! "  if tsat > 32.95
                             | sf ← SFSATT(tsat)
                             | sfg ← SFGSATT(tsat)
                             | x ← quality_Th(tsat,h)
                             | s ← sf + x · sfg
```

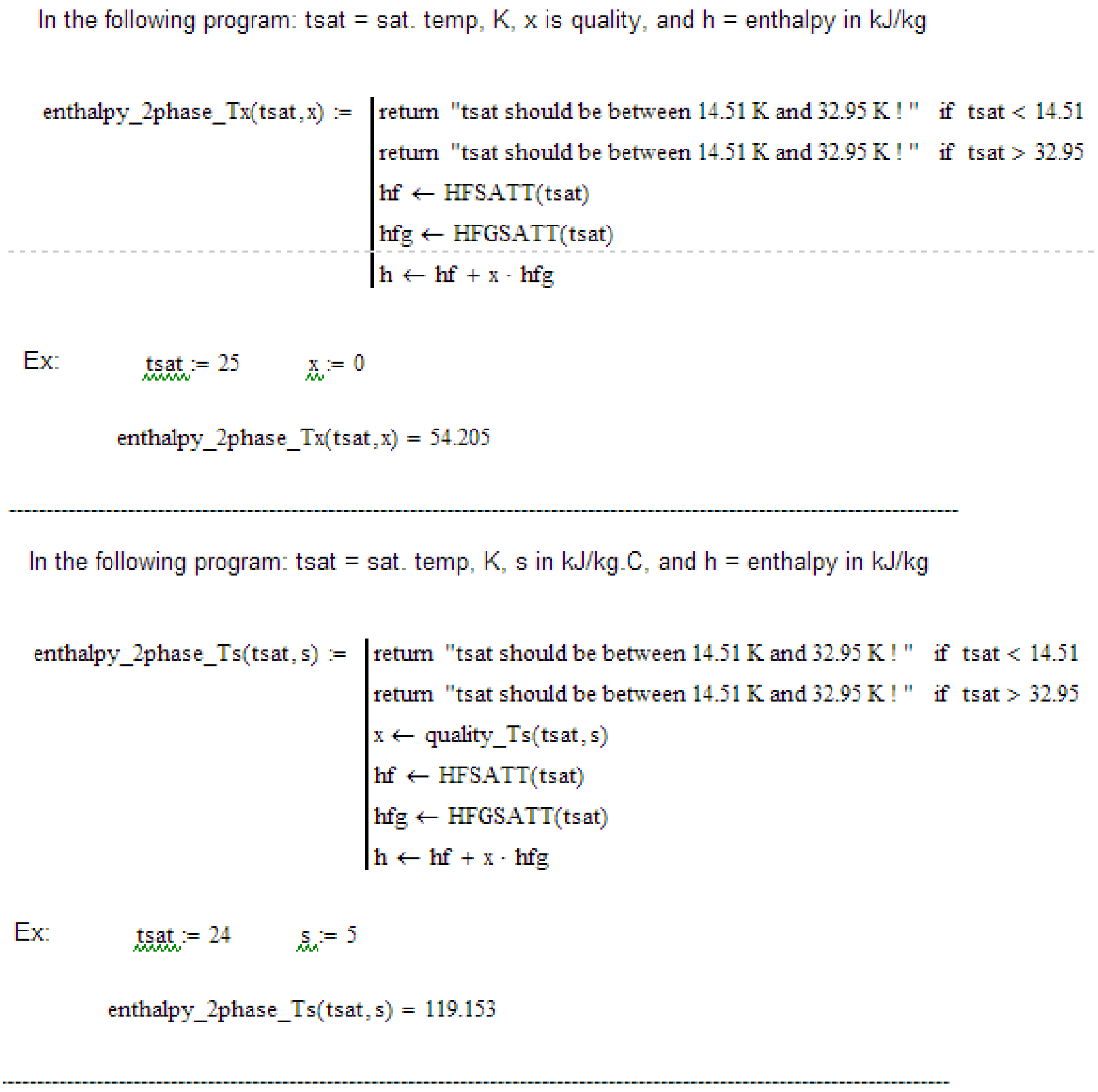
```
Ex:        tsat := 25        h := 400

        entropy_2phase_Th(tsat,h) = 16.078
```

==

In the following program: psat = sat. pr. in bar, x = quality, and h in kJ/kg

```
enthalpy_2phase_Px(psat,x) := | return "psat should be between 0.1 bar and 12.6 bar ! "  if psat < 0.1
                              | return "psat should be between 0.1 bar and 12.6 bar ! "  if psat > 12.6
                              | PSAT ← psat
                              | hf ← HFSATP(PSAT)
                              | hfg ← HFGSATP(PSAT)
                              | h ← hf + x · hfg
```

```
Ex:        psat := 2.25        x := 0.754

        enthalpy_2phase_Px(psat,x) = 355.692
```



In the following program: tsat = sat. temp, K, x is quality, and h = enthalpy in kJ/kg

```
enthalpy_2phase_Tx(tsat,x) := | return "tsat should be between 14.51 K and 32.95 K ! "  if tsat < 14.51
                              | return "tsat should be between 14.51 K and 32.95 K ! "  if tsat > 32.95
                              | hf ← HFSATT(tsat)
                              | hfg ← HFGSATT(tsat)
                              | h ← hf + x · hfg
```

Ex: tsat := 25   x := 0

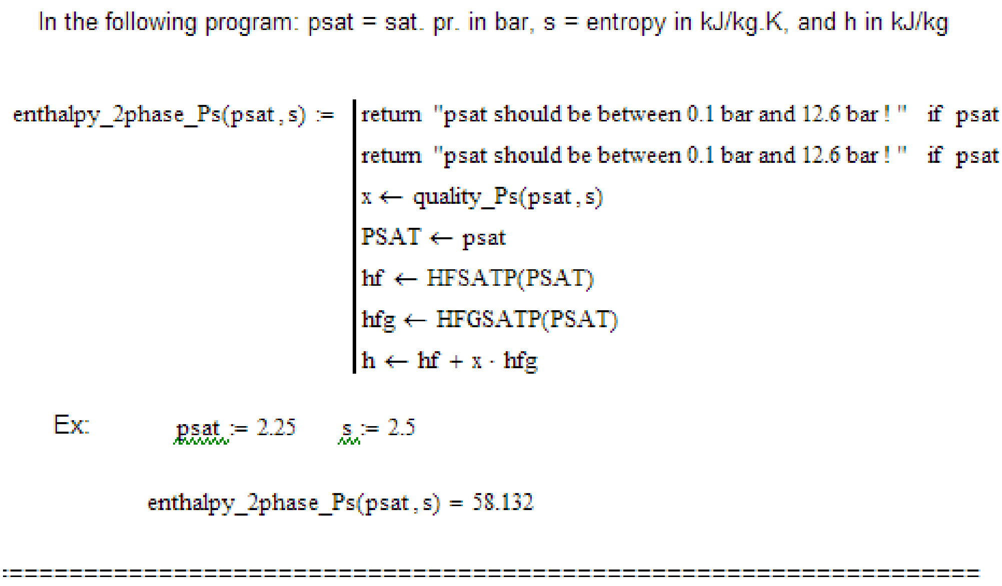
enthalpy_2phase_Tx(tsat,x) = 54.205

In the following program: tsat = sat. temp, K, s in kJ/kg.C, and h = enthalpy in kJ/kg

```
enthalpy_2phase_Ts(tsat,s) := | return "tsat should be between 14.51 K and 32.95 K ! "  if tsat < 14.51
                              | return "tsat should be between 14.51 K and 32.95 K ! "  if tsat > 32.95
                              | x ← quality_Ts(tsat,s)
                              | hf ← HFSATT(tsat)
                              | hfg ← HFGSATT(tsat)
                              | h ← hf + x · hfg
```

Ex: tsat := 24   s := 5

enthalpy_2phase_Ts(tsat,s) = 119.153



In the following program: psat = sat. pr. in bar, s = entropy in kJ/kg.K, and h in kJ/kg

```
enthalpy_2phase_Ps(psat, s) := | return "psat should be between 0.1 bar and 12.6 bar ! "  if psat < 0.1
                               | return "psat should be between 0.1 bar and 12.6 bar ! "  if psat > 12.6
                               | x ← quality_Ps(psat, s)
                               | PSAT ← psat
                               | hf ← HFSATP(PSAT)
                               | hfg ← HFGSATP(PSAT)
                               | h ← hf + x · hfg
```

Ex: psat := 2.25   s := 2.5

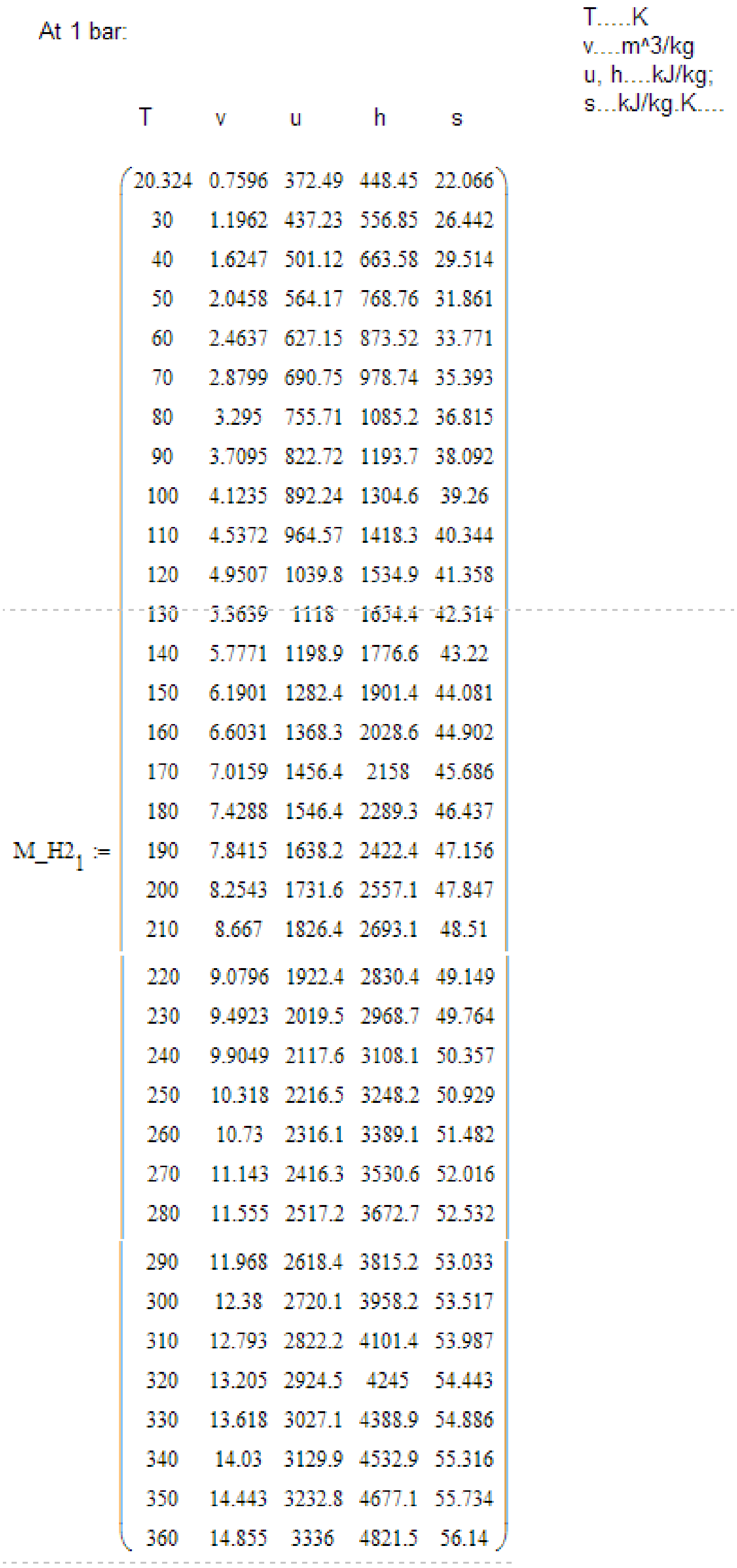
enthalpy_2phase_Ps(psat, s) = 58.132

===

Properties of superheated H2:

Isobaric data is collected for pressures, P = 1, 5…240 bar.
Now, properties at these pressures are written as Mathcad matrices.

Example for P = 1 bar only is shown below, to conserve space:



At 1 bar:

T.....K
v....m^3/kg
u, h....kJ/kg;
s...kJ/kg.K....

$$M\_H2_1 := \begin{array}{ccccc} T & v & u & h & s \\ \left(\begin{array}{ccccc} 20.324 & 0.7596 & 372.49 & 448.45 & 22.066 \\ 30 & 1.1962 & 437.23 & 556.85 & 26.442 \\ 40 & 1.6247 & 501.12 & 663.58 & 29.514 \\ 50 & 2.0458 & 564.17 & 768.76 & 31.861 \\ 60 & 2.4637 & 627.15 & 873.52 & 33.771 \\ 70 & 2.8799 & 690.75 & 978.74 & 35.393 \\ 80 & 3.295 & 755.71 & 1085.2 & 36.815 \\ 90 & 3.7095 & 822.72 & 1193.7 & 38.092 \\ 100 & 4.1235 & 892.24 & 1304.6 & 39.26 \\ 110 & 4.5372 & 964.57 & 1418.3 & 40.344 \\ 120 & 4.9507 & 1039.8 & 1534.9 & 41.358 \\ 130 & 5.3639 & 1118 & 1654.4 & 42.314 \\ 140 & 5.7771 & 1198.9 & 1776.6 & 43.22 \\ 150 & 6.1901 & 1282.4 & 1901.4 & 44.081 \\ 160 & 6.6031 & 1368.3 & 2028.6 & 44.902 \\ 170 & 7.0159 & 1456.4 & 2158 & 45.686 \\ 180 & 7.4288 & 1546.4 & 2289.3 & 46.437 \\ 190 & 7.8415 & 1638.2 & 2422.4 & 47.156 \\ 200 & 8.2543 & 1731.6 & 2557.1 & 47.847 \\ 210 & 8.667 & 1826.4 & 2693.1 & 48.51 \\ 220 & 9.0796 & 1922.4 & 2830.4 & 49.149 \\ 230 & 9.4923 & 2019.5 & 2968.7 & 49.764 \\ 240 & 9.9049 & 2117.6 & 3108.1 & 50.357 \\ 250 & 10.318 & 2216.5 & 3248.2 & 50.929 \\ 260 & 10.73 & 2316.1 & 3389.1 & 51.482 \\ 270 & 11.143 & 2416.3 & 3530.6 & 52.016 \\ 280 & 11.555 & 2517.2 & 3672.7 & 52.532 \\ 290 & 11.968 & 2618.4 & 3815.2 & 53.033 \\ 300 & 12.38 & 2720.1 & 3958.2 & 53.517 \\ 310 & 12.793 & 2822.2 & 4101.4 & 53.987 \\ 320 & 13.205 & 2924.5 & 4245 & 54.443 \\ 330 & 13.618 & 3027.1 & 4388.9 & 54.886 \\ 340 & 14.03 & 3129.9 & 4532.9 & 55.316 \\ 350 & 14.443 & 3232.8 & 4677.1 & 55.734 \\ 360 & 14.855 & 3336 & 4821.5 & 56.14 \end{array}\right) \end{array}$$

$$\text{temp1} := \left(M\_H2_1\right)^{\langle 0\rangle}$$

$$\text{spvol1} := \left(M\_H2_1\right)^{\langle 1\rangle} \qquad \text{enth1} := \left(M\_H2_1\right)^{\langle 3\rangle} \qquad \text{entrop1} := \left(M\_H2_1\right)^{\langle 4\rangle}$$

$$\text{HH21B}(T) := \text{linterp}(\text{temp1}, \text{enth1}, T) \qquad \text{ex:} \quad \text{HH21B}(340) = 4.533 \times 10^{3}$$

$$\text{SH21B}(T) := \text{linterp}(\text{temp1}, \text{entrop1}, T) \qquad \text{ex:} \quad \text{SH21B}(340) = 55.316$$

Now, using the above Data Matrices, write a Function to find enthalpy and entropy of superheated hydrogen as a function of P and T:

First, create a vector of all pressure values for which we have collected data Matrices:

$$\text{Pressures\_H2} := (1\ \ 5\ \ 10\ \ 15\ \ 20\ \ 30\ \ 40\ \ 60\ \ 80\ \ 100\ \ 120\ \ 140\ \ 160\ \ 180\ \ 200\ \ 220\ \ 240)$$

Now, write the Function just as we did for He and N2:

```
h_and_s_SuperheatH2(P, T) :=
    return "T < Tsat; use two-phase  Functions"  if  $T < TSAT(P) \wedge P < 12.96$
    return "P should be between 1 bar and 240 bar"  if  $P < 1 \vee P > 240$
    for  $k \in 0..16$
        if  $P = Pressures\_H2_{0,k}$
            $h \leftarrow linterp\left[\left(M\_H2_{Pressures\_H2_{0,k}}\right)^{\langle 0\rangle}, \left(M\_H2_{Pressures\_H2_{0,k}}\right)^{\langle 3\rangle}, T\right]$
            $s \leftarrow linterp\left[\left(M\_H2_{Pressures\_H2_{0,k}}\right)^{\langle 0\rangle}, \left(M\_H2_{Pressures\_H2_{0,k}}\right)^{\langle 4\rangle}, T\right]$
            return $\begin{pmatrix} \text{"Enthalpy (kJ/kg)"} & \text{"Entropy (kJ/kg.K)"} \\ h & s \end{pmatrix}$
        if  $P > Pressures\_H2_{0,k} \wedge P < Pressures\_H2_{0,k+1}$
            $PL \leftarrow Pressures\_H2_{0,k}$
            $PH \leftarrow Pressures\_H2_{0,k+1}$
            $A \leftarrow linterp\left[\left(M\_H2_{PL}\right)^{\langle 0\rangle}, \left(M\_H2_{PL}\right)^{\langle 3\rangle}, T\right]$
            $B \leftarrow linterp\left[\left(M\_H2_{PH}\right)^{\langle 0\rangle}, \left(M\_H2_{PH}\right)^{\langle 3\rangle}, T\right]$
            $h \leftarrow A + \left(\frac{P - PL}{PH - PL}\right) \cdot (B - A)$
            $C \leftarrow linterp\left[\left(M\_H2_{PL}\right)^{\langle 0\rangle}, \left(M\_H2_{PL}\right)^{\langle 4\rangle}, T\right]$
            $D \leftarrow linterp\left[\left(M\_H2_{PH}\right)^{\langle 0\rangle}, \left(M\_H2_{PH}\right)^{\langle 4\rangle}, T\right]$
            $s \leftarrow C + \left(\frac{P - PL}{PH - PL}\right) \cdot (D - C)$
            return $\begin{pmatrix} \text{"Enthalpy (kJ/kg)"} & \text{"Entropy (kJ/kg.K)"} \\ h & s \end{pmatrix}$
```

Ex: $P := 200$ $T := 70$

$$h\_and\_s\_SuperheatH2(P,T) = \begin{pmatrix} \text{"Enthalpy (kJ/kg)"} & \text{"Entropy (kJ/kg.K)"} \\ 770.25 & 10.855 \end{pmatrix}$$

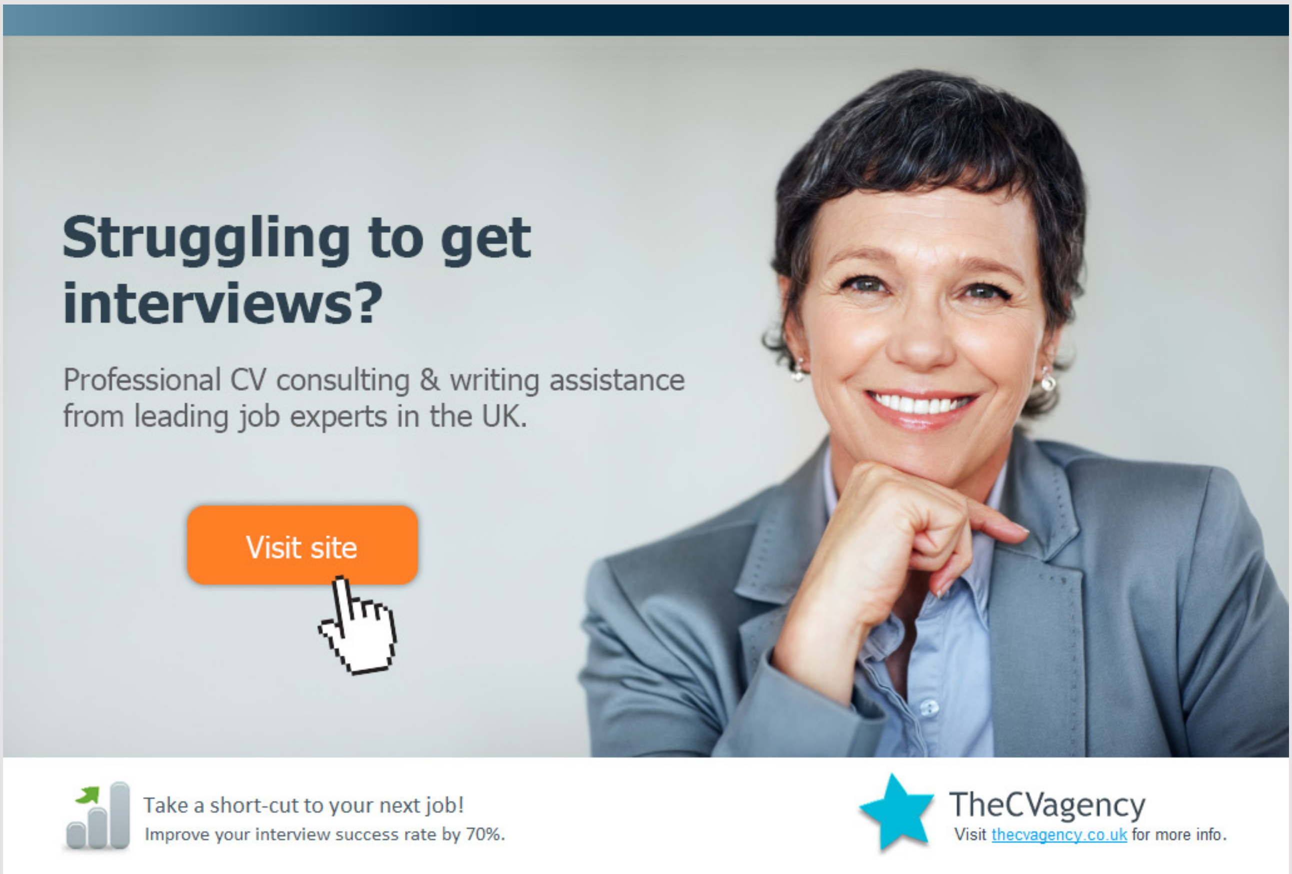
Now, write Functions for enthalpy and entropy in terms of P and T:

Remember: For H2: Pc =12.964 bar, Tc = 33.14 K, NBP = 20.369 K

```
enthalpy_H2(P,T) := | return "P should be between 1 bar and 240 bar"  if P < 1 ∨ P > 240
                    | tsat ← TSAT(P)  if P ≤ 12.96
                    | h ← h_and_s_SuperheatH2(P,T)_{1,0}  if (T ≥ tsat ∧ P ≤ 12.96)
                    | h ← h_and_s_SuperheatH2(P,T)_{1,0}  if (P > 12.96)
                    | (return "State point in two phase region--- use 2 phase Functions")  if (T < tsat ∧ P < 12.96)
```

Ex: $P := 10$ bar $T := 35$ K $TSAT(10.0) = 31.391$

$enthalpy\_H2(P,T) = 491.198$



```
entropy_H2(P,T) := | return "P should be between 1 bar and 240 bar"  if P < 1 ∨ P > 240
                   | tsat ← TSAT(P)  if P ≤ 12.96
                   | s ← h_and_s_SuperheatH2(P,T)1,1  if (T ≥ tsat ∧ P ≤ 12.96)
                   | s ← h_and_s_SuperheatH2(P,T)1,1  if (P > 12.96)
                   | (return "State point in two phase region--- use 2 phase Functions")  if (T < tsat ∧ P < 12.96)
```

Ex: $P := 10$ bar $T := 32$ K $TSAT(10) = 31.391$

$entropy\_H2(P,T) = 14.183$

===

Function to find h when P and s are known:

As a first step, get T when P and s are known:

$P := 16$ bar

$s := 44$ kJ/kg.C

$T := 300$ K....guess value

Given

$entropy\_H2(P,T) = s$

$Temp\_H2(P,s) := Find(T)$

$Temp\_H2(P,s) = 342.406$ K

Now, write the Function to get h:

```
enthalpy_H2_Ps(P,s) := | return "P should be between 1 bar and 240 bar"  if P < 1 ∨ P > 240
                       | T ← Temp_H2(P,s)
                       | h ← enthalpy_H2(P,T)
```

Ex: $P := 20$ bar $s := 25$ kJ/kg.C

$enthalpy\_H2\_Ps(P,s) = 1.125 \times 10^{3}$ kJ/kg

===

Function to find s when P and h are known:

As a first step, get T when P and h are known:

$P := 20$ bar $\quad h := 1800$ kJ/kg

$T := 120$ K....guess value

Given

$\mathrm{enthalpy\_H2}(P,T) = h$

$\mathrm{Temp\_H2}(P,h) := \mathrm{Find}(T)$

$\mathrm{Temp\_H2}(P,h) = 142.7$ K

$$\mathrm{entropy\_H2\_Ph}(P,h) := \left|\begin{array}{l} \text{return "P should be between 1 bar and 240 bar" if } P < 1 \vee P > 240 \\ T \leftarrow \mathrm{Temp\_H2}(P,h) \\ s \leftarrow \mathrm{entropy\_H2}(P,T) \end{array}\right.$$

Ex: $P := 20$ bar $\quad h := 1800$ kJ/kg

$\mathrm{entropy\_H2\_Ph}(P,h) = 30.981$

===

Function to find T when P and h are known:

$P := 20$ bar $\quad h := 1770$ kJ/kg

$T := 150$ K....guess value

Given

$\mathrm{enthalpy\_H2}(P,T) = h$

$\mathrm{Temp\_H2}(P,h) := \mathrm{Find}(T)$

$\mathrm{Temp\_H2}(P,h) = 140.345$ K

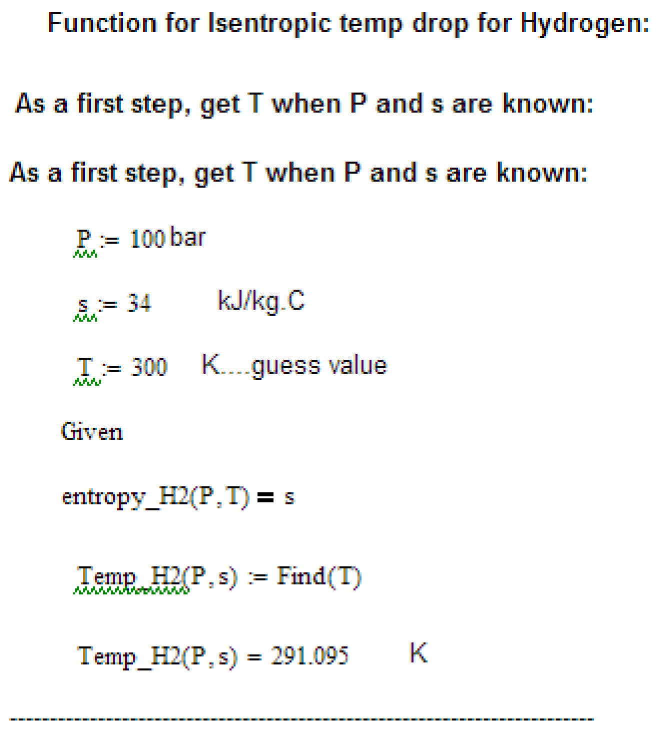
Function for Isenthalpic temp drop:

```
DT_isenthalp_H2(P_i,T_i,P_f) := | h_i ← enthalpy_H2(P_i,T_i)
                                | h_f ← h_i
                                | T_f ← Temp_H2(P_f,h_f)
                                | DT ← T_i – T_f
                                | ("P_i(bar)"  "P_f(bar)"  "T_i(K)"  "T_f(K)"  "DT(K)")
                                | (   P_i         P_f        T_i       T_f       DT   )
```

Ex:

$T\_i := 300$ K $\quad P\_f := 1$ bar $\quad P\_i := 200$

$$DT\_isenthalp\_H2(P\_i,T\_i,P\_f) = \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DT(K)"} \\ 200 & 1 & 300 & 307.172 & -7.172 \end{pmatrix}$$



Function for Isentropic temp drop for Hydrogen:

As a first step, get T when P and s are known:

As a first step, get T when P and s are known:

$P := 100$ bar

$s := 34$ kJ/kg.C

$T := 300$ K....guess value

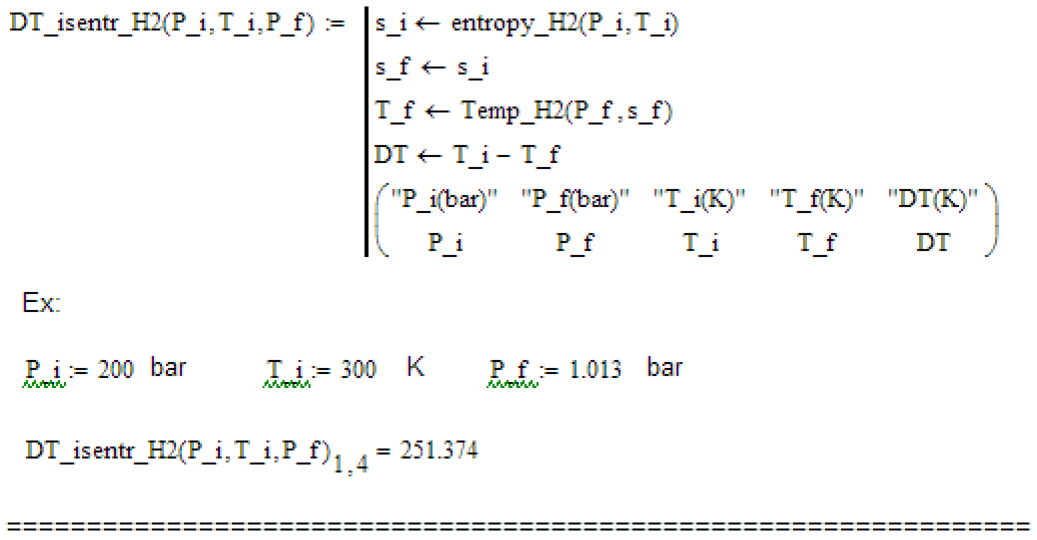
Given

$\text{entropy\_H2}(P,T) = s$

$\text{Temp\_H2}(P,s) := \text{Find}(T)$

$\text{Temp\_H2}(P,s) = 291.095$ K

Now, write the required Mathcad Function:



```
DT_isentr_H2(P_i,T_i,P_f) := | s_i ← entropy_H2(P_i,T_i)
                              | s_f ← s_i
                              | T_f ← Temp_H2(P_f,s_f)
                              | DT ← T_i − T_f
                              | ( "P_i(bar)"  "P_f(bar)"  "T_i(K)"  "T_f(K)"  "DT(K)" )
                              | (   P_i         P_f         T_i       T_f       DT    )
```

Ex:

$P\_i := 200$ bar $T\_i := 300$ K $P\_f := 1.013$ bar

$\text{DT\_isentr\_H2}(P\_i,T\_i,P\_f)_{1,4} = 251.374$

==

Note: Along similar lines, Mathcad Functions are written for properties of Argon, Methane, Fluorine and Oxygen, based on NIST data.

Use of these Functions is illustrated in Prob. 3.3.10.

Prob. 3.3.4 Write Mathcad Functions to determine temperature drop on isentropic expansion for various gases.

For isentropic expansion from initial pressure P_i and initial temp T_i to a final pressure of P_f, we have:

$$\frac{T\_f}{T\_i} = \left(\frac{P\_f}{P\_i}\right)^{\frac{k-1}{k}}$$

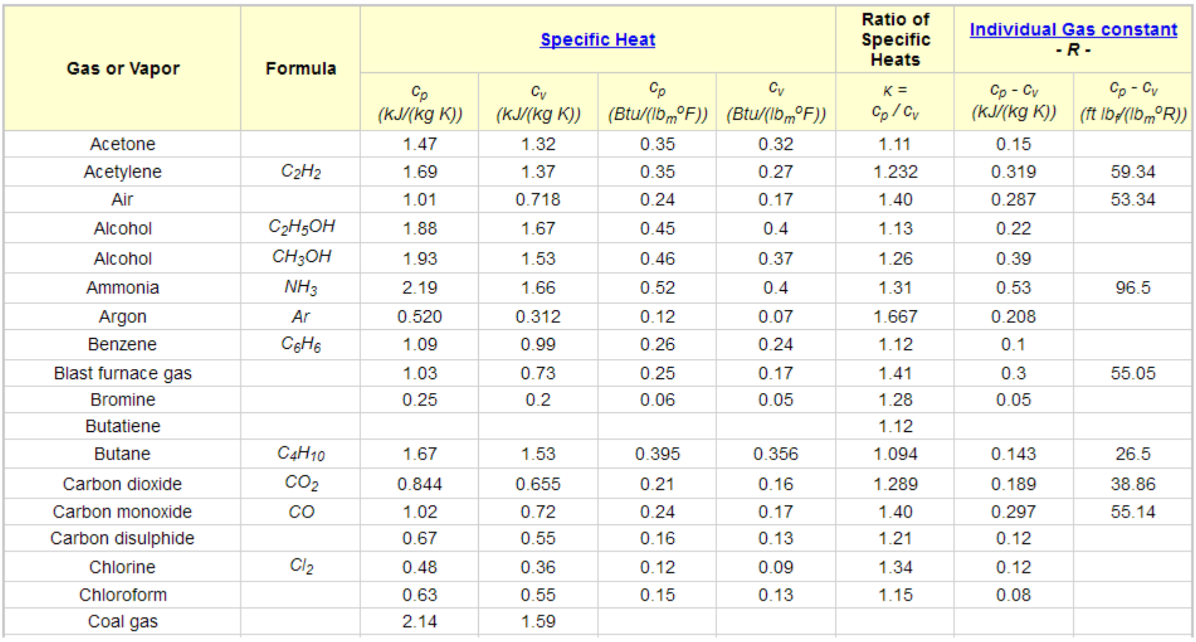
where k is the ratio of sp. heats for the gas concerned, i.e. k = cp/cv

T_f is the final temp.

DT = (T_i – T_f) is the temp drop.

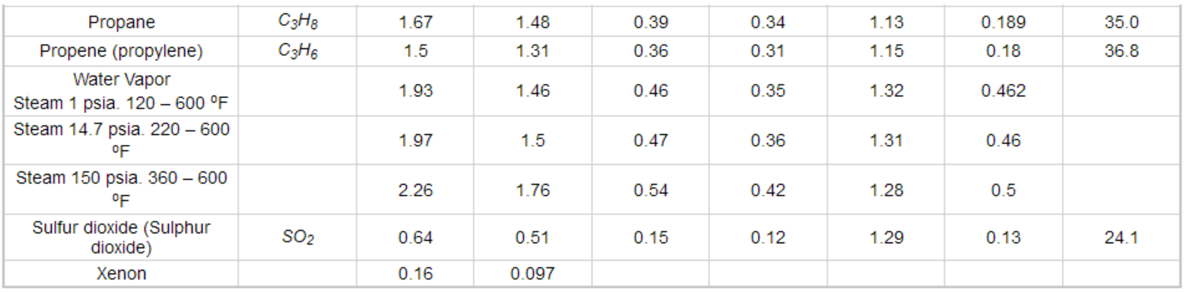
Now, for various gases, the ratios of sp. heats are given:

Ref: http://www.engineeringtoolbox.com/specific-heat-capacity-gases-d_159.html



Gas or Vapor	Formula	Specific Heat				Ratio of Specific Heats	Individual Gas constant - R -	
		c_p (kJ/(kg K))	c_v (kJ/(kg K))	c_p (Btu/(lb$_m$ ^{o}F))	c_v (Btu/(lb$_m$ ^{o}F))	$\kappa = c_p / c_v$	$c_p - c_v$ (kJ/(kg K))	$c_p - c_v$ (ft lb$_f$/(lb$_m$ ^{o}R))
Acetone		1.47	1.32	0.35	0.32	1.11	0.15	
Acetylene	C_2H_2	1.69	1.37	0.35	0.27	1.232	0.319	59.34
Air		1.01	0.718	0.24	0.17	1.40	0.287	53.34
Alcohol	C_2H_5OH	1.88	1.67	0.45	0.4	1.13	0.22	
Alcohol	CH_3OH	1.93	1.53	0.46	0.37	1.26	0.39	
Ammonia	NH_3	2.19	1.66	0.52	0.4	1.31	0.53	96.5
Argon	Ar	0.520	0.312	0.12	0.07	1.667	0.208	
Benzene	C_6H_6	1.09	0.99	0.26	0.24	1.12	0.1	
Blast furnace gas		1.03	0.73	0.25	0.17	1.41	0.3	55.05
Bromine		0.25	0.2	0.06	0.05	1.28	0.05	
Butatiene						1.12		
Butane	C_4H_{10}	1.67	1.53	0.395	0.356	1.094	0.143	26.5
Carbon dioxide	CO_2	0.844	0.655	0.21	0.16	1.289	0.189	38.86
Carbon monoxide	CO	1.02	0.72	0.24	0.17	1.40	0.297	55.14
Carbon disulphide		0.67	0.55	0.16	0.13	1.21	0.12	
Chlorine	Cl_2	0.48	0.36	0.12	0.09	1.34	0.12	
Chloroform		0.63	0.55	0.15	0.13	1.15	0.08	
Coal gas		2.14	1.59					

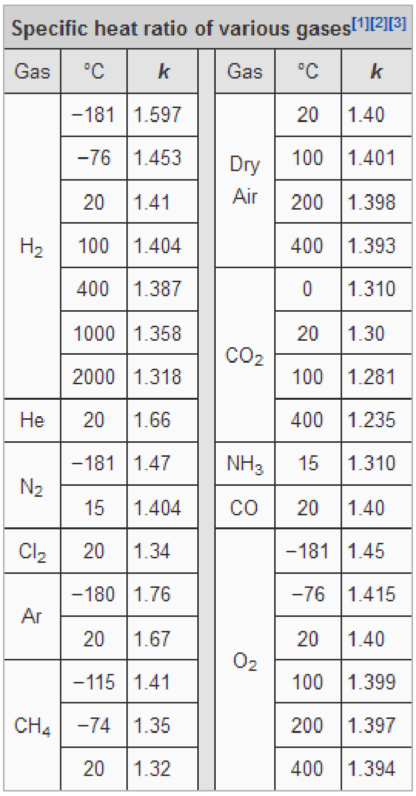
Combustion products		1		0.24				
Ethane	C_2H_6	1.75	1.48	0.39	0.32	1.187	0.276	51.5
Ether		2.01	1.95	0.48	0.47	1.03	0.06	
Ethylene	C_2H_4	1.53	1.23	0.4	0.33	1.240	0.296	55.08
Monochlorodifluoromethane, R-22						1.18		
Helium	He	5.19	3.12	1.25	0.75	1.667	2.08	386.3
Hexane						1.06		
Hydrochloric acid		0.795	0.567					
Hydrogen	H_2	14.32	10.16	3.42	2.43	1.405	4.12	765.9
Hydrogen Chloride	HCl	0.8	0.57	0.191	0.135	1.41	0.23	42.4
Hydrogen Sulfide	H_2S			0.243	0.187	1.32		45.2
Hydroxyl	OH	1.76	1.27			1.384	0.489	
Krypton		0.25	0.151					
Methane	CH_4	2.22	1.70	0.59	0.45	1.304	0.518	96.4
Methyl Chloride	CH_3Cl			0.240	0.200	1.20		30.6
Natural Gas		2.34	1.85	0.56	0.44	1.27	0.5	79.1
Neon		1.03	0.618			1.667	0.412	
Nitric Oxide	NO	0.995	0.718	0.23	0.17	1.386	0.277	
Nitrogen	N_2	1.04	0.743	0.25	0.18	1.400	0.297	54.99
Nitrogen tetroxide		4.69	4.6	1.12	1.1	1.02	0.09	
Nitrous oxide	N_2O	0.88	0.69	0.21	0.17	1.27	0.18	35.1
Oxygen	O_2	0.919	0.659	0.22	0.16	1.395	0.260	48.24
Pentane						1.07		



Propane	C_3H_8	1.67	1.48	0.39	0.34	1.13	0.189	35.0
Propene (propylene)	C_3H_6	1.5	1.31	0.36	0.31	1.15	0.18	36.8
Water Vapor Steam 1 psia. 120 – 600 ºF		1.93	1.46	0.46	0.35	1.32	0.462	
Steam 14.7 psia. 220 – 600 ºF		1.97	1.5	0.47	0.36	1.31	0.46	
Steam 150 psia. 360 – 600 ºF		2.26	1.76	0.54	0.42	1.28	0.5	
Sulfur dioxide (Sulphur dioxide)	SO_2	0.64	0.51	0.15	0.12	1.29	0.13	24.1
Xenon		0.16	0.097					

Of course, k varies with temp. For example, see below:

Ref: http://en.citizendium.org/wiki/Specific_heat_ratio

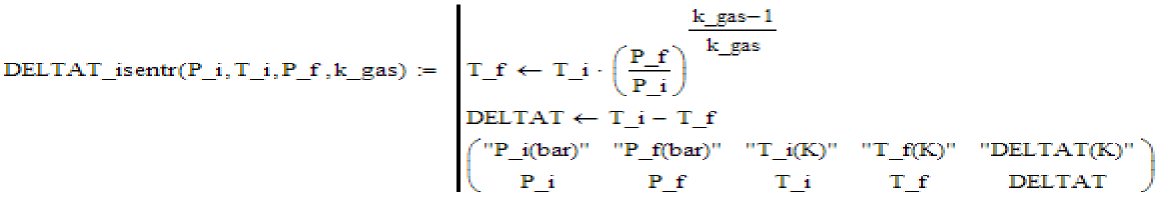


Specific heat ratio of various gases[1][2][3]

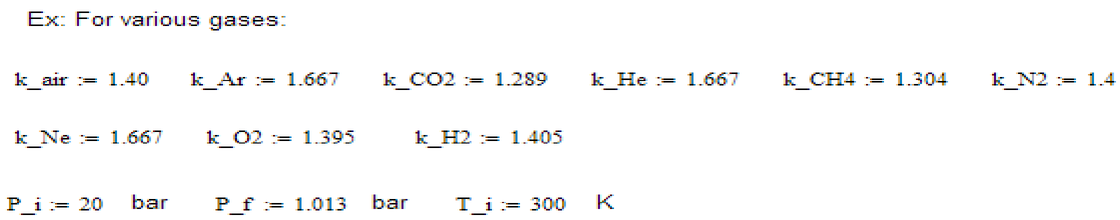
Gas	°C	*k*	Gas	°C	*k*
H_2	−181	1.597	Dry Air	20	1.40
	−76	1.453		100	1.401
	20	1.41		200	1.398
	100	1.404		400	1.393
	400	1.387	CO_2	0	1.310
	1000	1.358		20	1.30
	2000	1.318		100	1.281
He	20	1.66		400	1.235
N_2	−181	1.47	NH_3	15	1.310
	15	1.404	CO	20	1.40
Cl_2	20	1.34	O_2	−181	1.45
Ar	−180	1.76		−76	1.415
	20	1.67		20	1.40
CH_4	−115	1.41		100	1.399
	−74	1.35		200	1.397
	20	1.32		400	1.394

....

Now, the Mathcad Function is written:



$$\text{DELTAT\_isentr}(P\_i, T\_i, P\_f, k\_gas) := \left| \begin{array}{l} T\_f \leftarrow T\_i \cdot \left(\dfrac{P\_f}{P\_i}\right)^{\frac{k\_gas-1}{k\_gas}} \\ DELTAT \leftarrow T\_i - T\_f \\ \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DELTAT(K)"} \\ P\_i & P\_f & T\_i & T\_f & DELTAT \end{pmatrix} \end{array} \right.$$



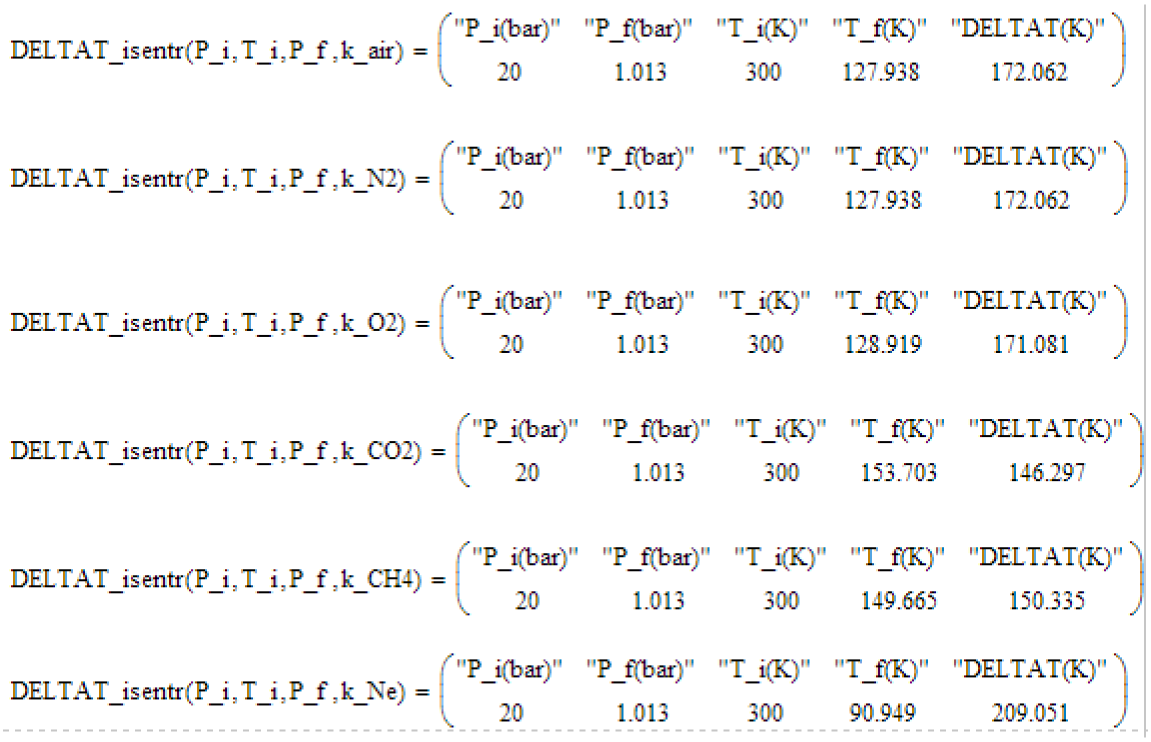
Ex: For various gases:

$k\_air := 1.40 \quad k\_Ar := 1.667 \quad k\_CO2 := 1.289 \quad k\_He := 1.667 \quad k\_CH4 := 1.304 \quad k\_N2 := 1.4$

$k\_Ne := 1.667 \quad k\_O2 := 1.395 \quad k\_H2 := 1.405$

$P\_i := 20$ bar $\quad P\_f := 1.013$ bar $\quad T\_i := 300$ K

Then, from the Mathcad Function written, we get:



$$DELTAT\_isentr(P\_i,T\_i,P\_f,k\_air) = \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DELTAT(K)"} \\ 20 & 1.013 & 300 & 127.938 & 172.062 \end{pmatrix}$$

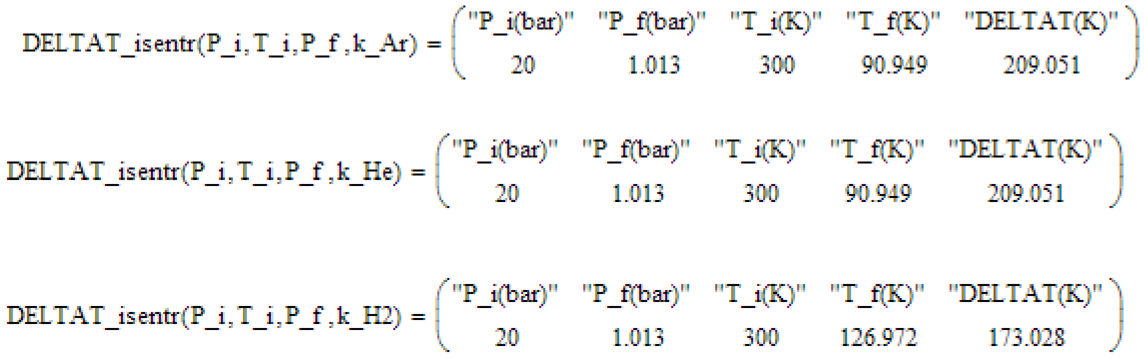
$$DELTAT\_isentr(P\_i,T\_i,P\_f,k\_N2) = \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DELTAT(K)"} \\ 20 & 1.013 & 300 & 127.938 & 172.062 \end{pmatrix}$$

$$DELTAT\_isentr(P\_i,T\_i,P\_f,k\_O2) = \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DELTAT(K)"} \\ 20 & 1.013 & 300 & 128.919 & 171.081 \end{pmatrix}$$

$$DELTAT\_isentr(P\_i,T\_i,P\_f,k\_CO2) = \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DELTAT(K)"} \\ 20 & 1.013 & 300 & 153.703 & 146.297 \end{pmatrix}$$

$$DELTAT\_isentr(P\_i,T\_i,P\_f,k\_CH4) = \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DELTAT(K)"} \\ 20 & 1.013 & 300 & 149.665 & 150.335 \end{pmatrix}$$

$$DELTAT\_isentr(P\_i,T\_i,P\_f,k\_Ne) = \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DELTAT(K)"} \\ 20 & 1.013 & 300 & 90.949 & 209.051 \end{pmatrix}$$

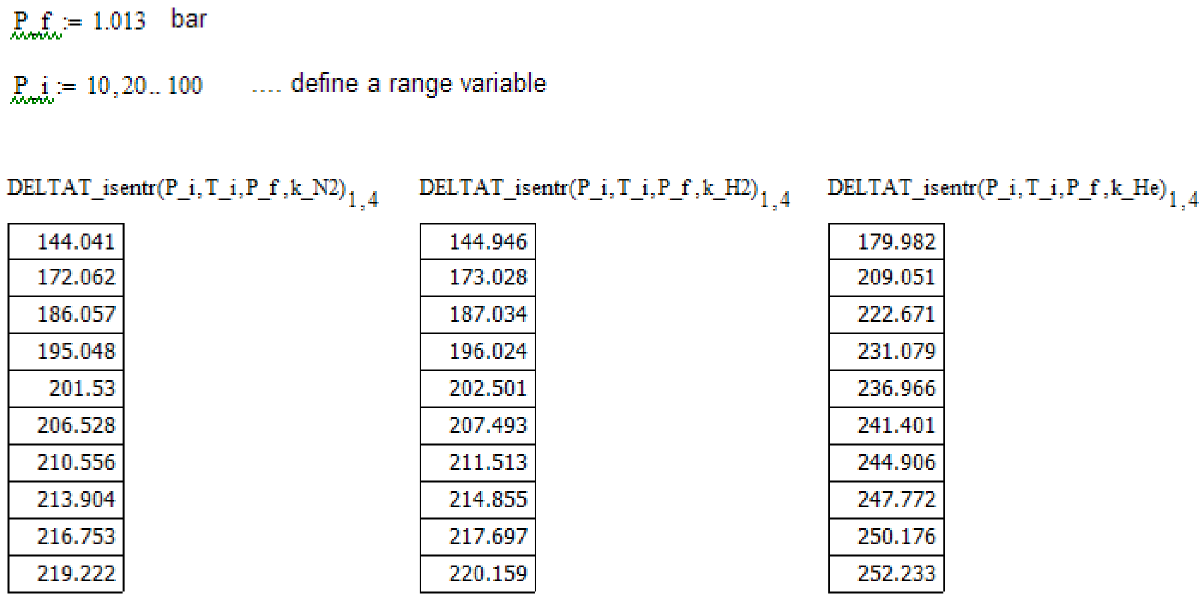


$$DELTAT\_isentr(P\_i,T\_i,P\_f,k\_Ar) = \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DELTAT(K)"} \\ 20 & 1.013 & 300 & 90.949 & 209.051 \end{pmatrix}$$

$$DELTAT\_isentr(P\_i,T\_i,P\_f,k\_He) = \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DELTAT(K)"} \\ 20 & 1.013 & 300 & 90.949 & 209.051 \end{pmatrix}$$

$$DELTAT\_isentr(P\_i,T\_i,P\_f,k\_H2) = \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DELTAT(K)"} \\ 20 & 1.013 & 300 & 126.972 & 173.028 \end{pmatrix}$$

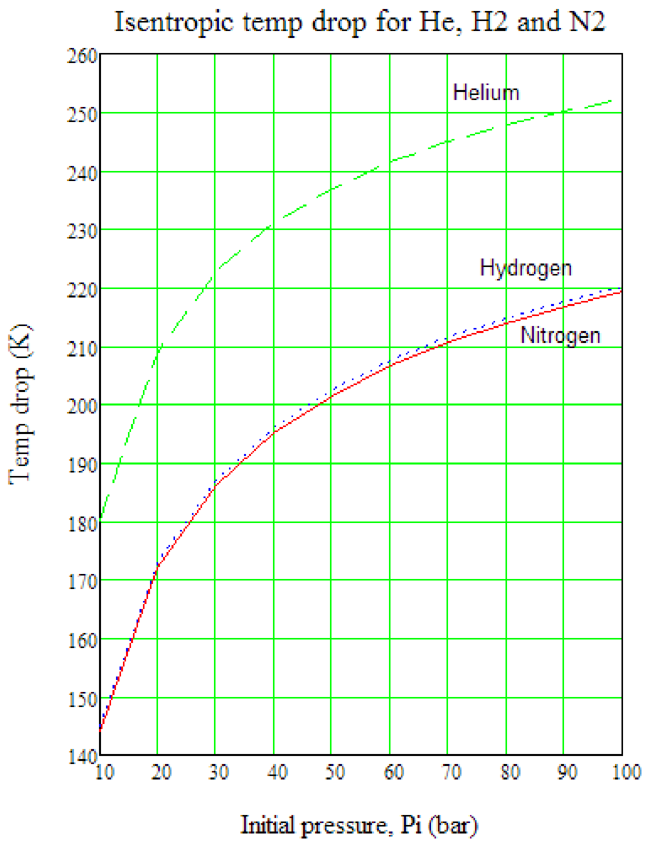
Plot DELTAT for N2, H2 and He as P_i varies from 10 bar to 100 bar final pressure being 1.013 bar:



P_f := 1.013 bar

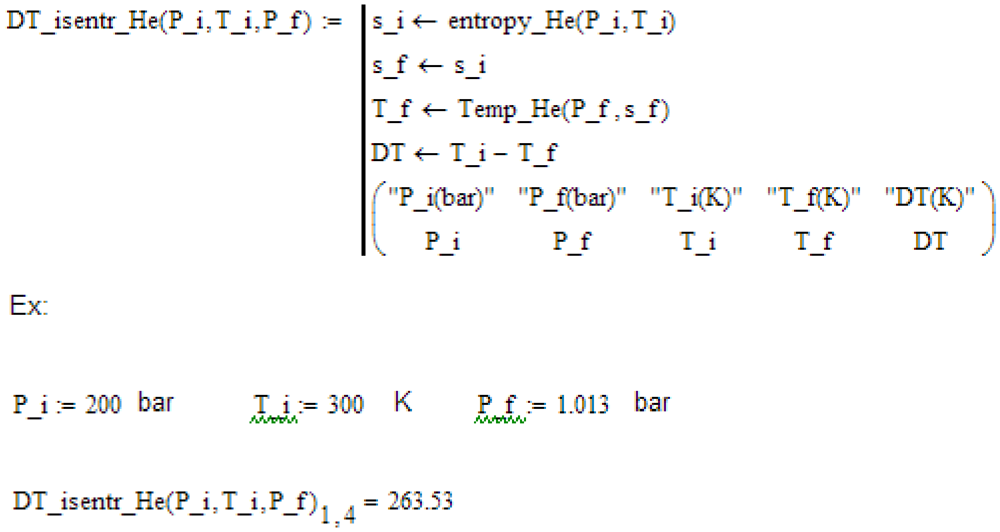
P_i := 10, 20 .. 100 .... define a range variable

$DELTAT\_isentr(P\_i, T\_i, P\_f, k\_N2)_{1,4}$	$DELTAT\_isentr(P\_i, T\_i, P\_f, k\_H2)_{1,4}$	$DELTAT\_isentr(P\_i, T\_i, P\_f, k\_He)_{1,4}$
144.041	144.946	179.982
172.062	173.028	209.051
186.057	187.034	222.671
195.048	196.024	231.079
201.53	202.501	236.966
206.528	207.493	241.401
210.556	211.513	244.906
213.904	214.855	247.772
216.753	217.697	250.176
219.222	220.159	252.233



Prob. 3.3.5 Write Mathcad Functions to determine temperature drop on isentropic expansion for helium using the NIST data.

Recollect: we have the Mathcad Function:

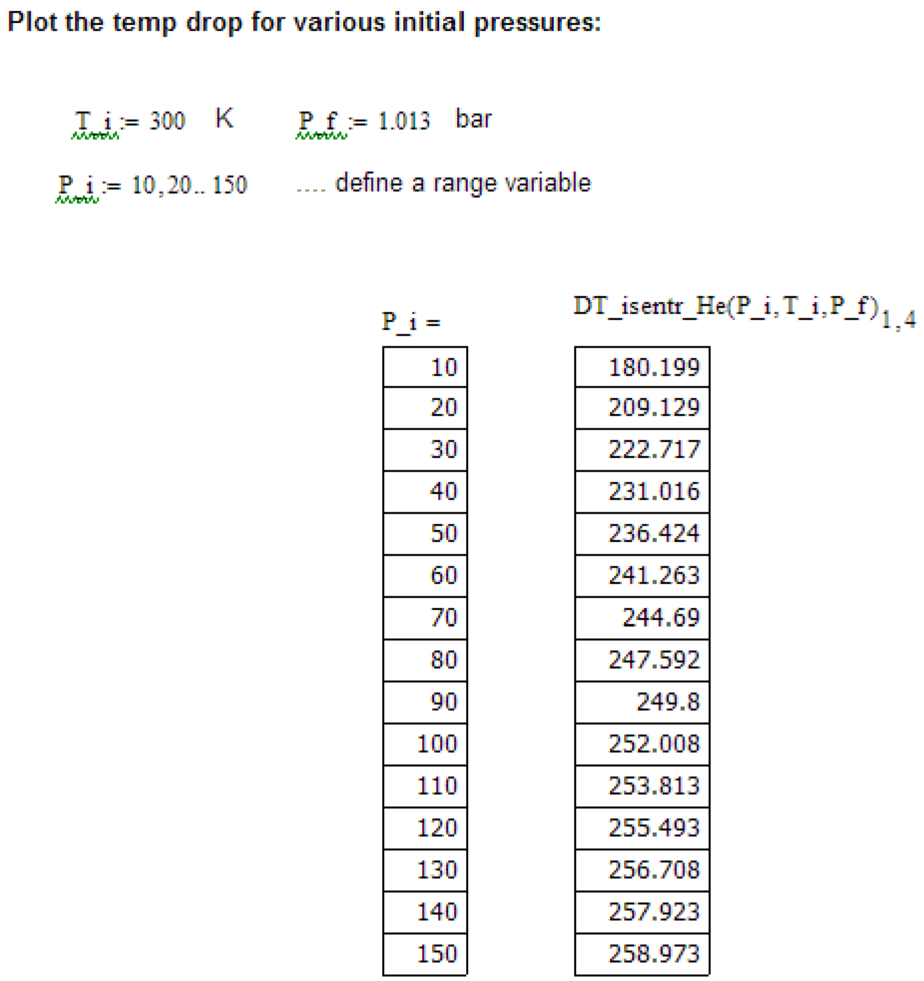


```
DT_isentr_He(P_i,T_i,P_f) := | s_i ← entropy_He(P_i,T_i)
                             | s_f ← s_i
                             | T_f ← Temp_He(P_f,s_f)
                             | DT ← T_i − T_f
                             | ( "P_i(bar)"  "P_f(bar)"  "T_i(K)"  "T_f(K)"  "DT(K)" )
                             | (   P_i         P_f         T_i       T_f       DT    )
```

Ex:

$P\_i := 200$ bar $\quad T\_i := 300$ K $\quad P\_f := 1.013$ bar

$DT\_isentr\_He(P\_i,T\_i,P\_f)_{1,4} = 263.53$

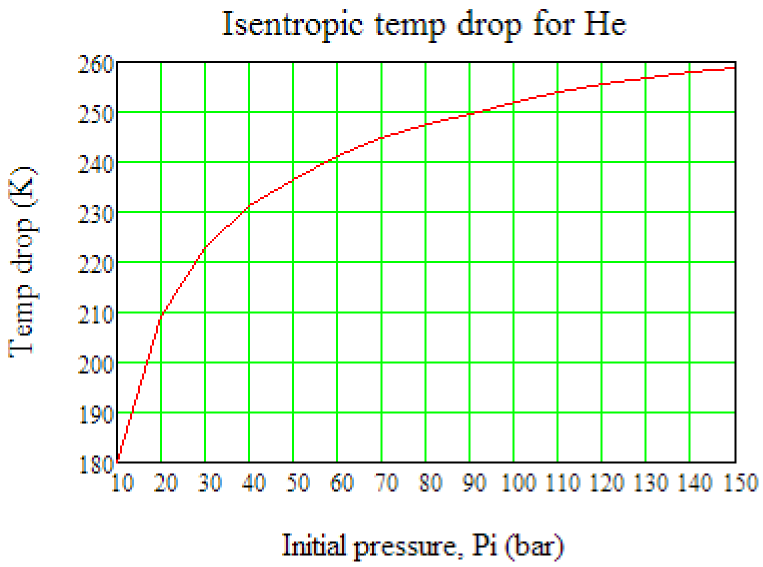


Plot the temp drop for various initial pressures:

$T\_i := 300$ K $\quad P\_f := 1.013$ bar

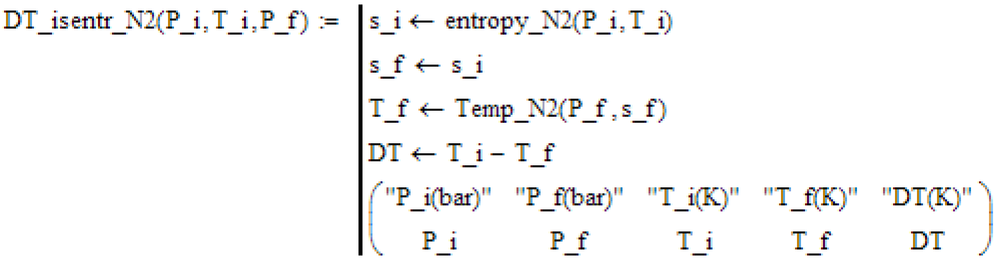
$P\_i := 10, 20 .. 150$ .... define a range variable

$P\_i =$	$DT\_isentr\_He(P\_i, T\_i, P\_f)_{1,4}$
10	180.199
20	209.129
30	222.717
40	231.016
50	236.424
60	241.263
70	244.69
80	247.592
90	249.8
100	252.008
110	253.813
120	255.493
130	256.708
140	257.923
150	258.973



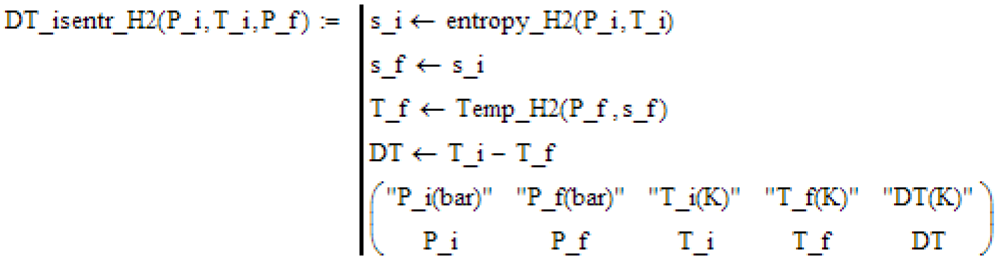
Prob. 3.3.6 Write Mathcad Functions to determine temperature drop on isentropic expansion for Nitrogen and Hydrogen using the NIST data.

Again, recollect that we have Mathcad Functions for N2 and H2:



```
DT_isentr_N2(P_i,T_i,P_f) := | s_i ← entropy_N2(P_i,T_i)
                             | s_f ← s_i
                             | T_f ← Temp_N2(P_f,s_f)
                             | DT ← T_i − T_f
                             | ⎛"P_i(bar)"  "P_f(bar)"  "T_i(K)"  "T_f(K)"  "DT(K)"⎞
                             | ⎝   P_i         P_f        T_i       T_f       DT   ⎠
```

And,



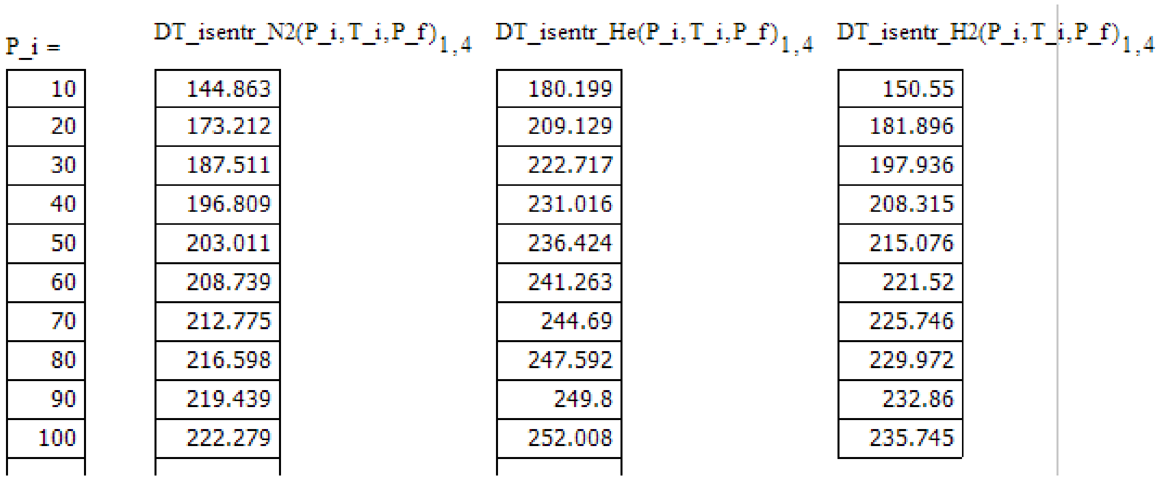
```
DT_isentr_H2(P_i,T_i,P_f) := | s_i ← entropy_H2(P_i,T_i)
                             | s_f ← s_i
                             | T_f ← Temp_H2(P_f,s_f)
                             | DT ← T_i − T_f
                             | ⎛"P_i(bar)"  "P_f(bar)"  "T_i(K)"  "T_f(K)"  "DT(K)"⎞
                             | ⎝   P_i         P_f        T_i       T_f       DT   ⎠
```

Now, plot the temp drops for isentropic expansion fron various starting pressures to a final pressure P_f = 1.013 bar:

T_i := 300 K   P_f := 1.013 bar

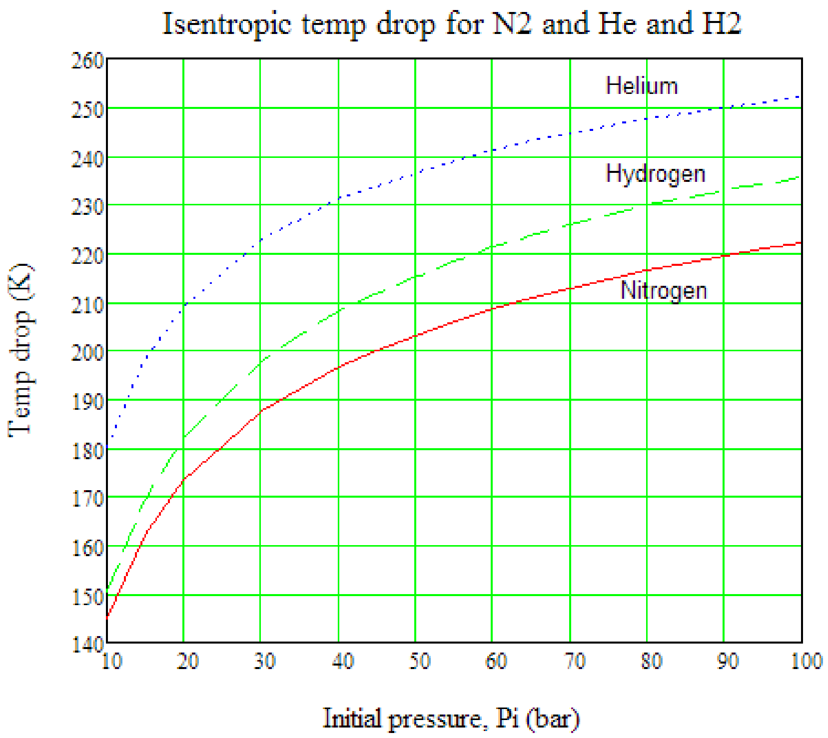
P_i := 10, 20.. 100   .... define a range variable

We get:

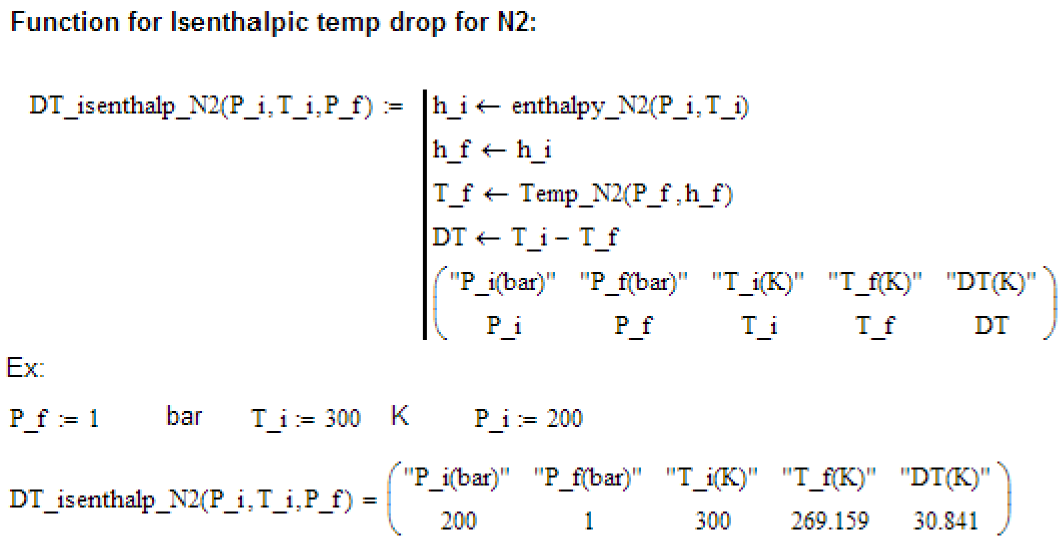


P_i =	DT_isentr_N2(P_i,T_i,P_f)$_{1,4}$	DT_isentr_He(P_i,T_i,P_f)$_{1,4}$	DT_isentr_H2(P_i,T_i,P_f)$_{1,4}$
10	144.863	180.199	150.55
20	173.212	209.129	181.896
30	187.511	222.717	197.936
40	196.809	231.016	208.315
50	203.011	236.424	215.076
60	208.739	241.263	221.52
70	212.775	244.69	225.746
80	216.598	247.592	229.972
90	219.439	249.8	232.86
100	222.279	252.008	235.745

And, now plot the results:



Prob. 3.3.7 Write Mathcad Function to determine temperature drop on isenthalpic expansion for Nitrogen using the NIST data.



Function for Isenthalpic temp drop for N2:

```
DT_isenthalp_N2(P_i,T_i,P_f) := | h_i ← enthalpy_N2(P_i,T_i)
                                | h_f ← h_i
                                | T_f ← Temp_N2(P_f,h_f)
                                | DT ← T_i − T_f
                                | ( "P_i(bar)"  "P_f(bar)"  "T_i(K)"  "T_f(K)"  "DT(K)" )
                                | (   P_i         P_f         T_i       T_f       DT    )
```

Ex:

$P\_f := 1$ bar $\quad T\_i := 300$ K $\quad P\_i := 200$

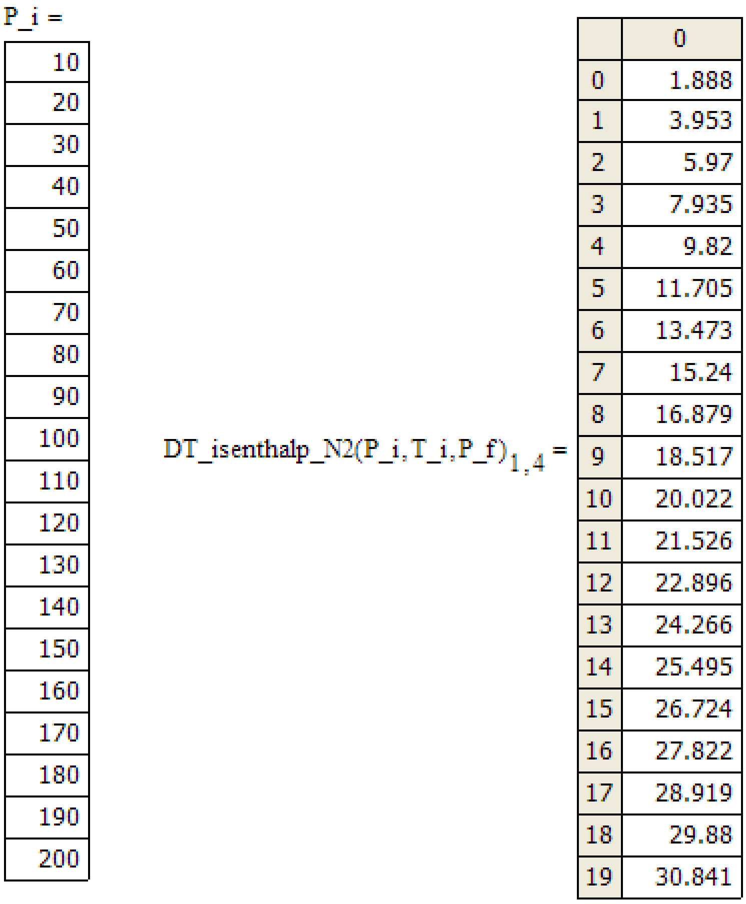
$$DT\_isenthalp\_N2(P\_i,T\_i,P\_f) = \begin{pmatrix} \text{"P\_i(bar)"} & \text{"P\_f(bar)"} & \text{"T\_i(K)"} & \text{"T\_f(K)"} & \text{"DT(K)"} \\ 200 & 1 & 300 & 269.159 & 30.841 \end{pmatrix}$$

Now, plot the temp drops for various starting pressures, final pressure being P_f = 1.013 bar:

P_i := 10, 20 .. 20(....define a range variable

P_f := 1 bar T_i := 30(K

We get:



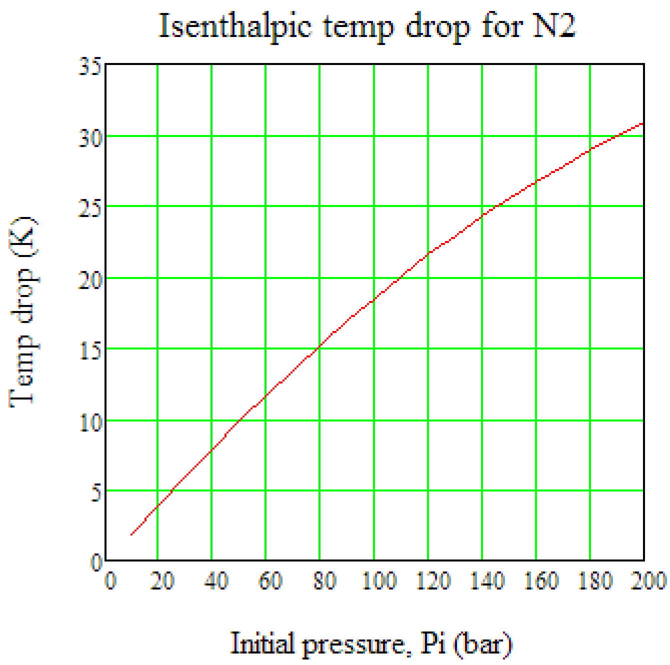
P_i =

10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200

$DT\_isenthalp\_N2(P\_i, T\_i, P\_f)_{1,4} =$

	0
0	1.888
1	3.953
2	5.97
3	7.935
4	9.82
5	11.705
6	13.473
7	15.24
8	16.879
9	18.517
10	20.022
11	21.526
12	22.896
13	24.266
14	25.495
15	26.724
16	27.822
17	28.919
18	29.88
19	30.841

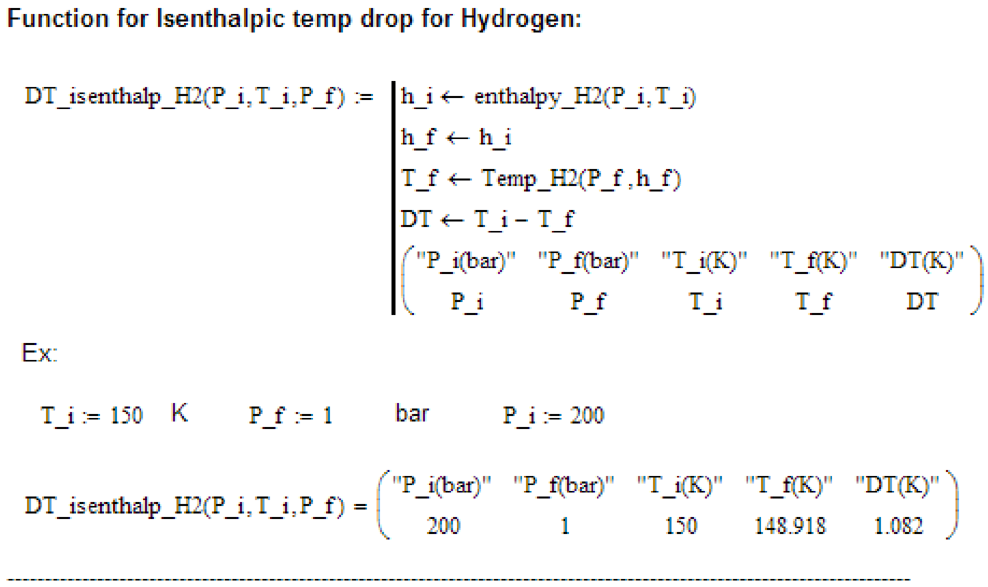
And, now plot the results:



Prob. 3.3.8 Write Mathcad Functions to determine temperature drops on isenthalpic expansion for Helium and Hydrogen using the NIST data. For helium starting temp is 20 K and for hydrogen starting temp is 80 K. Final pressure for each case is 1.013 bar.

Note: For H2 and He the Max. inversion temps are 202 K and 40 K respectively, and, the temp before isenthalpic expansion should be below these value for each gas.

Again, recollect that we have Mathcad Functions for He and H2:

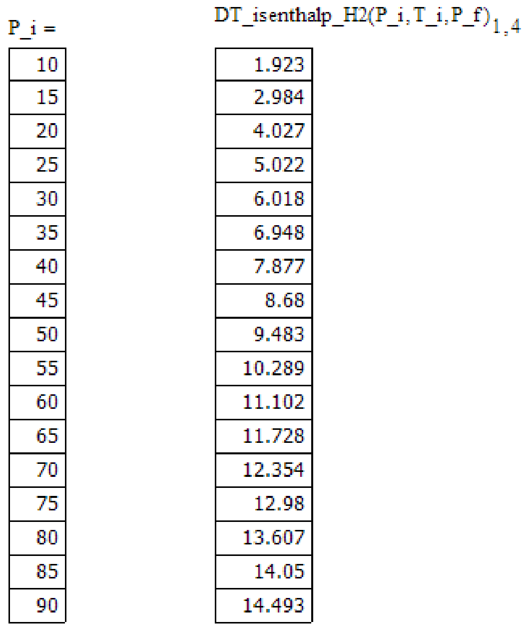


Function for Isenthalpic temp drop for Hydrogen:

```
DT_isenthalp_H2(P_i,T_i,P_f) := | h_i ← enthalpy_H2(P_i,T_i)
                                | h_f ← h_i
                                | T_f ← Temp_H2(P_f,h_f)
                                | DT ← T_i − T_f
                                | ("P_i(bar)"  "P_f(bar)"  "T_i(K)"  "T_f(K)"  "DT(K)")
                                | (   P_i         P_f         T_i       T_f       DT  )
Ex:

 T_i := 150    K        P_f := 1         bar         P_i := 200

DT_isenthalp_H2(P_i,T_i,P_f) = ("P_i(bar)"  "P_f(bar)"  "T_i(K)"  "T_f(K)"  "DT(K)")
                               (   200          1          150     148.918    1.082 )
----------------------------------------------------------------------------------------------------------------
```

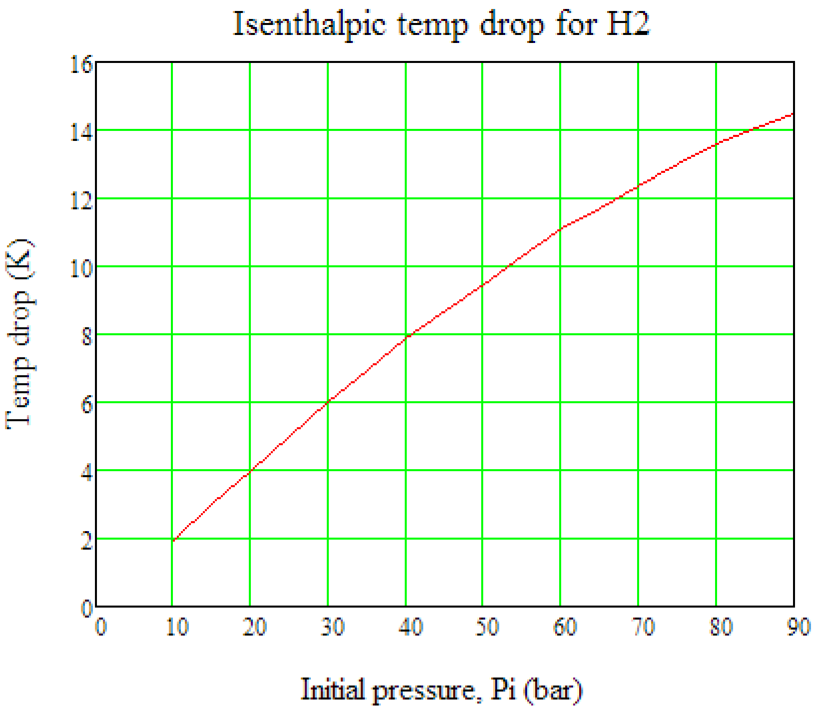
Now, plot the temp drop for various initial pressures:

We get:



P_i =	DT_isenthalp_H2(P_i,T_i,P_f)$_{1,4}$
10	1.923
15	2.984
20	4.027
25	5.022
30	6.018
35	6.948
40	7.877
45	8.68
50	9.483
55	10.289
60	11.102
65	11.728
70	12.354
75	12.98
80	13.607
85	14.05
90	14.493

Now, plot the results:



Mathcad Function for isenthalpic temp drop for helium:

Function for Isenthalpic temp drop for helium:

```
DT_isenthalp_He(P_i,T_i,P_f) := | h_i ← enthalpy_He(P_i,T_i)
                                | h_f ← h_i
                                | T_f ← Temp_He(P_f,h_f)
                                | DT ← T_i − T_f
                                | ( "P_i(bar)"  "P_f(bar)"  "T_i(K)"  "T_f(K)"  "DT(K)" )
                                | (    P_i         P_f        T_i       T_f       DT    )
```

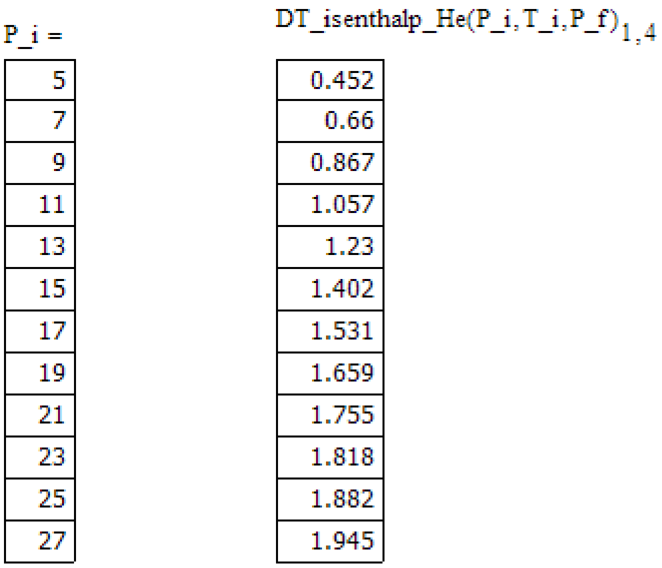
Ex:

```
T_i := 30     K        P_f := 1      bar        P_i := 20     bar

DT_isenthalp_He(P_i,T_i,P_f) = ( "P_i(bar)"  "P_f(bar)"  "T_i(K)"  "T_f(K)"  "DT(K)" )
                               (    20          1          30       29.499    0.501  )
```

Now, plot temp drop for various starting pressures:

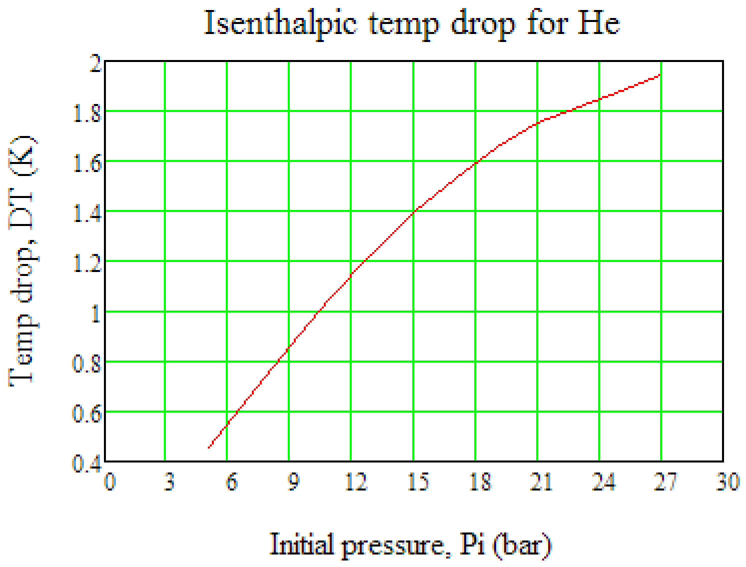
```
T_i := 20     K        P_f := 1.013   bar

P_i := 5,7..27         ...define a range variable
```

We get:



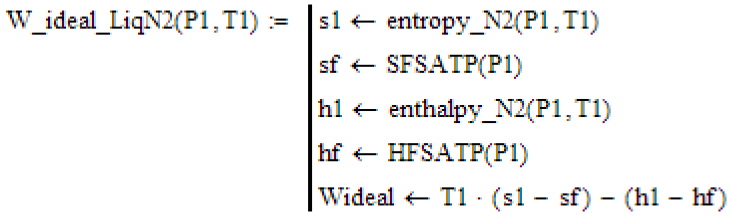
P_i =	DT_isenthalp_He(P_i, T_i, P_f)$_{1,4}$
5	0.452
7	0.66
9	0.867
11	1.057
13	1.23
15	1.402
17	1.531
19	1.659
21	1.755
23	1.818
25	1.882
27	1.945

And, now plot the results:

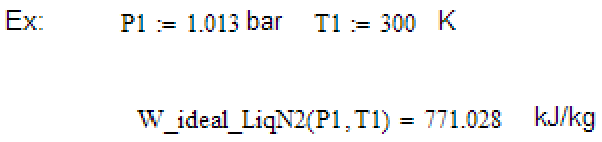


Prob. 3.3.9 Write Mathcad Functions to determine Ideal work of liquefaction for Nitrogen, Hydrogen and Helium, starting from initial pressure and temp of 1.013 bar (= 1 atm) and 300 K respectively.

Write a Mathcad Function to determine Ideal (or minimum) work of liquefaction for N2:



```
W_ideal_LiqN2(P1,T1) := | s1 ← entropy_N2(P1,T1)
                        | sf ← SFSATP(P1)
                        | h1 ← enthalpy_N2(P1,T1)
                        | hf ← HFSATP(P1)
                        | Wideal ← T1 · (s1 − sf) − (h1 − hf)
```

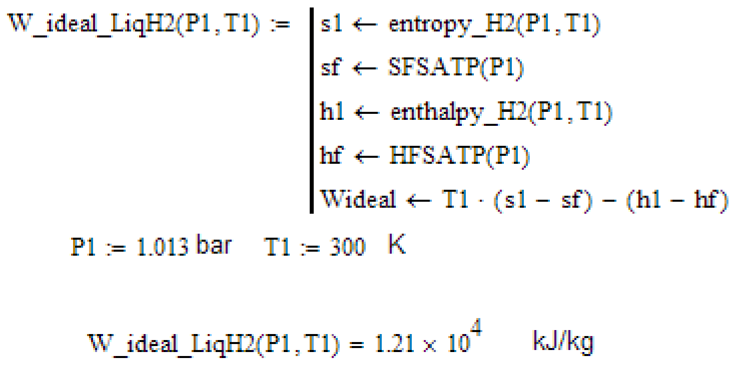


Ex: $P1 := 1.013$ bar $T1 := 300$ K

$W\_ideal\_LiqN2(P1,T1) = 771.028$ kJ/kg

……

Write a Mathcad Function to determine Ideal (or minimum) work of liquefaction for H2:



```
W_ideal_LiqH2(P1,T1) := | s1 ← entropy_H2(P1,T1)
                        | sf ← SFSATP(P1)
                        | h1 ← enthalpy_H2(P1,T1)
                        | hf ← HFSATP(P1)
                        | Wideal ← T1 · (s1 − sf) − (h1 − hf)
```

Ex: $P1 := 1.013$ bar $T1 := 300$ K

$W\_ideal\_LiqH2(P1,T1) = 1.21 \times 10^{4}$ kJ/kg

……

Write a Mathcad Function to determine Ideal (or minimum) work of liquefaction for He:

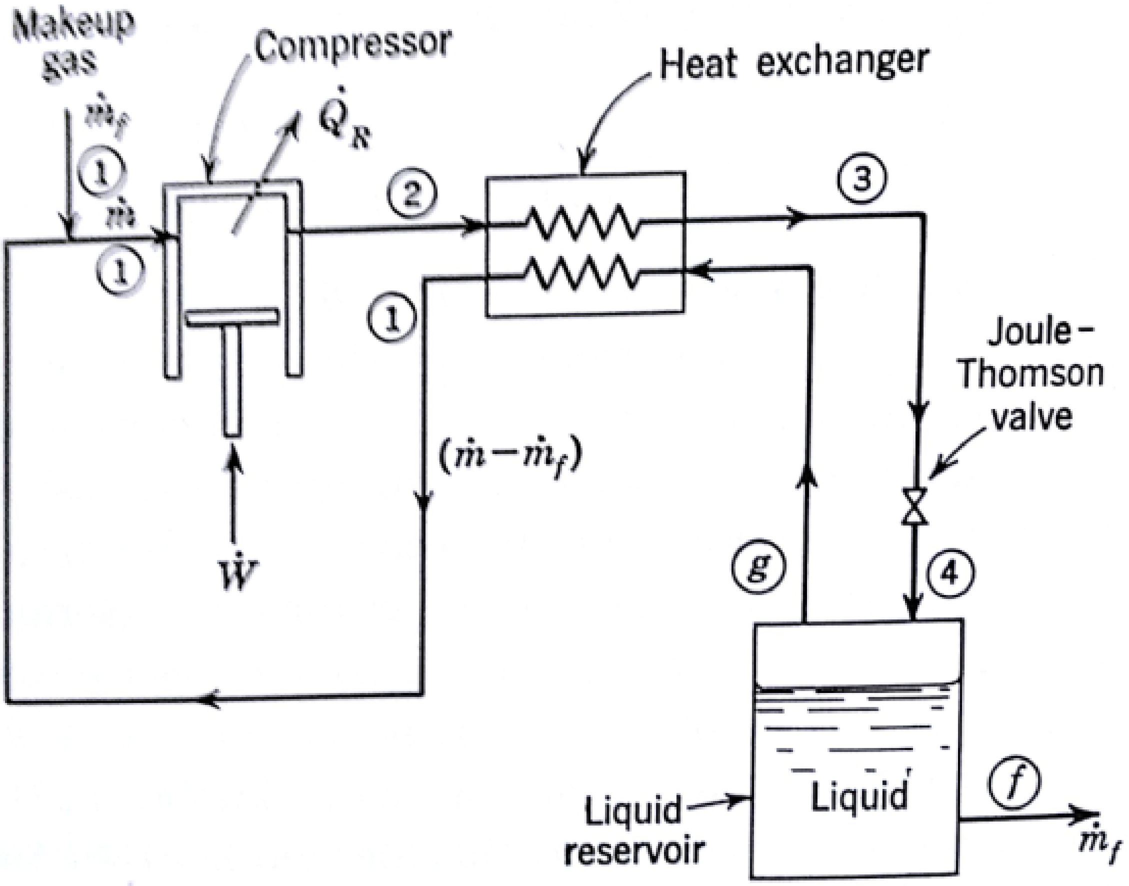
```
W_ideal_LiqHe(P1,T1) :=  | s1 ← entropy_He(P1,T1)
                         | sf ← SFSATP(P1)
                         | h1 ← enthalpy_He(P1,T1)
                         | hf ← HFSATP(P1)
                         | Wideal ← T1 · (s1 − sf) − (h1 − hf)
```

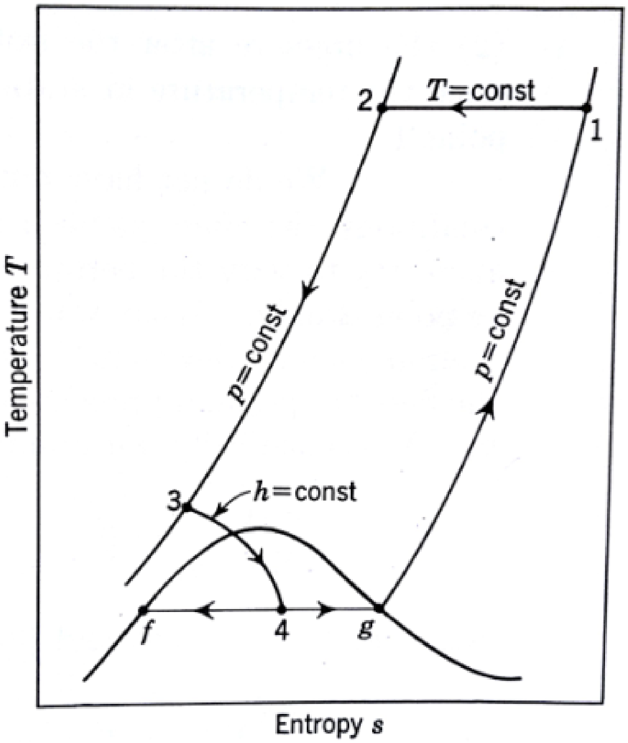
Ex: P1 := 1.013 bar T1 := 300 K

$$W\_ideal\_LiqHe(P1,T1) = 6.841 \times 10^{3} \quad kJ/kg$$

Prob. 3.3.10 Write Mathcad Functions to determine liquefaction fraction, y, work required per kg of gas compressed, work required per kg liquefied, Figure of Merit (FOM = Wideal / Work per kg liqid) for simple Linde-Hampson cycle for Nitrogen. Take the initial pressure and temp of 1.013 bar (= 1 atm) and 300 K respectively and the final pressure is 200 bar. Plot y, Wperkgliq and FOM as final pressure varies from 50 bar to 200 bar.

Compare these parameters for N2, Ar, CH4, F2 and O2 as P2 varies from 50 bar to 200 bar, with P1 = 1.013 bar, T1 = 300 K.





First, Mathcad program:

```
Ideal_LHcycle_N2(P1,T1,P2) :=
```

```
| s1 ← entropy_N2(P1,T1)
| s2 ← entropy_N2(P2,T1)
| h1 ← enthalpy_N2(P1,T1)
| h2 ← enthalpy_N2(P2,T1)
| hf ← HFSATP(P1)
| Wideal ← W_ideal_LiqN2(P1,T1)
|      h1 − h2
| y ← ―――――――
|      h1 − hf
| Wperkggas ← T1 · (s1 − s2) − (h1 − h2)
|                Wperkggas
| Wperkgliq ← ―――――――――
|                    y
|           Wideal
| FOM ← ―――――――――
|         Wperkgliq
| ( "P1(bar)"  "T1(K)"  "P2(bar)"  "Liq. frn. (y)"  "Wperkggas (kJ/kg)"  "Wperkgliq(kJ/kg)"  "Wideal(kJ/kgliq)"  "FOM(=Wideal/Wperkgliq)" )
| (    P1        T1        P2            y               Wperkggas             Wperkgliq              Wideal                   FOM           )
```

Output of the program gives: P1, T1, P2, y, Wperkggas, Wperkhliq, Wideal and FOM.

```
Ex:      P1 := 1.013 bar    T1 := 300  K
         P2 := 200   bar

Ideal_LHcycle_N2(P1,T1,P2) =
```

$$\left(\begin{array}{cccccccc}\text{"P1(bar)"} & \text{"T1(K)"} & \text{"P2(bar)"} & \text{"Liq. frn. (y)"} & \text{"Wperkggas (kJ/kg)"} & \text{"Wperkgliq(kJ/kg)"} & \text{"Wideal(kJ/kgliq)"} & \text{"FOM(=Wideal/Wperkgliq)"} \\ 1.013 & 300 & 200 & 0.074 & 472.259 & 6.377 \times 10^{3} & 771.028 & 0.121\end{array}\right)$$

i.e.

$y := \text{Ideal\_LHcycle\_N2}(P1,T1,P2)_{1,3}$ i.e. $y = 0.074$

$\text{Wperkggas} := \text{Ideal\_LHcycle\_N2}(P1,T1,P2)_{1,4}$ i.e. $\text{Wperkggas} = 472.259$ kJ/kg

$\text{Wperkgliq} := \text{Ideal\_LHcycle\_N2}(P1,T1,P2)_{1,5}$ i.e. $\text{Wperkgliq} = 6.377 \times 10^{3}$ kJ/kg

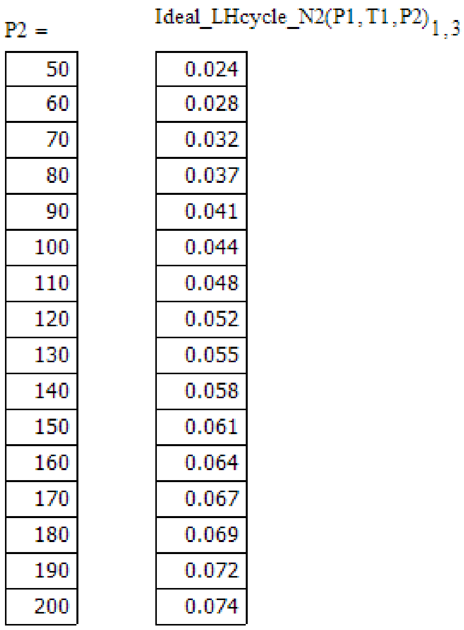
$\text{Wideal} := \text{Ideal\_LHcycle\_N2}(P1,T1,P2)_{1,6}$ i.e. $\text{Wideal} = 771.028$ kJ/kg

$\text{FOM} := \text{Ideal\_LHcycle\_N2}(P1,T1,P2)_{1,7}$ i.e. $\text{FOM} = 0.121$

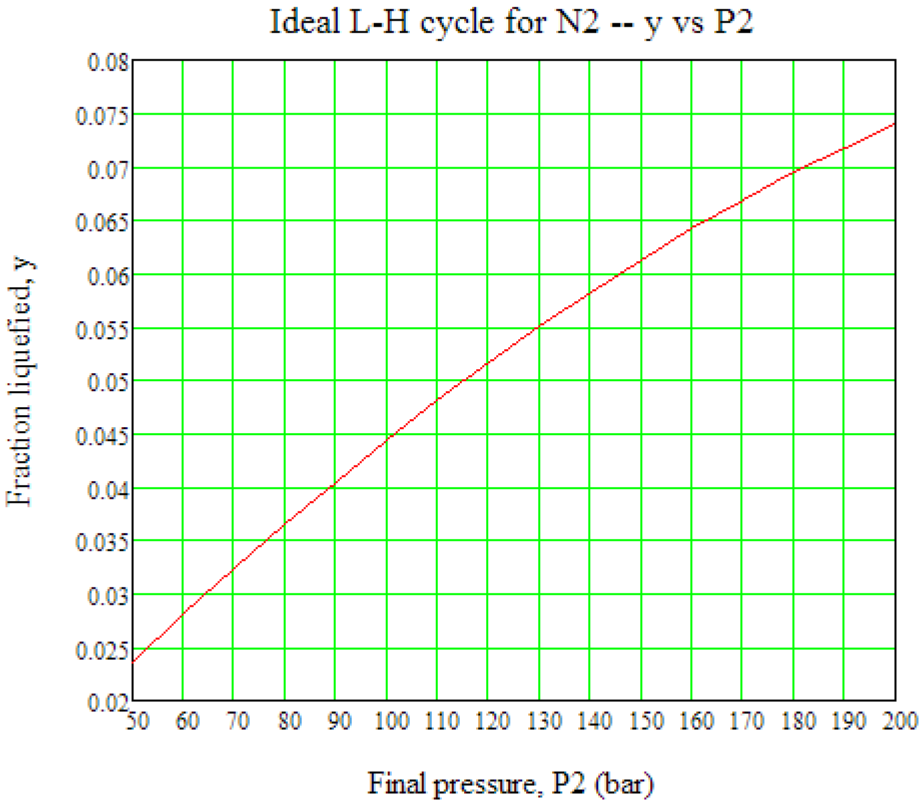
To plot y as the pressure P2 varies from 50 to 200 bar:

$P2 := 50, 60 .. 200$ ...define a range variable

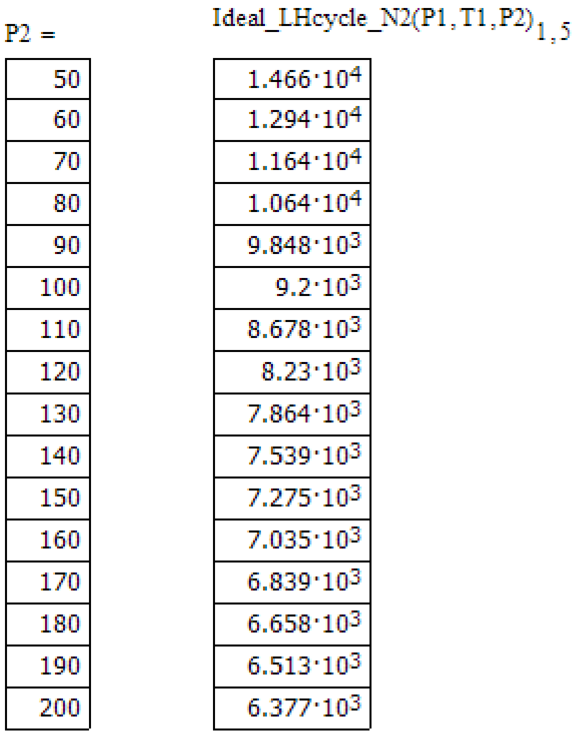
$P1 := 1.013$ bar $T1 := 300$ K



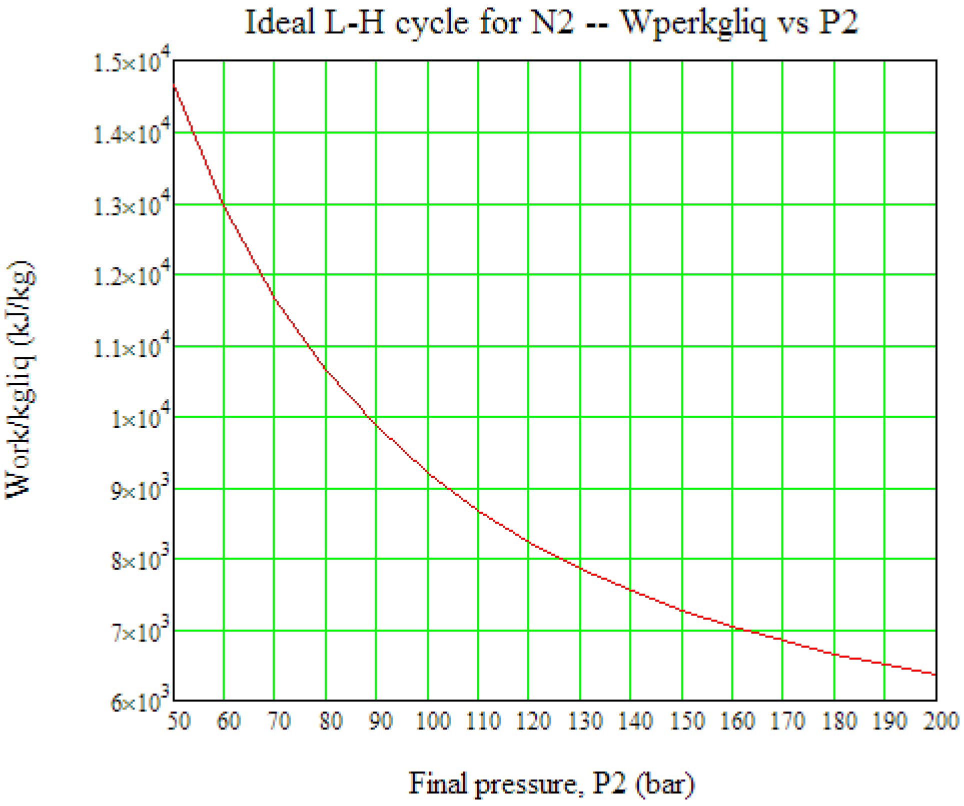
P2 =	$\mathrm{Ideal\_LHcycle\_N2}(P1, T1, P2)_{1,3}$
50	0.024
60	0.028
70	0.032
80	0.037
90	0.041
100	0.044
110	0.048
120	0.052
130	0.055
140	0.058
150	0.061
160	0.064
170	0.067
180	0.069
190	0.072
200	0.074



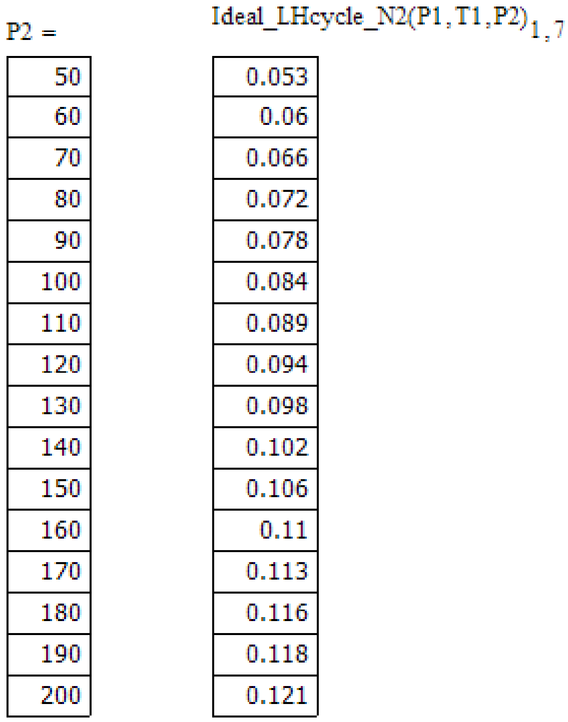
To plot Wperkgliq vs P2:



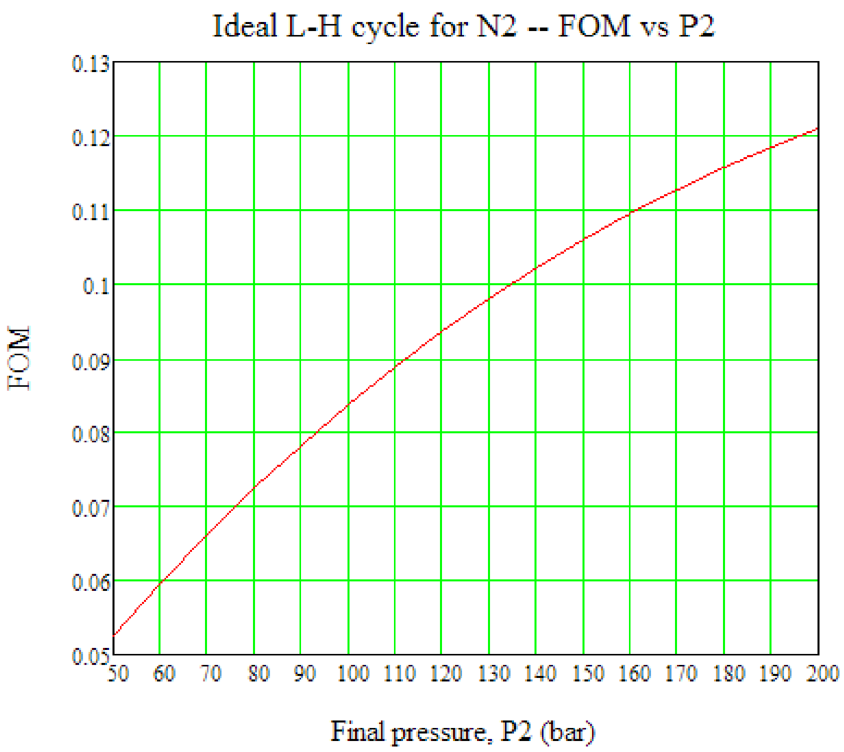
P2 =	Ideal_LHcycle_N2(P1,T1,P2)$_{1,5}$
50	$1.466 \cdot 10^4$
60	$1.294 \cdot 10^4$
70	$1.164 \cdot 10^4$
80	$1.064 \cdot 10^4$
90	$9.848 \cdot 10^3$
100	$9.2 \cdot 10^3$
110	$8.678 \cdot 10^3$
120	$8.23 \cdot 10^3$
130	$7.864 \cdot 10^3$
140	$7.539 \cdot 10^3$
150	$7.275 \cdot 10^3$
160	$7.035 \cdot 10^3$
170	$6.839 \cdot 10^3$
180	$6.658 \cdot 10^3$
190	$6.513 \cdot 10^3$
200	$6.377 \cdot 10^3$



To plot FOM vs P2:



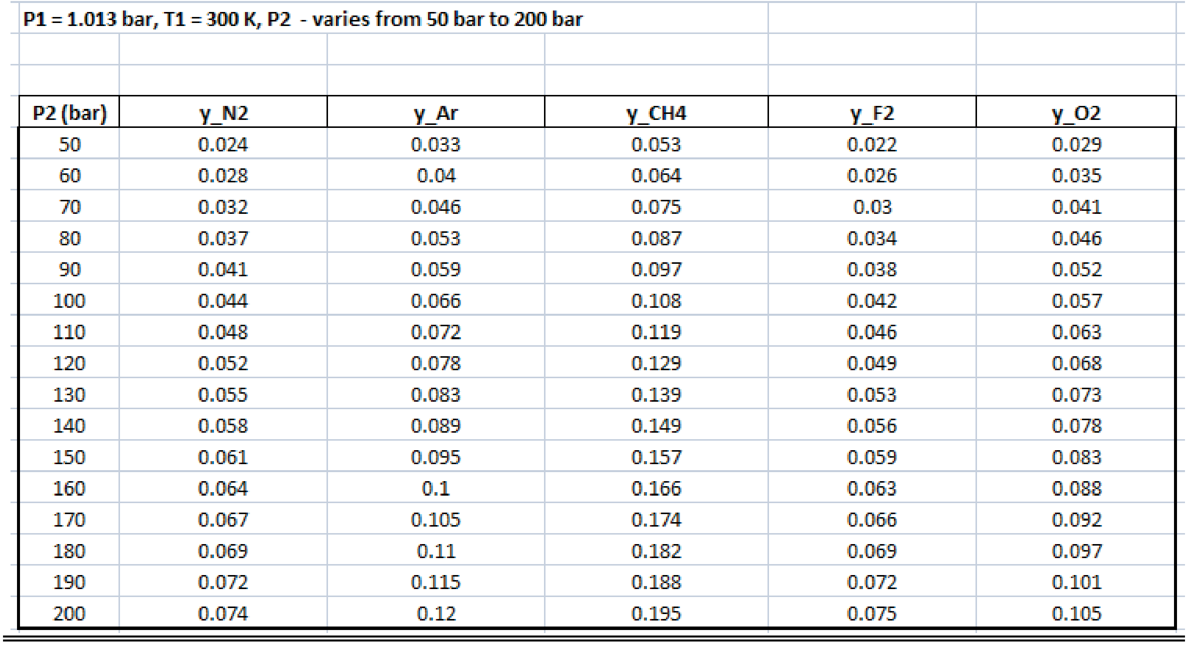
P2 =	$\text{Ideal\_LHcycle\_N2}(P1,T1,P2)_{1,7}$
50	0.053
60	0.06
70	0.066
80	0.072
90	0.078
100	0.084
110	0.089
120	0.094
130	0.098
140	0.102
150	0.106
160	0.11
170	0.113
180	0.116
190	0.118
200	0.121



Now, compare the results for y, Wperkgiq and FOM for N2, Ar, CH4, F2 and O2 as P2 varies from 50 bar to 200 bar, with P1 = 1.013 bar, T1 = 300 K:

Similar Mathcad programs are written for these fluids, and the results are summarized below:

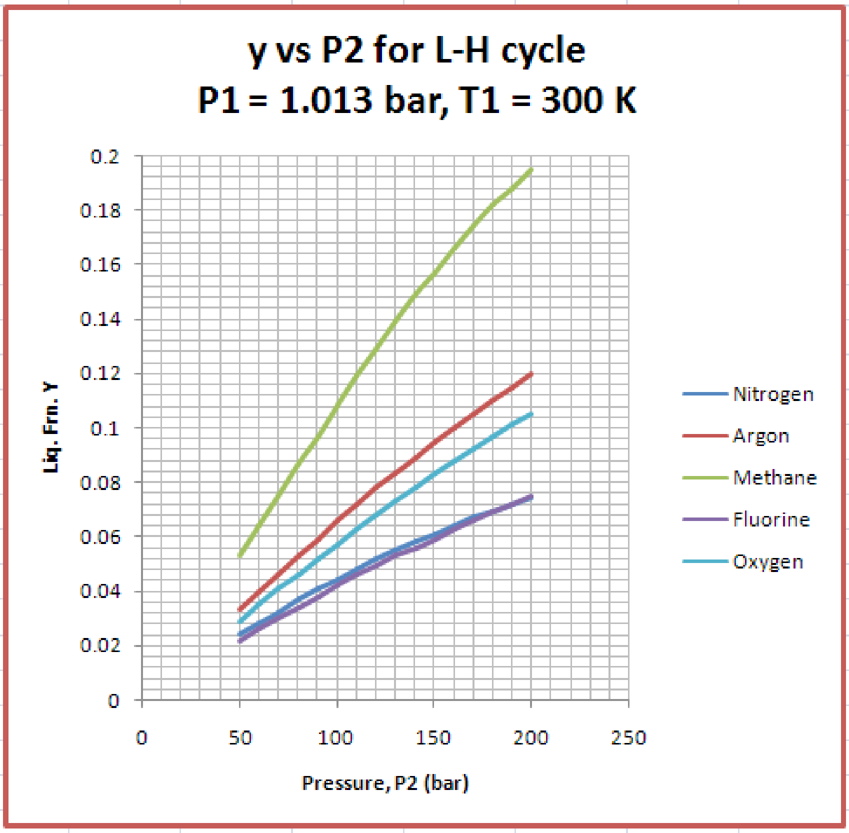
Variation of Liq. Fraction, y:



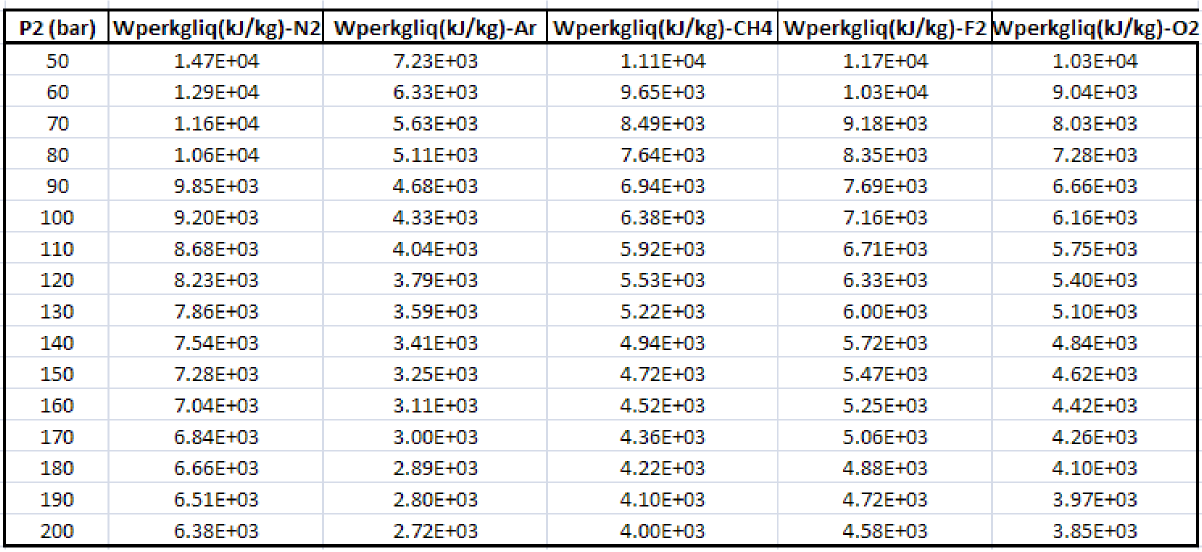
P1 = 1.013 bar, T1 = 300 K, P2 - varies from 50 bar to 200 bar

P2 (bar)	y_N2	y_Ar	y_CH4	y_F2	y_O2
50	0.024	0.033	0.053	0.022	0.029
60	0.028	0.04	0.064	0.026	0.035
70	0.032	0.046	0.075	0.03	0.041
80	0.037	0.053	0.087	0.034	0.046
90	0.041	0.059	0.097	0.038	0.052
100	0.044	0.066	0.108	0.042	0.057
110	0.048	0.072	0.119	0.046	0.063
120	0.052	0.078	0.129	0.049	0.068
130	0.055	0.083	0.139	0.053	0.073
140	0.058	0.089	0.149	0.056	0.078
150	0.061	0.095	0.157	0.059	0.083
160	0.064	0.1	0.166	0.063	0.088
170	0.067	0.105	0.174	0.066	0.092
180	0.069	0.11	0.182	0.069	0.097
190	0.072	0.115	0.188	0.072	0.101
200	0.074	0.12	0.195	0.075	0.105

And, the plot:

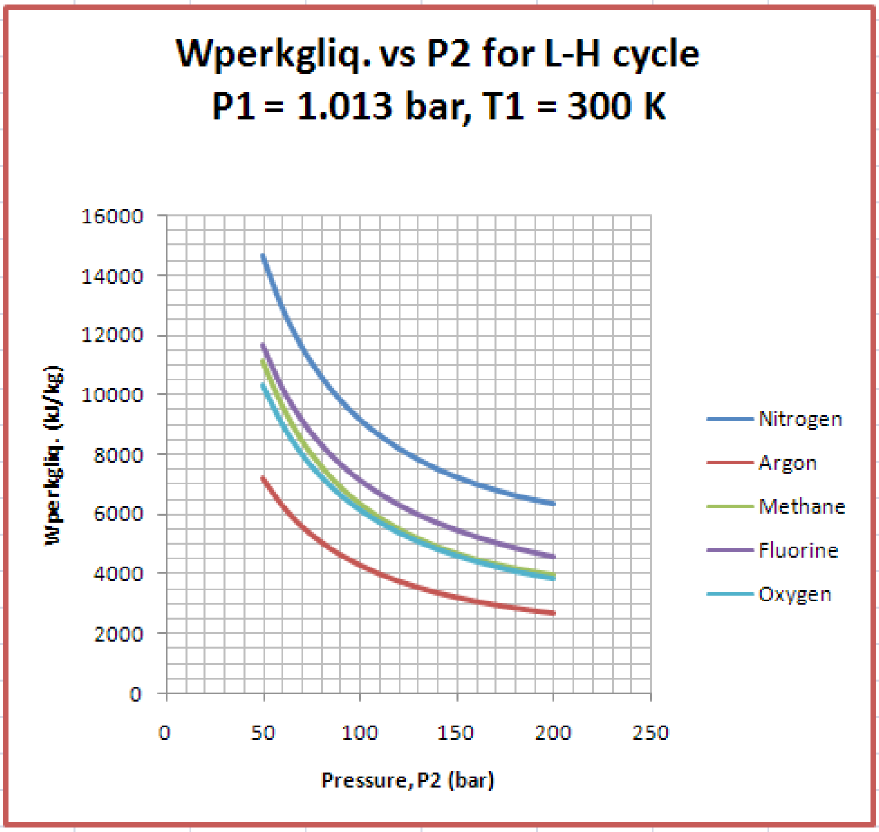


Variation of Wperkgliq.:



P2 (bar)	Wperkgliq(kJ/kg)-N2	Wperkgliq(kJ/kg)-Ar	Wperkgliq(kJ/kg)-CH4	Wperkgliq(kJ/kg)-F2	Wperkgliq(kJ/kg)-O2
50	1.47E+04	7.23E+03	1.11E+04	1.17E+04	1.03E+04
60	1.29E+04	6.33E+03	9.65E+03	1.03E+04	9.04E+03
70	1.16E+04	5.63E+03	8.49E+03	9.18E+03	8.03E+03
80	1.06E+04	5.11E+03	7.64E+03	8.35E+03	7.28E+03
90	9.85E+03	4.68E+03	6.94E+03	7.69E+03	6.66E+03
100	9.20E+03	4.33E+03	6.38E+03	7.16E+03	6.16E+03
110	8.68E+03	4.04E+03	5.92E+03	6.71E+03	5.75E+03
120	8.23E+03	3.79E+03	5.53E+03	6.33E+03	5.40E+03
130	7.86E+03	3.59E+03	5.22E+03	6.00E+03	5.10E+03
140	7.54E+03	3.41E+03	4.94E+03	5.72E+03	4.84E+03
150	7.28E+03	3.25E+03	4.72E+03	5.47E+03	4.62E+03
160	7.04E+03	3.11E+03	4.52E+03	5.25E+03	4.42E+03
170	6.84E+03	3.00E+03	4.36E+03	5.06E+03	4.26E+03
180	6.66E+03	2.89E+03	4.22E+03	4.88E+03	4.10E+03
190	6.51E+03	2.80E+03	4.10E+03	4.72E+03	3.97E+03
200	6.38E+03	2.72E+03	4.00E+03	4.58E+03	3.85E+03

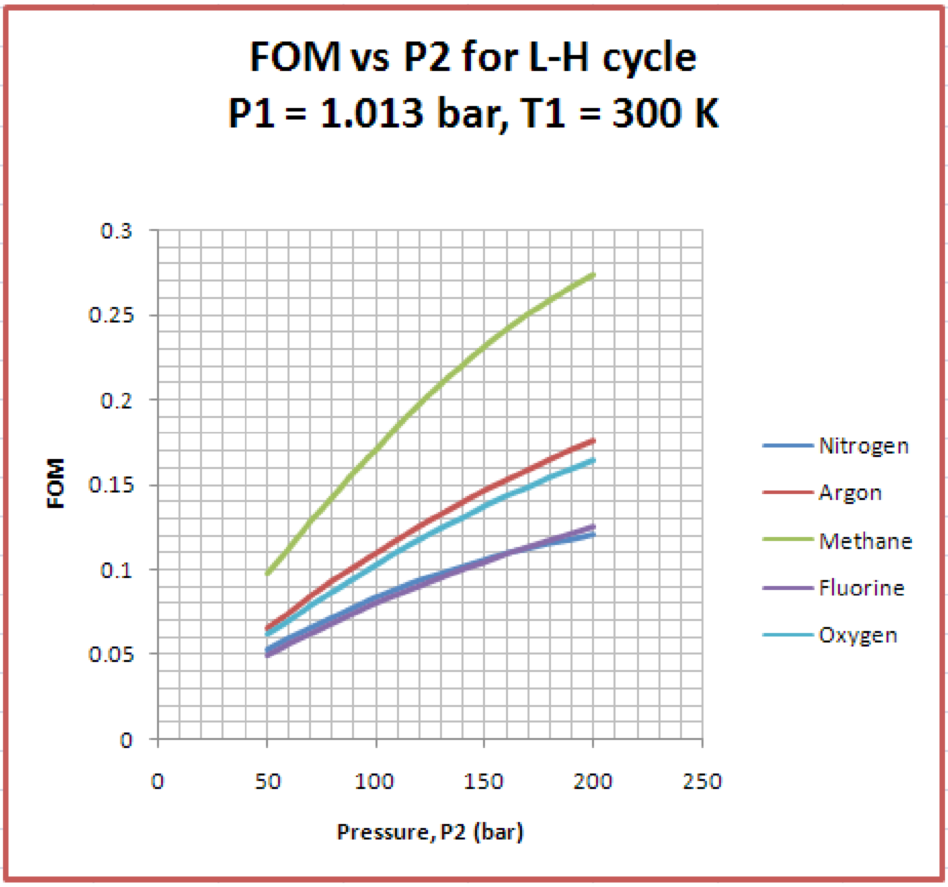
And, the plot:



Variation of FOM:

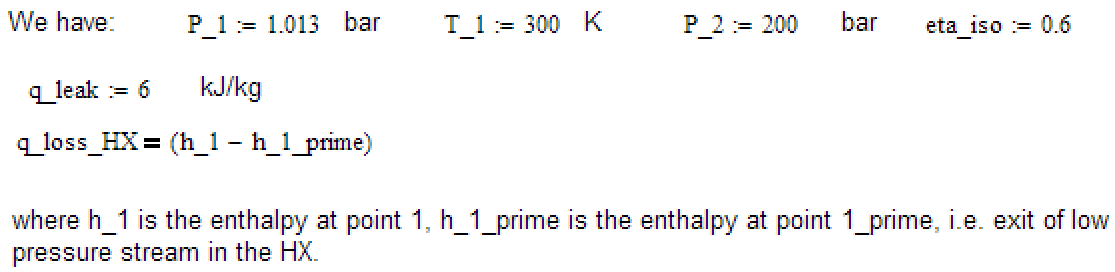
P2 (bar)	FOM_II-N2	FOM_II-Ar	FOM_II-CH4	FOM_II-F2	FOM_II-O2
50	0.053	0.066	0.098	0.049	0.062
60	0.06	0.075	0.113	0.056	0.07
70	0.066	0.085	0.129	0.062	0.079
80	0.072	0.094	0.143	0.068	0.087
90	0.078	0.102	0.158	0.074	0.095
100	0.084	0.11	0.171	0.08	0.103
110	0.089	0.118	0.185	0.085	0.111
120	0.094	0.126	0.198	0.09	0.118
130	0.098	0.133	0.21	0.095	0.125
140	0.102	0.14	0.221	0.1	0.131
150	0.106	0.147	0.232	0.104	0.138
160	0.11	0.153	0.242	0.109	0.144
170	0.113	0.159	0.251	0.113	0.149
180	0.116	0.165	0.259	0.117	0.155
190	0.118	0.171	0.267	0.121	0.16
200	0.121	0.176	0.274	0.125	0.165

And, the plot:



Prob. 3.3.11 In the above problem, let the HX be not 100% efficient, i.e. the warm end temp difference ΔT = 5 deg (i.e. low pressure gas exits the HX at 295 K), heat losses to the system, qleak = 6 kJ/kg, and the isothermal efficiency of compressor is eta_iso = 0.6. Then, find the liquefaction fraction, y, work required per kg of gas compressed, work required per kg liquefied, Figure of Merit (FOM = Wideal / Work per kg liqid) for simple Linde-Hampson cycle for Nitrogen. Take the initial pressure and temp as 1.013 bar (= 1 atm) and 300 K respectively and the final pressure is 200 bar. Plot y, Wperkgliq and FOM as final pressure varies from 50 bar to 200 bar.

Solution:



We have: $P\_1 := 1.013$ bar $T\_1 := 300$ K $P\_2 := 200$ bar $eta\_iso := 0.6$

$q\_leak := 6$ kJ/kg

$q\_loss\_HX = (h\_1 - h\_1\_prime)$

where h_1 is the enthalpy at point 1, h_1_prime is the enthalpy at point 1_prime, i.e. exit of low pressure stream in the HX.

$$h\_1 := \mathrm{enthalpy\_N2}(P\_1, T\_1)$$

$$h\_1\_prime := \mathrm{enthalpy\_N2}(P\_1, T\_1 - 5)$$

$$h\_2 := \mathrm{enthalpy\_N2}(P\_2, T\_1)$$

$$h\_f := \mathrm{HFSATP}(P\_1)$$

$$s\_1 := \mathrm{entropy\_N2}(P\_1, T\_1)$$

$$s\_2 := \mathrm{entropy\_N2}(P\_2, T\_1)$$

Therefore:

$q\_loss\_HX := h\_1 - h\_1\_prime$ i.e. $q\_loss\_HX = 5.205$ kJ/kg

Now, applying the I Law to all components of the system, excepting the compressor, we get:

$$y\_realLH := \frac{h\_1 - h\_2 - q\_leak - q\_loss\_HX}{h\_1 - h\_f - q\_loss\_HX}$$

i.e. $y\_realLH = 0.049$ **....fraction liquefied ... Ans.**

Work required per kg of gas compressed:

$$\mathrm{W\_per\_kg\_gas} := \frac{\mathrm{T\_1} \cdot (\mathrm{s\_1} - \mathrm{s\_2}) - (\mathrm{h\_1} - \mathrm{h\_2})}{\mathrm{eta\_iso}}$$

i.e. $\mathrm{W\_per\_kg\_gas} = 787.099$ **kJ/kggas .... Ans.**

Work required per kg of liquid:

$$\mathrm{W\_per\_kg\_liq} := \frac{\mathrm{W\_per\_kg\_gas}}{\mathrm{y\_realLH}}$$

i.e. $\mathrm{W\_per\_kg\_liq} = 1.613 \times 10^{4}$ **kJ/kgliquid .... Ans.**

Also: $\mathrm{W\_per\_lit\_liq} := \mathrm{W\_per\_kg\_liq} \cdot 0.808$ where density of LN2 = 0.808 kg/lit

i.e. $\mathrm{W\_per\_lit\_liq} = 1.303 \times 10^{4}$ **kJ/litre of liquid .... Ans.**

$\mathrm{W\_ideal} := \mathrm{W\_ideal\_LiqN2}(\mathrm{P\_1}, \mathrm{T\_1})$ ...Ideal or min. work of liquefaction for N2

i.e. $\mathrm{W\_ideal} = 771.028$ kJ/kgliq

Therefore: $$\mathrm{FOM\_realLH} := \frac{\mathrm{W\_ideal}}{\mathrm{W\_per\_kg\_liq}}$$

i.e. $\mathrm{FOM\_realLH} = 0.048$ **...FOM of real L-H cycle ... Ans.**

Plot y_realLH, W_per_kg_liq and FOM_realLH against P_2, assuming that all other parameters remain the same:

To plot the results, first write the required parameters as functions of P_2:

$$\mathrm{s\_2}(\mathrm{P\_2}) := \mathrm{entropy\_N2}(\mathrm{P\_2}, \mathrm{T\_1})$$

$$\mathrm{h\_2}(\mathrm{P\_2}) := \mathrm{enthalpy\_N2}(\mathrm{P\_2}, \mathrm{T\_1})$$

$$\mathrm{y\_realLH}(\mathrm{P\_2}) := \frac{\mathrm{h\_1} - \mathrm{h\_2}(\mathrm{P\_2}) - \mathrm{q\_leak} - \mathrm{q\_loss\_HX}}{\mathrm{h\_1} - \mathrm{h\_f} - \mathrm{q\_loss\_HX}}$$

$$\mathrm{W\_per\_kg\_gas}(\mathrm{P\_2}) := \frac{\mathrm{T\_1} \cdot (\mathrm{s\_1} - \mathrm{s\_2}(\mathrm{P\_2})) - (\mathrm{h\_1} - \mathrm{h\_2}(\mathrm{P\_2}))}{\mathrm{eta\_iso}}$$

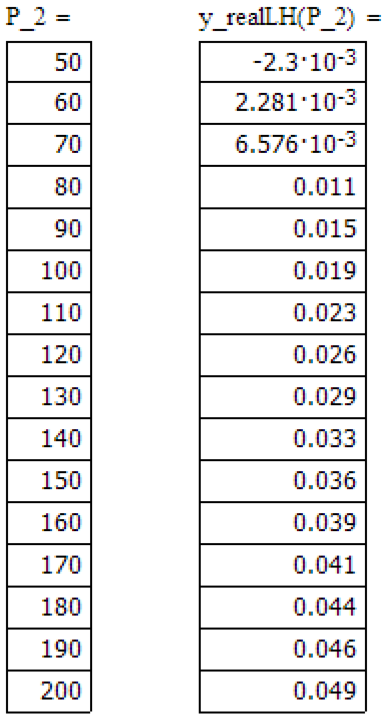
$$W\_per\_kg\_liq(P\_2) := \frac{W\_per\_kg\_gas(P\_2)}{y\_realLH(P\_2)}$$

$$FOM\_realLH(P\_2) := \frac{W\_ideal}{W\_per\_kg\_liq(P\_2)}$$

Now, we can plot the results as P_2 varies from 50 bar to 200 bar:

$P\_2 := 50, 60 .. 200$ ...define a range variable

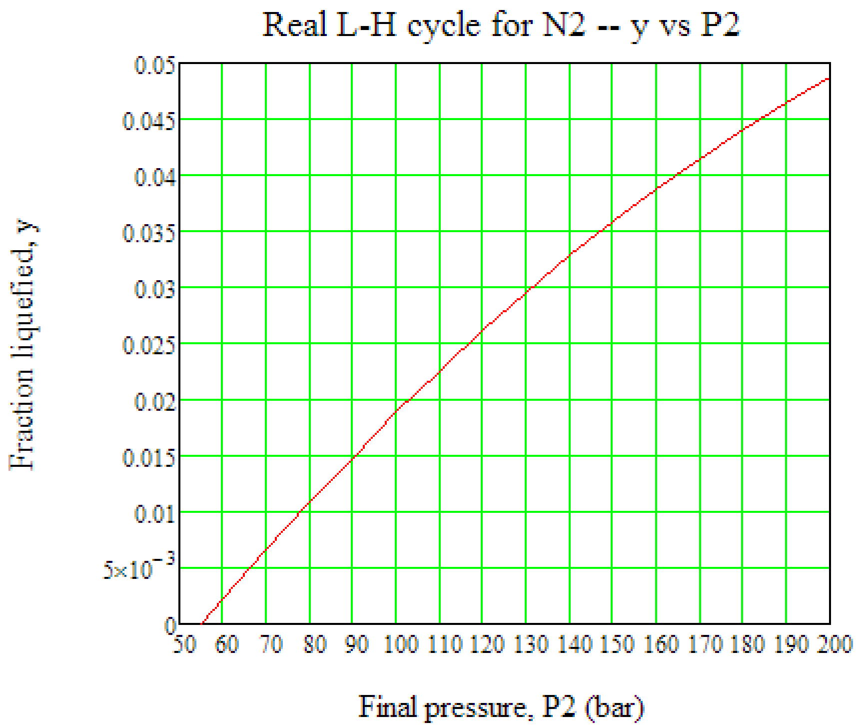
y_realLH vs P_2:



P_2 =	y_realLH(P_2) =
50	$-2.3 \cdot 10^{-3}$
60	$2.281 \cdot 10^{-3}$
70	$6.576 \cdot 10^{-3}$
80	0.011
90	0.015
100	0.019
110	0.023
120	0.026
130	0.029
140	0.033
150	0.036
160	0.039
170	0.041
180	0.044
190	0.046
200	0.049

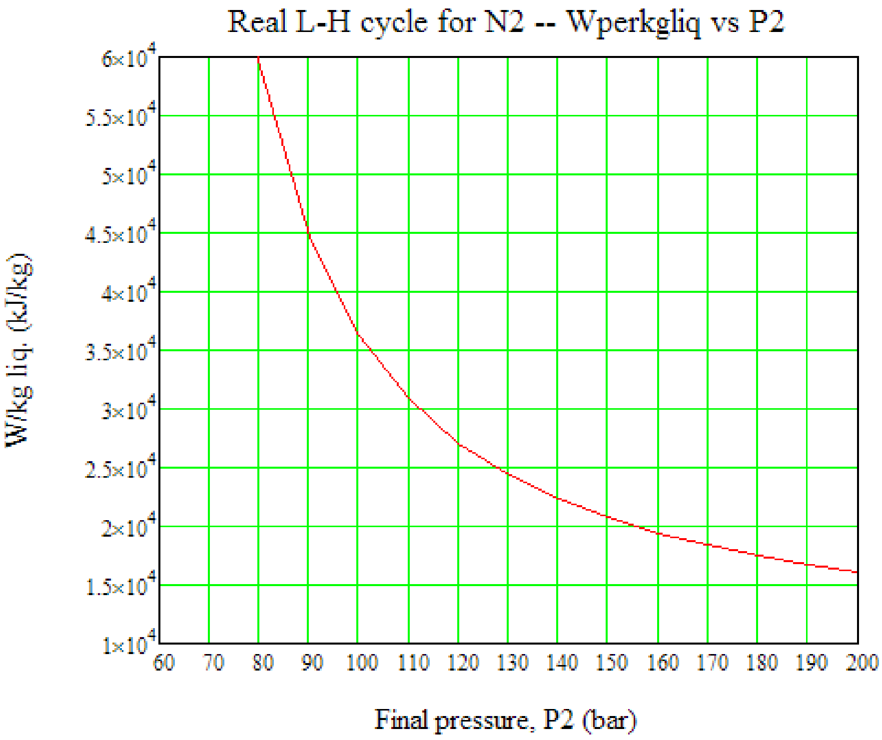
Note that at pressures P2 less than 50 bar, the fraction liquefied, y is -ve. i.e. At pressures P2 < 50 bar, there is no liquefaction.

Now, plot the result:

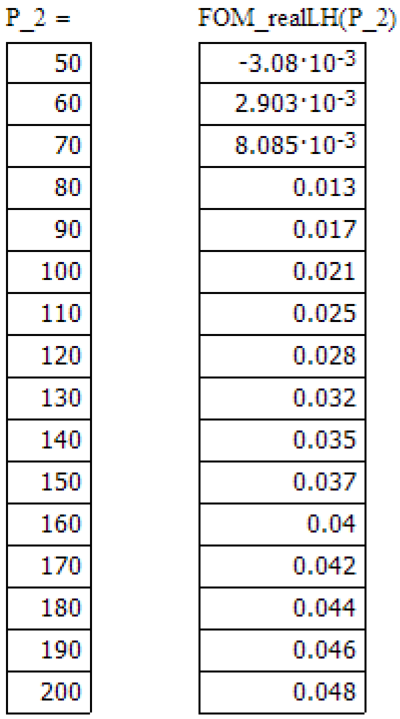


W_per_kg_liq vs P_2:

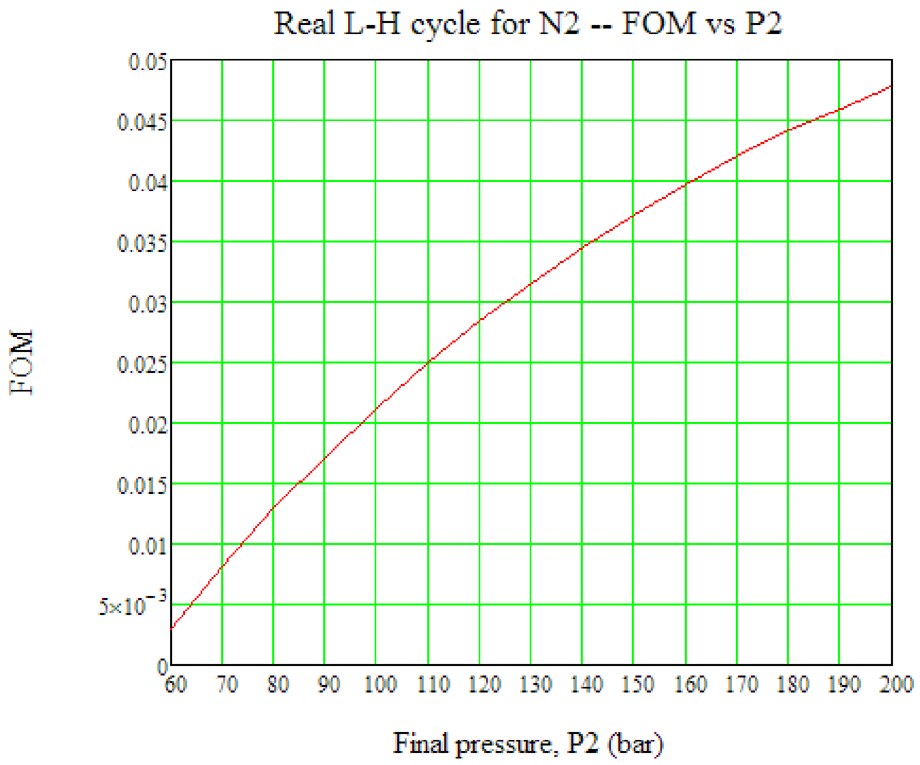
P_2 =	W_per_kg_liq(P_2)
50	$-2.503\cdot10^{5}$
60	$2.656\cdot10^{5}$
70	$9.537\cdot10^{4}$
80	$5.965\cdot10^{4}$
90	$4.477\cdot10^{4}$
100	$3.618\cdot10^{4}$
110	$3.09\cdot10^{4}$
120	$2.71\cdot10^{4}$
130	$2.443\cdot10^{4}$
140	$2.231\cdot10^{4}$
150	$2.073\cdot10^{4}$
160	$1.939\cdot10^{4}$
170	$1.837\cdot10^{4}$
180	$1.746\cdot10^{4}$
190	$1.676\cdot10^{4}$
200	$1.613\cdot10^{4}$



FOM vs P_2:



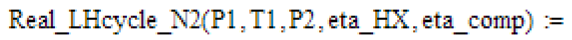
P_2 =	FOM_realLH(P_2)
50	$-3.08 \cdot 10^{-3}$
60	$2.903 \cdot 10^{-3}$
70	$8.085 \cdot 10^{-3}$
80	0.013
90	0.017
100	0.021
110	0.025
120	0.028
130	0.032
140	0.035
150	0.037
160	0.04
170	0.042
180	0.044
190	0.046
200	0.048



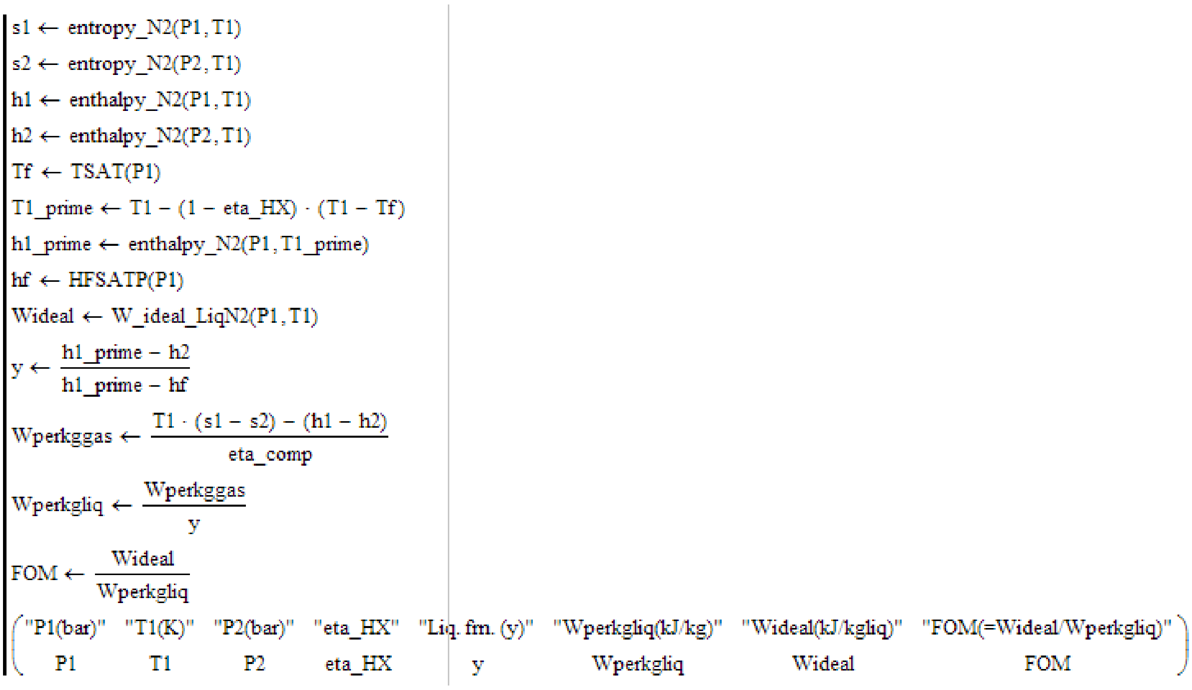
Prob. 3.3.12 If in a Linde-Hampson system, nitrogen gas is compressed from 1.013 bar to 200 bar with an overall effcy of compressor = 0.75, and the gas leaves the compressor at 300 K, find the efficiency of HX to get a liquid yield of 0.04. Also find the work required per unit mass liquefied [1].

Also, plot Liq. yield, y and FOM against eta_HX.

First, write a Mathcad Function to find the desired quantities:



```
Real_LHcycle_N2(P1,T1,P2,eta_HX,eta_comp) :=
```

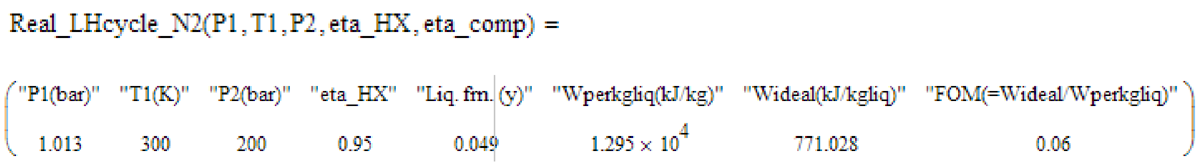


```
s1 ← entropy_N2(P1, T1)
s2 ← entropy_N2(P2, T1)
h1 ← enthalpy_N2(P1, T1)
h2 ← enthalpy_N2(P2, T1)
Tf ← TSAT(P1)
T1_prime ← T1 − (1 − eta_HX) · (T1 − Tf)
h1_prime ← enthalpy_N2(P1, T1_prime)
hf ← HFSATP(P1)
Wideal ← W_ideal_LiqN2(P1, T1)
y ← $\frac{h1\_prime - h2}{h1\_prime - hf}$
Wperkggas ← $\frac{T1 \cdot (s1 - s2) - (h1 - h2)}{eta\_comp}$
Wperkgliq ← $\frac{Wperkggas}{y}$
FOM ← $\frac{Wideal}{Wperkgliq}$
$\begin{pmatrix} \text{"P1(bar)"} & \text{"T1(K)"} & \text{"P2(bar)"} & \text{"eta\_HX"} & \text{"Liq. frn. (y)"} & \text{"Wperkgliq(kJ/kg)"} & \text{"Wideal(kJ/kgliq)"} & \text{"FOM(=Wideal/Wperkgliq)"} \\ P1 & T1 & P2 & eta\_HX & y & Wperkgliq & Wideal & FOM \end{pmatrix}$
```

Ex: P1 := 1.013 bar T1 := 300 K eta_HX := 0.95 eta_comp := 0.75

P2 := 200 bar

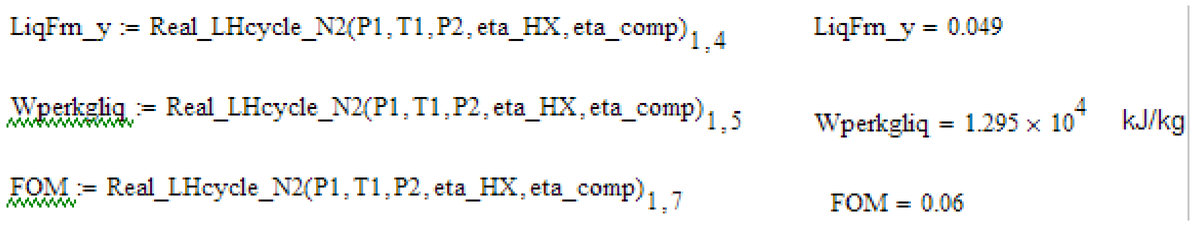
We get:



Real_LHcycle_N2(P1, T1, P2, eta_HX, eta_comp) =

$$\begin{pmatrix} \text{"P1(bar)"} & \text{"T1(K)"} & \text{"P2(bar)"} & \text{"eta\_HX"} & \text{"Liq. frn. (y)"} & \text{"Wperkgliq(kJ/kg)"} & \text{"Wideal(kJ/kgliq)"} & \text{"FOM(=Wideal/Wperkgliq)"} \\ 1.013 & 300 & 200 & 0.95 & 0.049 & 1.295 \times 10^{4} & 771.028 & 0.06 \end{pmatrix}$$

i.e.



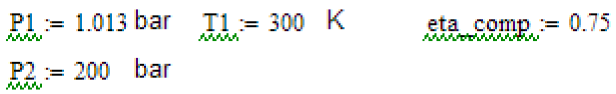
$LiqFrn\_y := Real\_LHcycle\_N2(P1, T1, P2, eta\_HX, eta\_comp)_{1,4}$ $LiqFrn\_y = 0.049$

$Wperkgliq := Real\_LHcycle\_N2(P1, T1, P2, eta\_HX, eta\_comp)_{1,5}$ $Wperkgliq = 1.295 \times 10^{4}$ kJ/kg

$FOM := Real\_LHcycle\_N2(P1, T1, P2, eta\_HX, eta\_comp)_{1,7}$ $FOM = 0.06$

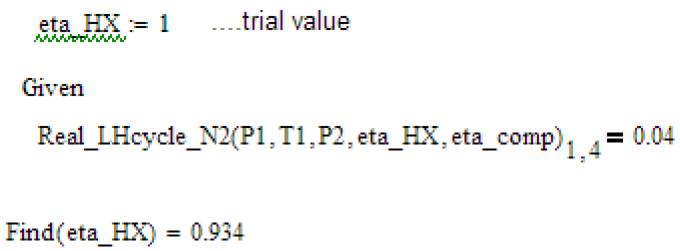
Now, find the value of eta_HX required to get a yield of 0.04, use the 'Solve block' of Mathcad:

We have:



P1 := 1.013 bar T1 := 300 K eta_comp := 0.75

P2 := 200 bar



eta_HX := 1 ....trial value

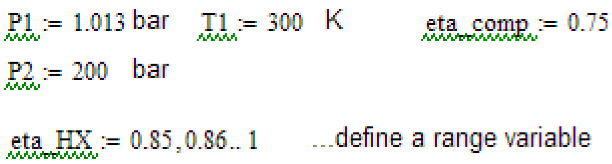
Given

$\text{Real\_LHcycle\_N2}(P1, T1, P2, \text{eta\_HX}, \text{eta\_comp})_{1,4} = 0.04$

$\text{Find}(\text{eta\_HX}) = 0.934$

i.e. eta_HX = 0.934 = 93.4% to get a value of y = 0.04…Ans.

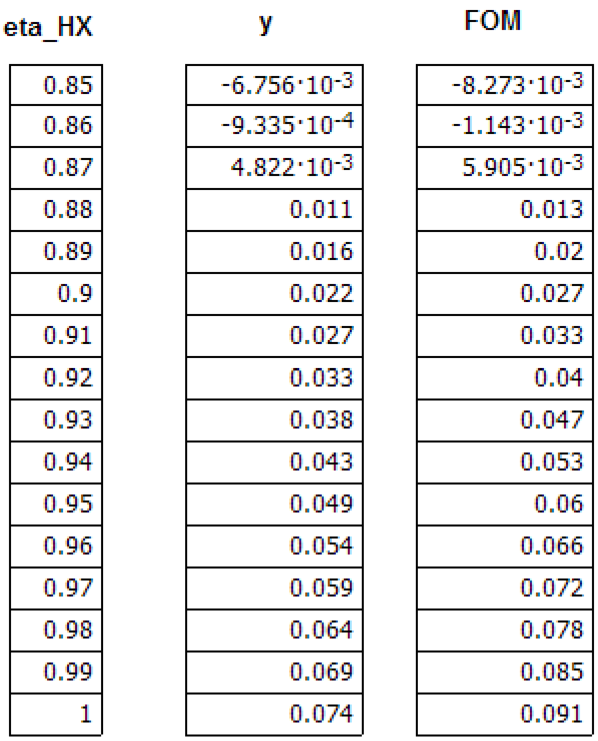
Plot y and FOM against eta_HX, other factors remaining the same:



P1 := 1.013 bar T1 := 300 K eta_comp := 0.75

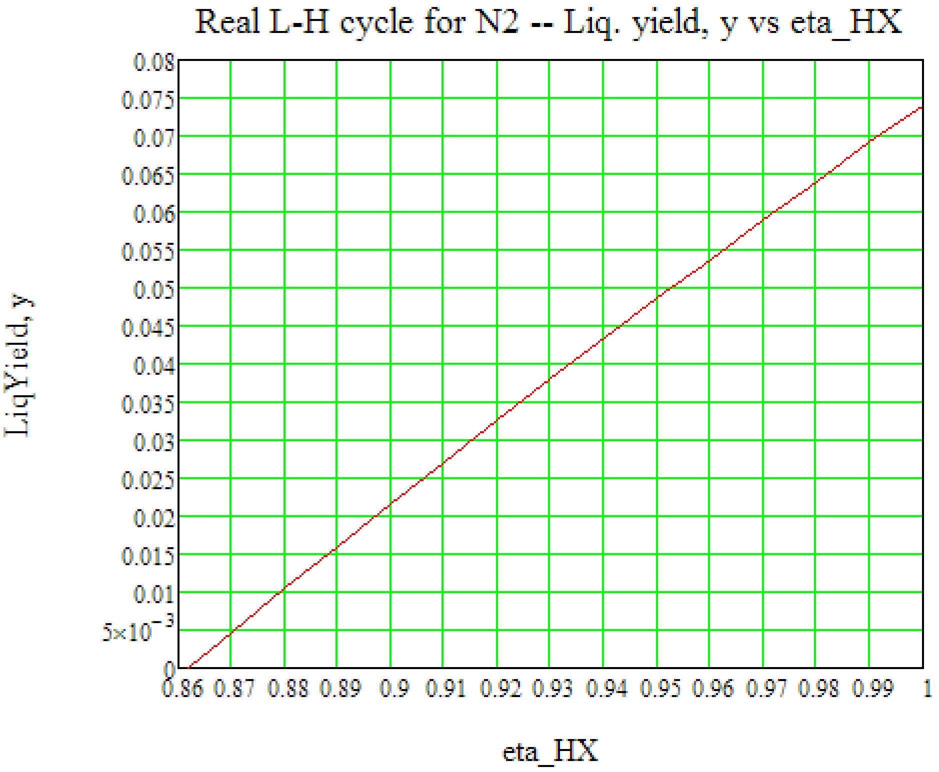
P2 := 200 bar

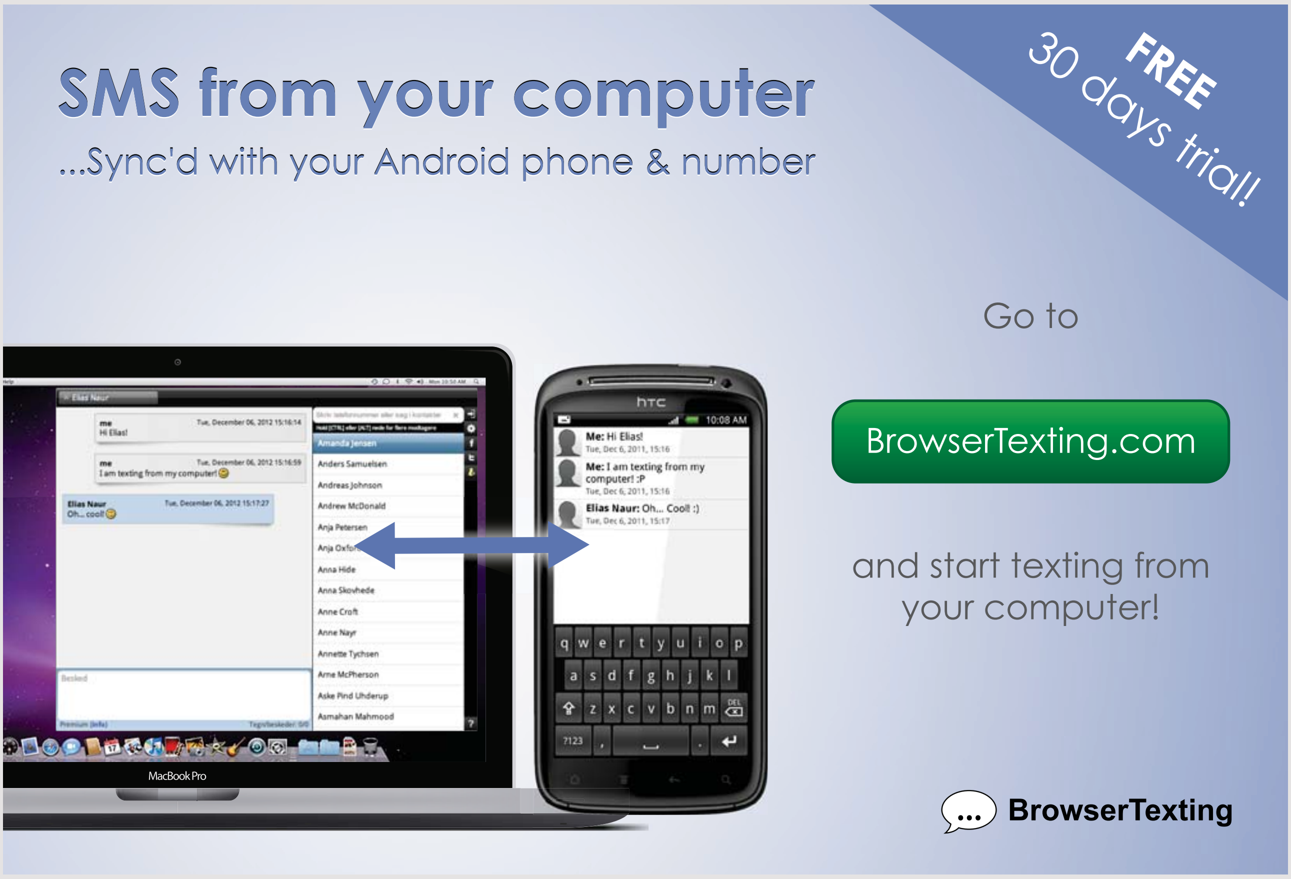
eta_HX := 0.85, 0.86.. 1 ...define a range variable



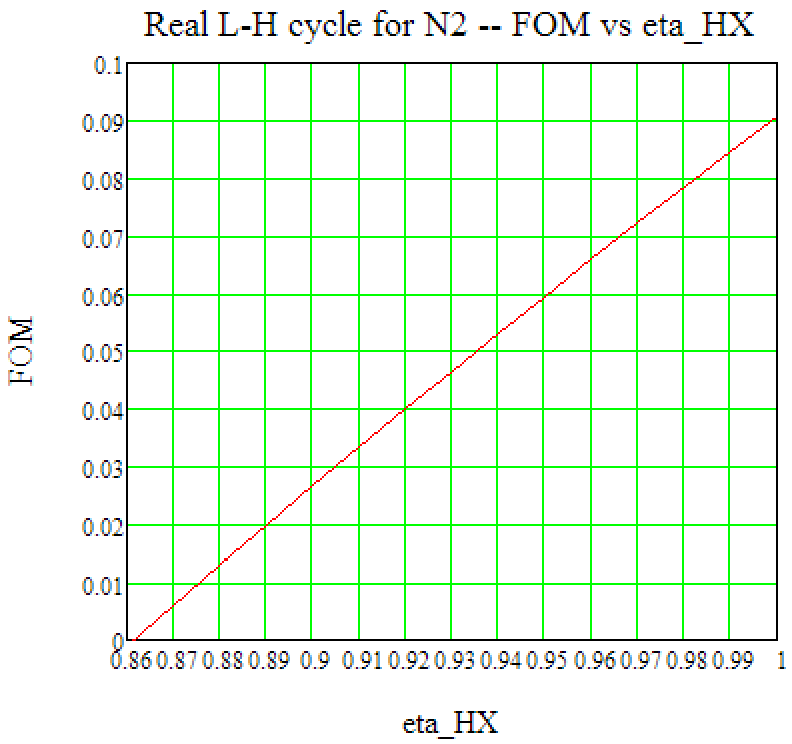
eta_HX	y	FOM
0.85	$-6.756\cdot10^{-3}$	$-8.273\cdot10^{-3}$
0.86	$-9.335\cdot10^{-4}$	$-1.143\cdot10^{-3}$
0.87	$4.822\cdot10^{-3}$	$5.905\cdot10^{-3}$
0.88	0.011	0.013
0.89	0.016	0.02
0.9	0.022	0.027
0.91	0.027	0.033
0.92	0.033	0.04
0.93	0.038	0.047
0.94	0.043	0.053
0.95	0.049	0.06
0.96	0.054	0.066
0.97	0.059	0.072
0.98	0.064	0.078
0.99	0.069	0.085
1	0.074	0.091

Note that at values of eta_HX < 0.86, the liquid yield is zero.


Real L-H cycle for N2 -- Liq. yield, y vs eta_HX
LiqYield, y
0.08
0.075
0.07
0.065
0.06
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
5×10⁻³
0
0.86
0.87
0.88
0.89
0.9
0.91
0.92
0.93
0.94
0.95
0.96
0.97
0.98
0.99
1
eta_HX



And,

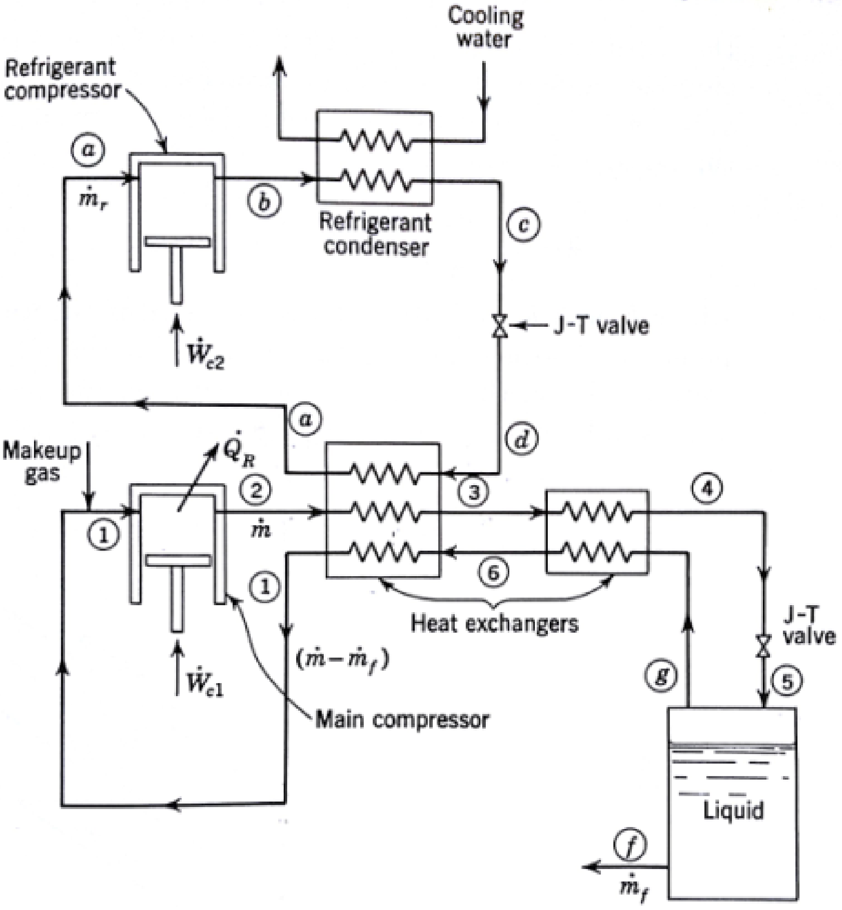


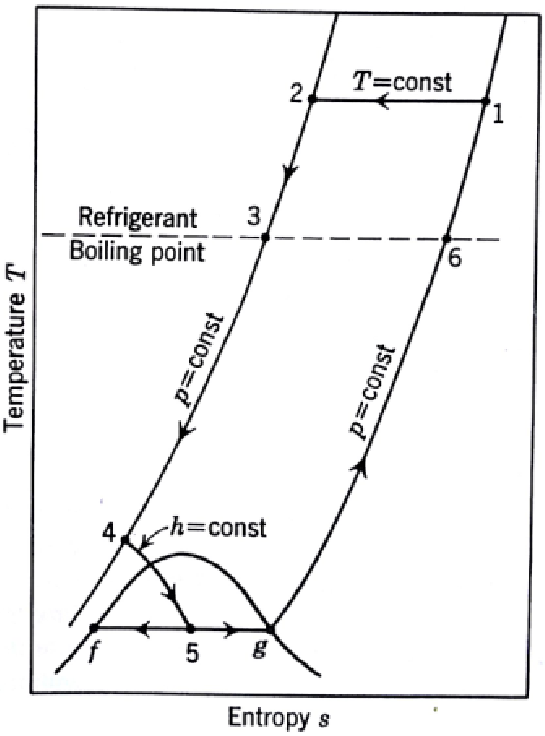
Prob. 3.3.13 Write Mathcad Functions to determine liquefaction fraction, y, work required per kg of gas compressed, work required per kg liquefied, Figure of Merit (FOM = Wideal / Work per kg liqid) for pre-cooled Linde-Hampson cycle for Nitrogen, including the adiabatic work of the refrigeration system. Take the initial pressure and temp of 1.013 bar (= 1 atm) and 300 K respectively and the final pressure is 200 bar. For the refrigerant circuit using F_12, we have:

refrigerant flow rate ratio = r (= m_r / m):

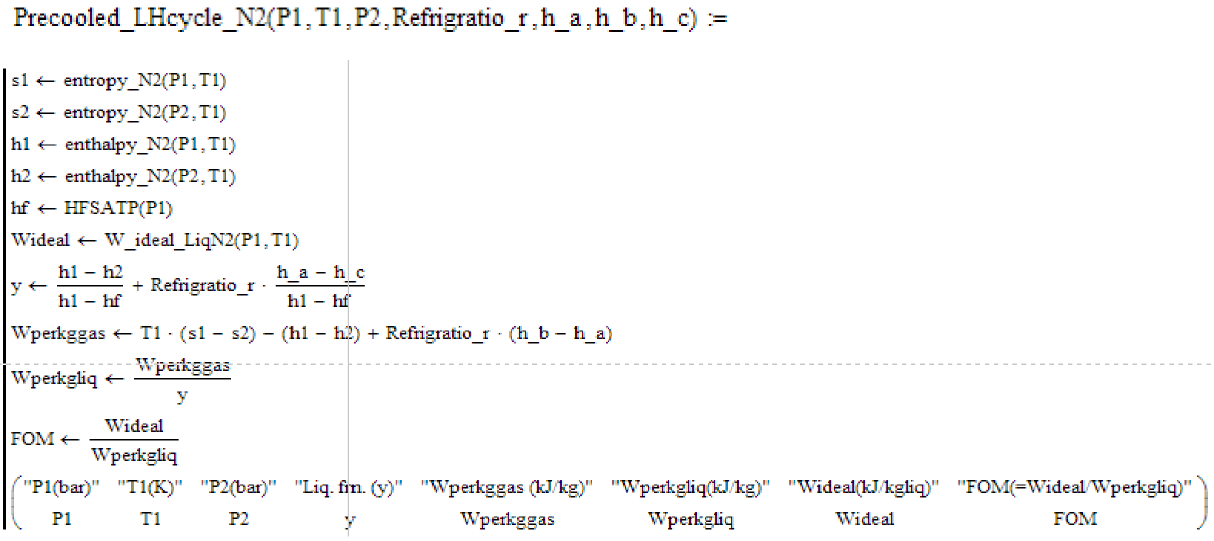
h_a = 204.5 kJ/kg, h_b = 242.1 kJ/kg and h_c = 55.9 kJ/kg.

Also, plot y, Wperkgliq and FOM as final pressure varies from 50 bar to 200 bar.


Cooling water
Refrigerant compressor
ⓐ
$\dot{m}_r$
ⓑ
Refrigerant condenser
ⓒ
J-T valve
$\dot{W}_{c2}$
ⓐ
ⓓ
Makeup gas
$\dot{Q}_R$
②
$\dot{m}$
①
③
④
①
⑥
Heat exchangers
$(\dot{m}-\dot{m}_f)$
$\dot{W}_{c1}$
Main compressor
J-T valve
ⓖ
⑤
Liquid
ⓕ
$\dot{m}_f$

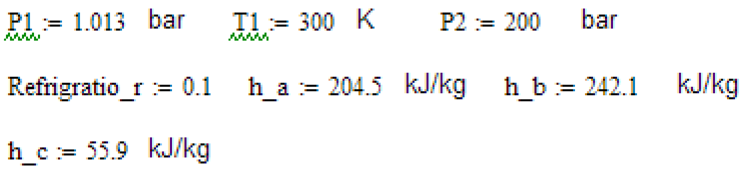

T=const
2
1
3
Refrigerant
Boiling point
6
p=const
p=const
Temperature T
4
h=const
f
5
g
Entropy s

First, write the Mathcad Function:



```
Precooled_LHcycle_N2(P1,T1,P2,Refrigratio_r,h_a,h_b,h_c) :=
    s1 ← entropy_N2(P1,T1)
    s2 ← entropy_N2(P2,T1)
    h1 ← enthalpy_N2(P1,T1)
    h2 ← enthalpy_N2(P2,T1)
    hf ← HFSATP(P1)
    Wideal ← W_ideal_LiqN2(P1,T1)
    y ← $\frac{h1 - h2}{h1 - hf} + Refrigratio\_r \cdot \frac{h\_a - h\_c}{h1 - hf}$
    Wperkggas ← T1 · (s1 − s2) − (h1 − h2) + Refrigratio_r · (h_b − h_a)
    Wperkgliq ← $\frac{Wperkggas}{y}$
    FOM ← $\frac{Wideal}{Wperkgliq}$
    ( "P1(bar)"  "T1(K)"  "P2(bar)"  "Liq. frn. (y)"  "Wperkggas (kJ/kg)"  "Wperkgliq(kJ/kg)"  "Wideal(kJ/kgliq)"  "FOM(=Wideal/Wperkgliq)" )
    (   P1         T1       P2          y                Wperkggas             Wperkgliq            Wideal               FOM                )
```

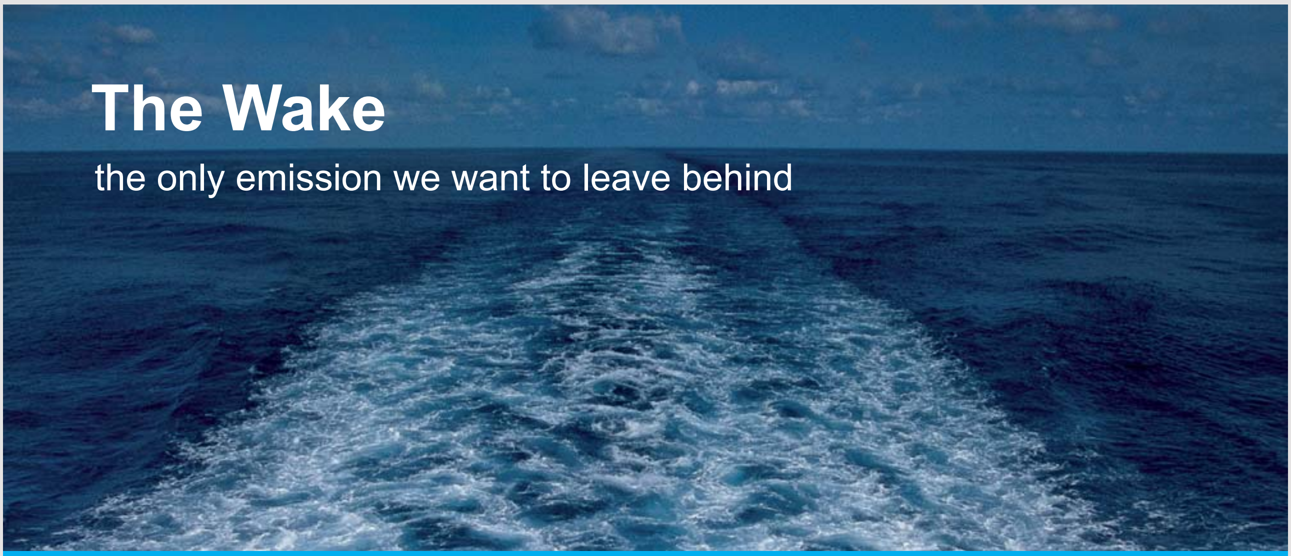
Now, solve the problem:



P1 := 1.013 bar T1 := 300 K P2 := 200 bar

Refrigratio_r := 0.1 h_a := 204.5 kJ/kg h_b := 242.1 kJ/kg

h_c := 55.9 kJ/kg



Precooled_LHcycle_N2(P1, T1, P2, Refrigratio_r, h_a, h_b, h_c) =

$$\begin{pmatrix} \text{"P1(bar)"} & \text{"T1(K)"} & \text{"P2(bar)"} & \text{"Liq. frn. (y)"} & \text{"Wperkggas (kJ/kg)"} & \text{"Wperkgliq(kJ/kg)"} & \text{"Wideal(kJ/kgliq)"} & \text{"FOM(=Wideal/Wperkgliq)"} \\ 1.013 & 300 & 200 & 0.108 & 476.019 & 4.394\times10^{3} & 771.028 & 0.175 \end{pmatrix}$$

i.e.

$y :=$ Precooled_LHcycle_N2(P1, T1, P2, Refrigratio_r, h_a, h_b, h_c)$_{1,3}$ i.e. $y = 0.108$

Wperkggas := Precooled_LHcycle_N2(P1, T1, P2, Refrigratio_r, h_a, h_b, h_c)$_{1,4}$ i.e. Wperkggas = 476.019 kJ/kg

Wperkgliq := Precooled_LHcycle_N2(P1, T1, P2, Refrigratio_r, h_a, h_b, h_c)$_{1,5}$ i.e. Wperkgliq = 4.394×10^{3} kJ/kg

Wideal := Precooled_LHcycle_N2(P1, T1, P2, Refrigratio_r, h_a, h_b, h_c)$_{1,6}$ i.e. Wideal = 771.028 kJ/kg

FOM := Precooled_LHcycle_N2(P1, T1, P2, Refrigratio_r, h_a, h_b, h_c)$_{1,7}$ i.e. FOM = 0.175

Now, to calculate for Refrigratio_r = 0 to 0.15, for P2 = 50, 100, 150, 200 1nd 220 bar:

$P1 := 1.013$ bar $T1 := 300$ K

$h\_a := 204.5$ kJ/kg $h\_b := 242.1$ kJ/kg

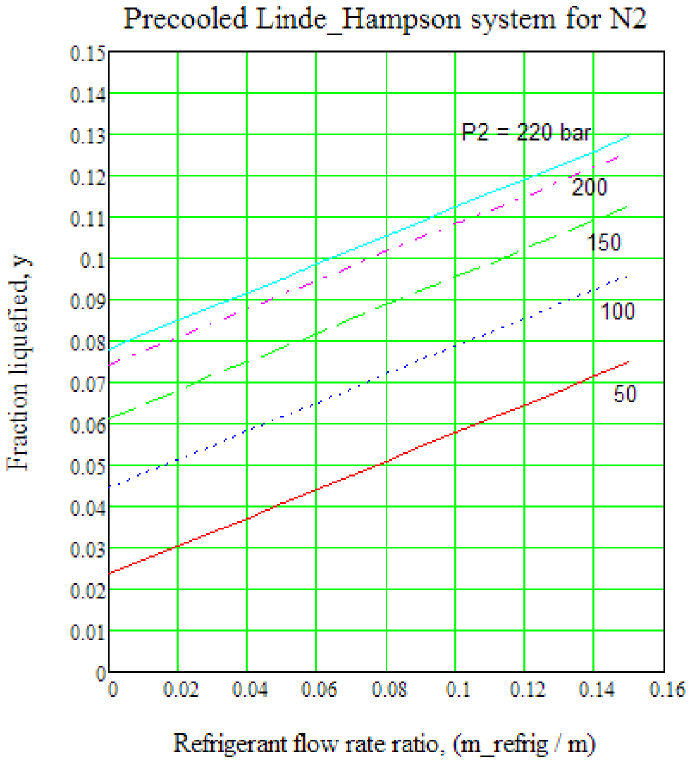
$h\_c := 55.9$ kJ/kg

Refrigratio_r := 0, 0.01 .. 0.15 ....define a range variable

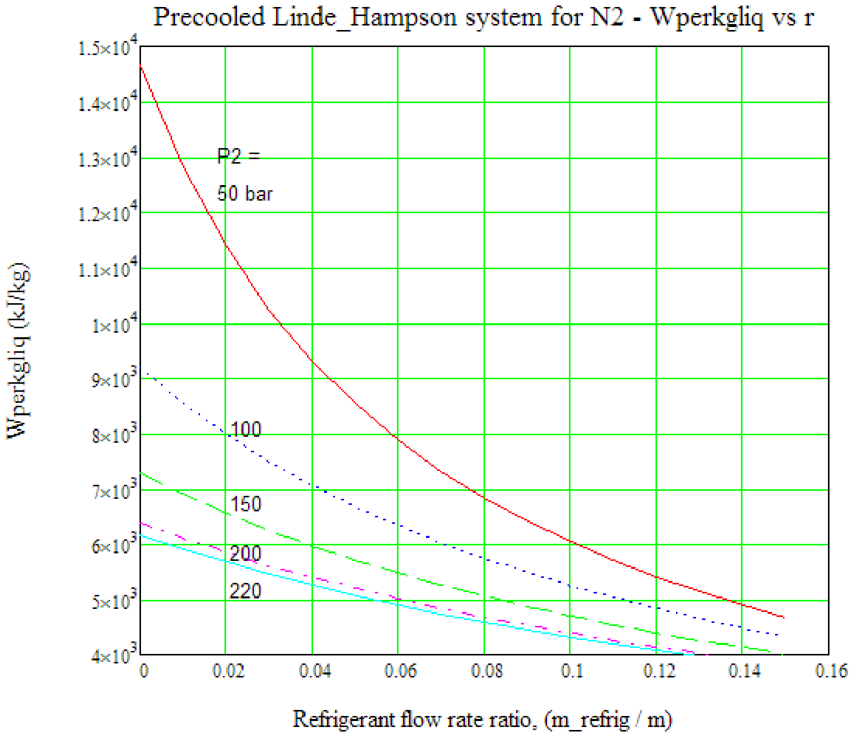
And, the results:

Refrigratio_r	P2 = 50	100	150	200	220 bar
0	0.024	0.044	0.061	0.074	0.078
0.01	0.027	0.048	0.065	0.077	0.081
0.02	0.03	0.051	0.068	0.081	0.085
0.03	0.034	0.055	0.072	0.084	0.088
0.04	0.037	0.058	0.075	0.088	0.092
0.05	0.041	0.062	0.078	0.091	0.095
0.06	0.044	0.065	0.082	0.095	0.099
0.07	0.048	0.068	0.085	0.098	0.102
0.08	0.051	0.072	0.089	0.101	0.105
0.09	0.054	0.075	0.092	0.105	0.109
0.1	0.058	0.079	0.095	0.108	0.112
0.11	0.061	0.082	0.099	0.112	0.116
0.12	0.065	0.086	0.102	0.115	0.119
0.13	0.068	0.089	0.106	0.119	0.123
0.14	0.072	0.092	0.109	0.122	0.126
0.15	0.075	0.096	0.113	0.125	0.129

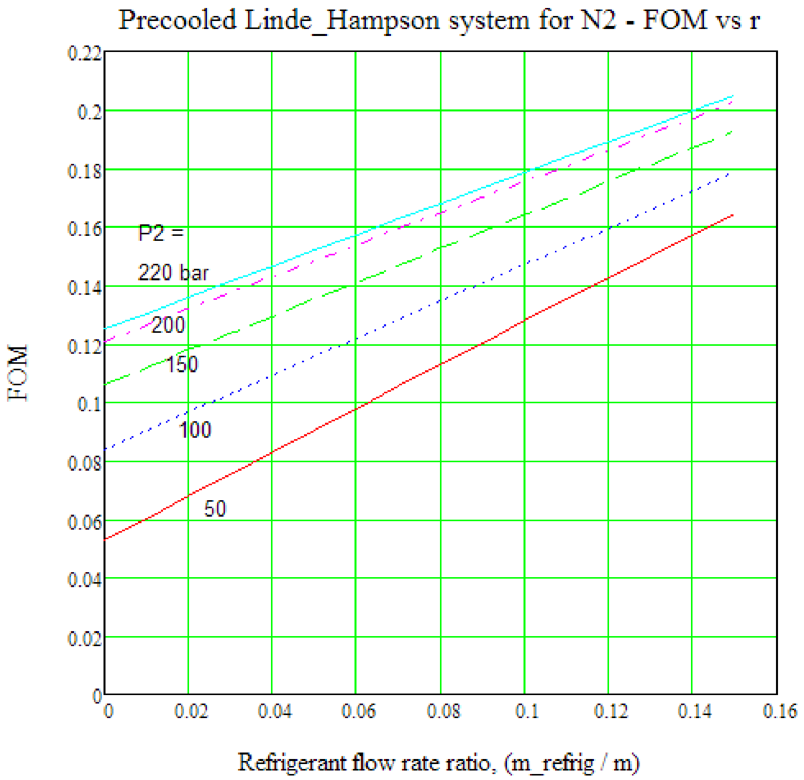
Now, plot the results:



Similarly,


Precooled Linde_Hampson system for N2 - Wperkgliq vs r
1.5×10^4
1.4×10^4
1.3×10^4
1.2×10^4
1.1×10^4
1×10^4
9×10^3
8×10^3
7×10^3
6×10^3
5×10^3
4×10^3
Wperkgliq (kJ/kg)
P2 =
50 bar
100
150
200
220
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
Refrigerant flow rate ratio, (m_refrig / m)

And,



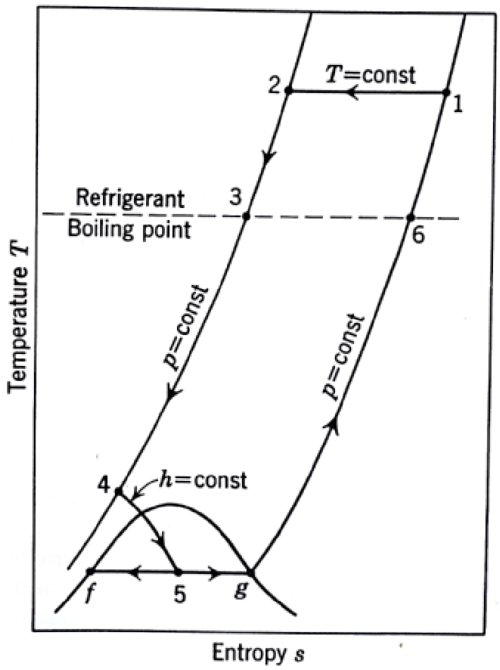
……

Prob. 3.3.14 Write Mathcad Function to determine liquefaction fraction, y, work required per kg of gas compressed, work required per kg liquefied, Figure of Merit (FOM = Wideal / Work per kg liqid) for pre-cooled Linde-Hampson cycle for Hydrogen, including the adiabatic work of the pre-cooling refrigeration system. Take the initial pressure and temp of 1.013 bar (= 1 atm) and 300 K respectively and the final pressure is 200 bar. Pre-cooling is with LN2 at a temp of 70K, and we have:

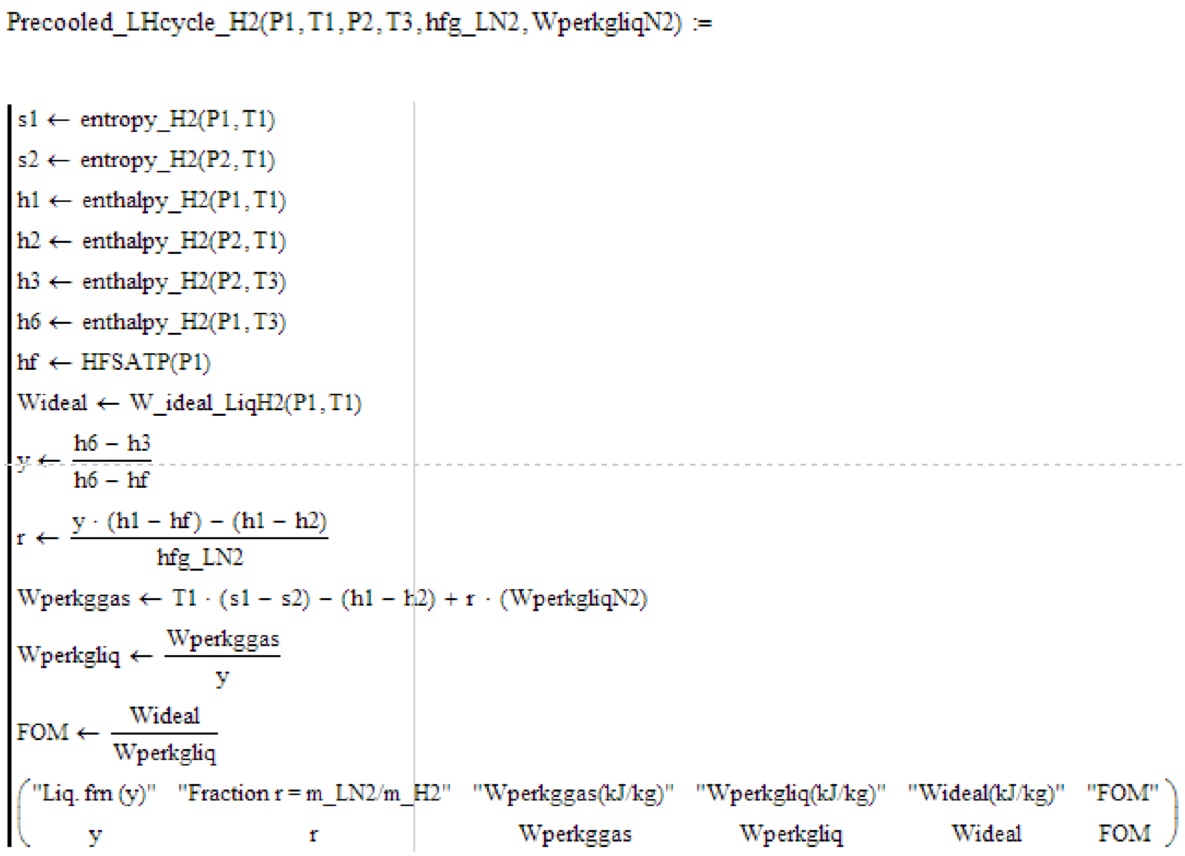
At 70 K, for LN2: hfg = 208 kJ/kg, WperkgliqN2 = 4500 kJ/kgliqN2

Find: LN2flow rate ratio = r (= mf / mH2)

Also, plot y, Wperkgliq and FOM as final pressure varies from 50 bar to 200 bar.



Mathcad Function for the ideal system, when efficiencies of compressor and heat exchangers are 100%, and *only* the latent heat of boiling LN2 is used in pre-cooling:



```
Precooled_LHcycle_H2(P1,T1,P2,T3,hfg_LN2,WperkgliqN2) :=
    s1 ← entropy_H2(P1,T1)
    s2 ← entropy_H2(P2,T1)
    h1 ← enthalpy_H2(P1,T1)
    h2 ← enthalpy_H2(P2,T1)
    h3 ← enthalpy_H2(P2,T3)
    h6 ← enthalpy_H2(P1,T3)
    hf ← HFSATP(P1)
    Wideal ← W_ideal_LiqH2(P1,T1)
    $y \leftarrow \frac{h6 - h3}{h6 - hf}$
    $r \leftarrow \frac{y \cdot (h1 - hf) - (h1 - h2)}{hfg\_LN2}$
    Wperkggas ← T1 · (s1 − s2) − (h1 − h2) + r · (WperkgliqN2)
    $Wperkgliq \leftarrow \frac{Wperkggas}{y}$
    $FOM \leftarrow \frac{Wideal}{Wperkgliq}$
    ( "Liq. frn (y)"  "Fraction r = m_LN2/m_H2"  "Wperkggas(kJ/kg)"  "Wperkgliq(kJ/kg)"  "Wideal(kJ/kg)"  "FOM" )
    (      y                    r                     Wperkggas           Wperkgliq             Wideal         FOM  )
```

Ex: $P1 := 1.013$ bar $T1 := 300$ K $P2 := 120$ bar $T3 := 70$ K

$hfg\_LN2 := 208$ kJ/kg

$WperkgliqN2 := 4500$ kJ/kg

We get:

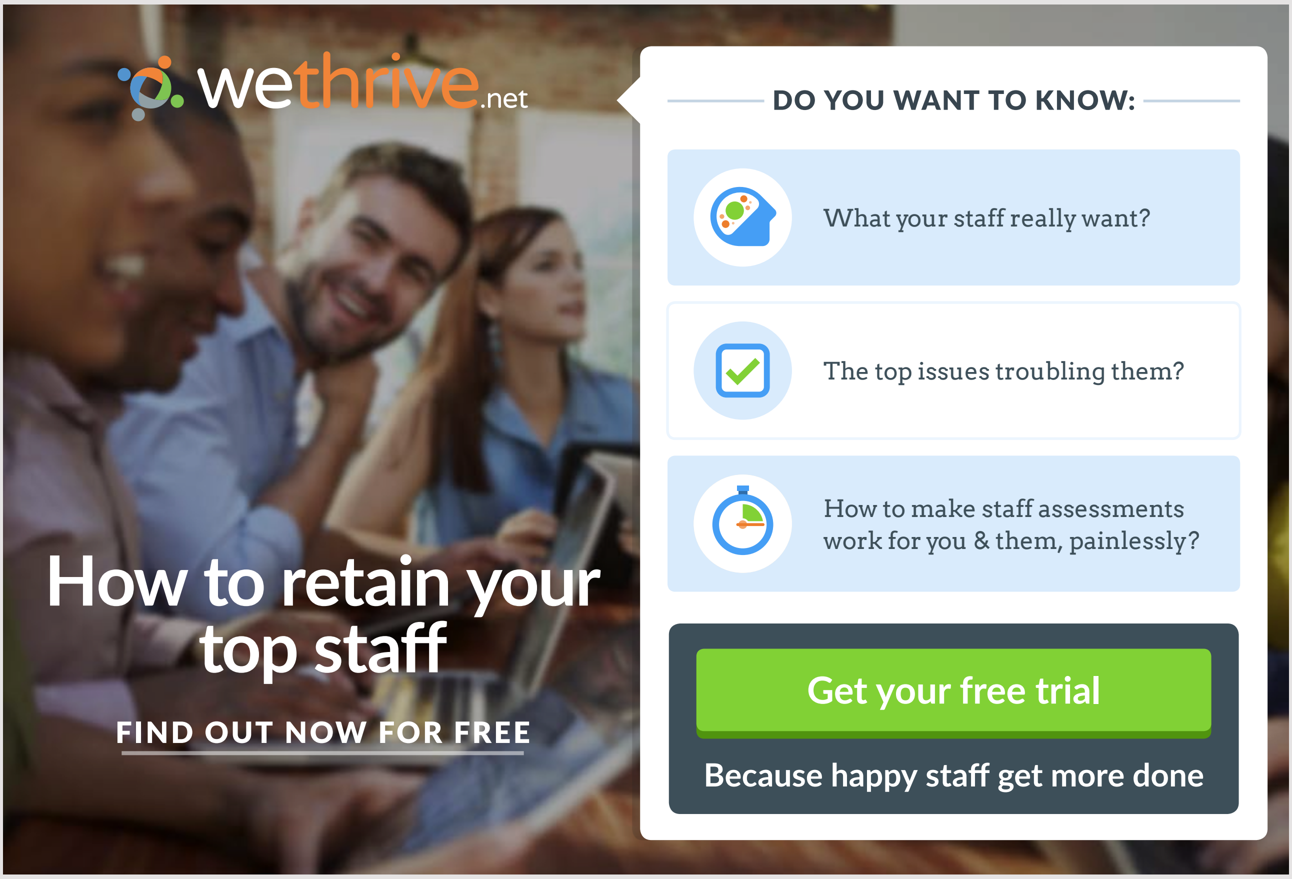
$$Precooled\_LHcycle\_H2(P1,T1,P2,T3,hfg\_LN2,WperkgliqN2) =$$

$$\begin{pmatrix} \text{"Liq. frn (y)"} & \text{"Fraction r = m\_LN2/m\_H2"} & \text{"Wperkggas(kJ/kg)"} & \text{"Wperkgliq(kJ/kg)"} & \text{"Wideal(kJ/kg)"} & \text{"FOM"} \\ 0.222 & 4.493 & 2.622 \times 10^{4} & 1.183 \times 10^{5} & 1.21 \times 10^{4} & 0.102 \end{pmatrix}$$

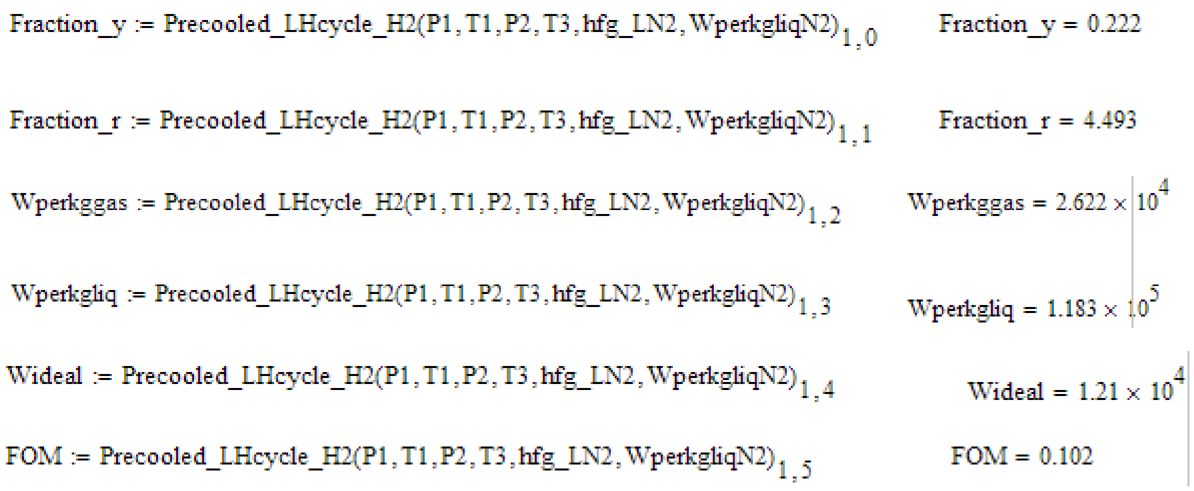
Thus:

Liq. Fraction = y = 0.222; Wperkgliq = 1.183 * 10^5 kJ/kg; FOM = 0.102, and

LN2 required for pre-cooling = 4.493 kg per kg of H2 compressed.



Also, we have:



$\text{Fraction\_y} := \text{Precooled\_LHcycle\_H2}(P1, T1, P2, T3, \text{hfg\_LN2}, \text{WperkgliqN2})_{1,0}$ $\text{Fraction\_y} = 0.222$

$\text{Fraction\_r} := \text{Precooled\_LHcycle\_H2}(P1, T1, P2, T3, \text{hfg\_LN2}, \text{WperkgliqN2})_{1,1}$ $\text{Fraction\_r} = 4.493$

$\text{Wperkggas} := \text{Precooled\_LHcycle\_H2}(P1, T1, P2, T3, \text{hfg\_LN2}, \text{WperkgliqN2})_{1,2}$ $\text{Wperkggas} = 2.622 \times 10^{4}$

$\text{Wperkgliq} := \text{Precooled\_LHcycle\_H2}(P1, T1, P2, T3, \text{hfg\_LN2}, \text{WperkgliqN2})_{1,3}$ $\text{Wperkgliq} = 1.183 \times 10^{5}$

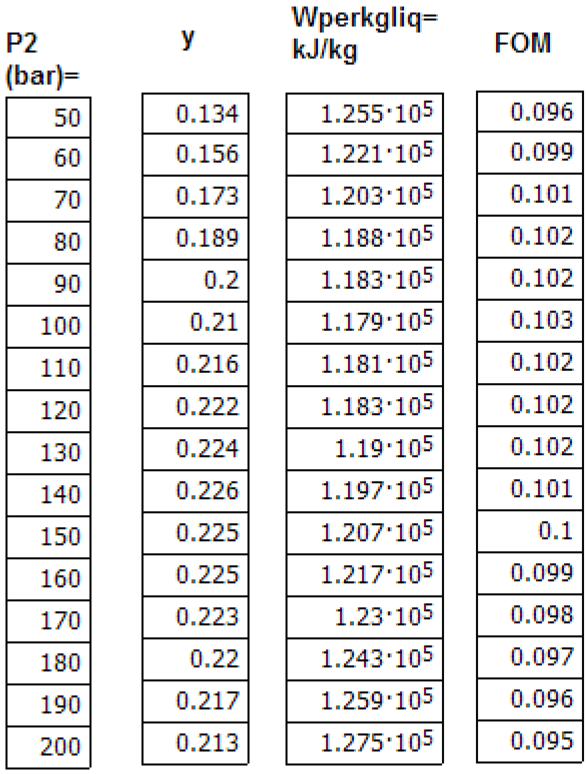
$\text{Wideal} := \text{Precooled\_LHcycle\_H2}(P1, T1, P2, T3, \text{hfg\_LN2}, \text{WperkgliqN2})_{1,4}$ $\text{Wideal} = 1.21 \times 10^{4}$

$\text{FOM} := \text{Precooled\_LHcycle\_H2}(P1, T1, P2, T3, \text{hfg\_LN2}, \text{WperkgliqN2})_{1,5}$ $\text{FOM} = 0.102$

Now, to plot y, Wperkgliq and FOM against P2:

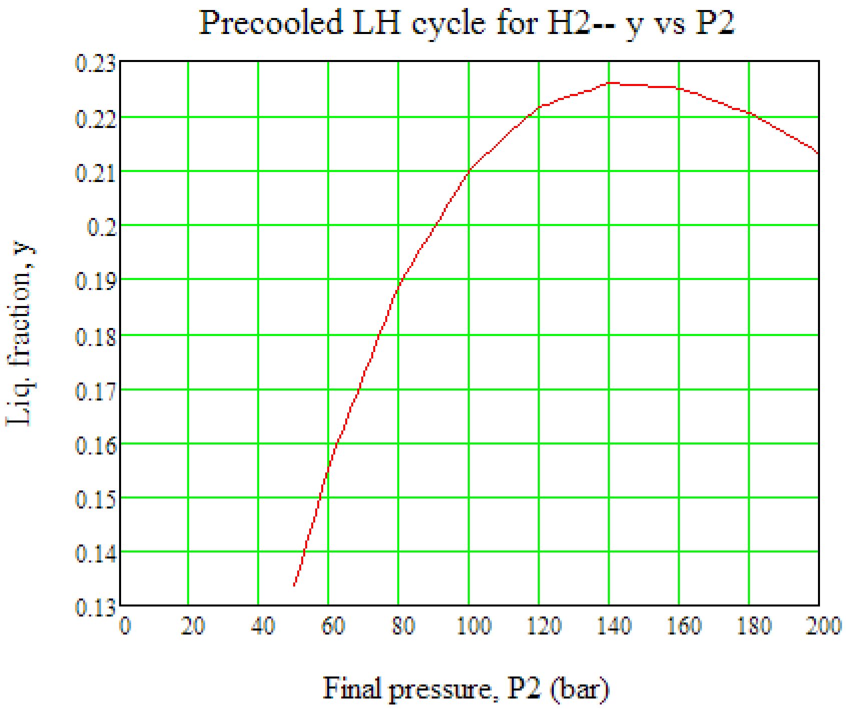
$P2 := 50, 60 .. 200$ ...define a range variable

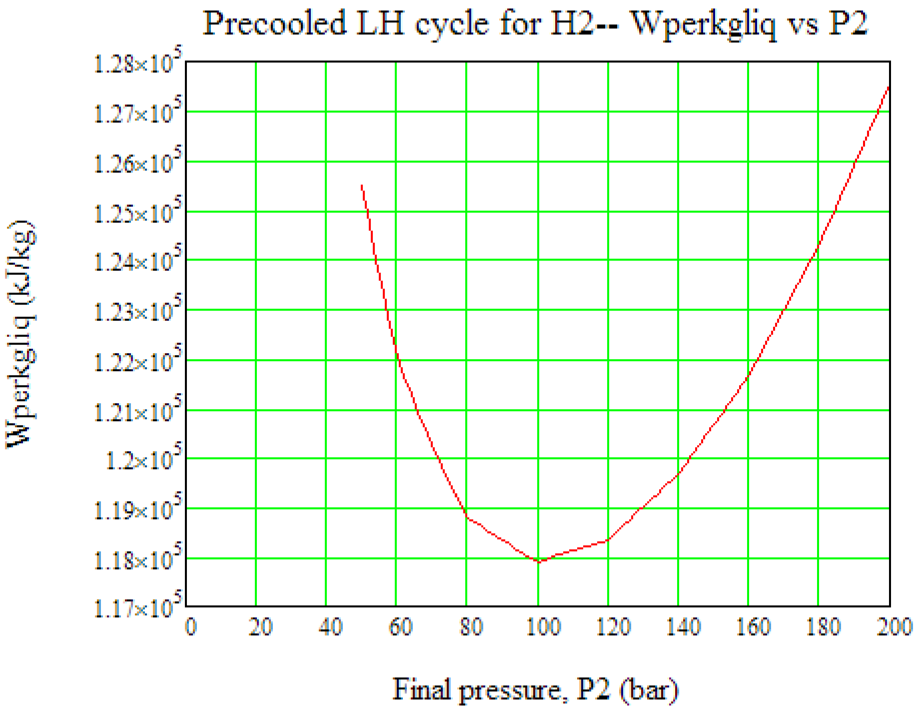
We get:

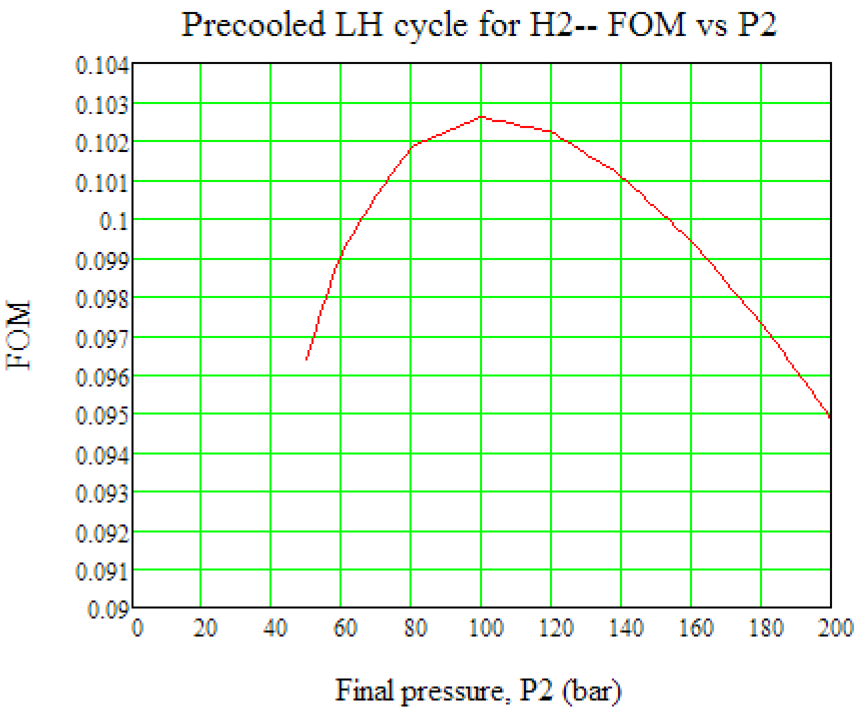


P2 (bar)=	y	Wperkgliq= kJ/kg	FOM
50	0.134	$1.255 \cdot 10^{5}$	0.096
60	0.156	$1.221 \cdot 10^{5}$	0.099
70	0.173	$1.203 \cdot 10^{5}$	0.101
80	0.189	$1.188 \cdot 10^{5}$	0.102
90	0.2	$1.183 \cdot 10^{5}$	0.102
100	0.21	$1.179 \cdot 10^{5}$	0.103
110	0.216	$1.181 \cdot 10^{5}$	0.102
120	0.222	$1.183 \cdot 10^{5}$	0.102
130	0.224	$1.19 \cdot 10^{5}$	0.102
140	0.226	$1.197 \cdot 10^{5}$	0.101
150	0.225	$1.207 \cdot 10^{5}$	0.1
160	0.225	$1.217 \cdot 10^{5}$	0.099
170	0.223	$1.23 \cdot 10^{5}$	0.098
180	0.22	$1.243 \cdot 10^{5}$	0.097
190	0.217	$1.259 \cdot 10^{5}$	0.096
200	0.213	$1.275 \cdot 10^{5}$	0.095

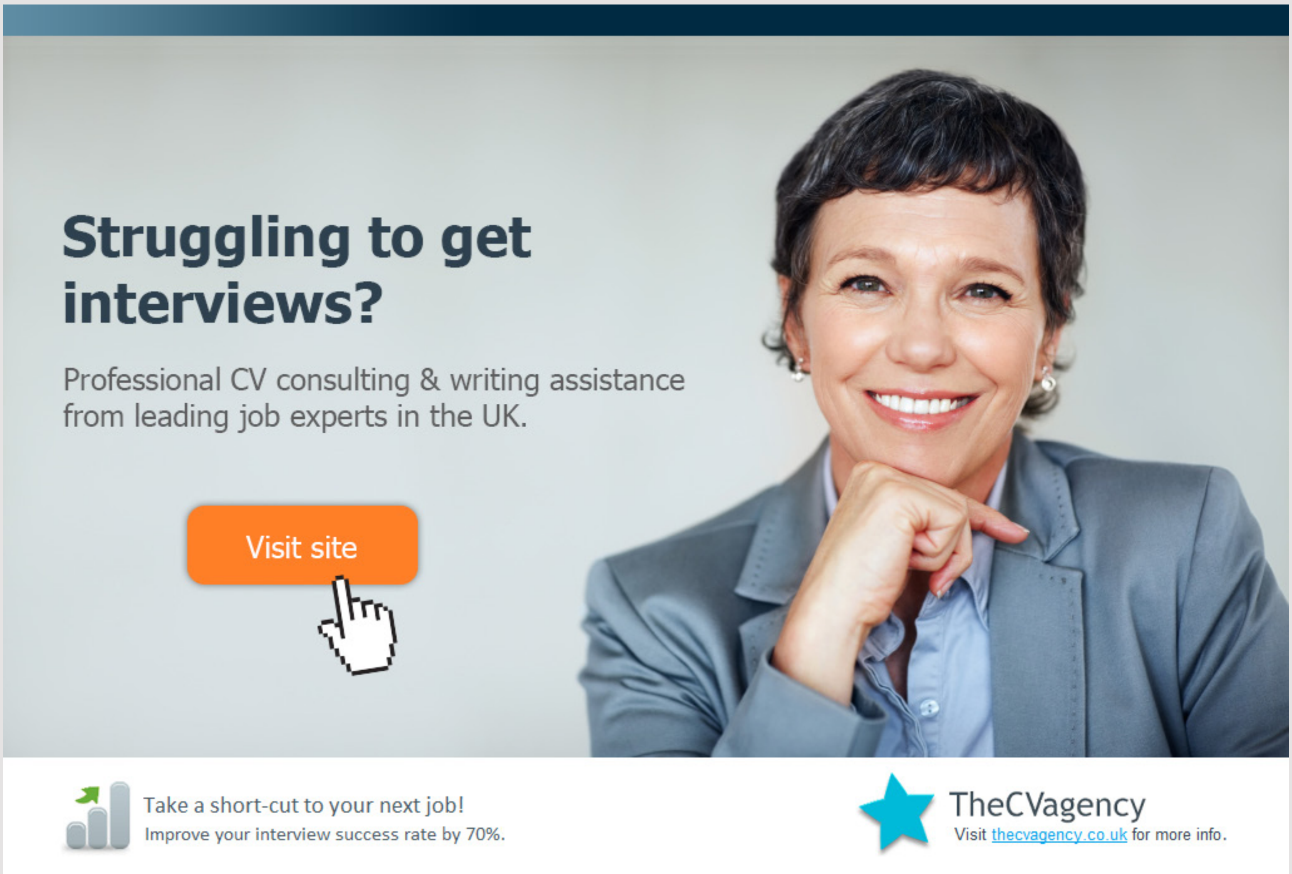
Now, plot the results:





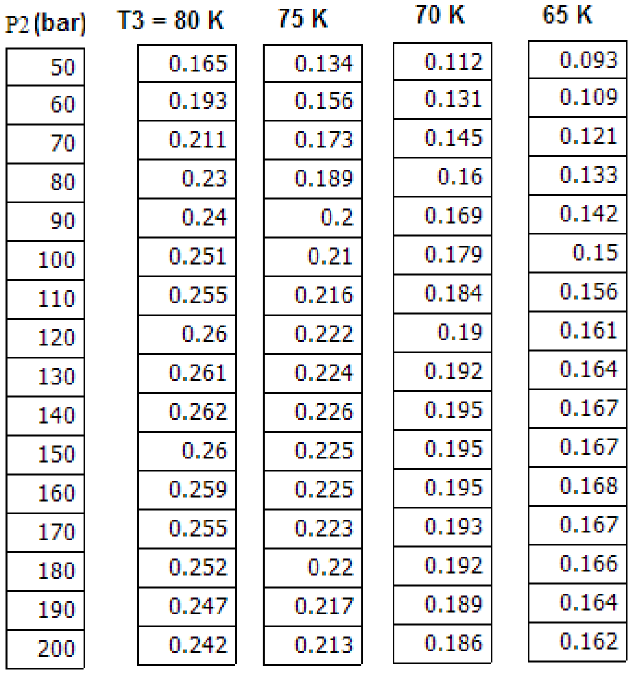


Precooled LH cycle for H2-- FOM vs P2



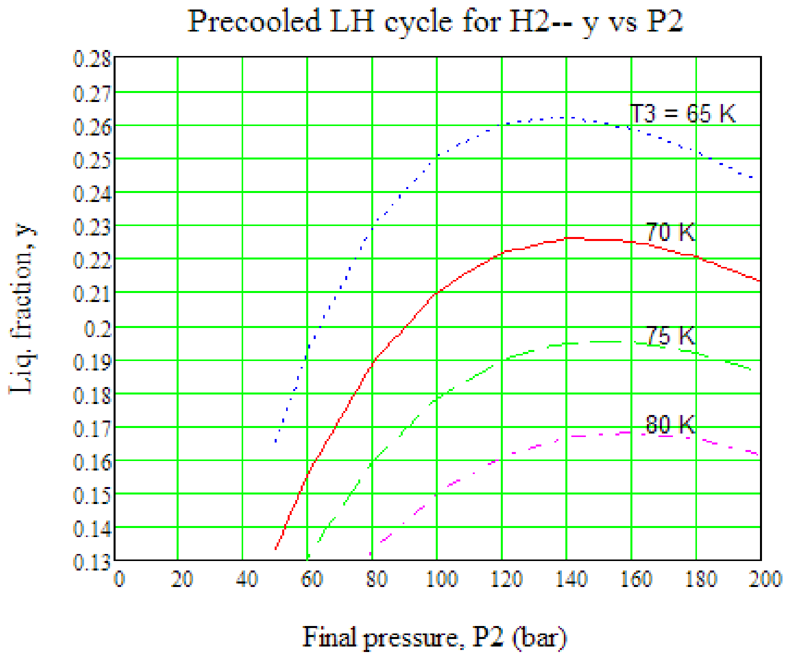
Effect of changing T3 on y:

Values of y at various pressures for different values of T3:



P2 (bar)	T3 = 80 K	75 K	70 K	65 K
50	0.165	0.134	0.112	0.093
60	0.193	0.156	0.131	0.109
70	0.211	0.173	0.145	0.121
80	0.23	0.189	0.16	0.133
90	0.24	0.2	0.169	0.142
100	0.251	0.21	0.179	0.15
110	0.255	0.216	0.184	0.156
120	0.26	0.222	0.19	0.161
130	0.261	0.224	0.192	0.164
140	0.262	0.226	0.195	0.167
150	0.26	0.225	0.195	0.167
160	0.259	0.225	0.195	0.168
170	0.255	0.223	0.193	0.167
180	0.252	0.22	0.192	0.166
190	0.247	0.217	0.189	0.164
200	0.242	0.213	0.186	0.162

Now, plot the results:

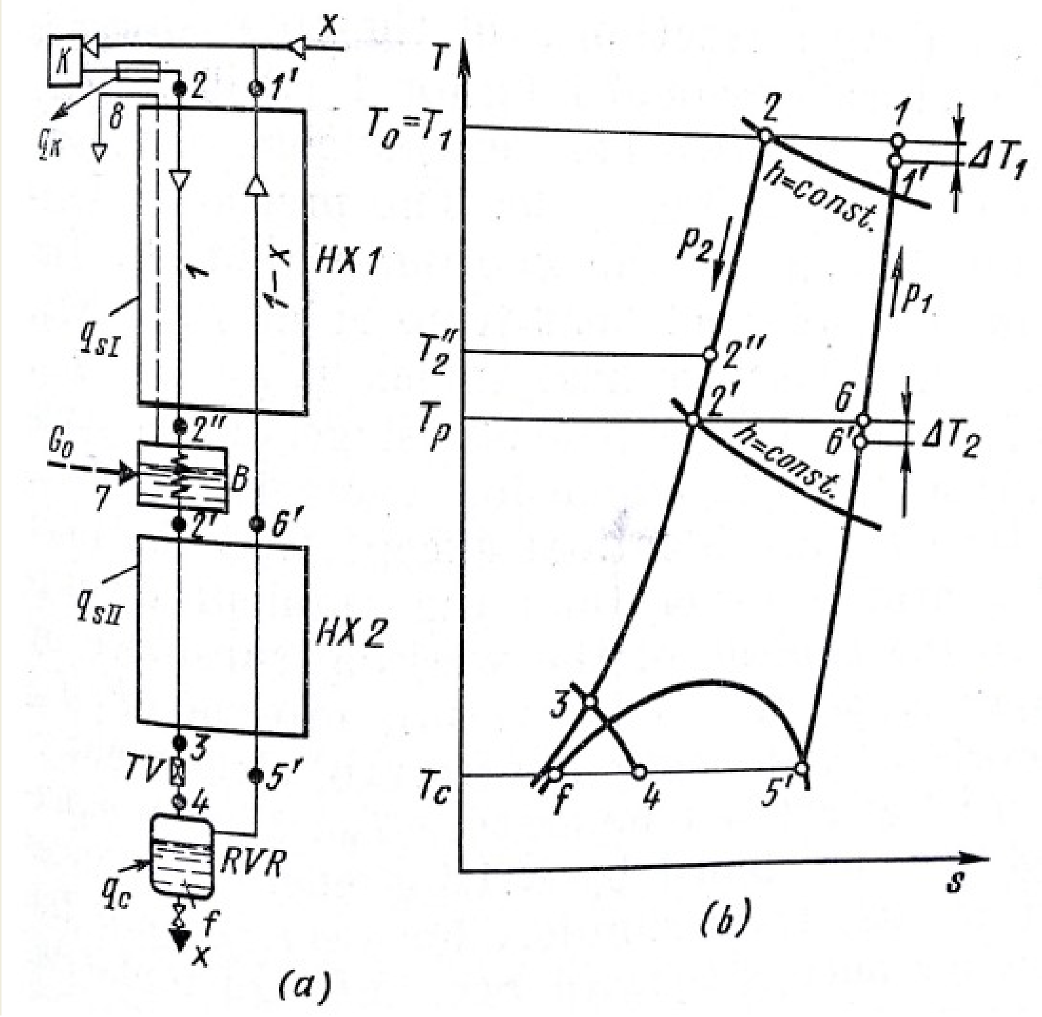


Note: Liquid fraction y is a maximum at about 140 bar, i.e. approximately the inversion pressure. Also, the pre-cooling temp strongly affects the liquid fraction y.

==

Prob. 3.3.15 Now, consider a practical LN2 pre-cooled, Linde_Hampson liquefaction cycle for Hydrogen: Let the HX1 and HX2 have less than 100% effectiveness, i.e. warm end temp difference for HX1 = Δ T1 = 15 deg. And warm end temp difference for HX2 = Δ T2 = 3 deg. Let the external heat leaks be q_leak1 = 6 kJ/kg and q_leak2 = 6 kJ/kg.
Take the initial pressure and temp of 1.013 bar (= 1 atm) and 300 K respectively and the final pressure is 200 bar. Isoth. effcy of compressor = eta_iso = 0.6
Pre-cooling is with LN2 at a temp of 70K, and we have:
At 70 K, for LN2: hfg = 208 kJ/kg, WperkgliqN2 = 4500 kJ/kgliqN2
Find: LN2flow rate ratio = G0 (= mLN2 / mH2gas)
Also, plot y, Wperkgliq, COP and FOM as final pressure varies from 50 bar to 200 bar.

Schematic diagram and the T-s diagram for liquefaction of H2 with LN2 pre-cooling is shown below [29]:



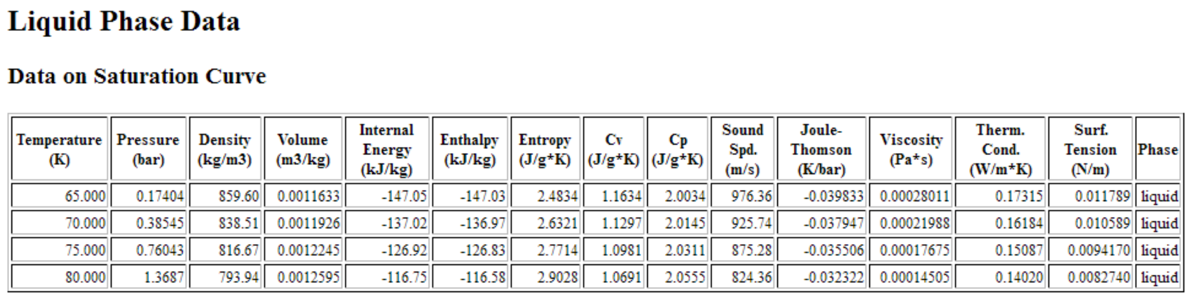
Note: In the above fig. x is the liquid fraction of H2 compressed.

Here, LN2 at 70 K (i.e. at a low pressure Psat of 0.38545 bar) is used for pre-cooling; the Nitrogen vapor passes through the first HX so that the sensible heat of vapor is also utilized.

Thus, the total cooling provided by Nitrogen stream is: (h8 – h7) kJ/kg of N2. Enthalpy h8 is at P = 0.38545 bar and T = 300 K, and h7 = hf = enthalpy at Tsat = 70 K and quality = 0.

See the Tables below (from NIST data):

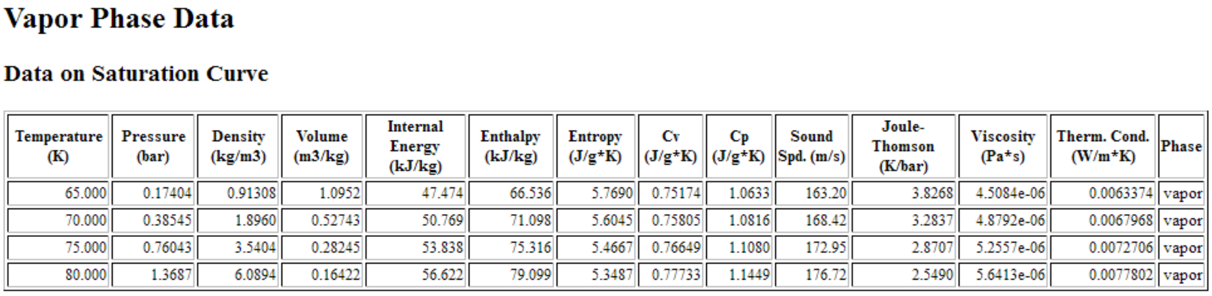
For Nitrogen (NIST data):



Liquid Phase Data

Data on Saturation Curve

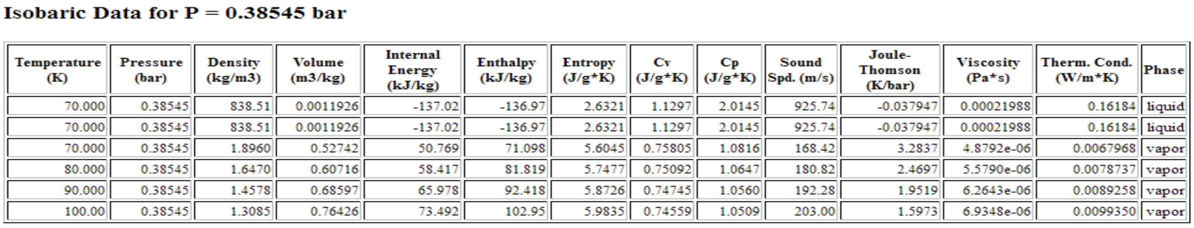
Temperature (K)	Pressure (bar)	Density (kg/m3)	Volume (m3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (J/g*K)	Cv (J/g*K)	Cp (J/g*K)	Sound Spd. (m/s)	Joule-Thomson (K/bar)	Viscosity (Pa*s)	Therm. Cond. (W/m*K)	Surf. Tension (N/m)	Phase
65.000	0.17404	859.60	0.0011633	-147.05	-147.03	2.4834	1.1634	2.0034	976.36	-0.039833	0.00028011	0.17315	0.011789	liquid
70.000	0.38545	838.51	0.0011926	-137.02	-136.97	2.6321	1.1297	2.0145	925.74	-0.037947	0.00021988	0.16184	0.010589	liquid
75.000	0.76043	816.67	0.0012245	-126.92	-126.83	2.7714	1.0981	2.0311	875.28	-0.035506	0.00017675	0.15087	0.0094170	liquid
80.000	1.3687	793.94	0.0012595	-116.75	-116.58	2.9028	1.0691	2.0555	824.36	-0.032322	0.00014505	0.14020	0.0082740	liquid



Vapor Phase Data

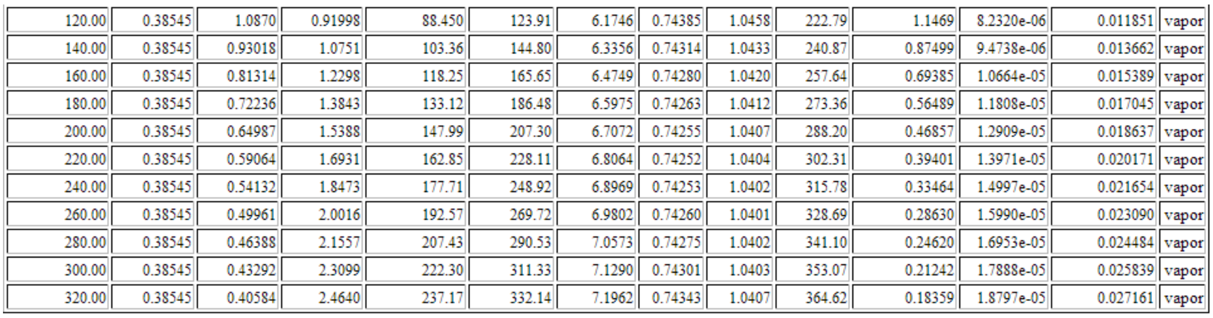
Data on Saturation Curve

Temperature (K)	Pressure (bar)	Density (kg/m3)	Volume (m3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (J/g*K)	Cv (J/g*K)	Cp (J/g*K)	Sound Spd. (m/s)	Joule-Thomson (K/bar)	Viscosity (Pa*s)	Therm. Cond. (W/m*K)	Phase
65.000	0.17404	0.91308	1.0952	47.474	66.536	5.7690	0.75174	1.0633	163.20	3.8268	4.5084e-06	0.0063374	vapor
70.000	0.38545	1.8960	0.52743	50.769	71.098	5.6045	0.75805	1.0816	168.42	3.2837	4.8792e-06	0.0067968	vapor
75.000	0.76043	3.5404	0.28245	53.838	75.316	5.4667	0.76649	1.1080	172.95	2.8707	5.2557e-06	0.0072706	vapor
80.000	1.3687	6.0894	0.16422	56.622	79.099	5.3487	0.77733	1.1449	176.72	2.5490	5.6413e-06	0.0077802	vapor



Isobaric Data for P = 0.38545 bar

Temperature (K)	Pressure (bar)	Density (kg/m3)	Volume (m3/kg)	Internal Energy (kJ/kg)	Enthalpy (kJ/kg)	Entropy (J/g*K)	Cv (J/g*K)	Cp (J/g*K)	Sound Spd. (m/s)	Joule-Thomson (K/bar)	Viscosity (Pa*s)	Therm. Cond. (W/m*K)	Phase
70.000	0.38545	838.51	0.0011926	-137.02	-136.97	2.6321	1.1297	2.0145	925.74	-0.037947	0.00021988	0.16184	liquid
70.000	0.38545	838.51	0.0011926	-137.02	-136.97	2.6321	1.1297	2.0145	925.74	-0.037947	0.00021988	0.16184	liquid
70.000	0.38545	1.8960	0.52742	50.769	71.098	5.6045	0.75805	1.0816	168.42	3.2837	4.8792e-06	0.0067968	vapor
80.000	0.38545	1.6470	0.60716	58.417	81.819	5.7477	0.75092	1.0647	180.82	2.4697	5.5790e-06	0.0078737	vapor
90.000	0.38545	1.4578	0.68597	65.978	92.418	5.8726	0.74745	1.0560	192.28	1.9519	6.2643e-06	0.0089258	vapor
100.00	0.38545	1.3085	0.76426	73.492	102.95	5.9835	0.74559	1.0509	203.00	1.5973	6.9348e-06	0.0099350	vapor

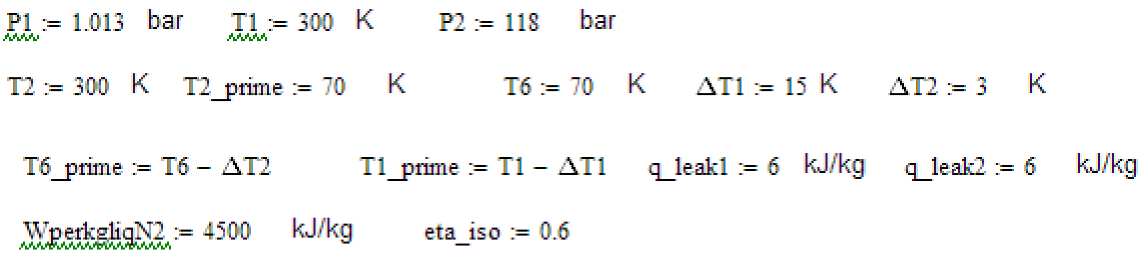


120.00	0.38545	1.0870	0.91998	88.450	123.91	6.1746	0.74385	1.0458	222.79	1.1469	8.2320e-06	0.011851	vapor
140.00	0.38545	0.93018	1.0751	103.36	144.80	6.3356	0.74314	1.0433	240.87	0.87499	9.4738e-06	0.013662	vapor
160.00	0.38545	0.81314	1.2298	118.25	165.65	6.4749	0.74280	1.0420	257.64	0.69385	1.0664e-05	0.015389	vapor
180.00	0.38545	0.72236	1.3843	133.12	186.48	6.5975	0.74263	1.0412	273.36	0.56489	1.1808e-05	0.017045	vapor
200.00	0.38545	0.64987	1.5388	147.99	207.30	6.7072	0.74255	1.0407	288.20	0.46857	1.2909e-05	0.018637	vapor
220.00	0.38545	0.59064	1.6931	162.85	228.11	6.8064	0.74252	1.0404	302.31	0.39401	1.3971e-05	0.020171	vapor
240.00	0.38545	0.54132	1.8473	177.71	248.92	6.8969	0.74253	1.0402	315.78	0.33464	1.4997e-05	0.021654	vapor
260.00	0.38545	0.49961	2.0016	192.57	269.72	6.9802	0.74260	1.0401	328.69	0.28630	1.5990e-05	0.023090	vapor
280.00	0.38545	0.46388	2.1557	207.43	290.53	7.0573	0.74275	1.0402	341.10	0.24620	1.6953e-05	0.024484	vapor
300.00	0.38545	0.43292	2.3099	222.30	311.33	7.1290	0.74301	1.0403	353.07	0.21242	1.7888e-05	0.025839	vapor
320.00	0.38545	0.40584	2.4640	237.17	332.14	7.1962	0.74343	1.0407	364.62	0.18359	1.8797e-05	0.027161	vapor

Thus, for Nitrogen, h8 = 311.33 kJ/kg at T = 300 K, P = 0.38545 bar; and, h7 = hf at 70 K =

-137.97 kJ/kg.

We have, by data:



P1 := 1.013 bar   T1 := 300 K   P2 := 118 bar

T2 := 300 K   T2_prime := 70 K   T6 := 70 K ΔT1 := 15 K ΔT2 := 3 K

T6_prime := T6 − ΔT2   T1_prime := T1 − ΔT1   q_leak1 := 6 kJ/kg   q_leak2 := 6 kJ/kg

WperkgliqN2 := 4500 kJ/kg   eta_iso := 0.6

Then:

$h1 := enthalpy\_H2(P1, T1)$ i.e. $h1 = 3.958 \times 10^{3}$ kJ/kg

$h1\_prime := enthalpy\_H2(P1, T1\_prime)$ i.e. $h1\_prime = 3.744 \times 10^{3}$ kJ/kg

$h2 := enthalpy\_H2(P2, T2)$ i.e. $h2 = 4.015 \times 10^{3}$ kJ/kg

$h2\_prime := enthalpy\_H2(P2, T2\_prime)$ i.e. $h2\_prime = 762.861$ kJ/kg

$s1 := entropy\_H2(P1, T1)$ i.e. $s1 = 53.495$ kJ/kg.K

$s2 := entropy\_H2(P2, T1)$ i.e. $s2 = 33.75$ kJ/kg.K

$h6 := enthalpy\_H2(P1, T6)$ i.e. $h6 = 978.702$ kJ/kg

$h6\_prime := enthalpy\_H2(P1, T6\_prime)$ i.e. $h6\_prime = 947.133$ kJ/kg

$hf := HFSATP(P1)$ i.e. $hf = -0.396$ kJ/kg

$hg := HGSATP(P1)$ i.e. $hg = 448.18$ kJ/kg

$h5\_prime := hg$ i.e. $h5\_prime = 448.18$ kJ/kg

For N2: $h8 := 311.33$ kJ/kg at $T = 300$ K, $P = 0.38545$ bar

$h7 := -137.97$ kJ/kg = hf at 70 K for LN2

For liq. fraction, x: Applying I Law to the (HX2 + JT valve + liquid reservoir) taken together:

$$h2\_prime + q\_leak2 = x \cdot hf + (1 - x) \cdot h6\_prime$$

i.e. $$x = \frac{(h6\_prime - h2\_prime) - q\_leak2}{(h6\_prime - hf)}$$

Therefore:

$$LiqFraction\_x := \frac{(h6\_prime - h2\_prime) - q\_leak2}{(h6\_prime - hf)}$$

i.e. $LiqFraction\_x = 0.188$ **...Ans.**

For LN2 fraction G0: Applying I Law to (HX1 + LN2 cooler):

$$h2 + G0 \cdot h7 + (1 - x) \cdot h6\_prime + q\_leak1 = h2\_prime + (1 - x) \cdot h1\_prime + G0 \cdot h8$$

i.e. $$G0 \cdot (h8 - h7) = x \cdot (h1\_prime - h6\_prime) + h2 + h6\_prime - h1\_prime + q\_leak1$$

i.e. $$G0 := \frac{x \cdot (h1\_prime - h6\_prime) + h2 + h6\_prime - h1\_prime + q\_leak1}{h8 - h7}$$ ...kg of LN2/kg H2 compressed

i.e. $G0 = 2.724$ ...kg of LN2/kg H2 compressed ... Ans.

Therefore, LN2 requirement per kg of liquid H2, adding 25% extra to account for losses:

$$m_{LN2} := \frac{1.25 \cdot G0}{LiqFraction\_x}$$ ...kg of LN2/kg H2 liquefied.

i.e. $m_{LN2} = 18.095$ ...kg of LN2/kg H2 liquefied ... Ans.

Work requirement per kg of H2 compressed, with isoth effcy = 0.6:

$$Wperkggas := \frac{T1 \cdot (s1 - s2) - (h1 - h2)}{eta\_iso} + 1.25 \cdot G0 \cdot WperkgliqN2$$ **kJ/kggas compressed**

i.e. $Wperkggas = 2.529 \times 10^{4}$ **kJ/kggas compressed,.. includes 25% extra LN2 for losses**

Work requirement per kg of H2 liquefied, with isoth effcy = 0.6:

$$Wperkgliq := \frac{Wperkggas}{LiqFraction\_x}$$ kJ/kggas liquefied

i.e. $Wperkgliq = 1.344 \times 10^{5}$ **kJ/kggas liquefied.... Ans.**

Coeff of Performance, COP:

$$COP := \frac{LiqFraction\_x \cdot (h1 - hf)}{\frac{T1 \cdot (s1 - s2) - (h1 - h2)}{eta\_iso} + 1.25 \cdot G0 \cdot WperkgliqN2}$$

i.e. $COP = 0.029$ **...Ans.**

Ideal or min. work of liquefaction:

$$Wideal := W\_ideal\_LiqH2(P1, T1)$$ kJ/kgliqH2

i.e. $Wideal = 1.21 \times 10^{4}$ kJ/kgliqH2

Figure of Merit, FOM:

$$FOM := \frac{Wideal}{Wperkgliq}$$

i.e. $FOM = 0.09$ **...Ans.**

Plot LiqFraction_x, Wperkgliq, COP and FOM against P2:

First, write the relevant quantities as Functions of P2:

$\mathrm{T6\_prime(T6)} := \mathrm{T6} - \Delta \mathrm{T2}$ $\qquad \mathrm{T6\_prime(T6)} = 67$

$\mathrm{h2(P2)} := \mathrm{enthalpy\_H2(P2, T2)}$ kJ/kg $\qquad \mathrm{h2(P2)} = 4.015 \times 10^{3}$

$\mathrm{h2\_prime(P2)} := \mathrm{enthalpy\_H2(P2, T2\_prime)}$ kJ/kg $\qquad \mathrm{h2\_prime(P2)} = 762.861$

$\mathrm{h6\_prime(T6)} := \mathrm{enthalpy\_H2(P1, T6\_prime(T6))}$ kJ/kg $\qquad \mathrm{h6\_prime(T6)} = 947.133$

$\mathrm{s2(P2)} := \mathrm{entropy\_H2(P2, T1)}$ kJ/kg.K $\qquad \mathrm{s2(P2)} = 33.75$

$\mathrm{T2\_prime(T6)} := \mathrm{T6}$ K $\qquad \mathrm{T2\_prime(T6)} = 70$

$$\mathrm{LiqFraction\_x(P2, T6)} := \frac{(\mathrm{h6\_prime(T6)} - \mathrm{h2\_prime(P2)}) - \mathrm{q\_leak2}}{(\mathrm{h6\_prime(T6)} - \mathrm{hf})}$$

i.e. $\mathrm{LiqFraction\_x(P2, T6)} = 0.188$

$$\mathrm{G0(P2, T6)} := \frac{\mathrm{x} \cdot (\mathrm{h1\_prime} - \mathrm{h6\_prime(T6)}) + \mathrm{h2(P2)} + \mathrm{h6\_prime(T6)} - \mathrm{h1\_prime} + \mathrm{q\_leak1}}{\mathrm{h8} - \mathrm{h7}}$$

i.e. $\mathrm{G0(P2, T6)} = 2.724$ **...kg of LN2/kg H2 compressed**

$$m_{\mathrm{LN2}}\mathrm{(P2, T6)} := \frac{1.25 \cdot \mathrm{G0(P2, T6)}}{\mathrm{LiqFraction\_x(P2, T6)}}$$

...kg of LN2/kg H2 liquefied.

i.e. $m_{\mathrm{LN2}}\mathrm{(P2, T6)} = 18.095$

$$\mathrm{Wperkggas(P2, T6)} := \frac{\mathrm{T1} \cdot (\mathrm{s1} - \mathrm{s2(P2)}) - (\mathrm{h1} - \mathrm{h2(P2)})}{\mathrm{eta\_iso}} + 1.25 \cdot \mathrm{G0(P2, T6)} \cdot \mathrm{WperkgliqN2}$$

i.e. $\mathrm{Wperkggas(P2, T6)} = 2.529 \times 10^{4}$ **kJ/kggas compressed,.. includes 25% extra LN2 for losses**

$$\mathrm{Wperkgliq(P2, T6)} := \frac{\mathrm{Wperkggas(P2, T6)}}{\mathrm{LiqFraction\_x(P2, T6)}}$$

kJ/kggas liquefied

i.e. $\mathrm{Wperkgliq(P2, T6)} = 1.344 \times 10^{5}$

$$\mathrm{COP(P2, T6)} := \frac{\mathrm{LiqFraction\_x(P2, T6)} \cdot (\mathrm{h1} - \mathrm{hf})}{\dfrac{\mathrm{T1} \cdot (\mathrm{s1} - \mathrm{s2(P2)}) - (\mathrm{h1} - \mathrm{h2(P2)})}{\mathrm{eta\_iso}} + 1.25 \cdot \mathrm{G0(P2, T6)} \cdot \mathrm{WperkgliqN2}}$$

i.e. $\mathrm{COP(P2, T6)} = 0.029$

Figure of Merit, FOM:

$$\mathrm{FOM}(P2, T6) := \frac{\mathrm{Wideal}}{\mathrm{Wperkgliq}(P2, T6)}$$

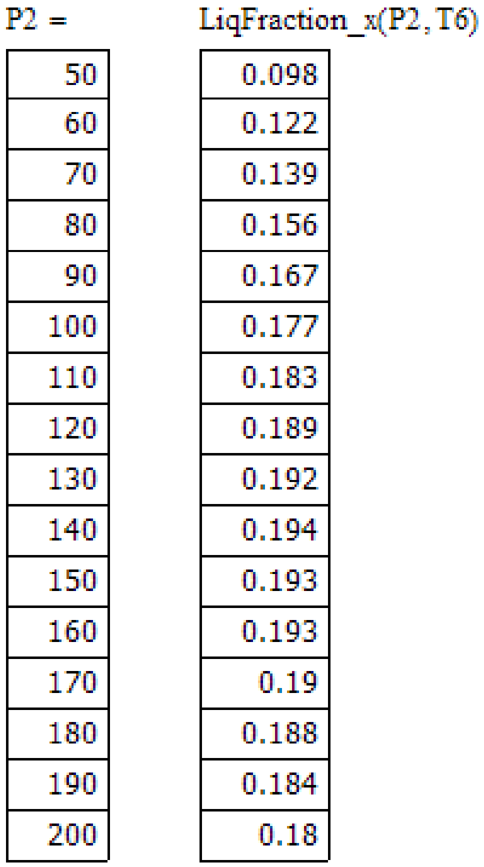
i.e. $\mathrm{FOM}(P2, T6) = 0.09$

Now, plot the various quantities:

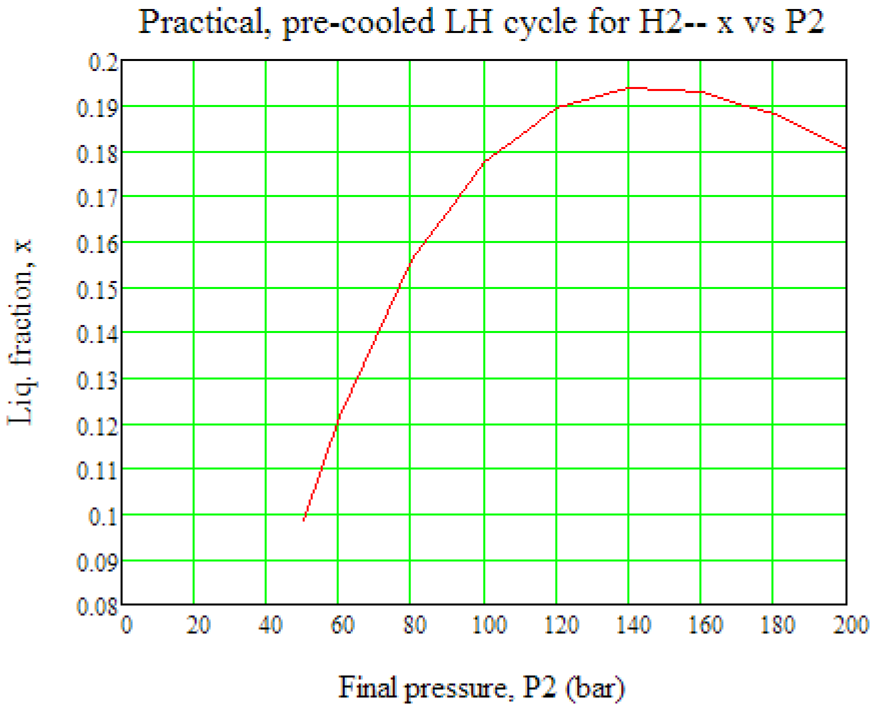
$T6 := 70$ K

$P2 := 50, 60 .. 200$ ... define a range variable

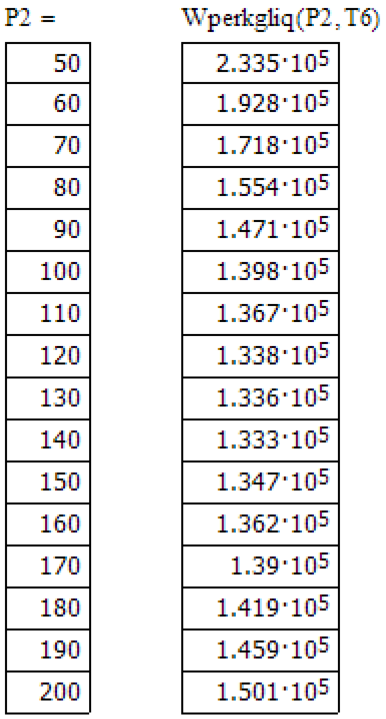
LiqFraction vs P2:



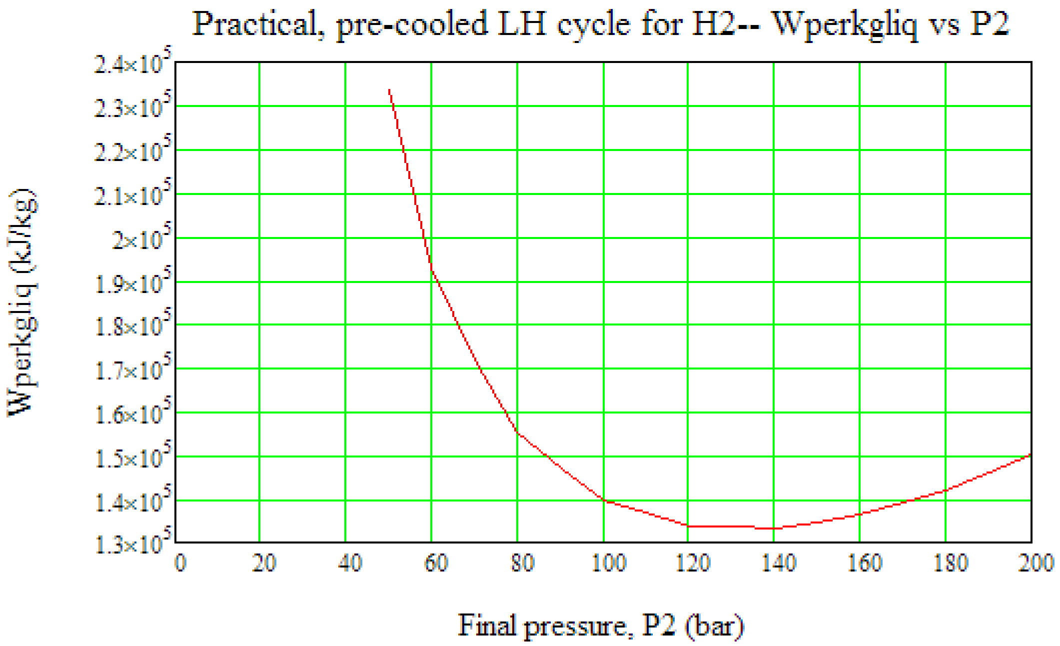
P2 =	LiqFraction_x(P2, T6)
50	0.098
60	0.122
70	0.139
80	0.156
90	0.167
100	0.177
110	0.183
120	0.189
130	0.192
140	0.194
150	0.193
160	0.193
170	0.19
180	0.188
190	0.184
200	0.18



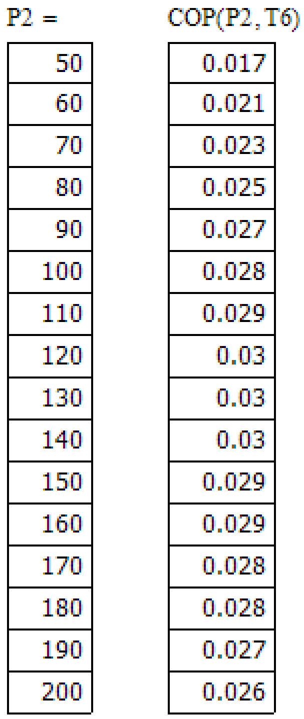
Wperkgliq vs P2:



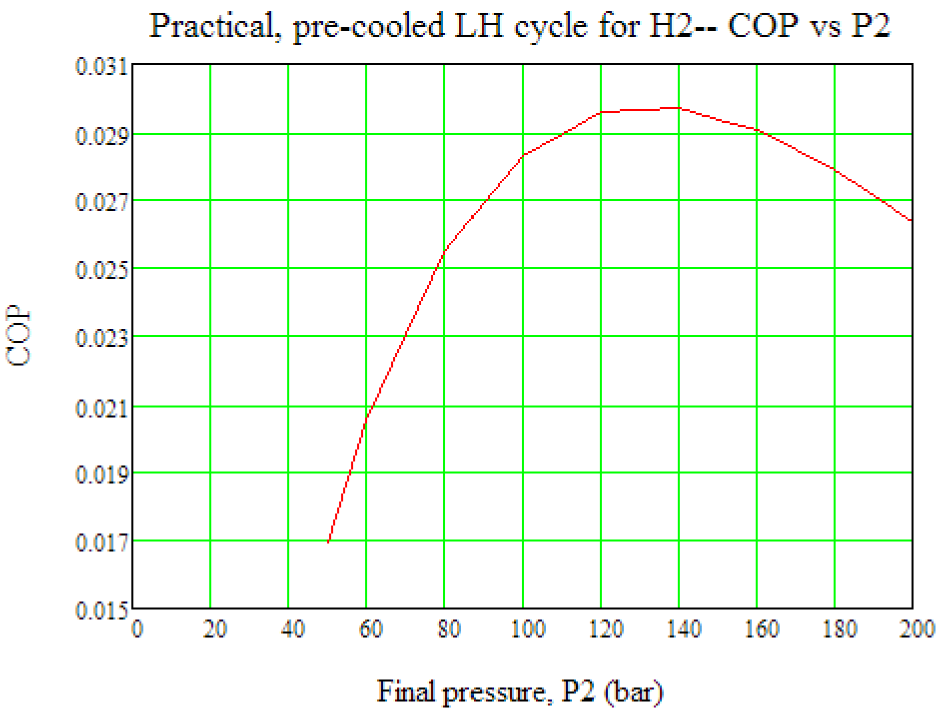
P2 =	Wperkgliq(P2, T6)
50	$2.335\cdot10^5$
60	$1.928\cdot10^5$
70	$1.718\cdot10^5$
80	$1.554\cdot10^5$
90	$1.471\cdot10^5$
100	$1.398\cdot10^5$
110	$1.367\cdot10^5$
120	$1.338\cdot10^5$
130	$1.336\cdot10^5$
140	$1.333\cdot10^5$
150	$1.347\cdot10^5$
160	$1.362\cdot10^5$
170	$1.39\cdot10^5$
180	$1.419\cdot10^5$
190	$1.459\cdot10^5$
200	$1.501\cdot10^5$



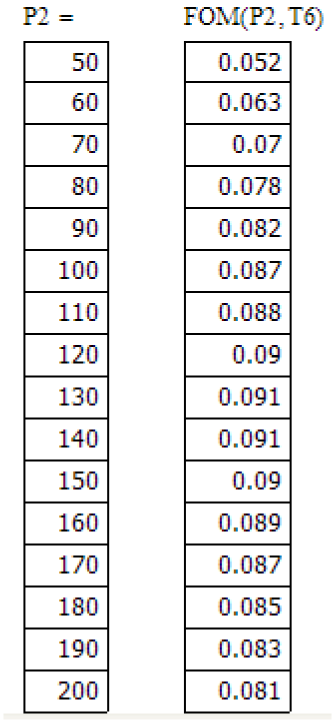
COP vs P2:



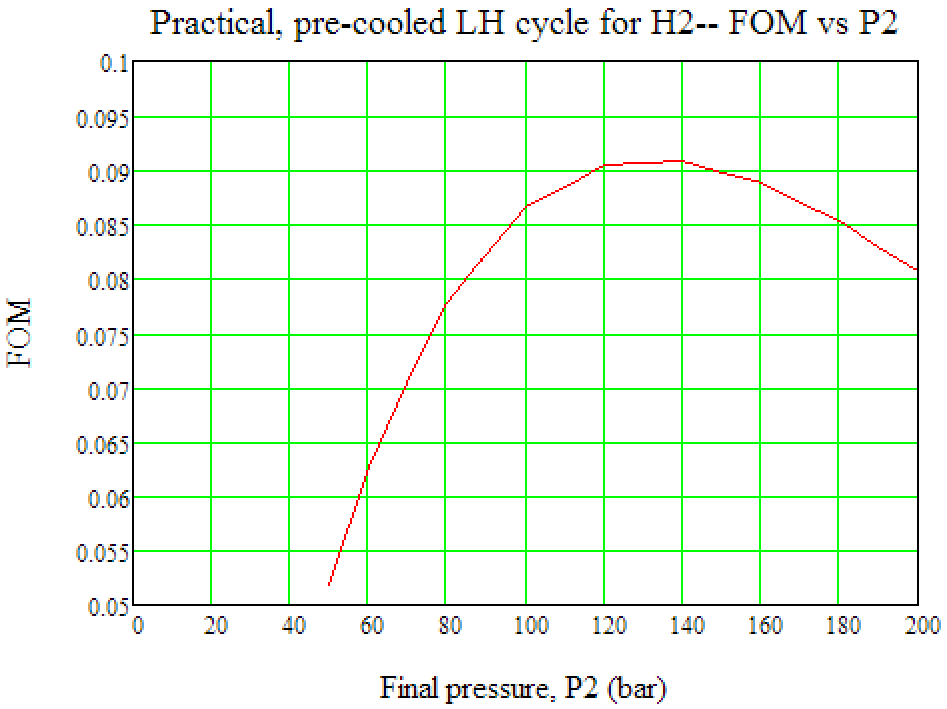
P2 =	COP(P2, T6)
50	0.017
60	0.021
70	0.023
80	0.025
90	0.027
100	0.028
110	0.029
120	0.03
130	0.03
140	0.03
150	0.029
160	0.029
170	0.028
180	0.028
190	0.027
200	0.026



FOM vs P2:

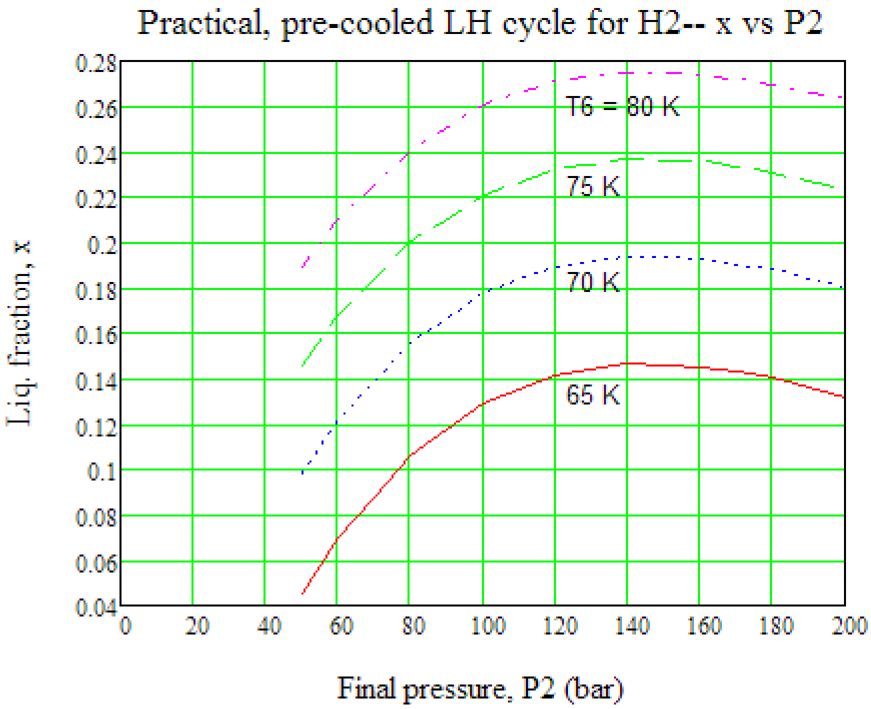


P2 =	FOM(P2, T6)
50	0.052
60	0.063
70	0.07
80	0.078
90	0.082
100	0.087
110	0.088
120	0.09
130	0.091
140	0.091
150	0.09
160	0.089
170	0.087
180	0.085
190	0.083
200	0.081

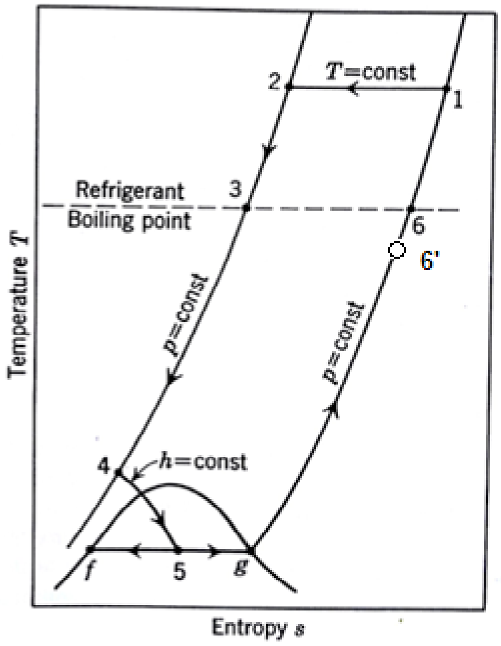


Also plot the effect of pre-cooling temp (T6) on Liq. fraction:

P2 (bar)	T6 = 65K	70K	75K	80K
50	0.045	0.098	0.146	0.189
60	0.07	0.122	0.168	0.21
70	0.088	0.139	0.184	0.225
80	0.106	0.156	0.2	0.241
90	0.118	0.167	0.211	0.251
100	0.129	0.177	0.221	0.26
110	0.135	0.183	0.227	0.266
120	0.142	0.189	0.232	0.271
130	0.144	0.192	0.234	0.273
140	0.146	0.194	0.236	0.275
150	0.146	0.193	0.236	0.275
160	0.145	0.193	0.235	0.274
170	0.143	0.19	0.233	0.272
180	0.14	0.188	0.231	0.27
190	0.136	0.184	0.227	0.266
200	0.132	0.18	0.224	0.263


Practical, pre-cooled LH cycle for H2-- x vs P2
Liq. fraction, x
Final pressure, P2 (bar)
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0
20
40
60
80
100
120
140
160
180
200
T6 = 80 K
75 K
70 K
65 K

Prob. 3.3.16 In a LN2 pre-cooled Linde-Hampson system for H2, the lower heat exchanger (between the LN2 bath and the LH2 receiver) has an effectiveness of 0.9. The high pressure stream enters the HX at 60.8 bar and 70 K, and the low pressure stream enters the HX at 1.013 bar and sat. vapor conditions. Determine the liquid yield 'y' for this system [1]. Also, plot y as final pressure varies from 50 bar to 200 bar.



Data:

$P1 := 1.013$ bar $T1 := 300$ K $P2 := 60.8$ bar

$T3 := 70$ K $T6 := 70$ K $\varepsilon := 0.9$ ....effectiveness of HX2

$Tsat := TSAT(P1)$ i.e $Tsat = 20.314$ K

$\Delta T := (1 - \varepsilon) \cdot (T3 - Tsat)$ ...warm end temp difference of HX2

i.e. $\Delta T = 4.969$ K

$T6\_prime := T6 - \Delta T$ K i.e. $T6\_prime = 65.031$ K

Now, we have:

$h3 := enthalpy\_H2(P2, T3)$ i.e. $h3 = 824.57$ kJ/kg

$h6 := enthalpy\_H2(P1, T6)$ i.e. $h6 = 978.702$ kJ/kg

$h6\_prime := enthalpy\_H2(P1, T6\_prime)$ i.e. $h6\_prime = 926.417$ kJ/kg

$hf := HFSATP(P1)$ i.e. $hf = -0.396$ kJ/kg

We have, for liquid yield, y:

$y := \frac{h6\_prime - h3}{h6\_prime - hf}$ i.e. $y = 0.11$ **...Ans.**

To plot y against P2, assuming that effectiveness of HX remains the constant at ε = 0.9:

First, write the relevant quantities as functions of P2:

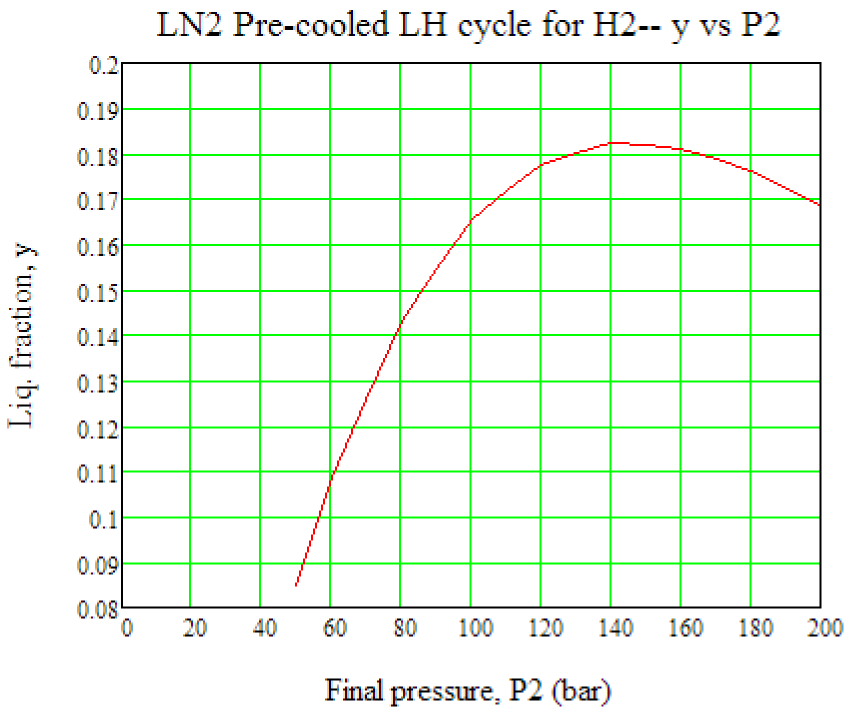
$h6\_prime(P2) := enthalpy\_H2(P1, T6\_prime)$

$h3(P2) := enthalpy\_H2(P2, T3)$

$y(P2) := \frac{h6\_prime(P2) - h3(P2)}{h6\_prime(P2) - hf}$

$P2 := 50, 60..200$ ...define a range variable

P2 =	y(P2) =
50	0.085
60	0.108
70	0.126
80	0.143
90	0.154
100	0.166
110	0.172
120	0.178
130	0.18
140	0.182
150	0.182
160	0.181
170	0.179
180	0.176
190	0.172
200	0.168


LN2 Pre-cooled LH cycle for H2-- y vs P2
Liq. fraction, y
0.2
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
0
20
40
60
80
100
120
140
160
180
200
Final pressure, P2 (bar)

Find the pressure at which liquid yield will be 0.15:

Use the 'Solve block' of Mathcad:

$P2 := 50$ ...trial value

Given

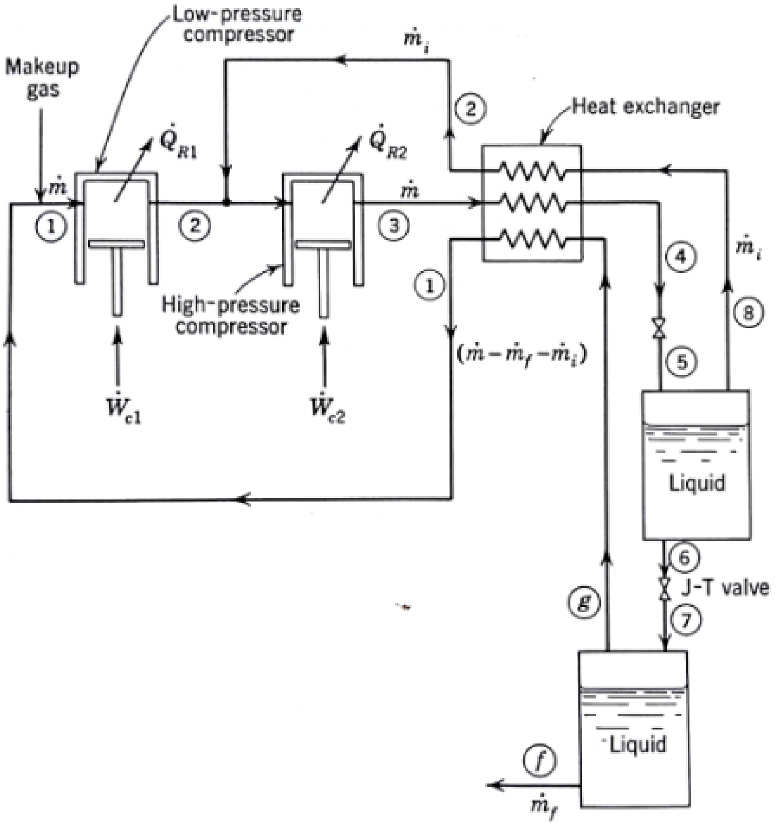
$y(P2) = 0.15$

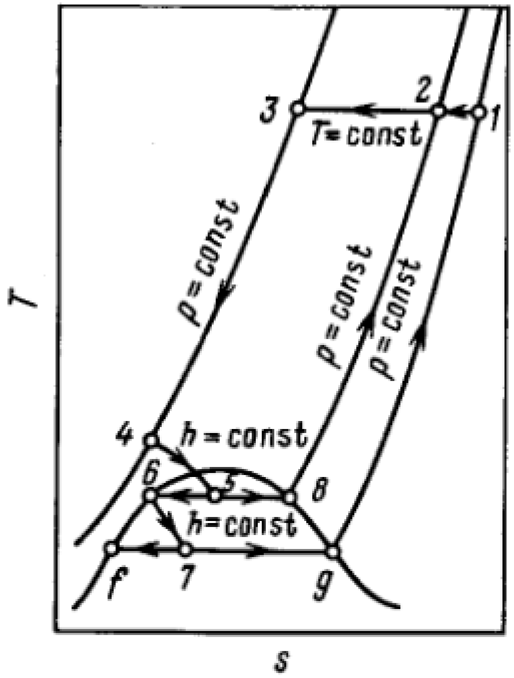
$\mathrm{Find}(P2) = 86.017$ ....pressure at which yield will be 0.15 .... Ans.

Note: Observe the ease with which Mathcad performs this 'trial and error' calculation.

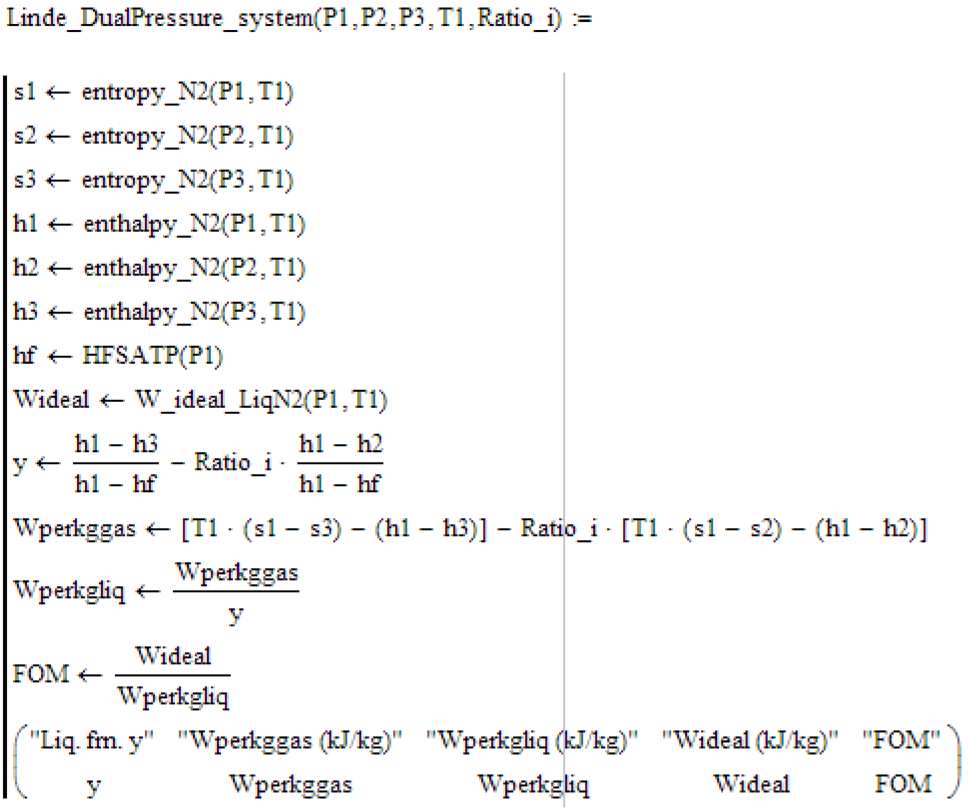
==

Prob. 3.3.17 Write a Mathcad Function to determine liquefaction fraction, y, work required per kg of gas compressed, work required per kg liquefied, Figure of Merit (FOM = Wideal / Work per kg liqid) for Linde dual pressure system, for Nitrogen. Take the initial pressure and temp of 1.013 bar (= 1 atm) and 300 K respectively and the final pressure is 200 bar. Intermediate pressure is 50 bar, and intermediate pressure flow ratio is = m_i / m = 0.8. Plot y, Wperkgliq and FOM as final pressure varies from 50 bar to 200 bar. Also, plot the Wperkgliq against P2 for intermediate flow ratio I = 0.5, 0.6, 0.7 and 0.8.



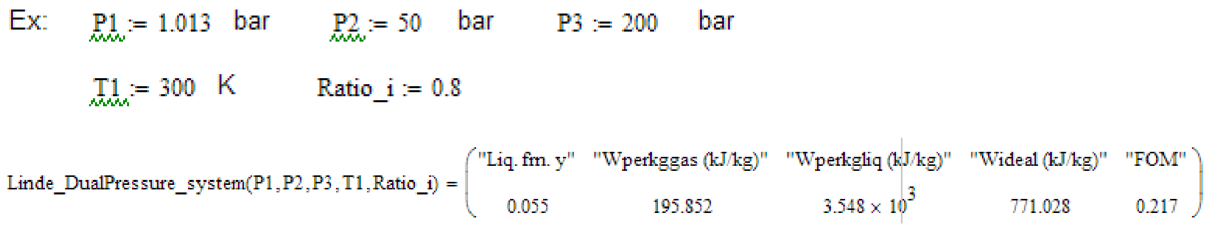


Mathcad Function:



```
Linde_DualPressure_system(P1,P2,P3,T1,Ratio_i) :=
    s1 ← entropy_N2(P1,T1)
    s2 ← entropy_N2(P2,T1)
    s3 ← entropy_N2(P3,T1)
    h1 ← enthalpy_N2(P1,T1)
    h2 ← enthalpy_N2(P2,T1)
    h3 ← enthalpy_N2(P3,T1)
    hf ← HFSATP(P1)
    Wideal ← W_ideal_LiqN2(P1,T1)
    $y \leftarrow \frac{h1 - h3}{h1 - hf} - Ratio\_i \cdot \frac{h1 - h2}{h1 - hf}$
    $Wperkggas \leftarrow [T1 \cdot (s1 - s3) - (h1 - h3)] - Ratio\_i \cdot [T1 \cdot (s1 - s2) - (h1 - h2)]$
    $Wperkgliq \leftarrow \frac{Wperkggas}{y}$
    $FOM \leftarrow \frac{Wideal}{Wperkgliq}$
    $\begin{pmatrix} \text{"Liq. frn. y"} & \text{"Wperkggas (kJ/kg)"} & \text{"Wperkgliq (kJ/kg)"} & \text{"Wideal (kJ/kg)"} & \text{"FOM"} \\ y & Wperkggas & Wperkgliq & Wideal & FOM \end{pmatrix}$
```

Now, apply this Function for the problem:

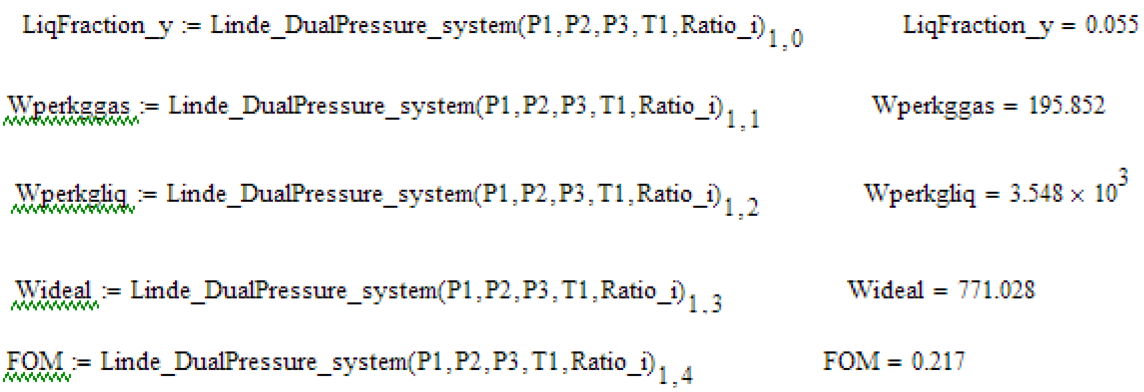


Ex: $P1 := 1.013$ bar $P2 := 50$ bar $P3 := 200$ bar

$T1 := 300$ K $Ratio\_i := 0.8$

$$\mathrm{Linde\_DualPressure\_system}(P1,P2,P3,T1,Ratio\_i) = \begin{pmatrix} \text{"Liq. frn. y"} & \text{"Wperkggas (kJ/kg)"} & \text{"Wperkgliq (kJ/kg)"} & \text{"Wideal (kJ/kg)"} & \text{"FOM"} \\ 0.055 & 195.852 & 3.548 \times 10^{3} & 771.028 & 0.217 \end{pmatrix}$$

Thus, we have:



$LiqFraction\_y := \mathrm{Linde\_DualPressure\_system}(P1,P2,P3,T1,Ratio\_i)_{1,0}$ $LiqFraction\_y = 0.055$

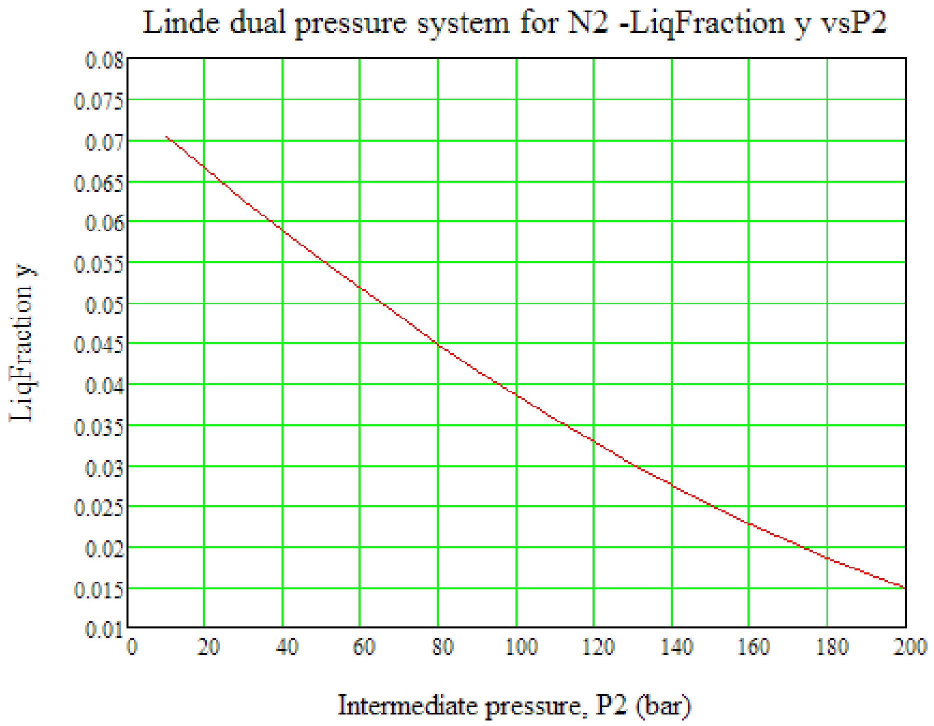
$Wperkggas := \mathrm{Linde\_DualPressure\_system}(P1,P2,P3,T1,Ratio\_i)_{1,1}$ $Wperkggas = 195.852$

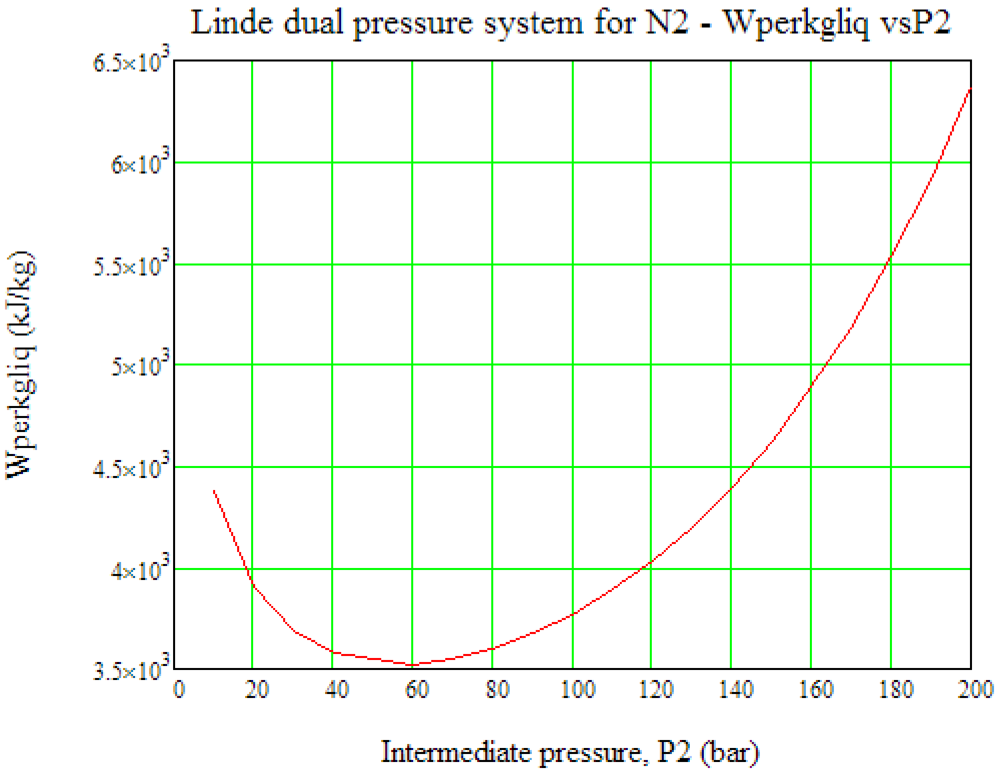
$Wperkgliq := \mathrm{Linde\_DualPressure\_system}(P1,P2,P3,T1,Ratio\_i)_{1,2}$ $Wperkgliq = 3.548 \times 10^{3}$

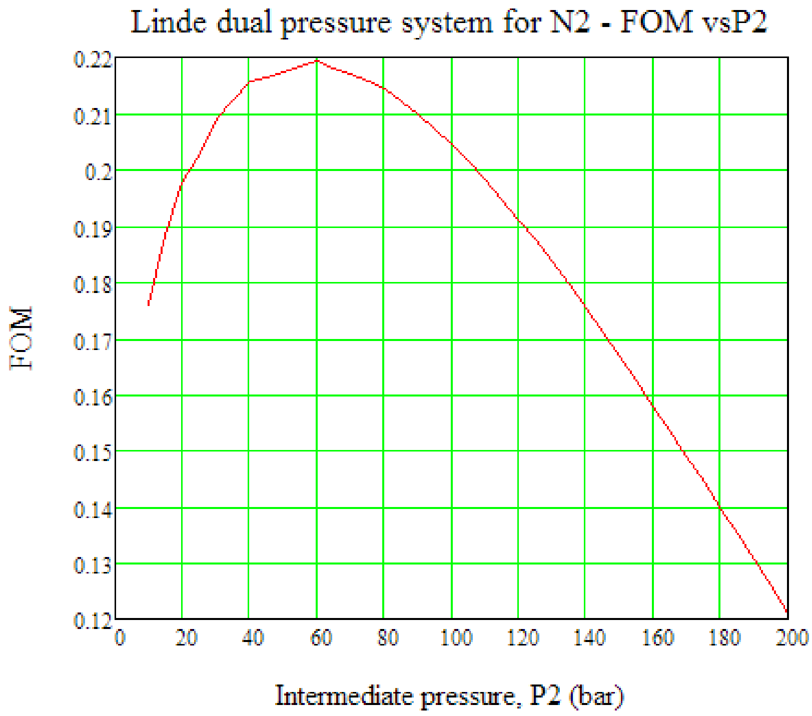
$Wideal := \mathrm{Linde\_DualPressure\_system}(P1,P2,P3,T1,Ratio\_i)_{1,3}$ $Wideal = 771.028$

$FOM := \mathrm{Linde\_DualPressure\_system}(P1,P2,P3,T1,Ratio\_i)_{1,4}$ $FOM = 0.217$

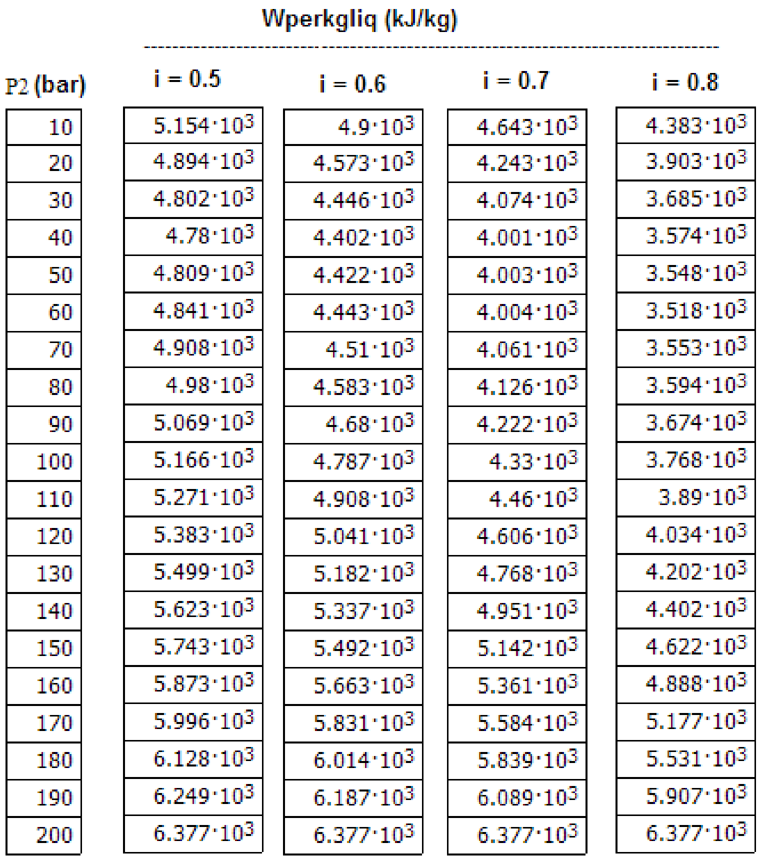
To plot LiqFraction_y against P2:







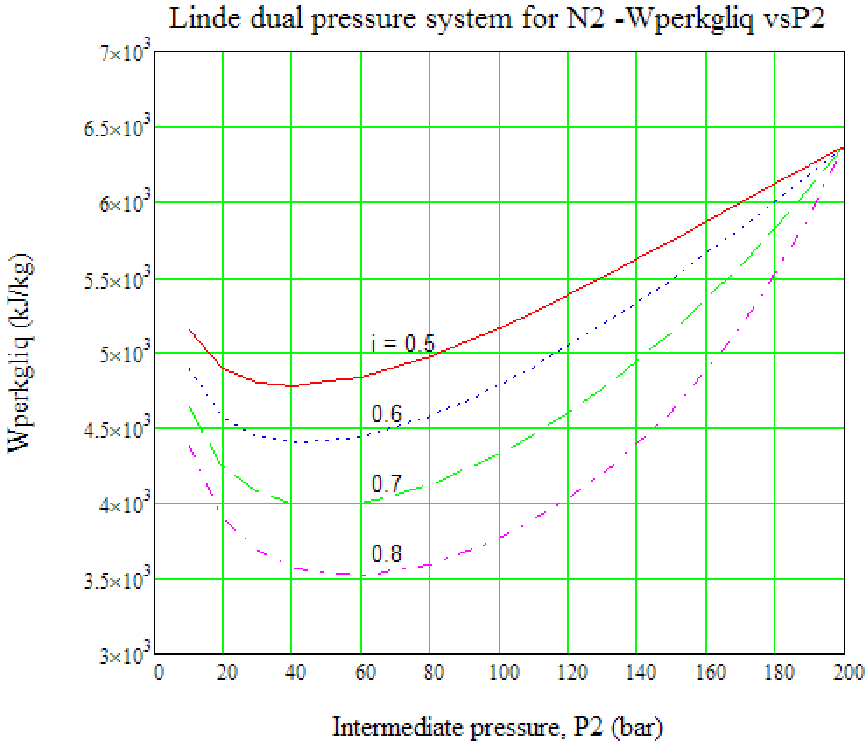
Plot Wperkgliq against P2 for Ratio_i = 0.5, 0.6, 0.7 and 0.8:



Wperkgliq (kJ/kg)

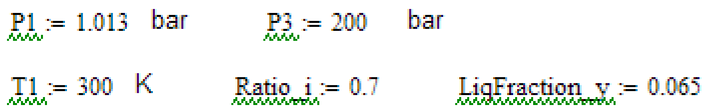
P2 (bar)	i = 0.5	i = 0.6	i = 0.7	i = 0.8
10	$5.154\cdot10^{3}$	$4.9\cdot10^{3}$	$4.643\cdot10^{3}$	$4.383\cdot10^{3}$
20	$4.894\cdot10^{3}$	$4.573\cdot10^{3}$	$4.243\cdot10^{3}$	$3.903\cdot10^{3}$
30	$4.802\cdot10^{3}$	$4.446\cdot10^{3}$	$4.074\cdot10^{3}$	$3.685\cdot10^{3}$
40	$4.78\cdot10^{3}$	$4.402\cdot10^{3}$	$4.001\cdot10^{3}$	$3.574\cdot10^{3}$
50	$4.809\cdot10^{3}$	$4.422\cdot10^{3}$	$4.003\cdot10^{3}$	$3.548\cdot10^{3}$
60	$4.841\cdot10^{3}$	$4.443\cdot10^{3}$	$4.004\cdot10^{3}$	$3.518\cdot10^{3}$
70	$4.908\cdot10^{3}$	$4.51\cdot10^{3}$	$4.061\cdot10^{3}$	$3.553\cdot10^{3}$
80	$4.98\cdot10^{3}$	$4.583\cdot10^{3}$	$4.126\cdot10^{3}$	$3.594\cdot10^{3}$
90	$5.069\cdot10^{3}$	$4.68\cdot10^{3}$	$4.222\cdot10^{3}$	$3.674\cdot10^{3}$
100	$5.166\cdot10^{3}$	$4.787\cdot10^{3}$	$4.33\cdot10^{3}$	$3.768\cdot10^{3}$
110	$5.271\cdot10^{3}$	$4.908\cdot10^{3}$	$4.46\cdot10^{3}$	$3.89\cdot10^{3}$
120	$5.383\cdot10^{3}$	$5.041\cdot10^{3}$	$4.606\cdot10^{3}$	$4.034\cdot10^{3}$
130	$5.499\cdot10^{3}$	$5.182\cdot10^{3}$	$4.768\cdot10^{3}$	$4.202\cdot10^{3}$
140	$5.623\cdot10^{3}$	$5.337\cdot10^{3}$	$4.951\cdot10^{3}$	$4.402\cdot10^{3}$
150	$5.743\cdot10^{3}$	$5.492\cdot10^{3}$	$5.142\cdot10^{3}$	$4.622\cdot10^{3}$
160	$5.873\cdot10^{3}$	$5.663\cdot10^{3}$	$5.361\cdot10^{3}$	$4.888\cdot10^{3}$
170	$5.996\cdot10^{3}$	$5.831\cdot10^{3}$	$5.584\cdot10^{3}$	$5.177\cdot10^{3}$
180	$6.128\cdot10^{3}$	$6.014\cdot10^{3}$	$5.839\cdot10^{3}$	$5.531\cdot10^{3}$
190	$6.249\cdot10^{3}$	$6.187\cdot10^{3}$	$6.089\cdot10^{3}$	$5.907\cdot10^{3}$
200	$6.377\cdot10^{3}$	$6.377\cdot10^{3}$	$6.377\cdot10^{3}$	$6.377\cdot10^{3}$

Now, plot the results:



Prob. 3.3.18 In a Linde dual pressure system, for Nitrogen, the gas is compressed from an initial pressure and temp of 1.013 bar and 300 K respectively to a final pressure is 200 bar. Find out the intermediate pressure P2, if the intermediate pressure flow ratio is = m_i / m = 0.7 and the liquefied fraction, y = 0.065 [1].

Solution:

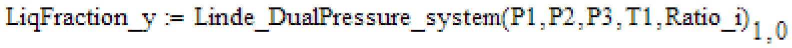


P1 := 1.013 bar P3 := 200 bar

T1 := 300 K Ratio_i := 0.7 LiqFraction_y := 0.065

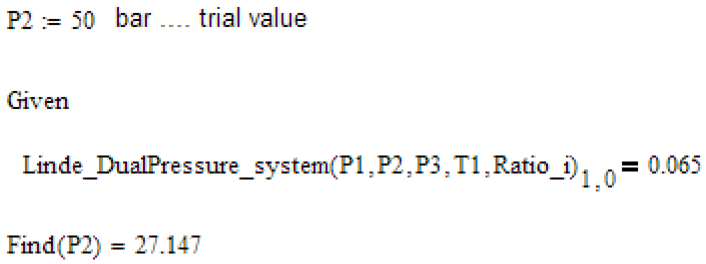
We find the intermediate pressure very easily by using the 'Solve block' of Mathcad and the Mathcad Function for Linde dual pressure system written above:

Note that LiqFraction_y is given as:



LiqFraction_y := Linde_DualPressure_system(P1,P2,P3,T1,Ratio_i)$_{1,0}$

Start with a trial value for P2:



P2 := 50 bar .... trial value

Given

Linde_DualPressure_system(P1,P2,P3,T1,Ratio_i)$_{1,0}$ = 0.065

Find(P2) = 27.147

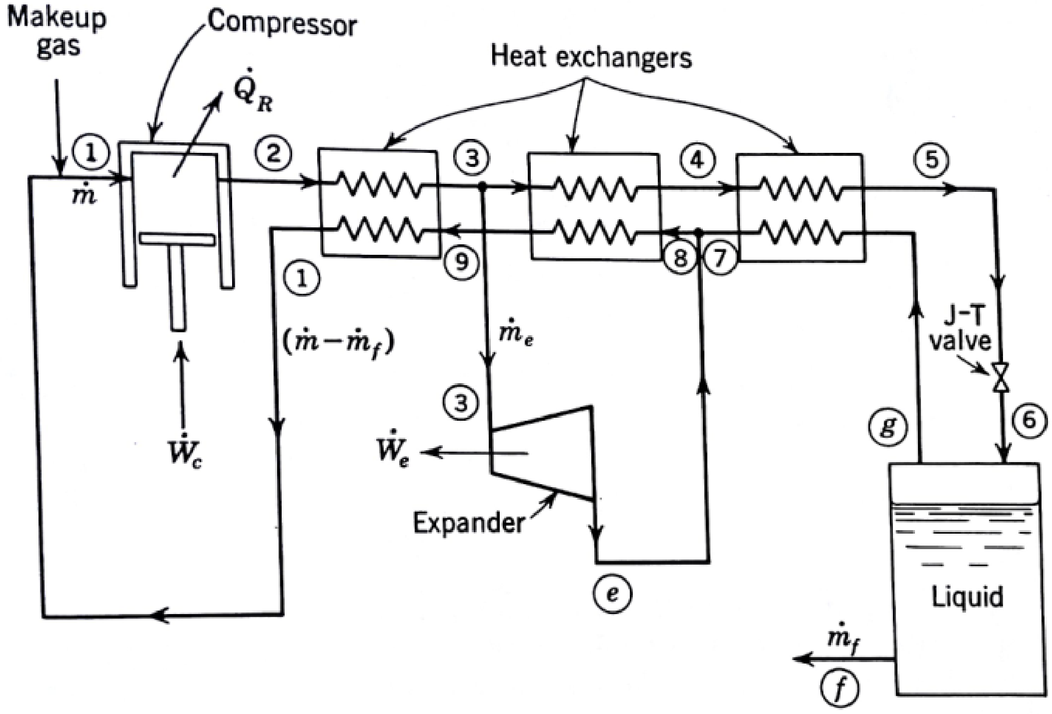
i.e. Intermediate pressure, P2 = 27.147 bar…Ans.

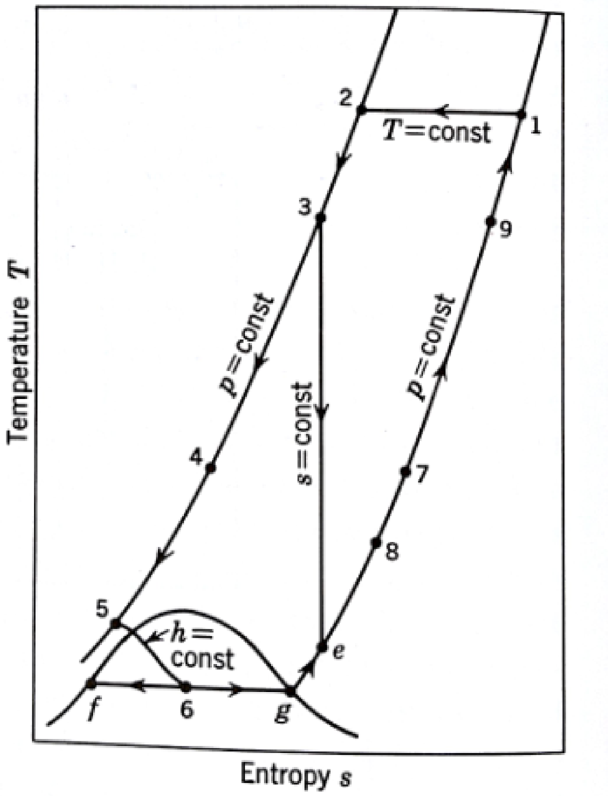
Prob. 3.3.19 (a). Write a Mathcad Function to calculate liquid yield and FOM of an Ideal Claude system.

(b). Then, determine the liquid yield, work output of expander per unit mass compressed, net work requirement of the system per unit mass liquefied, and the FOM assuming that the expander work is utilized to aid in the compression. Data for the Claude system is given below: Nitrogen is the working fluid, the system operates from 1.013 bar (1 atm) and 295 K to 50 bar. At 50 bar and 270 K, 60% of the main flow is diverted to the reversible adiabatic expander. The remainder flows through the heat exchangers and expands through the expansion valve to 1.013 bar [1].

Also plot the liq. Fraction y and net work required per unit mass liquefied against expander flow rate ratio from P2 = 30 bar to 70bar, other parameters remaining the same.

Following figures show the schematic diagram and the T-s diagram for Claude system:





Solution:

First, write the Mathcad Function for Ideal Claude system for N2:

```
IdealClaudeSystem_N2(P1,P2,T1,T3,FlowRatio_x) :=
```

$s1 \leftarrow \text{entropy\_N2}(P1, T1)$

$s2 \leftarrow \text{entropy\_N2}(P2, T1)$

$s3 \leftarrow \text{entropy\_N2}(P2, T3)$

$se \leftarrow s3$

$h1 \leftarrow \text{enthalpy\_N2}(P1, T1)$

$h2 \leftarrow \text{enthalpy\_N2}(P2, T1)$

$h3 \leftarrow \text{enthalpy\_N2}(P2, T3)$

$he \leftarrow \text{enthalpy\_N2\_Ps}(P1, se)$

$hf \leftarrow \text{HFSATP}(P1)$

$\text{Wideal} \leftarrow \text{W\_ideal\_LiqN2}(P1, T1)$

$$y \leftarrow \frac{h1 - h2}{h1 - hf} + \text{FlowRatio\_x} \cdot \frac{h3 - he}{h1 - hf}$$

$\text{Wnet\_perkggas} \leftarrow [T1 \cdot (s1 - s2) - (h1 - h2)] - \text{FlowRatio\_x} \cdot (h3 - he)$

$$\text{Wnet\_perkgliq} \leftarrow \frac{\text{Wnet\_perkggas}}{y}$$

$$\text{FOM} \leftarrow \frac{\text{Wideal}}{\text{Wnet\_perkgliq}}$$

$\text{Wexp\_perkggas} \leftarrow \text{FlowRatio\_x} \cdot (h3 - he)$

$$\begin{pmatrix} \text{"LiqFraction-y"} & \text{"Wexp\_perkggas(kJ/kg)"} & \text{"Wnet\_perkggas(kJ/kg)"} & \text{"Wnet\_perkgliq(kJ/kg)"} & \text{"Wideal(kJ/kg)"} & \text{"FOM"} \\ y & \text{Wexp\_perkggas} & \text{Wnet\_perkggas} & \text{Wnet\_perkgliq} & \text{Wideal} & \text{FOM} \end{pmatrix}$$

Now, solve the problem:

Data:

$P1 := 1.013$ bar $\quad P2 := 50$ bar $\quad T1 := 295$ K $\quad T3 := 270$ K

$\text{FlowRatio\_x} := 0.6$

And, we get:

$\text{IdealClaudeSystem\_N2}(P1, P2, T1, T3, \text{FlowRatio\_x}) =$

$$\begin{pmatrix} \text{"LiqFraction-y"} & \text{"Wexp\_perkggas(kJ/kg)"} & \text{"Wnet\_perkggas(kJ/kg)"} & \text{"Wnet\_perkgliq(kJ/kg)"} & \text{"Wideal(kJ/kg)"} & \text{"FOM"} \\ 0.276 & 107.59 & 231.969 & 840.655 & 751.023 & 0.893 \end{pmatrix}$$

i.e.

$\text{LiqFraction\_y} := \text{IdealClaudeSystem\_N2}(P1, P2, T1, T3, \text{FlowRatio\_x})_{1,0} \qquad \text{LiqFraction\_y} = 0.276$

Work of expander per kg gas compressed in compressor =

$Wexp\_perkggas := IdealClaudeSystem\_N2(P1,P2,T1,T3,FlowRatio\_x)_{1,1}$ $\quad Wexp\_perkggas = 107.59$

$Wnet\_perkggas := IdealClaudeSystem\_N2(P1,P2,T1,T3,FlowRatio\_x)_{1,2}$ $\quad Wnet\_perkggas = 231.969$

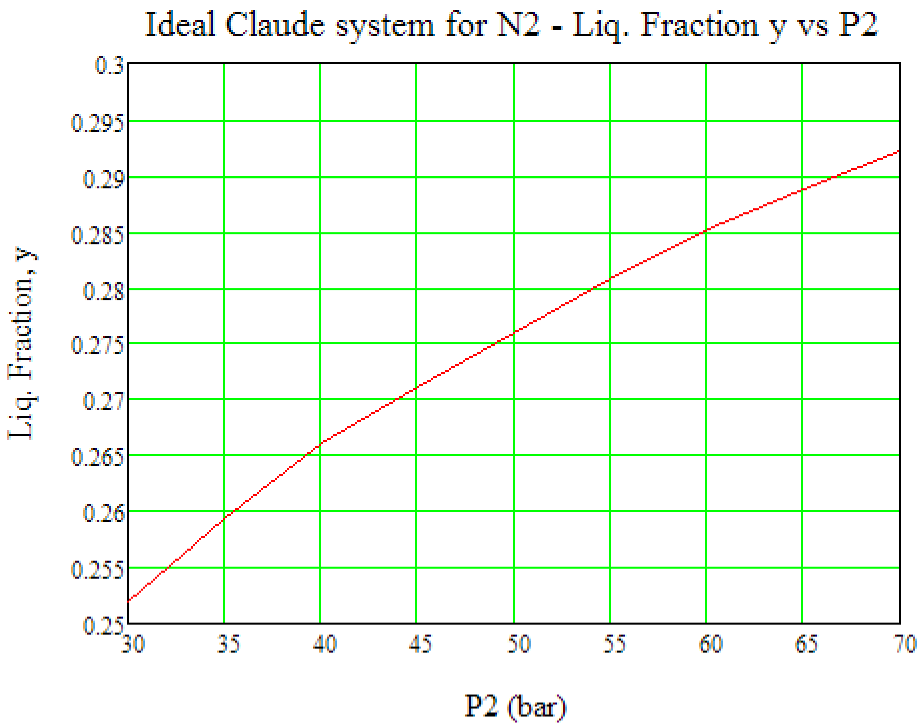
$Wnet\_perkgliq := IdealClaudeSystem\_N2(P1,P2,T1,T3,FlowRatio\_x)_{1,3}$ $\quad Wnet\_perkgliq = 840.655$

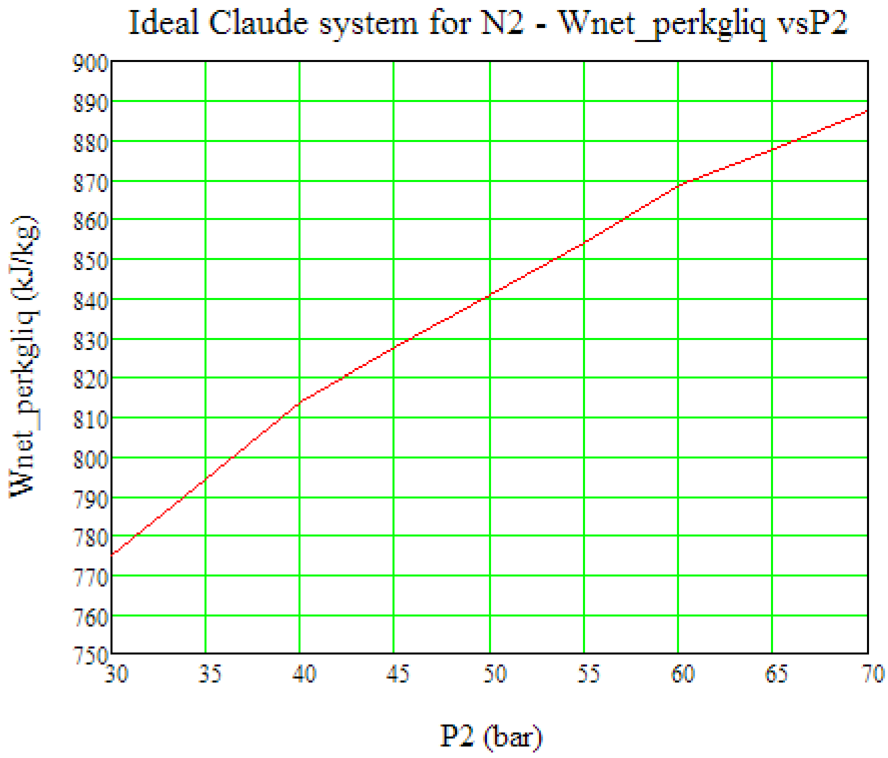
$Wideal := IdealClaudeSystem\_N2(P1,P2,T1,T3,FlowRatio\_x)_{1,4}$ $\quad Wideal = 751.023$

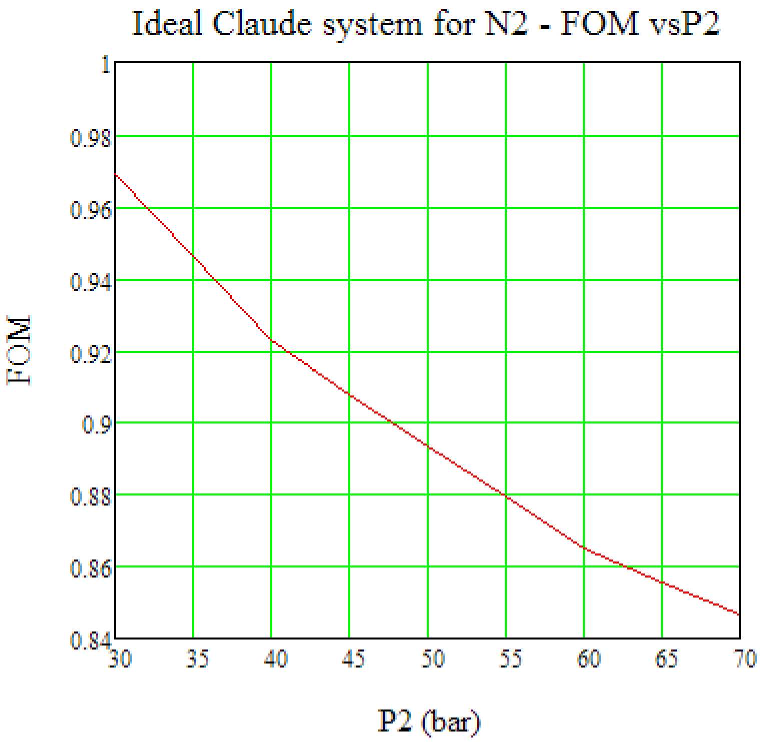
$FOM := IdealClaudeSystem\_N2(P1,P2,T1,T3,FlowRatio\_x)_{1,5}$ $\quad FOM = 0.893$

To plot LiqFraction-y and Wnet_perkgliq for various P2:

$P2 := 30, 35 .. 70$ ..define a range variable




Ideal Claude system for N2 - Wnet_perkgliq vsP2
Wnet_perkgliq (kJ/kg)
900
890
880
870
860
850
840
830
820
810
800
790
780
770
760
750
30
35
40
45
50
55
60
65
70
P2 (bar)



Prob. 3.3.20 In an Ideal Claude system for Nitrogen, the system operates from 101.3 kPa (1 atm) and 293 K to 4.05 MPa. At 4.05 MPa and 240 K, fraction 'x' of the main flow is diverted to the reversible adiabatic expander. Remainder flows through the heat exchangers and expands through the expansion valve to 101.3 kPa. Determine the expander flow rate ratio (x) for a liquid yield of 0.2.[1].

Data:

$P1 := 1.013$ bar $P2 := 40.5$ bar $T1 := 293$ K $T3 := 240$ K

$y := 0.2$ ...liq. fraction

FlowRatio_x is to be found out:

Use the 'Solve Block' of Mathcad, along with the Function written above:

$FlowRatio\_x := 0.2$ ...trial value

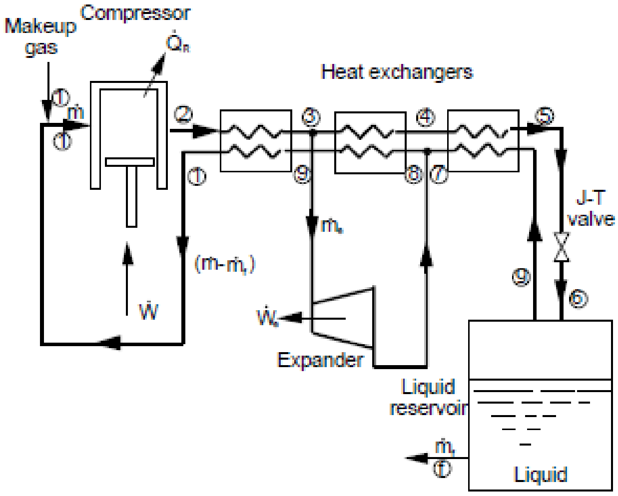
Given

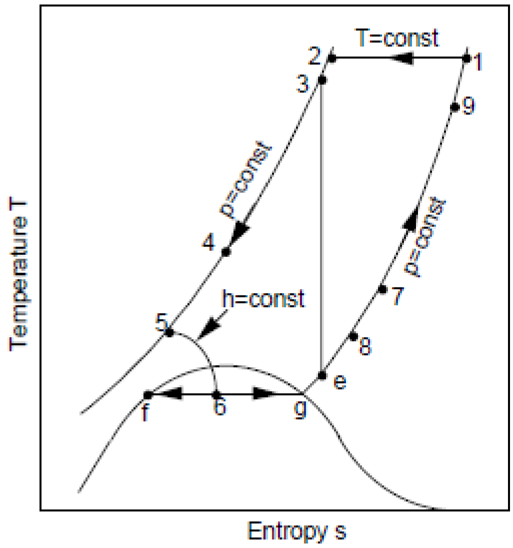
$y = IdealClaudeSystem\_N2(P1, P2, T1, T3, FlowRatio\_x)_{1,0}$

$Find(FlowRatio\_x) = 0.499$

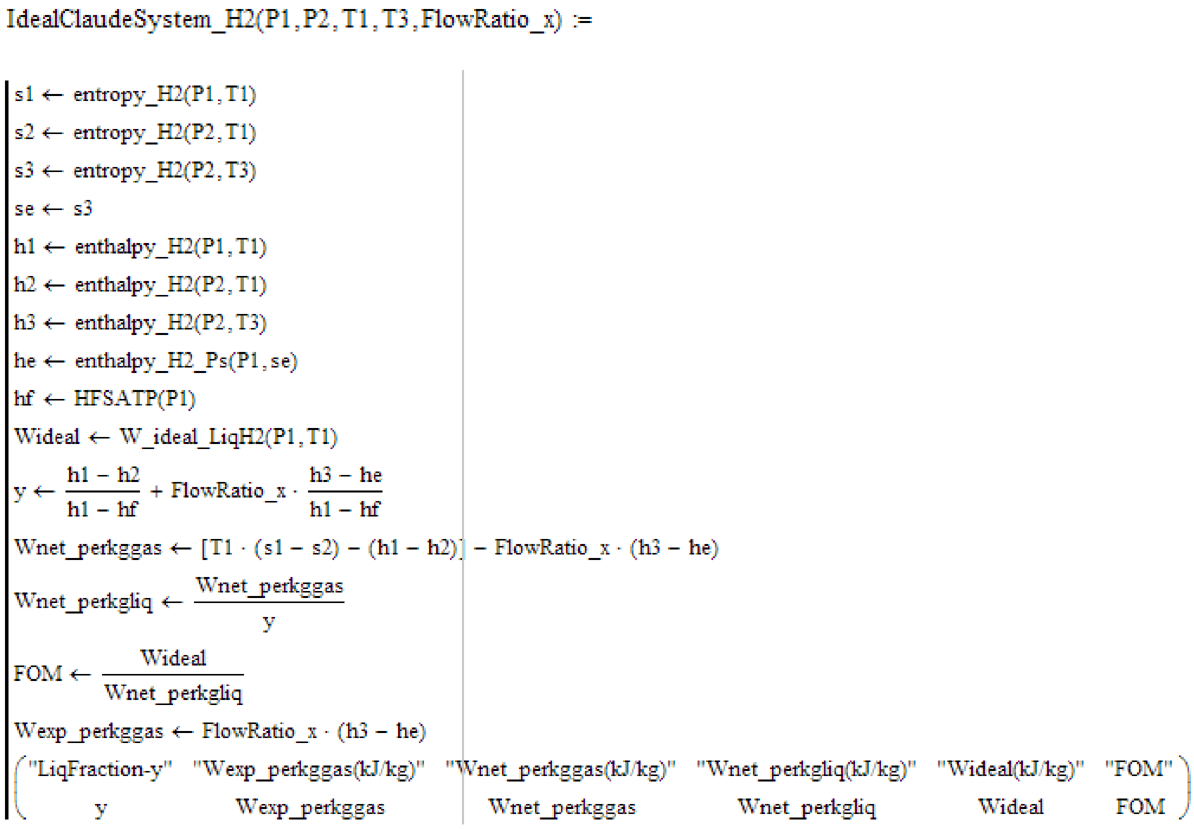
i.e. Flow ratio through the expander is = 0.499…Ans.

Prob. 3.3.21 In an Ideal Claude system for hydrogen, the system operates from 101.3 kPa (1 atm) and 293 K to 4.05 MPa. At 4.05 MPa and 180 K, fraction 'x' = 50% of the main flow is diverted to the reversible adiabatic expander. Remainder flows through the heat exchangers and expands through the expansion valve to 101.3 kPa. Determine the liquid yield, work per unit mass liquefied (assuming that the expander work is utilized in compression), and the FOM of the system. [1].





First, write the Mathcad Function:



```
$\text{IdealClaudeSystem\_H2(P1, P2, T1, T3, FlowRatio\_x)} :=$
    $s1 \leftarrow \text{entropy\_H2(P1, T1)}$
    $s2 \leftarrow \text{entropy\_H2(P2, T1)}$
    $s3 \leftarrow \text{entropy\_H2(P2, T3)}$
    $se \leftarrow s3$
    $h1 \leftarrow \text{enthalpy\_H2(P1, T1)}$
    $h2 \leftarrow \text{enthalpy\_H2(P2, T1)}$
    $h3 \leftarrow \text{enthalpy\_H2(P2, T3)}$
    $he \leftarrow \text{enthalpy\_H2\_Ps(P1, se)}$
    $hf \leftarrow \text{HFSATP(P1)}$
    $\text{Wideal} \leftarrow \text{W\_ideal\_LiqH2(P1, T1)}$
    $y \leftarrow \frac{h1 - h2}{h1 - hf} + \text{FlowRatio\_x} \cdot \frac{h3 - he}{h1 - hf}$
    $\text{Wnet\_perkggas} \leftarrow [T1 \cdot (s1 - s2) - (h1 - h2)] - \text{FlowRatio\_x} \cdot (h3 - he)$
    $\text{Wnet\_perkgliq} \leftarrow \frac{\text{Wnet\_perkggas}}{y}$
    $\text{FOM} \leftarrow \frac{\text{Wideal}}{\text{Wnet\_perkgliq}}$
    $\text{Wexp\_perkggas} \leftarrow \text{FlowRatio\_x} \cdot (h3 - he)$
    $\begin{pmatrix} \text{"LiqFraction-y"} & \text{"Wexp\_perkggas(kJ/kg)"} & \text{"Wnet\_perkggas(kJ/kg)"} & \text{"Wnet\_perkgliq(kJ/kg)"} & \text{"Wideal(kJ/kg)"} & \text{"FOM"} \\ y & \text{Wexp\_perkggas} & \text{Wnet\_perkggas} & \text{Wnet\_perkgliq} & \text{Wideal} & \text{FOM} \end{pmatrix}$
```

Now, solve the problem:

$P1 := 1.013$ bar $\quad P2 := 40.5$ bar $\quad T1 := 293$ K $\quad T3 := 180$ K

$FlowRatio\_x := 0.5$

Then, from the Function written above:

$IdealClaudeSystem\_H2(P1,P2,T1,T3,FlowRatio\_x) =$

$$\begin{pmatrix} \text{"LiqFraction-y"} & \text{"Wexp\_perkggas(kJ/kg)"} & \text{"Wnet\_perkggas(kJ/kg)"} & \text{"Wnet\_perkgliq(kJ/kg)"} & \text{"Wideal(kJ/kg)"} & \text{"FOM"} \\ 0.197 & 775.942 & 3.716 \times 10^{3} & 1.889 \times 10^{4} & 1.172 \times 10^{4} & 0.621 \end{pmatrix}$$

i.e.

$LiqFraction\_y := IdealClaudeSystem\_H2(P1,P2,T1,T3,FlowRatio\_x)_{1,0} \qquad LiqFraction\_y = 0.197$

Work of expander per kg gas compressed in compressor =

$Wexp\_perkggas := IdealClaudeSystem\_H2(P1,P2,T1,T3,FlowRatio\_x)_{1,1} \qquad Wexp\_perkggas = 775.942$

$Wnet\_perkggas := IdealClaudeSystem\_H2(P1,P2,T1,T3,FlowRatio\_x)_{1,2} \qquad Wnet\_perkggas = 3.716 \times 10^{3}$

$Wnet\_perkgliq := IdealClaudeSystem\_H2(P1,P2,T1,T3,FlowRatio\_x)_{1,3} \qquad Wnet\_perkgliq = 1.889 \times 10^{4}$

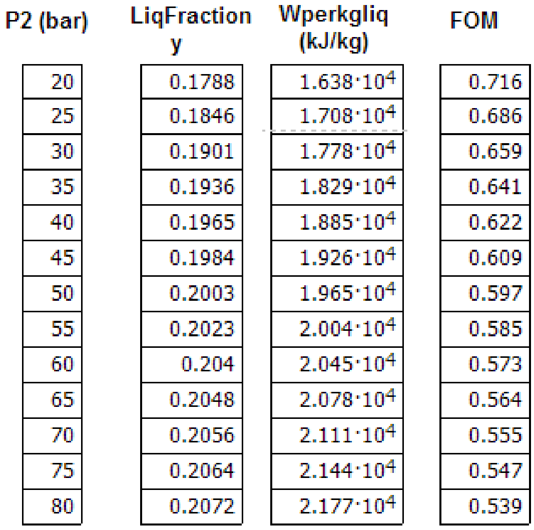
$Wideal := IdealClaudeSystem\_H2(P1,P2,T1,T3,FlowRatio\_x)_{1,4} \qquad Wideal = 1.172 \times 10^{4}$

$FOM := IdealClaudeSystem\_H2(P1,P2,T1,T3,FlowRatio\_x)_{1,5} \qquad FOM = 0.621$

Plot y, Wperkgliq and FOM for P2 = 20 to 80 bar:

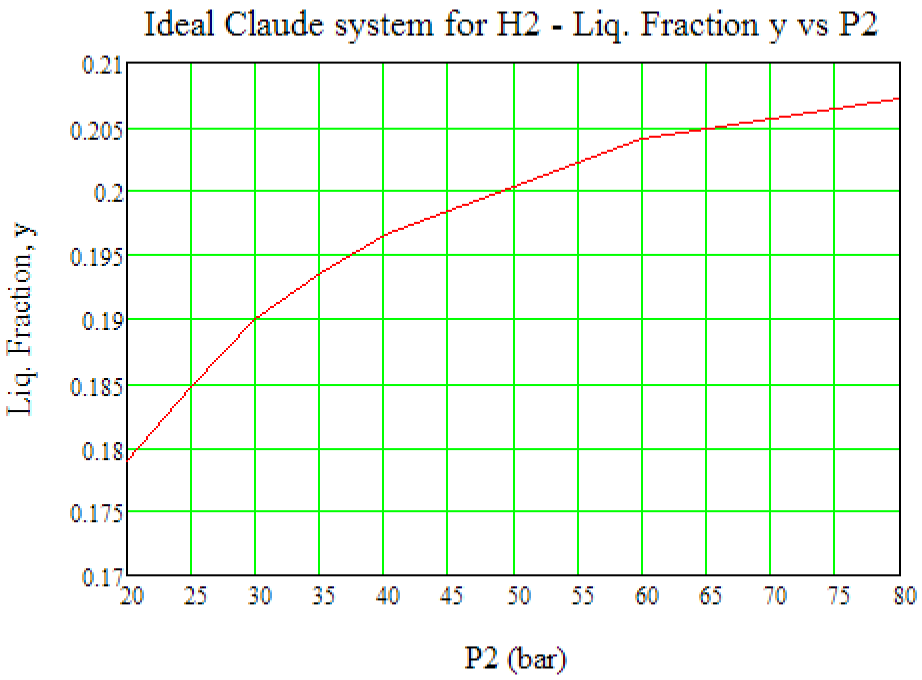
$P2 := 20, 25 .. 80$ ....define a range variable

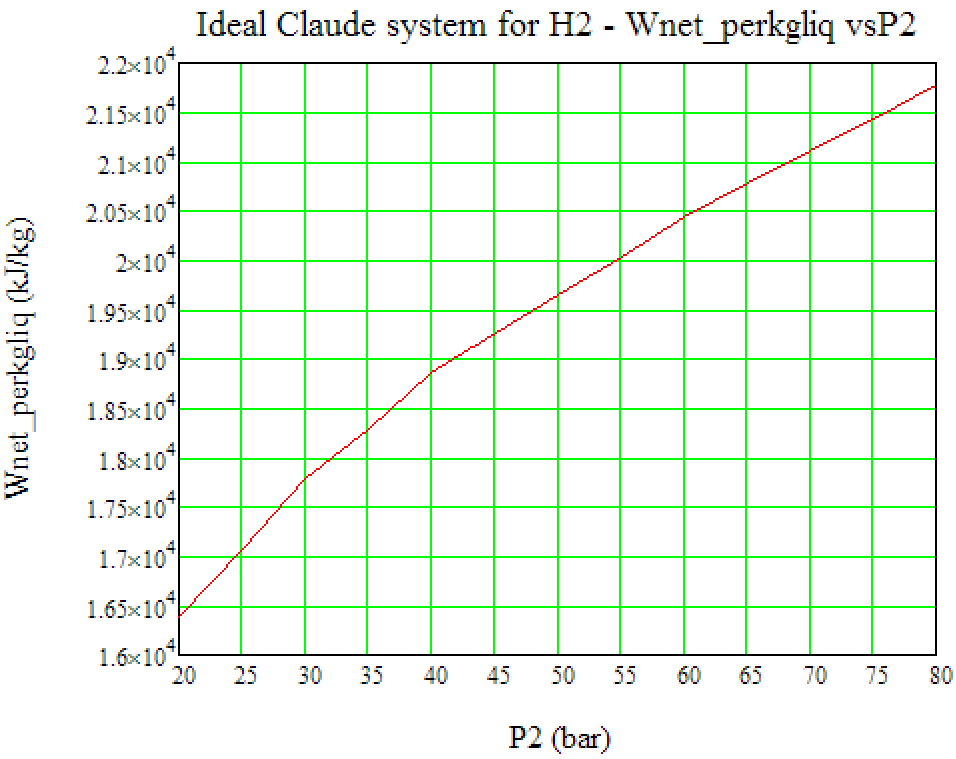
We get:



P2 (bar)	LiqFraction y	Wperkgliq (kJ/kg)	FOM
20	0.1788	$1.638 \cdot 10^4$	0.716
25	0.1846	$1.708 \cdot 10^4$	0.686
30	0.1901	$1.778 \cdot 10^4$	0.659
35	0.1936	$1.829 \cdot 10^4$	0.641
40	0.1965	$1.885 \cdot 10^4$	0.622
45	0.1984	$1.926 \cdot 10^4$	0.609
50	0.2003	$1.965 \cdot 10^4$	0.597
55	0.2023	$2.004 \cdot 10^4$	0.585
60	0.204	$2.045 \cdot 10^4$	0.573
65	0.2048	$2.078 \cdot 10^4$	0.564
70	0.2056	$2.111 \cdot 10^4$	0.555
75	0.2064	$2.144 \cdot 10^4$	0.547
80	0.2072	$2.177 \cdot 10^4$	0.539

And, plot the results:




Ideal Claude system for H2 - Wnet_perkgliq vsP2
Wnet_perkgliq (kJ/kg)
2.2×10^4
2.15×10^4
2.1×10^4
2.05×10^4
2×10^4
1.95×10^4
1.9×10^4
1.85×10^4
1.8×10^4
1.75×10^4
1.7×10^4
1.65×10^4
1.6×10^4
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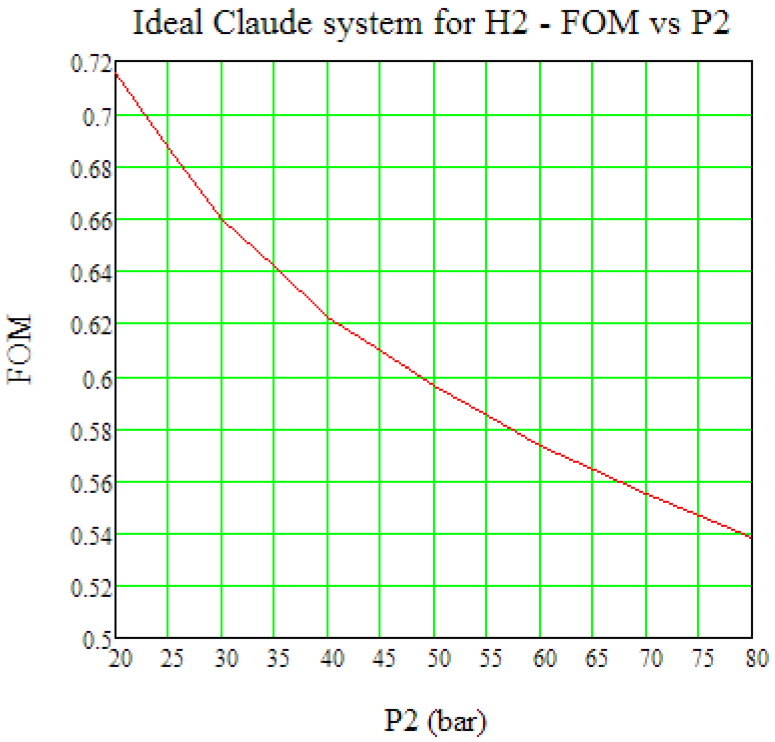
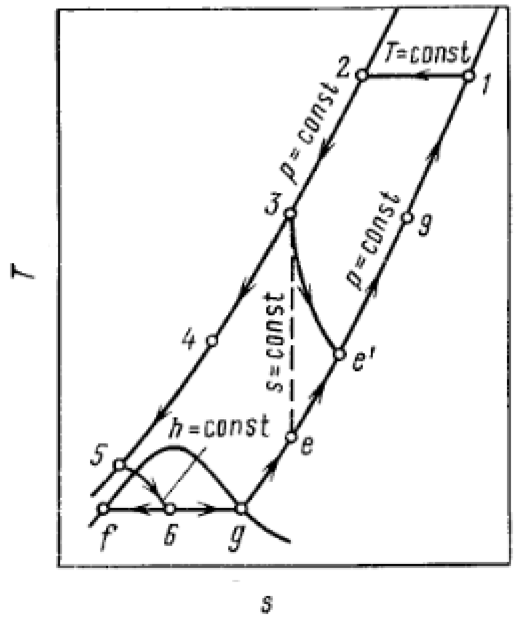
55
60
65
70
75
80
P2 (bar)



Prob. 3.3.22 Data for a Claude system is given below:
Nitrogen is the working fluid, the system operates from 101.3 kPa (1 atm) and 293 K to 70 bar. At 70 bar and 270 K, 40% of the main flow is diverted to the reversible adiabatic expander. Remainder flows through the heat exchangers and expands through the expansion valve to 101.3 kPa. Assume that the expander has an adiabatic effcy of 80% and a mech. effcy of 90%. Compressor has an overall effcy of 75% and all heat exchangers are 100% effective. After compression, air is after-cooled to 293 K. Determine the liquid yield, work output of expander per unit mass compressed and the net work requirement of the system per unit mass liquefied, assuming that the expander work is utilized to aid in the compression. [1].

When the efficiencies of compressor and expander are less than 100%, with the effectiveness of heat exchangers remaining 100%:

Temp-entropy diagram for this case is:



And, summary of formulas applicable is given below:
Compressor work per unit mass of gas liquefied:

$$-\frac{\dot W_c}{\dot m_f}=\frac{1}{y\eta_{c0}}\left[T_1(s_1-s_2)-(h_1-h_2)\right].$$

Actual enthalpy change across the expander:

$$h_3-h'_e=\eta_{ad}(h_3-h_e).$$

Liquid yield:

$$y=\frac{h_1-h_2}{h_1-h_f}+x\eta_{ad}\frac{h_3-h_e}{h_1-h_f}.$$

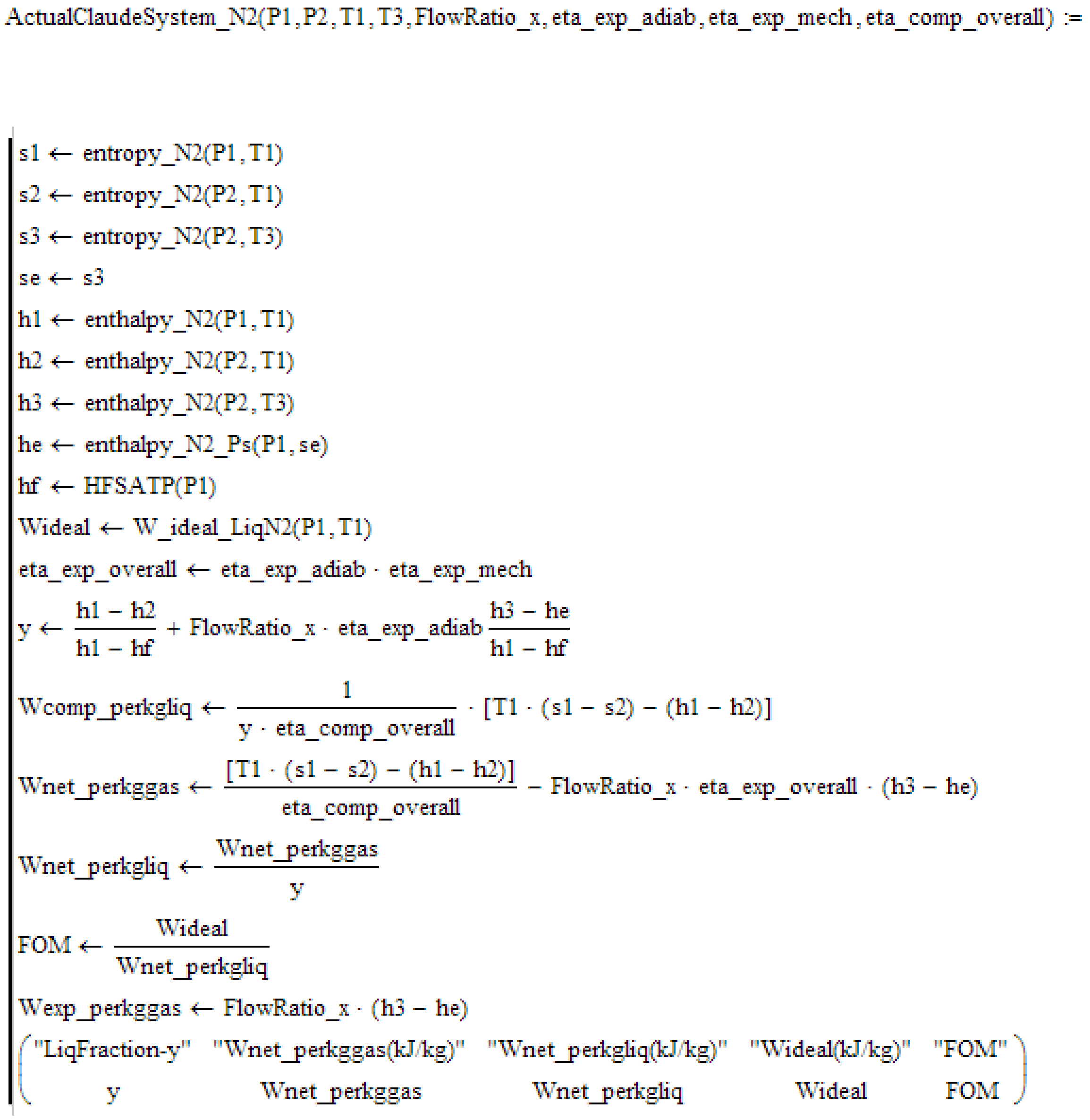
Net work requirement per unit mass compressed, when expander work is utilized:

$$\dot W/\dot m=-\dot W_c/\dot m-x\eta_{e,m}(h_3-h'_e)$$

Or,

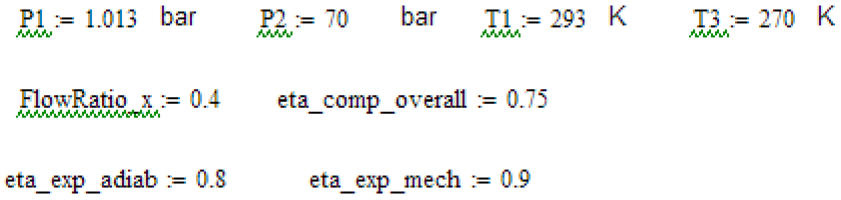
$$-\frac{\dot W}{\dot m}=\frac{T_1(s_1-s_2)-(h_1-h_2)}{\eta_{c0}}-x\eta_{e0}(h_3-h_e),$$

Mathcad Function:



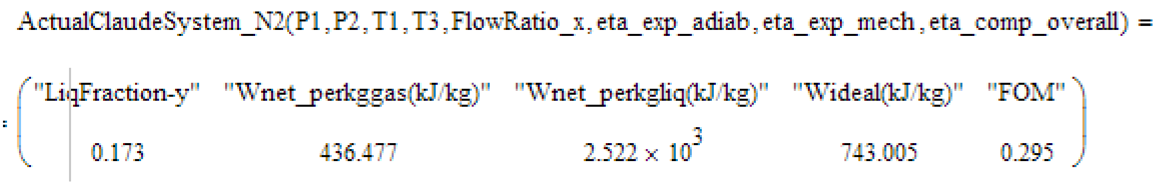
```
ActualClaudeSystem_N2(P1, P2, T1, T3, FlowRatio_x, eta_exp_adiab, eta_exp_mech, eta_comp_overall) :=
    s1 ← entropy_N2(P1, T1)
    s2 ← entropy_N2(P2, T1)
    s3 ← entropy_N2(P2, T3)
    se ← s3
    h1 ← enthalpy_N2(P1, T1)
    h2 ← enthalpy_N2(P2, T1)
    h3 ← enthalpy_N2(P2, T3)
    he ← enthalpy_N2_Ps(P1, se)
    hf ← HFSATP(P1)
    Wideal ← W_ideal_LiqN2(P1, T1)
    eta_exp_overall ← eta_exp_adiab · eta_exp_mech
    $y \leftarrow \frac{h1 - h2}{h1 - hf} + FlowRatio\_x \cdot eta\_exp\_adiab \frac{h3 - he}{h1 - hf}$
    $Wcomp\_perkgliq \leftarrow \frac{1}{y \cdot eta\_comp\_overall} \cdot [T1 \cdot (s1 - s2) - (h1 - h2)]$
    $Wnet\_perkggas \leftarrow \frac{[T1 \cdot (s1 - s2) - (h1 - h2)]}{eta\_comp\_overall} - FlowRatio\_x \cdot eta\_exp\_overall \cdot (h3 - he)$
    $Wnet\_perkgliq \leftarrow \frac{Wnet\_perkggas}{y}$
    $FOM \leftarrow \frac{Wideal}{Wnet\_perkgliq}$
    Wexp_perkggas ← FlowRatio_x · (h3 − he)
    ( "LiqFraction-y"  "Wnet_perkggas(kJ/kg)"  "Wnet_perkgliq(kJ/kg)"  "Wideal(kJ/kg)"  "FOM" )
    (       y              Wnet_perkggas            Wnet_perkgliq           Wideal          FOM  )
```

Now, solve the problem:



```
P1 := 1.013  bar      P2 := 70   bar      T1 := 293  K      T3 := 270  K

FlowRatio_x := 0.4      eta_comp_overall := 0.75

eta_exp_adiab := 0.8      eta_exp_mech := 0.9
```

And, we get:



ActualClaudeSystem_N2(P1,P2,T1,T3,FlowRatio_x,eta_exp_adiab,eta_exp_mech,eta_comp_overall) =

$$\begin{pmatrix} \text{"LiqFraction-y"} & \text{"Wnet\_perkggas(kJ/kg)"} & \text{"Wnet\_perkgliq(kJ/kg)"} & \text{"Wideal(kJ/kg)"} & \text{"FOM"} \\ 0.173 & 436.477 & 2.522 \times 10^{3} & 743.005 & 0.295 \end{pmatrix}$$

i.e.

Liq. Fraction, y = 0.173…Ans.

Wnet_perkg of gas compressed = 436.477 kJ/kg…Ans.

Wnet_perkg of liquid = 2522 kJ/kg…Ans.

ideal = 743.005 kJ/kg of liq.…Ans.

Figure of Merit = FOM = Wideal / Wnet_perkg of liquid = 0.295…Ans.

Plot liq. yield and FOM against adiab. effcy of expander:

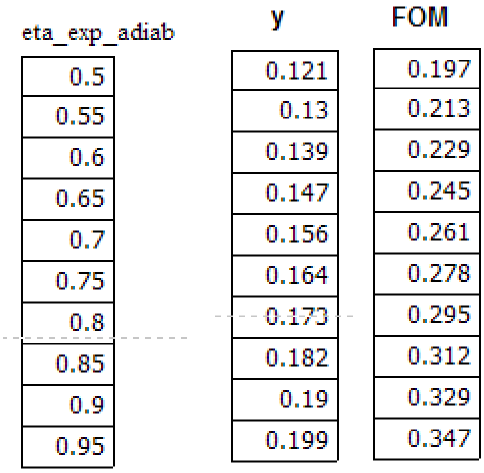
We have:

```
LiqFraction_y := ActualClaudeSystem_N2(P1,P2,T1,T3,FlowRatio_x,eta_exp_adiab,eta_exp_mech,eta_comp_overall)1,0
```

And,

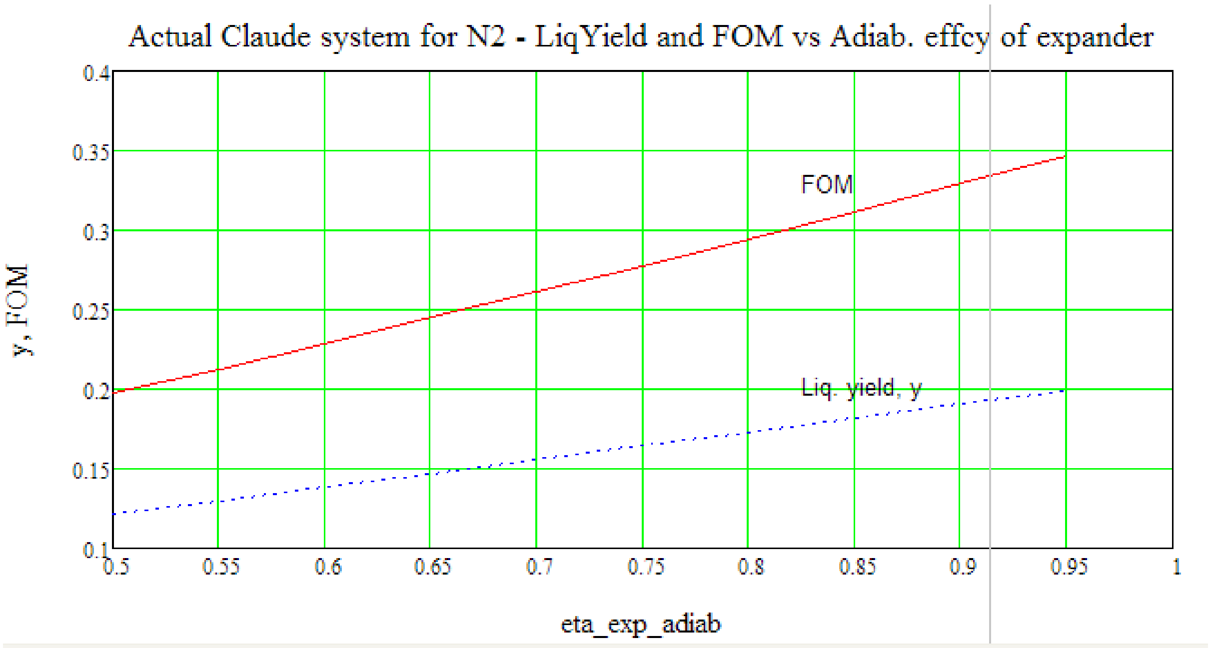
```
FOM := ActualClaudeSystem_N2(P1,P2,T1,T3,FlowRatio_x,eta_exp_adiab,eta_exp_mech,eta_comp_overall)1,4

eta_exp_adiab := 0.5,0.55..0.95    ...define a range variable
```

We get:



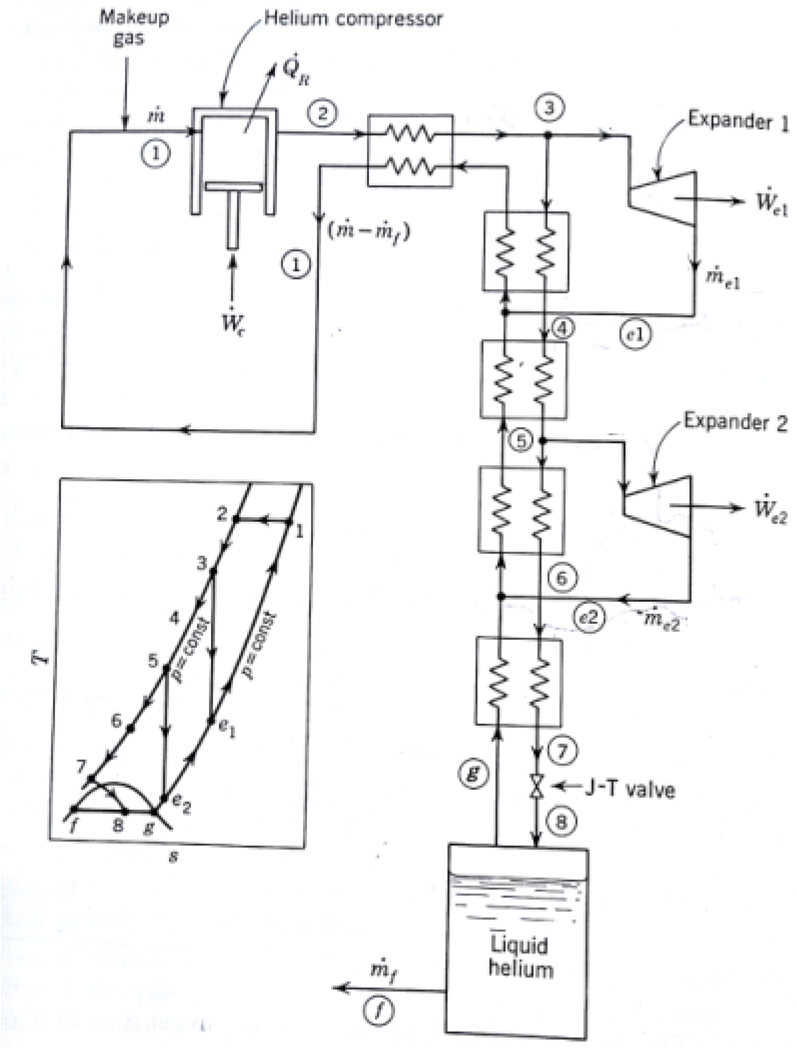
eta_exp_adiab	y	FOM
0.5	0.121	0.197
0.55	0.13	0.213
0.6	0.139	0.229
0.65	0.147	0.245
0.7	0.156	0.261
0.75	0.164	0.278
0.8	0.173	0.295
0.85	0.182	0.312
0.9	0.19	0.329
0.95	0.199	0.347

Now, plot the results:



“**Prob. 3.3.23** Write a Mathcad Function to calculate various parameters of interest for an Ideal Collins liquefier with two expanders, with the expander works being (and not being) utilized in compression.

(b) An Ideal two – expander Collins helium liquefaction system has expander flow rate ratios of x1 = 0.25 and x2 = 0.5. Helium gas enters the compressor at 101.3 kPa and 300 K and is compressed to 1.42 MPa. The condition of the gas at the inlet to the first expander is 1.42 MPa and 60 K and the condition of the gas at the inlet to the second expander is 15 K and 1.42 MPa. Both expanders are reversible and adiabatic. Determine the liquid yield, work per unit mass liquefied, and the Figure of Merit, assuming that: (a) expander work is not utilized in compression, (b) expander work is utilized in compressing the gas. [1].”



First, write the Mathcad Function:

```
Ideal_Collins_cycle(P1,P2,T1,T3,T5,x1,x2) :=

  s1 ← entropy_He(P1,T1)
  s2 ← entropy_He(P2,T1)
  s3 ← entropy_He(P2,T3)
  s5 ← entropy_He(P2,T5)
  se1 ← s3
  se2 ← s5
  sf ← SFSATP(P1)
  h1 ← enthalpy_He(P1,T1)
  h2 ← enthalpy_He(P2,T1)
  h3 ← enthalpy_He(P2,T3)
  h5 ← enthalpy_He(P2,T5)
  hf ← HFSATP(P1)
  he1 ← enthalpy_He_Ps(P1,se1)
  he2 ← enthalpy_He_Ps(P1,se2)
  y ← $\frac{h1 - h2}{h1 - hf} + x1 \cdot \frac{h3 - he1}{h1 - hf} + x2 \cdot \frac{h5 - he2}{h1 - hf}$
  Wperkggas_noexpwork ← T1 · (s1 − s2) − (h1 − h2)
  Wperkgliq_noexpwork ← $\frac{Wperkggas\_noexpwork}{y}$
  Wideal ← T1 · (s1 − sf) − (h1 − hf)
  FOM_noexpwork ← $\frac{Wideal}{Wperkgliq\_noexpwork}$
  Wperkggas_withexpwork ← T1 · (s1 − s2) − (h1 − h2) − x1 · (h3 − he1) − x2 · (h5 − he2)
  Wperkgliq_withexpwork ← $\frac{Wperkggas\_withexpwork}{y}$
  FOM_withexpwork ← $\frac{Wideal}{Wperkgliq\_withexpwork}$
  ( "LiqFrn.(y)"  "Wideal(kJ/kgliq)"  "Wperkgliq_noexpwork(kJ/kg)"  "Wperkgliq_withexpwork(kJ/kg)"  "FOM_noexpwork"  "FOM_withexpwork" )
  (     y              Wideal            Wperkgliq_noexpwork             Wperkgliq_withexpwork          FOM_noexpwork    FOM_withexpwork  )
```

Now, solve the problem:

Data:

P1 := 1.013 bar T1 := 300 K P2 := 14.2 bar T3 := 60 K T5 := 15 K

x1 := 0.25 x2 := 0.5

And, using the Function written above:

Ideal_Collins_cycle(P1,P2,T1,T3,T5,x1,x2) =

$$\begin{pmatrix} \text{"LiqFrn.(y)"} & \text{"Wideal(kJ/kgliq)"} & \text{"Wperkgliq\_noexpwork(kJ/kg)"} & \text{"Wperkgliq\_withexpwork(kJ/kg)"} & \text{"FOM\_noexpwork"} & \text{"FOM\_withexpwork"} \\ 0.044 & 6.841 \times 10^{3} & 3.767 \times 10^{4} & 3.601 \times 10^{4} & 0.182 & 0.19 \end{pmatrix}$$

i.e. we have:

$$\text{LiqFrn\_y} := \text{Ideal\_Collins\_cycle}(P1,P2,T1,T3,T5,x1,x2)_{1,0} \qquad \text{LiqFrn\_y} = 0.044$$

$$\text{Wperkgliq\_noexpwork} := \text{Ideal\_Collins\_cycle}(P1,P2,T1,T3,T5,x1,x2)_{1,2} \qquad \text{Wperkgliq\_noexpwork} = 3.767 \times 10^{4}$$

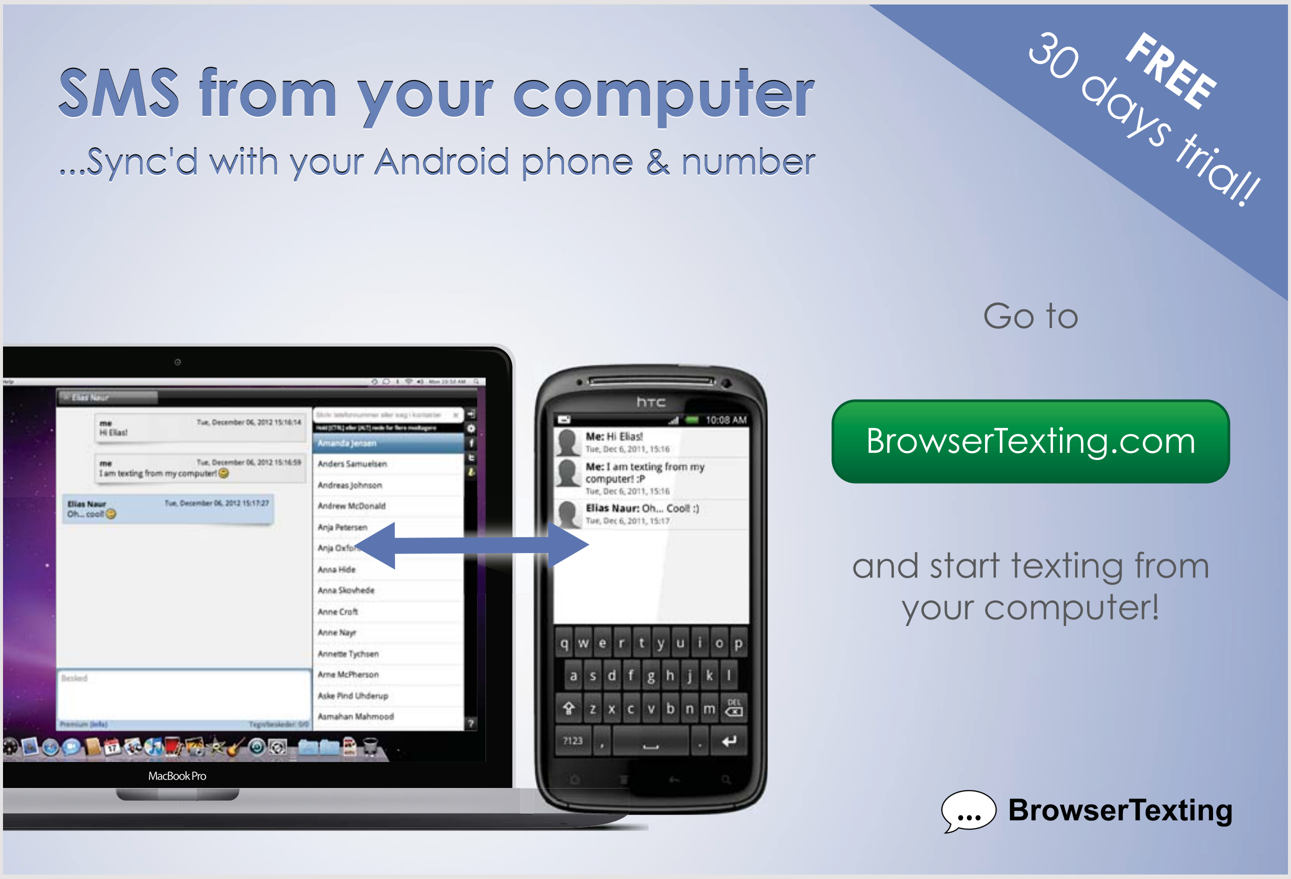
$$\text{Wperkgliq\_withexpwork} := \text{Ideal\_Collins\_cycle}(P1,P2,T1,T3,T5,x1,x2)_{1,3} \qquad \text{Wperkgliq\_withexpwork} = 3.601 \times 10^{4}$$

$$\text{FOM\_noexpwork} := \text{Ideal\_Collins\_cycle}(P1,P2,T1,T3,T5,x1,x2)_{1,4} \qquad \text{FOM\_noexpwork} = 0.182$$

$$\text{FOM\_withexpwork} := \text{Ideal\_Collins\_cycle}(P1,P2,T1,T3,T5,x1,x2)_{1,5} \qquad \text{FOM\_withexpwork} = 0.19$$

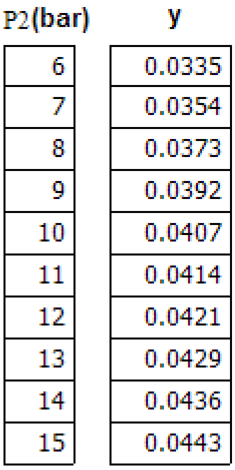
Thus:

a) **When Expander work is utilized in compression:**
Liquid yield = y = 0.044 = 4.4%…Ans.
Work per unit mass liquefied = 36010 kJ/kg…Ans.
Figure of Merit = 0.19…Ans.

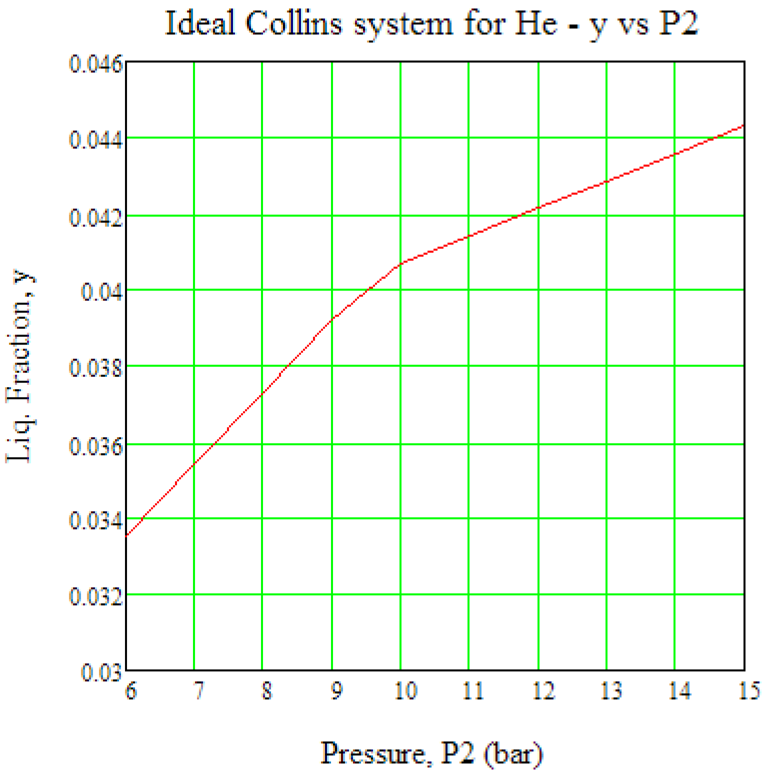


b) **When Expander work is *not utilized* in compression:**
Liquid yield = y = 0.044 = 4.4%…Ans.
Work per unit mass liquefied = 37670 kJ/kg…Ans.
Figure of Merit = 0.182…Ans.

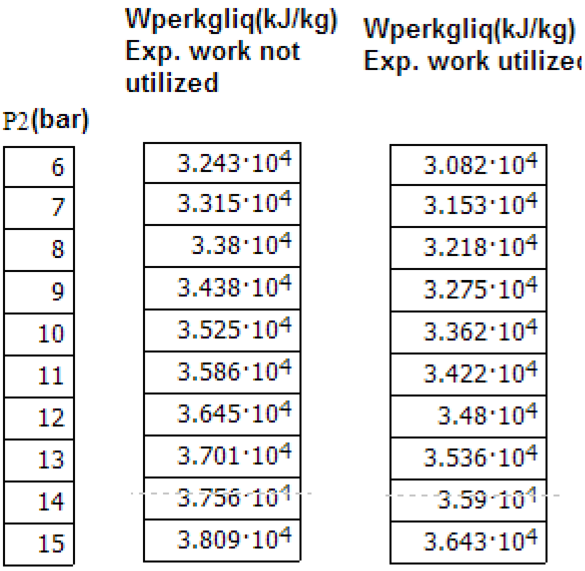
Now, plot y, Wperkgliq and FOM for P2 = 6 bar to 15 bar:



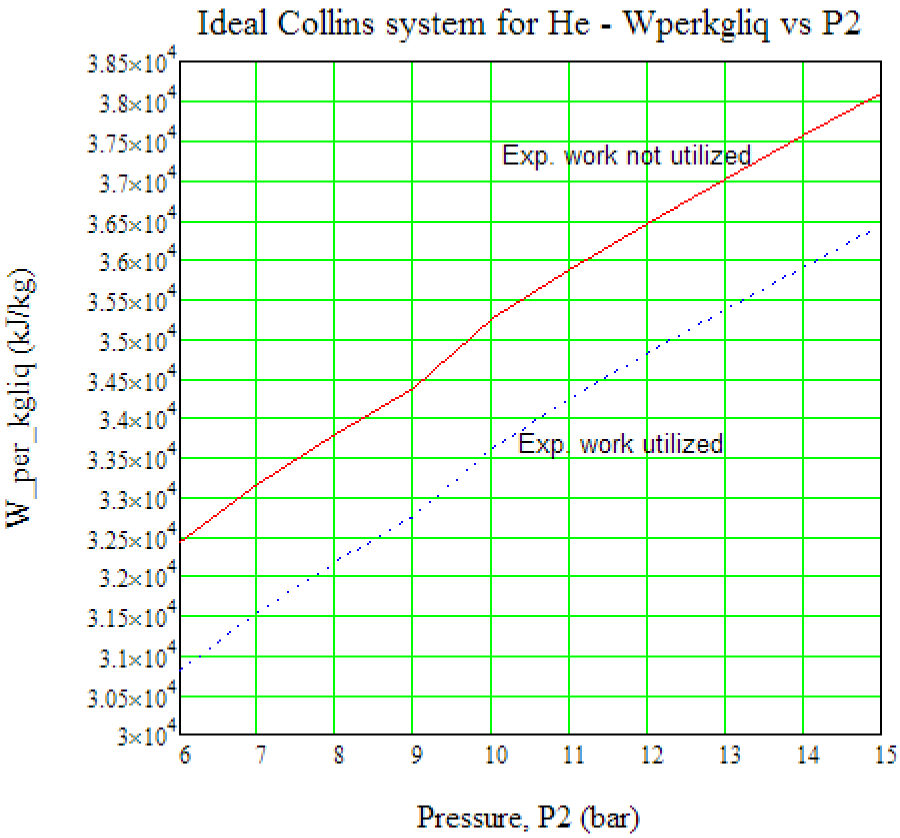
P2(bar)	y
6	0.0335
7	0.0354
8	0.0373
9	0.0392
10	0.0407
11	0.0414
12	0.0421
13	0.0429
14	0.0436
15	0.0443



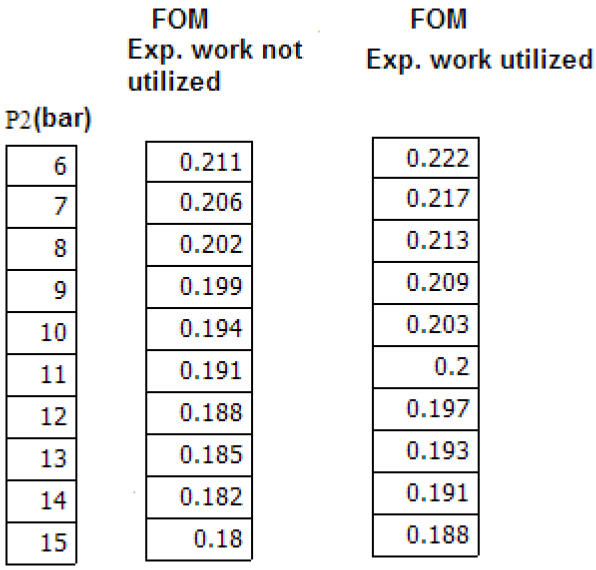
Work per kg liquid:



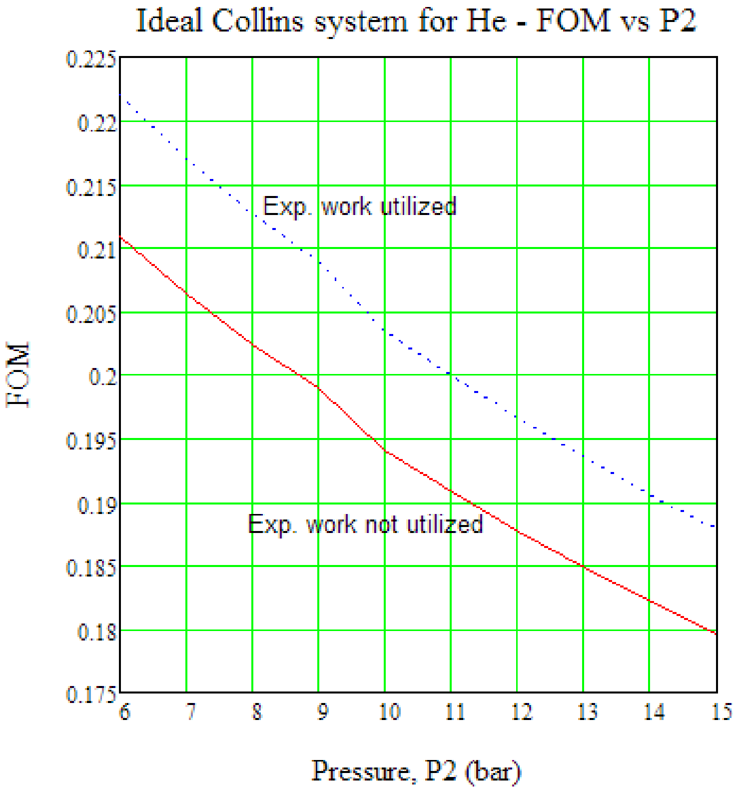
P2(bar)	Wperkgliq(kJ/kg) Exp. work not utilized	Wperkgliq(kJ/kg) Exp. work utilize
6	$3.243\cdot10^4$	$3.082\cdot10^4$
7	$3.315\cdot10^4$	$3.153\cdot10^4$
8	$3.38\cdot10^4$	$3.218\cdot10^4$
9	$3.438\cdot10^4$	$3.275\cdot10^4$
10	$3.525\cdot10^4$	$3.362\cdot10^4$
11	$3.586\cdot10^4$	$3.422\cdot10^4$
12	$3.645\cdot10^4$	$3.48\cdot10^4$
13	$3.701\cdot10^4$	$3.536\cdot10^4$
14	$3.756\cdot10^4$	$3.59\cdot10^4$
15	$3.809\cdot10^4$	$3.643\cdot10^4$



FOM:

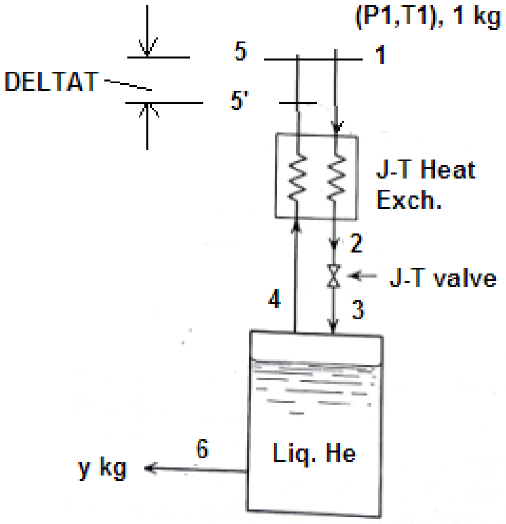


P2(bar)	FOM Exp. work not utilized	FOM Exp. work utilized
6	0.211	0.222
7	0.206	0.217
8	0.202	0.213
9	0.199	0.209
10	0.194	0.203
11	0.191	0.2
12	0.188	0.197
13	0.185	0.193
14	0.182	0.191
15	0.18	0.188



Prob. 3.3.24 Write a Mathcad Function to find the liquid yield for Linde Hampson system when the temperatures at the top of the J_T heat exchanger, effectiveness of J-T HX are known. Then, plot Liquid yield, y vs P1 (i.e. the high pressure) for temperatures at the top of the J_T HX, T1 = 6, 10, 15 and 20 K, with eta_HX = 0.9, 0.95 and 1.

Fig. below shows the final stage of a Helium liquefier, i.e. the J_T Heat exchanger, J-T valve, and the liquid reservoir.



We write the Mathcad Function for liquid yield, y, with the notations used in the above fig:

Mathcad Function:

```
LindeHampson_He_LiqYield_y(P1,T1,P5,eta_HX) :=
    h1 ← enthalpy_He(P1,T1)
    Tf ← TSAT(P5)
    hf ← HFSATP(P5)
    DELTAT ← (1 − eta_HX) · (T1 − Tf)
    T5_prime ← T1 − DELTAT
    h5_prime ← enthalpy_He(P5,T5_prime)
    y ← $\frac{h5\_prime - h1}{h5\_prime - hf}$
    $\begin{pmatrix} "P\_high(bar)" & "P\_low(bar)" & "T1(K)" & "eta\_HX" & "LiqYield,y" \\ P1 & P5 & T1 & eta\_HX & y \end{pmatrix}$
```

Ex:

P1 := 20 bar   T1 := 10 K   P5 := 1.013 bar   eta_HX := 0.95

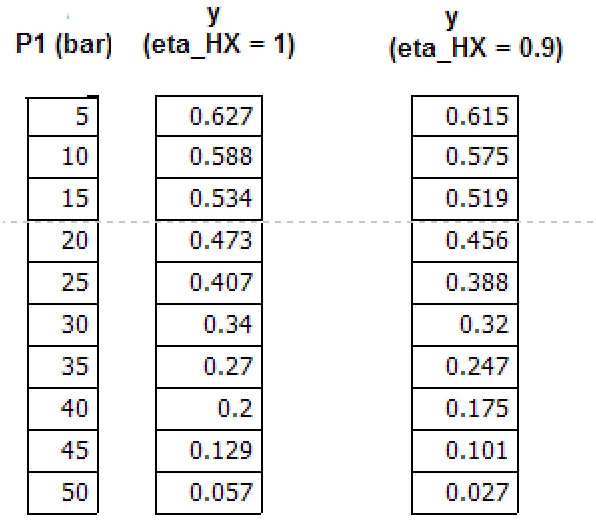
$$\text{LindeHampson\_He\_LiqYield\_y}(P1,T1,P5,\text{eta\_HX}) = \begin{pmatrix} "P\_high(bar)" & "P\_low(bar)" & "T1(K)" & "eta\_HX" & "LiqYield,y" \\ 20 & 1.013 & 10 & 0.95 & 0.243 \end{pmatrix}$$

i.e. $\text{LiqYield\_y} := \text{LindeHampson\_He\_LiqYield\_y}(P1,T1,P5,\text{eta\_HX})_{1,4}$ $\text{LiqYield\_y} = 0.243$

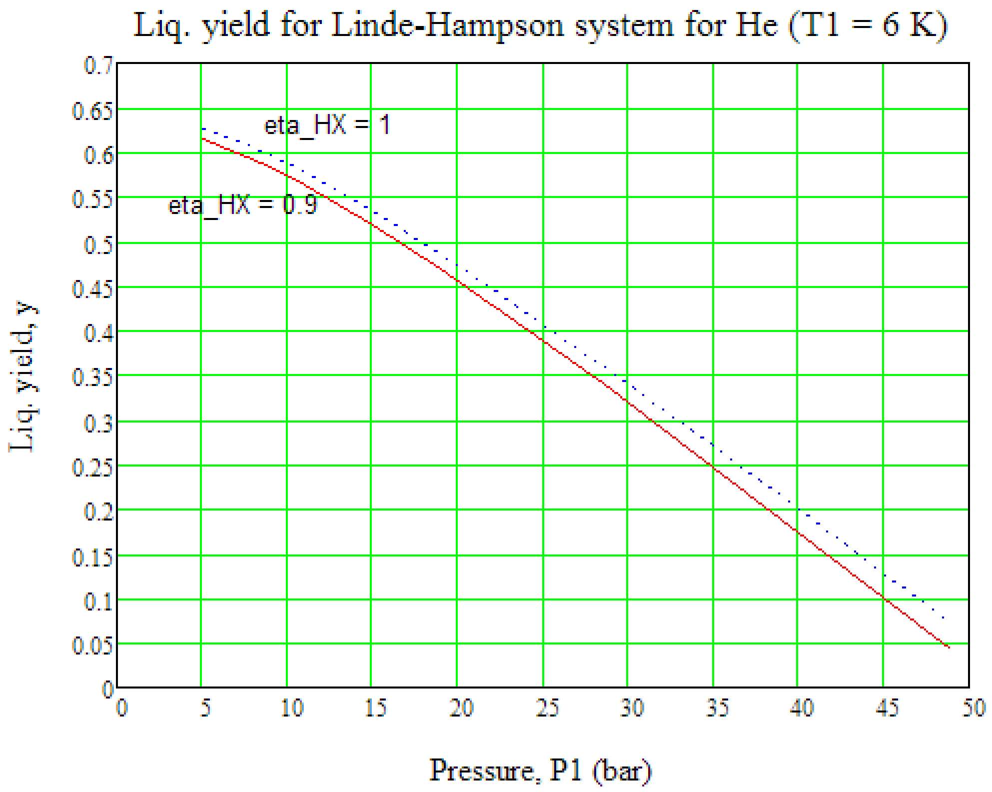
To plot LiqYield, y vs P1 for T1 = 6, 10 and 15 K, at eta_HX = 0.9, 0.95 and 1.0:

P1 := 5, 10 .. 50   ....define a range variable

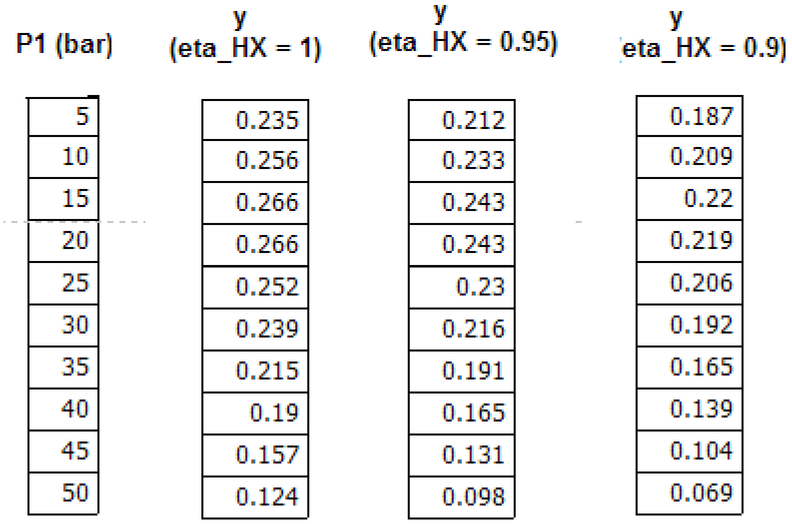
T1 = 6 K:



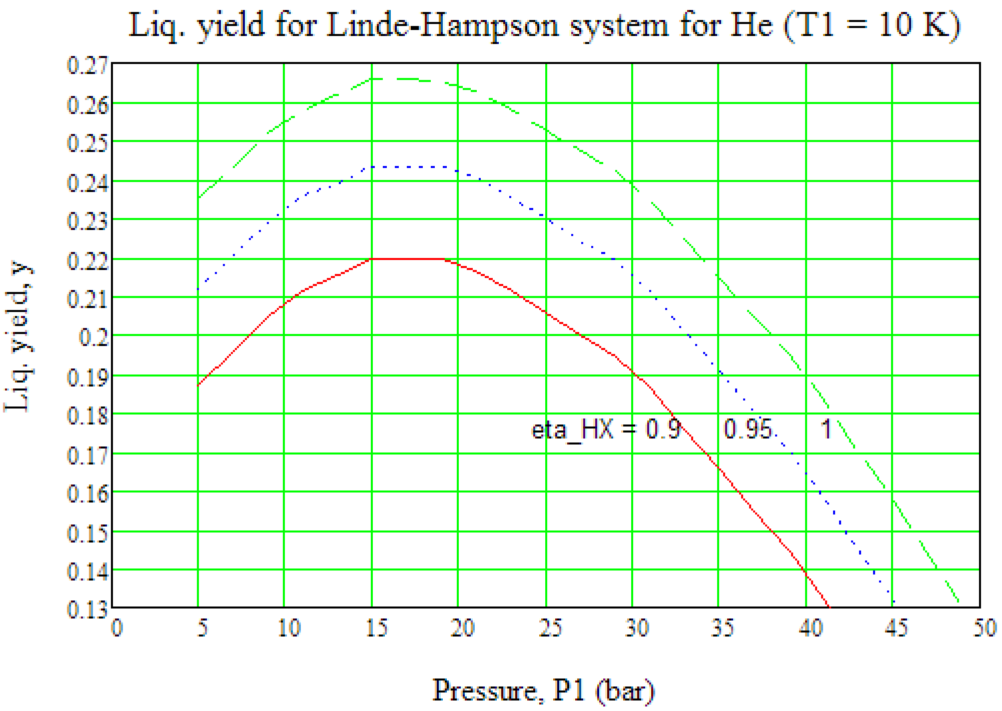
P1 (bar)	y (eta_HX = 1)	y (eta_HX = 0.9)
5	0.627	0.615
10	0.588	0.575
15	0.534	0.519
20	0.473	0.456
25	0.407	0.388
30	0.34	0.32
35	0.27	0.247
40	0.2	0.175
45	0.129	0.101
50	0.057	0.027



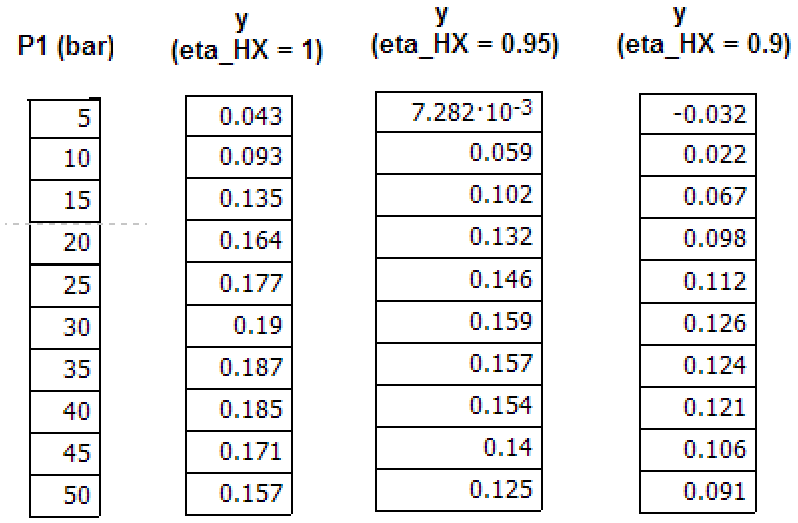
T1 = 10 K:



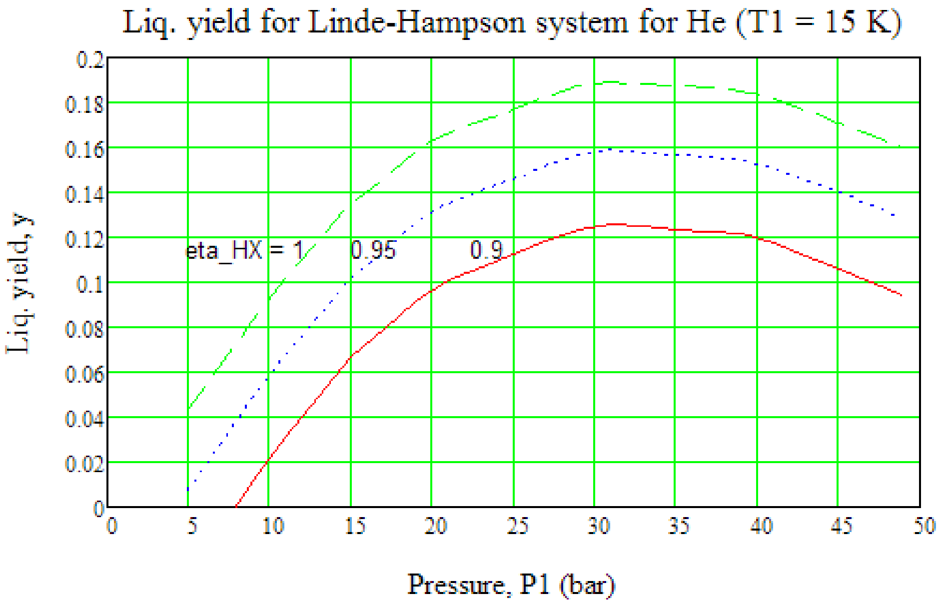
P1 (bar)	y (eta_HX = 1)	y (eta_HX = 0.95)	y (eta_HX = 0.9)
5	0.235	0.212	0.187
10	0.256	0.233	0.209
15	0.266	0.243	0.22
20	0.266	0.243	0.219
25	0.252	0.23	0.206
30	0.239	0.216	0.192
35	0.215	0.191	0.165
40	0.19	0.165	0.139
45	0.157	0.131	0.104
50	0.124	0.098	0.069



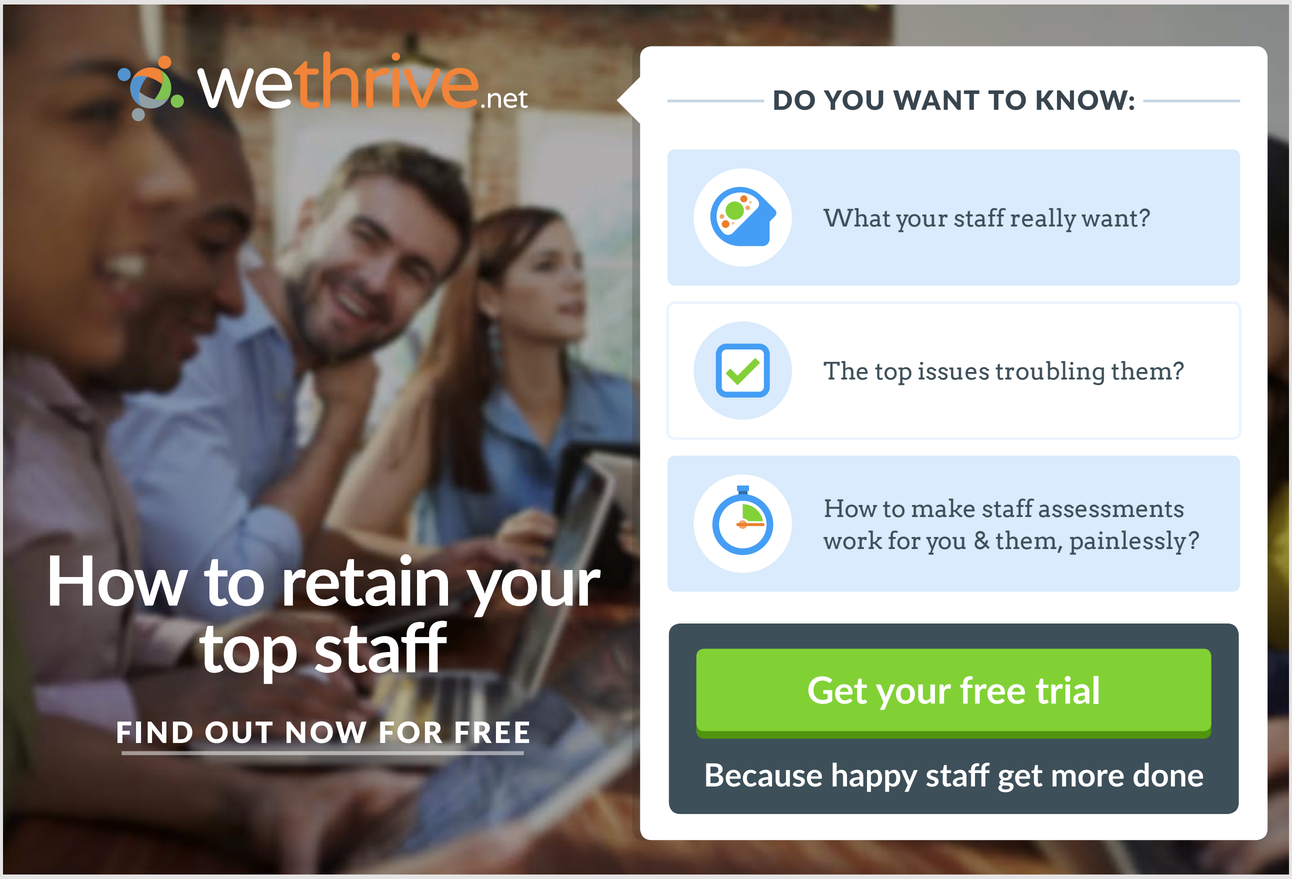
T1 = 15 K:



P1 (bar)	y (eta_HX = 1)	y (eta_HX = 0.95)	y (eta_HX = 0.9)
5	0.043	$7.282\cdot10^{-3}$	-0.032
10	0.093	0.059	0.022
15	0.135	0.102	0.067
20	0.164	0.132	0.098
25	0.177	0.146	0.112
30	0.19	0.159	0.126
35	0.187	0.157	0.124
40	0.185	0.154	0.121
45	0.171	0.14	0.106
50	0.157	0.125	0.091


Liq. yield for Linde-Hampson system for He (T1 = 15 K)
Liq. yield, y
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
eta_HX = 1
0.95
0.9
0
5
10
15
20
25
30
35
40
45
50
Pressure, P1 (bar)

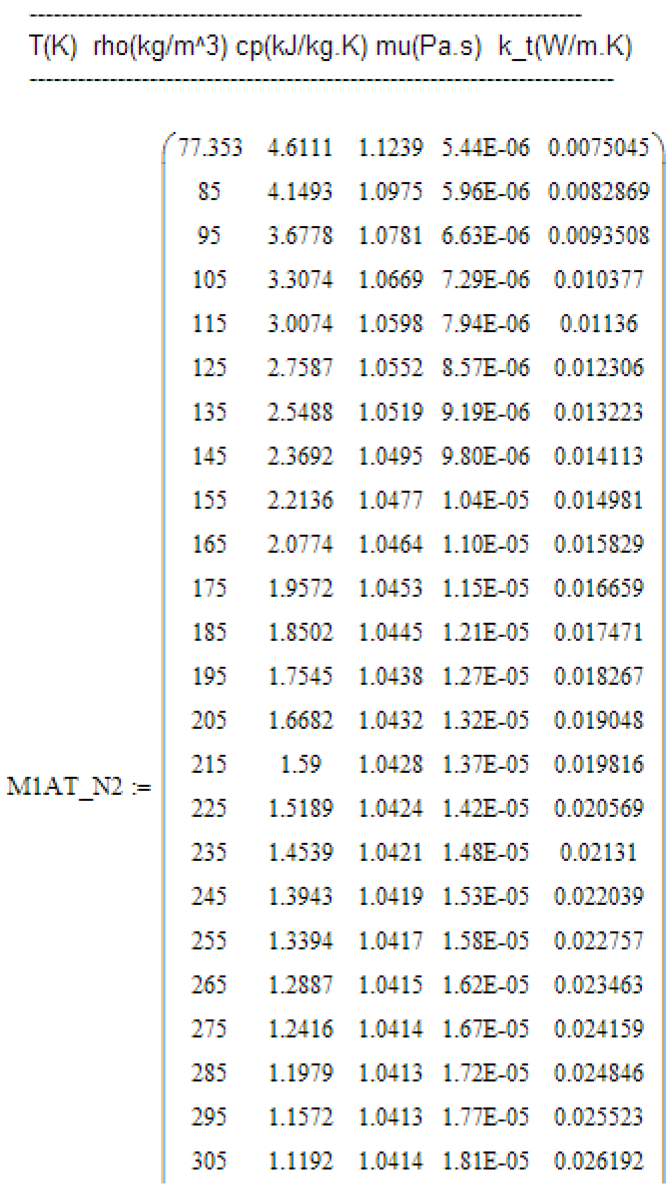
==



Prob. 3.3.25 Determine the inside heat transfer coeff (hc) and friction factor (f) for flow of nitrogen gas at 150 K and 101.3 kPa inside a 12 mm inside dia smooth tube that is coiled in a 600 mm dia helix. The tube wall has a temp of 160 K and the mass flow rate of nitrogen gas is 30 g/s.
Also, plot the variation of hc and f as m_dot varies from 10 g/s to 70 g/s [1].

First, write Mathcad Functions to determine the properties of Nitrogen gas at 1 atm (i.e. 1.013 bar), using the data from NIST [28]:

For Gaseous N2 at 1.013 bar (1 atm.)…data from NIST:



M1AT_N2 :=

T(K)	rho(kg/m^3)	cp(kJ/kg.K)	mu(Pa.s)	k_t(W/m.K)
77.353	4.6111	1.1239	5.44E-06	0.0075045
85	4.1493	1.0975	5.96E-06	0.0082869
95	3.6778	1.0781	6.63E-06	0.0093508
105	3.3074	1.0669	7.29E-06	0.010377
115	3.0074	1.0598	7.94E-06	0.01136
125	2.7587	1.0552	8.57E-06	0.012306
135	2.5488	1.0519	9.19E-06	0.013223
145	2.3692	1.0495	9.80E-06	0.014113
155	2.2136	1.0477	1.04E-05	0.014981
165	2.0774	1.0464	1.10E-05	0.015829
175	1.9572	1.0453	1.15E-05	0.016659
185	1.8502	1.0445	1.21E-05	0.017471
195	1.7545	1.0438	1.27E-05	0.018267
205	1.6682	1.0432	1.32E-05	0.019048
215	1.59	1.0428	1.37E-05	0.019816
225	1.5189	1.0424	1.42E-05	0.020569
235	1.4539	1.0421	1.48E-05	0.02131
245	1.3943	1.0419	1.53E-05	0.022039
255	1.3394	1.0417	1.58E-05	0.022757
265	1.2887	1.0415	1.62E-05	0.023463
275	1.2416	1.0414	1.67E-05	0.024159
285	1.1979	1.0413	1.72E-05	0.024846
295	1.1572	1.0413	1.77E-05	0.025523
305	1.1192	1.0414	1.81E-05	0.026192

$$\begin{pmatrix} 315 & 1.0836 & 1.0415 & 1.86E-05 & 0.026852 \\ 325 & 1.0502 & 1.0416 & 1.90E-05 & 0.027505 \\ 335 & 1.0188 & 1.0419 & 1.95E-05 & 0.028151 \\ 345 & 0.98919 & 1.0421 & 1.99E-05 & 0.02879 \\ 355 & 0.96128 & 1.0425 & 2.03E-05 & 0.029422 \\ 365 & 0.93491 & 1.0429 & 2.08E-05 & 0.030049 \end{pmatrix}$$

Now, write individual properties in separate vectors:

$temp1AT\_N2 := M1AT\_N2^{\langle 0 \rangle}$ $rho1AT\_N2 := M1AT\_N2^{\langle 1 \rangle}$

$cp1AT\_N2 := M1AT\_N2^{\langle 2 \rangle}$ $mu1AT\_N2 := M1AT\_N2^{\langle 3 \rangle}$

$thcond1AT\_N2 := M1AT\_N2^{\langle 4 \rangle}$

Now, write Mathcad Functions, using simple linear interpolation:

Ex:

$rho\_1AT\_N2(T) := linterp(temp1AT\_N2, rho1AT\_N2, T)$ $rho\_1AT\_N2(150) = 2.291$ kg/m^3

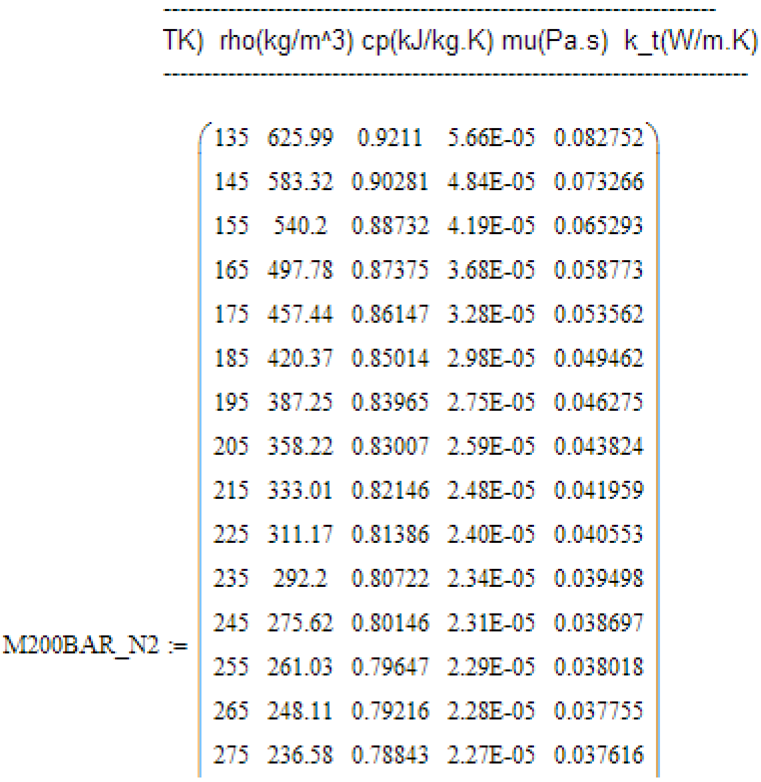
$cp\_1AT\_N2(T) := linterp(temp1AT\_N2, cp1AT\_N2, T)$ $cp\_1AT\_N2(150) = 1.049$ kJ/kg.K

$mu\_1AT\_N2(T) := linterp(temp1AT\_N2, mu1AT\_N2, T)$ $mu\_1AT\_N2(150) = 1.01 \times 10^{-5}$ Pa.s = kg.s/m^2

$kt\_1AT\_N2(T) := linterp(temp1AT\_N2, thcond1AT\_N2, T)$ $kt\_1AT\_N2(150) = 0.015$ W/m.K

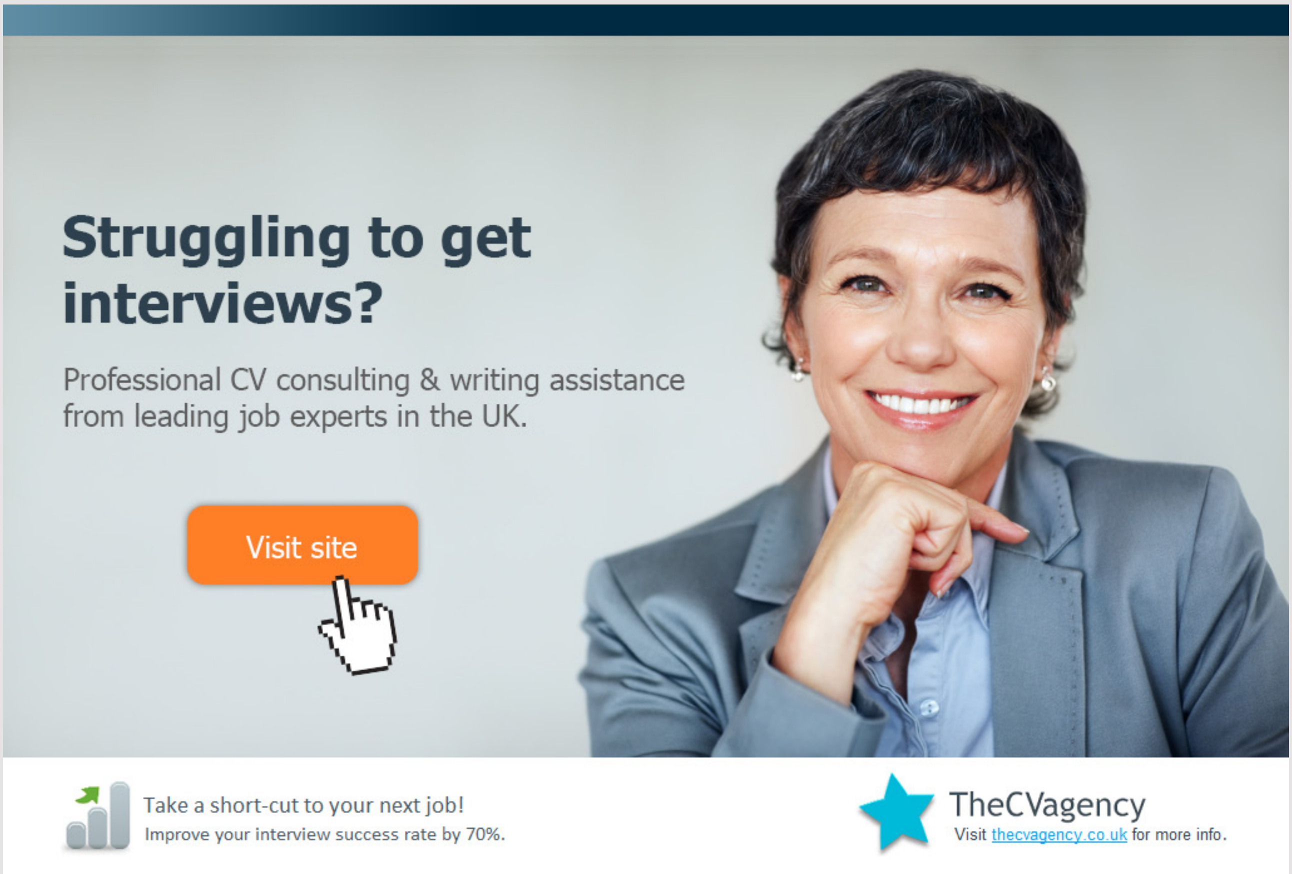
$$Prandtl\_1AT\_N2(T) := \frac{1000\, cp\_1AT\_N2(T) \cdot mu\_1AT\_N2(T)}{kt\_1AT\_N2(T)} \qquad Prandtl\_1AT\_N2(150) = 0.728$$

For Gaseous N2 at 200 bar:



```
---------------------------------------------------------------------
TK)  rho(kg/m^3) cp(kJ/kg.K) mu(Pa.s)  k_t(W/m.K)
-------------------------------------------------------------------------
```

$$M200BAR\_N2 := \begin{pmatrix} 135 & 625.99 & 0.9211 & 5.66E-05 & 0.082752 \\ 145 & 583.32 & 0.90281 & 4.84E-05 & 0.073266 \\ 155 & 540.2 & 0.88732 & 4.19E-05 & 0.065293 \\ 165 & 497.78 & 0.87375 & 3.68E-05 & 0.058773 \\ 175 & 457.44 & 0.86147 & 3.28E-05 & 0.053562 \\ 185 & 420.37 & 0.85014 & 2.98E-05 & 0.049462 \\ 195 & 387.25 & 0.83965 & 2.75E-05 & 0.046275 \\ 205 & 358.22 & 0.83007 & 2.59E-05 & 0.043824 \\ 215 & 333.01 & 0.82146 & 2.48E-05 & 0.041959 \\ 225 & 311.17 & 0.81386 & 2.40E-05 & 0.040553 \\ 235 & 292.2 & 0.80722 & 2.34E-05 & 0.039498 \\ 245 & 275.62 & 0.80146 & 2.31E-05 & 0.038697 \\ 255 & 261.03 & 0.79647 & 2.29E-05 & 0.038018 \\ 265 & 248.11 & 0.79216 & 2.28E-05 & 0.037755 \\ 275 & 236.58 & 0.78843 & 2.27E-05 & 0.037616 \end{pmatrix}$$



285	226.23	0.78519	2.28E-05	0.037573
295	216.88	0.78239	2.29E-05	0.037607
305	208.39	0.77996	2.30E-05	0.037702
315	200.63	0.77786	2.32E-05	0.037847
325	193.52	0.77606	2.34E-05	0.038034
335	186.96	0.77451	2.36E-05	0.038254
345	180.9	0.77321	2.39E-05	0.038504
355	175.27	0.77213	2.41E-05	0.038778
365	170.02	0.77125	2.44E-05	0.039073

Now, write individual properties in separate vectors:

$$\text{temp200B\_N2} := \text{M200BAR\_N2}^{\langle 0\rangle} \qquad \text{rho200B\_N2} := \text{M200BAR\_N2}^{\langle 1\rangle}$$

$$\text{cp200B\_N2} := \text{M200BAR\_N2}^{\langle 2\rangle} \qquad \text{mu200B\_N2} := \text{M200BAR\_N2}^{\langle 3\rangle}$$

$$\text{thcond200B\_N2} := \text{M200BAR\_N2}^{\langle 4\rangle}$$

And, now write the Functions:

Ex:

$$\text{rho\_200B\_N2}(T) := \text{linterp}(\text{temp200B\_N2}, \text{rho200B\_N2}, T) \qquad \text{rho\_200B\_N2}(150) = 561.76 \ \text{kg/m^3}$$

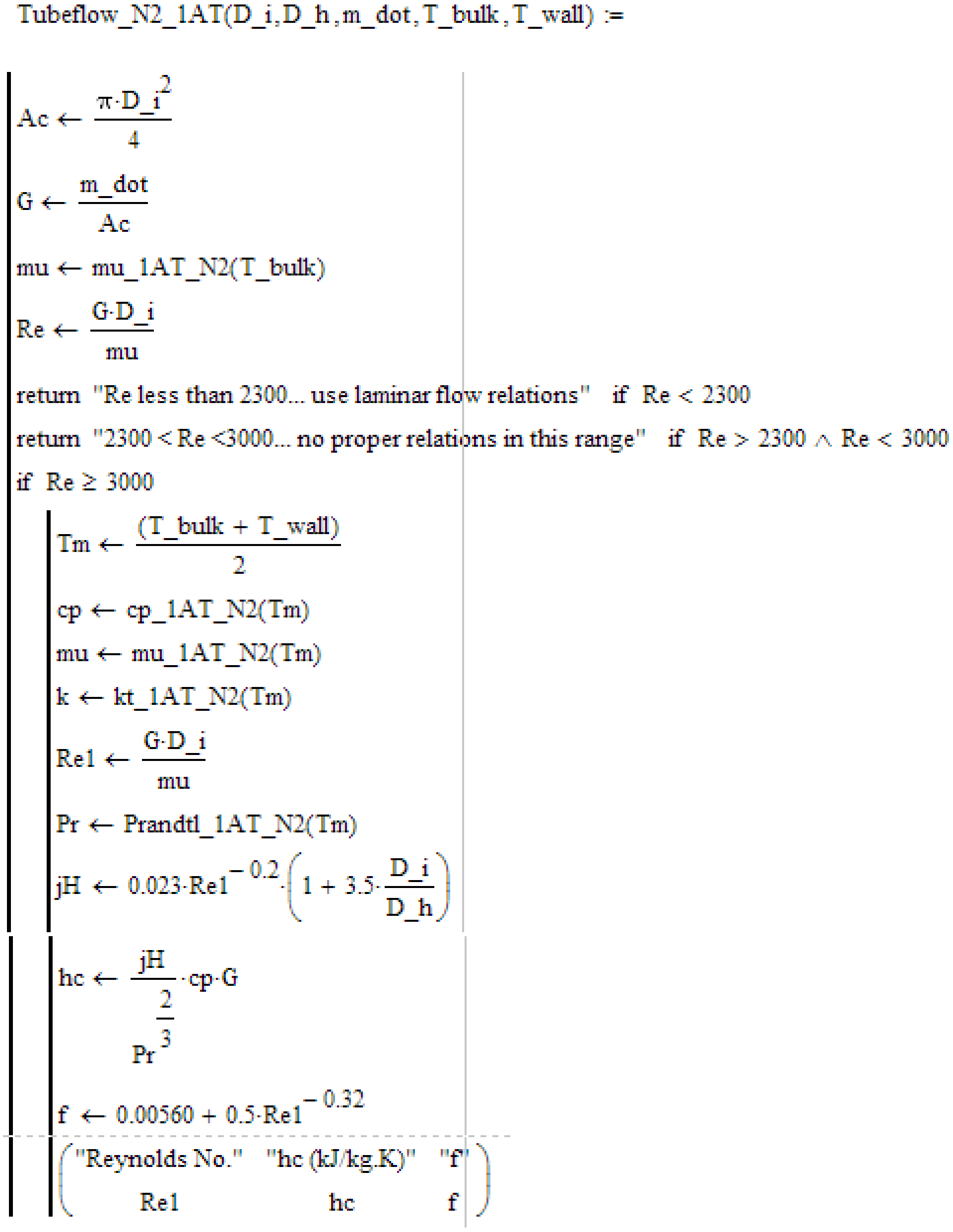
$$\text{cp\_200B\_N2}(T) := \text{linterp}(\text{temp200B\_N2}, \text{cp200B\_N2}, T) \qquad \text{cp\_200B\_N2}(150) = 0.895 \ \text{kJ/kg.K}$$

$$\text{mu\_200B\_N2}(T) := \text{linterp}(\text{temp200B\_N2}, \text{mu200B\_N2}, T) \qquad \text{mu\_200B\_N2}(150) = 4.515 \times 10^{-5} \ \text{Pa.s = kg.s/m^2}$$

$$\text{kt\_200B\_N2}(T) := \text{linterp}(\text{temp200B\_N2}, \text{thcond200B\_N2}, T) \qquad \text{kt\_200B\_N2}(150) = 0.069 \ \text{W/m.K}$$

$$\text{Prandtl\_200B\_N2}(T) := \frac{1000\,\text{cp\_200B\_N2}(T)\cdot\text{mu\_200B\_N2}(T)}{\text{kt\_200B\_N2}(T)} \qquad \text{Prandtl\_200B\_N2}(150) = 0.583$$

Now, write the required Function for turb. Flow through a tube:



```
Tubeflow_N2_1AT(D_i,D_h,m_dot,T_bulk,T_wall) :=
    $Ac \leftarrow \frac{\pi \cdot D\_i^2}{4}$
    $G \leftarrow \frac{m\_dot}{Ac}$
    $mu \leftarrow mu\_1AT\_N2(T\_bulk)$
    $Re \leftarrow \frac{G \cdot D\_i}{mu}$
    return "Re less than 2300... use laminar flow relations"   if  $Re < 2300$
    return "2300 < Re <3000... no proper relations in this range"   if  $Re > 2300 \wedge Re < 3000$
    if  $Re \geq 3000$
        $Tm \leftarrow \frac{(T\_bulk + T\_wall)}{2}$
        $cp \leftarrow cp\_1AT\_N2(Tm)$
        $mu \leftarrow mu\_1AT\_N2(Tm)$
        $k \leftarrow kt\_1AT\_N2(Tm)$
        $Re1 \leftarrow \frac{G \cdot D\_i}{mu}$
        $Pr \leftarrow Prandtl\_1AT\_N2(Tm)$
        $jH \leftarrow 0.023 \cdot Re1^{-0.2} \cdot \left(1 + 3.5 \cdot \frac{D\_i}{D\_h}\right)$
        $hc \leftarrow \frac{jH}{Pr^{\frac{2}{3}}} \cdot cp \cdot G$
        $f \leftarrow 0.00560 + 0.5 \cdot Re1^{-0.32}$
    $\begin{pmatrix} \text{"Reynolds No."} & \text{"hc (kJ/kg.K)"} & \text{"f"} \\ Re1 & hc & f \end{pmatrix}$
```

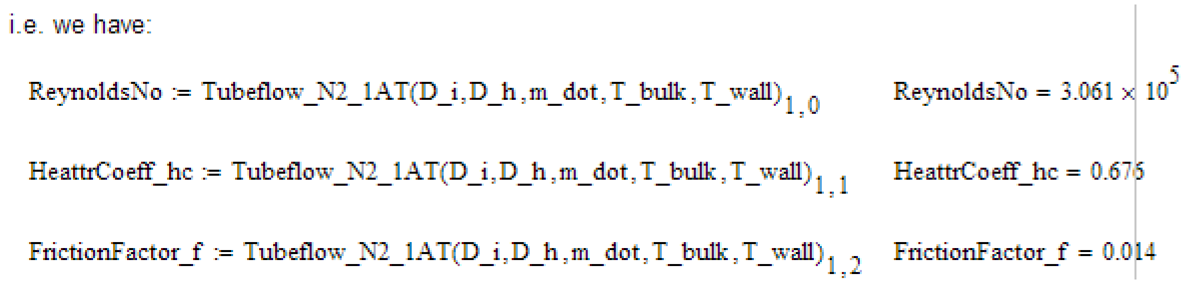
Ex:

$D\_i := 0.012$ m $D\_h := 0.6$ m $m\_dot := 0.03$ kg/s

$T\_bulk := 150$ K $T\_wall := 160$ K

And, using the Function written above, we get:

$$Tubeflow\_N2\_1AT(D\_i, D\_h, m\_dot, T\_bulk, T\_wall) = \begin{pmatrix} \text{"Reynolds No."} & \text{"hc (kJ/kg.K)"} & \text{"f"} \\ 3.0607 \times 10^5 & 0.6761 & 0.0144 \end{pmatrix}$$



i.e. we have:

$\text{ReynoldsNo} := \text{Tubeflow\_N2\_1AT}(\text{D\_i}, \text{D\_h}, \text{m\_dot}, \text{T\_bulk}, \text{T\_wall})_{1,0}$ $\quad \text{ReynoldsNo} = 3.061 \times 10^{5}$

$\text{HeattrCoeff\_hc} := \text{Tubeflow\_N2\_1AT}(\text{D\_i}, \text{D\_h}, \text{m\_dot}, \text{T\_bulk}, \text{T\_wall})_{1,1}$ $\quad \text{HeattrCoeff\_hc} = 0.676$

$\text{FrictionFactor\_f} := \text{Tubeflow\_N2\_1AT}(\text{D\_i}, \text{D\_h}, \text{m\_dot}, \text{T\_bulk}, \text{T\_wall})_{1,2}$ $\quad \text{FrictionFactor\_f} = 0.014$

Thus:

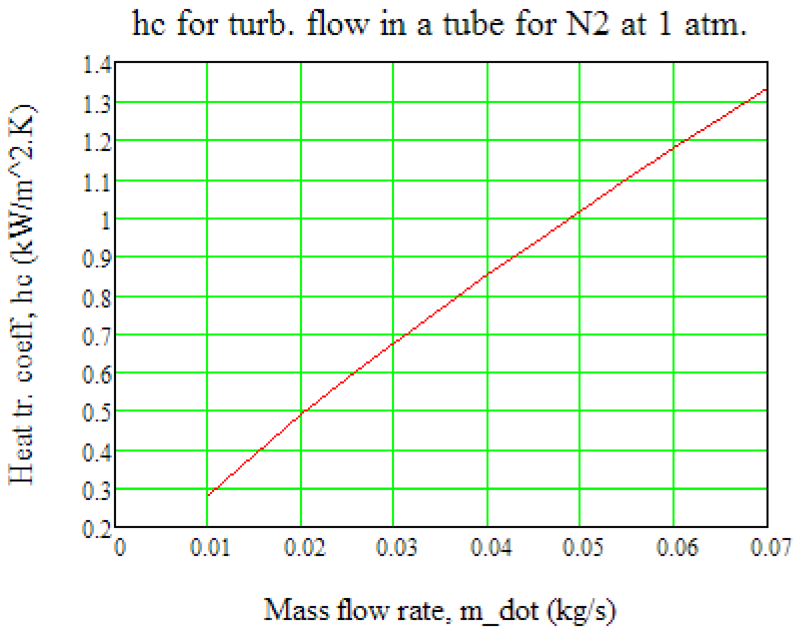
Heat transfer coeff. h_c = 676 W/m^2.K…Ans.

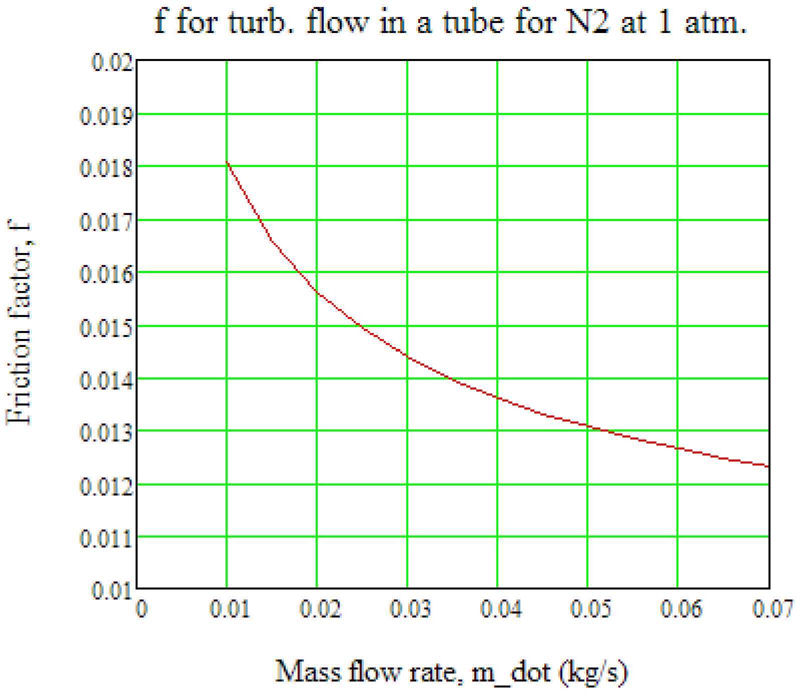
Friction factor, f = 0.014…Ans.

Now, plot the variation of hc and f as m_dot varies from 10 g/s to 70 g/s, other factors remaining the same:

$\text{m\_dot} := 0.01, 0.015 .. 0.07$ ...define a range variable

m_dot (kg/s)	hc (kW/m^2.K)	f
0.01	0.281	0.0181
0.015	0.388	0.0166
0.02	0.489	0.0156
0.025	0.584	0.0149
0.03	0.676	0.0144
0.035	0.765	0.014
0.04	0.851	0.0136
0.045	0.935	0.0133
0.05	1.017	0.0131
0.055	1.098	0.0128
0.06	1.177	0.0126
0.065	1.255	0.0125
0.07	1.332	0.0123



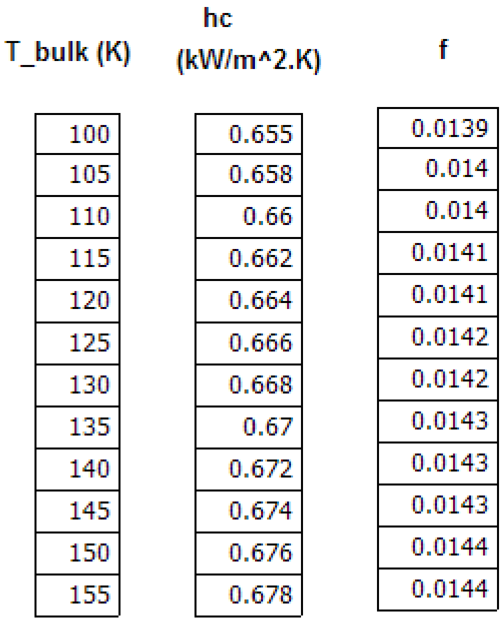


And, plot the variation of hc and f as T_bulk varies from 100 K to 155 K, other factors remaining the same:

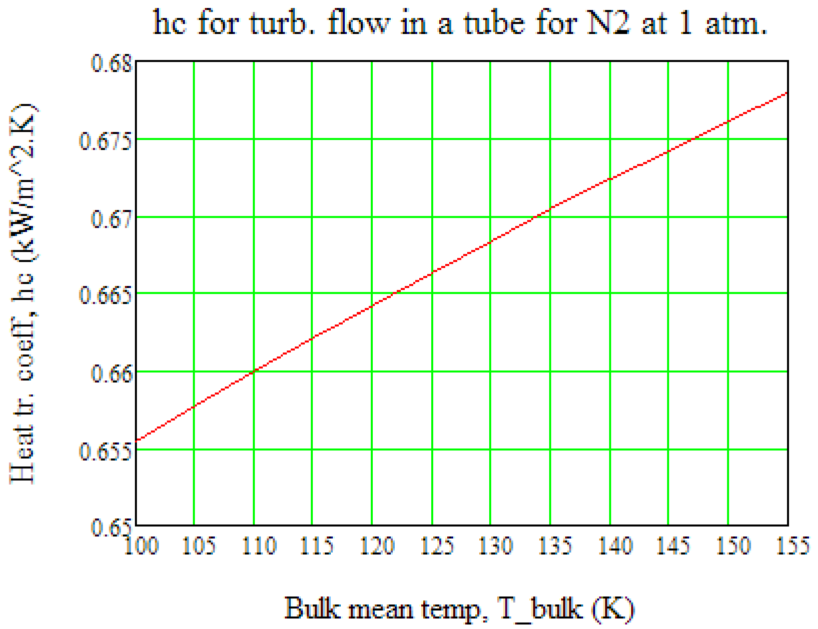
D_i := 0.012 m D_h := 0.6 m m_dot := 0.03 kg/s

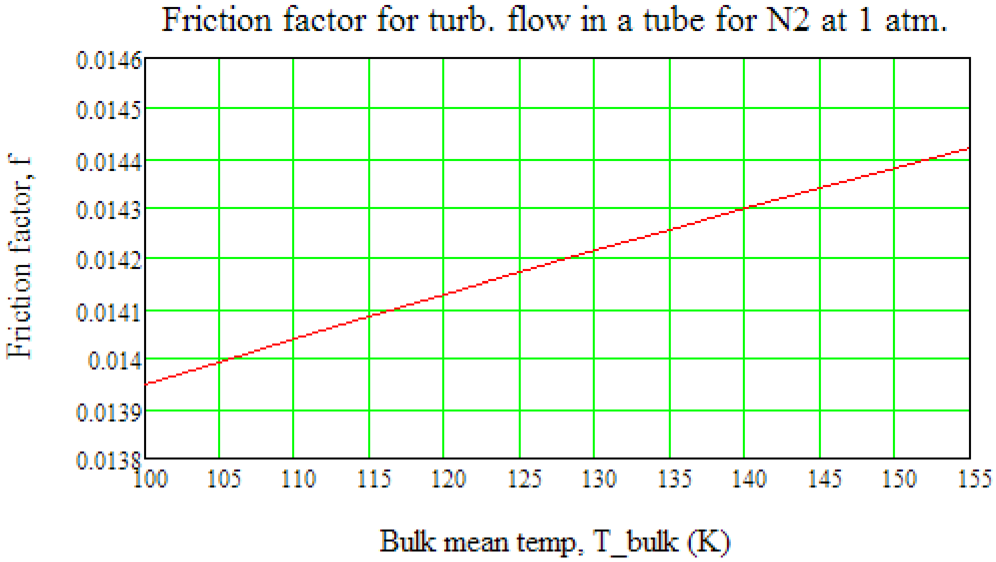
T_wall := 160 K

T_bulk := 100, 105 .. 155 ...define a range vriable



T_bulk (K)	hc (kW/m^2.K)	f
100	0.655	0.0139
105	0.658	0.014
110	0.66	0.014
115	0.662	0.0141
120	0.664	0.0141
125	0.666	0.0142
130	0.668	0.0142
135	0.67	0.0143
140	0.672	0.0143
145	0.674	0.0143
150	0.676	0.0144
155	0.678	0.0144





“**Prob. 3.3.26** A circular tube is constructed of copper (k_t = 50 W/m.K), has circular pin fins protruding from its outer surface. The circular tube has an outer dia of 25 mm. The pin fins are 1.25 mm in dia and 12.5 mm long. There are 2000 fins/m length of tube, and the fins are also of copper. The outside heat transfer coeff is 35 W/m^2.K and the tube temp is 100 K. Determine the fin effectiveness and the overall surface effectiveness.[1].”

Data:

$D_o := 0.025$ m $d_f := 0.00125$ m $L_f := 0.0125$ m $k_f := 50$ W/m.K

$L := 1$ m $N_f := 2000$ fins/m length $h_o := 35$ W/m^2.K $T_o := 100$ K

Calculations:

Fin parameter, M:

$P := \pi \cdot d_f$ m... perimeter of fin

$A_c := \dfrac{\pi \cdot d_f^{\,2}}{4}$ m^2.... cross-sectional area of fin

$M := \sqrt{\dfrac{h_o \cdot P}{k_f \cdot A_c}}$ i.e. $M = 47.329$ 1/m .... fin parameter

Therefore, fin effectiveness, η.f:

$$\eta_f := \frac{\tanh(M \cdot L_f)}{M \cdot L_f}$$

i.e. $\eta_f = 0.898$ **....fin effectiveness ... Ans.**

To calculate Overall surface area effectiveness:

$A\_sf := \pi \cdot d_f \cdot L_f$ m^2 .... surface area of one fin

i.e. $A\_sf = 4.909 \times 10^{-5}$ m^2 .... surface area of one fin

Total fin surface area, A_f:

$A_f := A\_sf \cdot N_f$ m^2

i.e. $A_f = 0.098$ **m^2....total fin surface area**

Total prime area, Ap:

$A_p := L \cdot \pi \cdot D_o - N_f \cdot \dfrac{\pi \cdot d_f^{\,2}}{4}$ m^2

i.e. $A_p = 0.076$ m^2....total prime area

Then, Overall surface area effectiveness:

$A_o := A_p + A_f$ m^2 .... sum of total prime area and fin area

i.e. $A_o = 0.174$ **m^2 .... sum of total prime area and fin area**

$$\eta_o := 1 - \left(\frac{A_f}{A_o}\right)\cdot\left(1 - \eta_f\right)$$ ...Overall surface area effectiveness

i.e. $\eta_o = 0.942$ ...Overall surface area effectiveness.... Ans.

Thus:

Fin effectiveness = 0.898 … Ans.

Overall surface area effectiveness = 0.942…Ans.

Prob. 3.3.27 Determine the overall heat transfer coeff in the previous problem if the inside dia of the tube is 22mm and is plain. Base the overall heat transfer coeff on the total outside surface area (i.e. fin area plus the prime area). Determine the total heat transfer rate if the tube is 15m long. The inside fluid temp is 140 K and the outside fluid temp is 106 K. The inside convective heat transfer coeff is 225 W/m^2.K. [1].

Data:

$D_o := 0.025$ m $D_i := 0.022$ m $T_o := 106$ K $T_i := 140$ K

$d_f := 0.00125$ m $L_f := 0.0125$ m $k_f := 50$ W/m.K

$L := 15$ m $N_f := 2000$ fins/m length $h_o := 35$ W/m^2.K

$h_i := 225$ W/m^2.K

$\eta_{oh} := 0.9423$ ...overall surface effectiveness ... calculated earlier

Calculations:

$A\_sf := \pi \cdot d_f \cdot L_f$ m^2 .... surface area of one fin

i.e. $A\_sf = 4.909 \times 10^{-5}$ m^2 .... surface area of one fin

Total fin surface area, A_f:

$A_f := A\_sf \cdot N_f$ m^2

i.e. $A_f = 0.098$ **m^2....total fin surface area**

Total prime area, Ap:

$$A_p := L \cdot \pi \cdot D_o - N_f \cdot \frac{\pi \cdot d_f^{\,2}}{4}$$ m^2

i.e. $A_p = 1.176$ m^2....total prime area

Then,total heat transfer area on the outside:

$A_{oh} := A_p + A_f$ m^2 .... sum of total prime area and fin area

i.e. $A_{oh} = 1.274$ m^2 .... sum of total prime area and fin area

And, total heat transfer area on the inside:

$A_{oc} := \pi \cdot D_i \cdot L$ m^2

i.e. $A_{oc} = 1.037$ m^2 .... heat tr. area on the inside

Therefore, overalll heat transfer coeff. U based on total outer area:

$$U_{oh} := \left(\frac{1}{\eta_{oh} \cdot h_o} + \frac{A_{oh}}{A_{oc}} \cdot \frac{1}{h_i}\right)^{-1}$$ W/m^2.K ... overall heat tr coeff based on total outer area

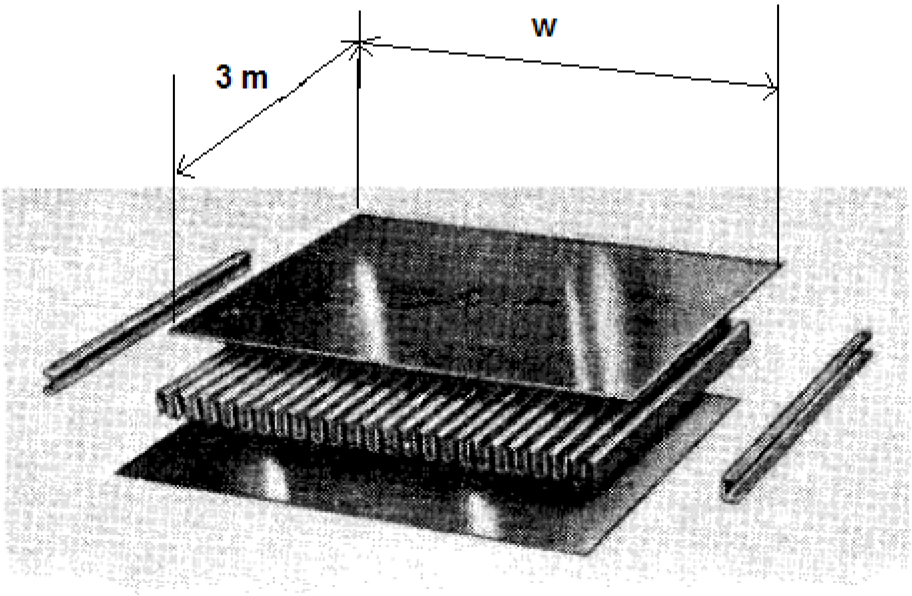
i.e. $U_{oh} = 27.947$ **W/m^2.K ... overall heat tr coeff based on total outer area .. Ans.**

And, total heat transfer rate:

$Q_{tot} := U_{oh} \cdot A_{oh} \cdot (T_i - T_o)$ W... total heat transfer rate

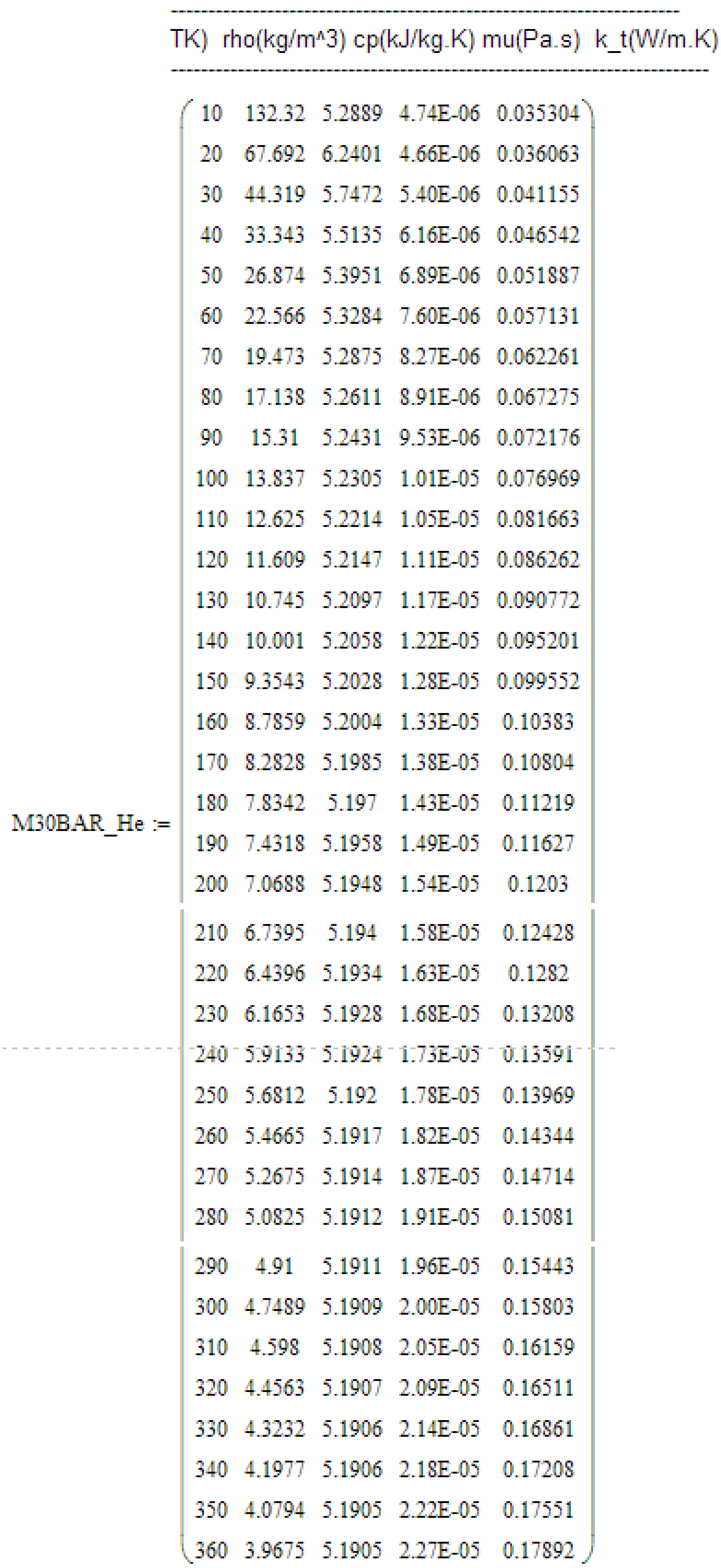
i.e. $Q_{tot} = 1.21 \times 10^3$ **W... total heat transfer rate ... Ans.**

Prob. 3.3.28 A plate – fin heat exchanger is constructed using straight fins, 7.87mm (0.31 in) high, 0.15mm (0.006 in) thick, with 492 fins/m (12.5 fins/in) width. The equivalent dia for the flow passage is 3mm (0.1181 in), the free-flow area for the exchanger is 0.07 m^2 (0.7535 ft^2), and the exchanger length is 3 m (9.84 ft). The fluid flowing is helium gas, which enters at 300 K and 3.0 MPa and leaves at 100 K. The mass flow rate of helium gas is 0.35 kg/s. The heat exchanger is constructed of aluminium (k_t = 150 W/m.K). Determine the convective heat transfer coeff. for this surface and the fin effectiveness. [1].



First, write Mathcad Functions to determine the properties of Helium gas at 30 bar, using the data from NIST [28]:

For Gaeous Helium at 30 bar:



M30BAR_He :=

TK)	rho(kg/m^3)	cp(kJ/kg.K)	mu(Pa.s)	k_t(W/m.K)
10	132.32	5.2889	4.74E-06	0.035304
20	67.692	6.2401	4.66E-06	0.036063
30	44.319	5.7472	5.40E-06	0.041155
40	33.343	5.5135	6.16E-06	0.046542
50	26.874	5.3951	6.89E-06	0.051887
60	22.566	5.3284	7.60E-06	0.057131
70	19.473	5.2875	8.27E-06	0.062261
80	17.138	5.2611	8.91E-06	0.067275
90	15.31	5.2431	9.53E-06	0.072176
100	13.837	5.2305	1.01E-05	0.076969
110	12.625	5.2214	1.05E-05	0.081663
120	11.609	5.2147	1.11E-05	0.086262
130	10.745	5.2097	1.17E-05	0.090772
140	10.001	5.2058	1.22E-05	0.095201
150	9.3543	5.2028	1.28E-05	0.099552
160	8.7859	5.2004	1.33E-05	0.10383
170	8.2828	5.1985	1.38E-05	0.10804
180	7.8342	5.197	1.43E-05	0.11219
190	7.4318	5.1958	1.49E-05	0.11627
200	7.0688	5.1948	1.54E-05	0.1203
210	6.7395	5.194	1.58E-05	0.12428
220	6.4396	5.1934	1.63E-05	0.1282
230	6.1653	5.1928	1.68E-05	0.13208
240	5.9133	5.1924	1.73E-05	0.13591
250	5.6812	5.192	1.78E-05	0.13969
260	5.4665	5.1917	1.82E-05	0.14344
270	5.2675	5.1914	1.87E-05	0.14714
280	5.0825	5.1912	1.91E-05	0.15081
290	4.91	5.1911	1.96E-05	0.15443
300	4.7489	5.1909	2.00E-05	0.15803
310	4.598	5.1908	2.05E-05	0.16159
320	4.4563	5.1907	2.09E-05	0.16511
330	4.3232	5.1906	2.14E-05	0.16861
340	4.1977	5.1906	2.18E-05	0.17208
350	4.0794	5.1905	2.22E-05	0.17551
360	3.9675	5.1905	2.27E-05	0.17892

Now, extract the separate data vectors:

$$\text{temp30B\_He} := \text{M30BAR\_He}^{\langle 0\rangle} \qquad \text{rho30B\_He} := \text{M30BAR\_He}^{\langle 1\rangle}$$

$$\text{cp30B\_He} := \text{M30BAR\_He}^{\langle 2\rangle} \qquad \text{mu30B\_He} := \text{M30BAR\_He}^{\langle 3\rangle}$$

$$\text{thcond30B\_He} := \text{M30BAR\_He}^{\langle 4\rangle}$$

And, now write the Functions:

$$\text{rho\_30B\_He}(T) := \text{linterp}(\text{temp30B\_He}, \text{rho30B\_He}, T) \qquad \text{rho\_30B\_He}(150) = 9.354$$ kg/m^3

$$\text{cp\_30B\_He}(T) := \text{linterp}(\text{temp30B\_He}, \text{cp30B\_He}, T) \qquad \text{cp\_30B\_He}(150) = 5.203$$ kJ/kg.K

$$\text{mu\_30B\_He}(T) := \text{linterp}(\text{temp30B\_He}, \text{mu30B\_He}, T) \qquad \text{mu\_30B\_He}(150) = 1.28 \times 10^{-5}$$ Pa.s = kg.s/m^2

$$\text{kt\_30B\_He}(T) := \text{linterp}(\text{temp30B\_He}, \text{thcond30B\_He}, T) \qquad \text{kt\_30B\_He}(150) = 0.1$$ W/m.K

$$\text{Prandtl\_30B\_He}(T) := \frac{1000\,\text{cp\_30B\_He}(T)\cdot\text{mu\_30B\_He}(T)}{\text{kt\_30B\_He}(T)} \qquad \text{Prandtl\_30B\_He}(150) = 0.669$$

Now, solve the problem:

Data:

$t_f := 0.00015$ m $\quad L_f := \dfrac{0.00787}{2}$ m .. divided by 2, since fin is supported at two ends

$N_f := 492$ fins/m width $\quad L := 3$ m.... length of HX

$D_e := 0.003$ m .... equiv. dia of flow passage

$A_{ff} := 0.07$ m^2 ... free flow area for the HX

$T_i := 300$ K $\quad T_e := 100$ K

$\text{mdot} := 0.35$ kg/s .... mass flow rate of He $\quad P1 := 30$ bar ... pressure of helium gas

$k_f := 150$ W/m.K ... th. cond. of fin material, viz. Al

Calculations:

$G := \dfrac{\text{mdot}}{A_{ff}}$ kg/m^2.s .... mass velocity

i.e. $\quad G = 5$ kg/m^2.s .... mass velocity

$T_b := \frac{(T_i + T_o)}{2}$ K .... mean bulk temp of He

i.e. $T_b = 200$ K .... mean bulk temp of He

Properties of Helium gas at 30 bar, and temp Tb:

$rho := rho\_30B\_He(T_b)$ $rho = 7.069$ kg/m^3

$cp := cp\_30B\_He(T_b)$ $cp = 5.195$ kJ/kg.K

$mu := mu\_30B\_He(T_b)$ $mu = 1.54 \times 10^{-5}$ Pa.s = kg.s/m^2

$k\_t := kt\_30B\_He(T_b)$ $k\_t = 0.12$ W/m.K

$Pr := Prandtl\_30B\_He(T_b)$ $Pr = 0.665$

Reynolds No.:

$Re := \frac{G \cdot D_e}{mu}$ i.e. $Re = 974.026$ ....Reynolds No.

Colburn j factor:

Ue the relation given in Ref.[1]

$jH := 0.0291 \cdot Re^{-0.24}$ i.e. $jH = 5.58 \times 10^{-3}$

Now, we have:

$$jH = \frac{h_c}{G \cdot cp} \cdot Pr^{\frac{2}{3}}$$

Therefore, heat transfer coeff. h_c:

$$h_c := \frac{jH \cdot G \cdot cp \cdot 1000}{Pr^{\frac{2}{3}}}$$

i.e. $h_c = 190.236$ **W/m^2.K .... Ans.**

To find fin effectiveness, ηf:

$P := 2 \cdot (L + t_f)$ m ... perimeter of fin

i.e. $P = 6$ m ... perimeter of fin

$A_c := L \cdot t_f$ m^2 ... cross-sectional area of fin

i.e. $A_c = 4.5 \times 10^{-4}$ m^2 ... cross-sectional area of fin

Fin parameter, M:

$M := \sqrt{\frac{h_c \cdot P}{k_f \cdot A_c}}$ i.e. $M = 130.041$ **1/m .... fin parameter**

Then, fin effectiveness is given by:

$$\eta_f := \frac{\tanh(M \cdot L_f)}{M \cdot L_f}$$

i.e. $\eta_f = 0.921$ **....fin effectiveness ... Ans.**

Plot hc and fin effectiveness as mass flow rate of helium varies from 0.2 kg/s to 0.8 kg/s, other quantities remaining unchanged:

Write the relevant quantities as functions of mdot:

$G(mdot) := \frac{mdot}{A_{ff}}$ kg/m^2.s .... mass velocity $G(mdot) = 5$

$Re(mdot) := \frac{G(mdot) \cdot D_e}{mu}$ i.e. $Re(mdot) = 974.026$

$jH(mdot) := 0.0291 \cdot Re(mdot)^{-0.24}$ i.e. $jH(mdot) = 5.58 \times 10^{-3}$

$h_c(mdot) := \frac{jH(mdot) \cdot G(mdot) \cdot cp \cdot 1000}{Pr^{\frac{2}{3}}}$ i.e. $h_c(mdot) = 190.236$

Fin parameter, M, and effectiveness, η f:

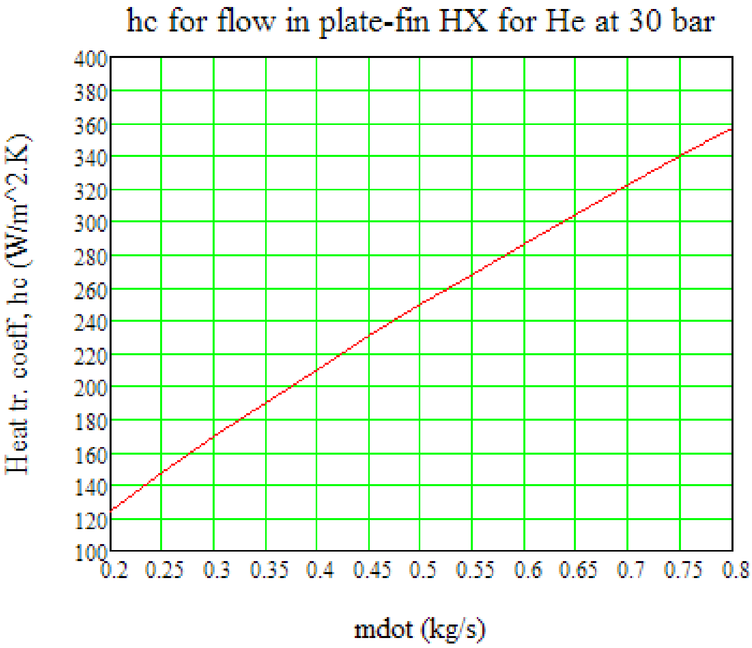
$M(mdot) := \sqrt{\frac{h_c(mdot) \cdot P}{k_f \cdot A_c}}$ i.e. $M(mdot) = 130.041$

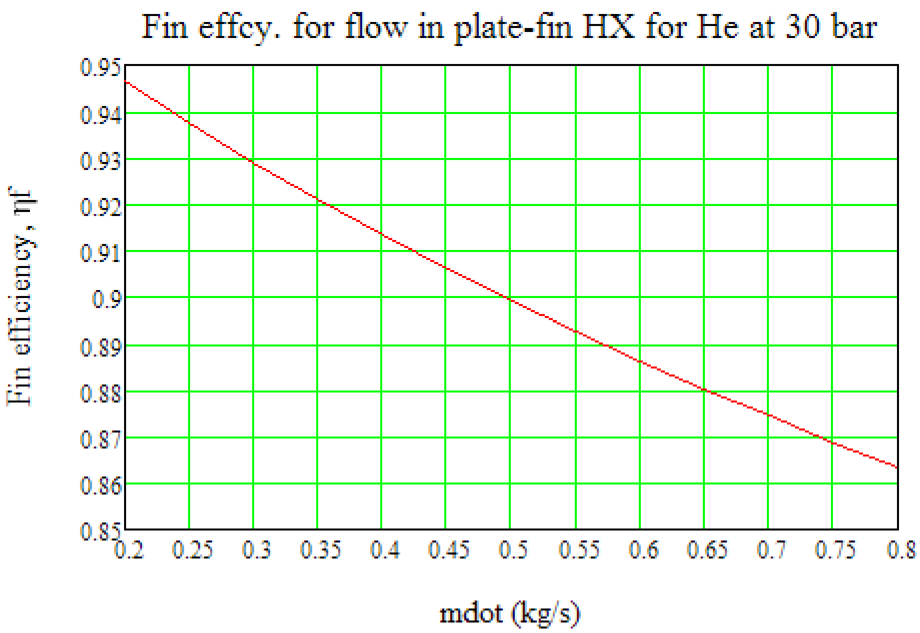
$\eta_f(mdot) := \frac{\tanh\left(M(mdot) \cdot L_f\right)}{M(mdot) \cdot L_f}$ i.e. $\eta_f(mdot) = 0.921$

Now, plot hc and ηf against mdot:

$mdot := 0.2, 0.25 .. 0.8$ ...define a range variable

$mdot =$	$h_c(mdot) =$	$\eta_f(mdot) =$
0.2	124.332	0.947
0.25	147.311	0.937
0.3	169.205	0.929
0.35	190.236	0.921
0.4	210.555	0.913
0.45	230.272	0.906
0.5	249.47	0.899
0.55	268.211	0.893
0.6	286.547	0.886
0.65	304.519	0.88
0.7	322.163	0.874
0.75	339.506	0.869
0.8	356.573	0.863


hc for flow in plate-fin HX for He at 30 bar
Heat tr. coeff, hc (W/m^2.K)
400
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
mdot (kg/s)

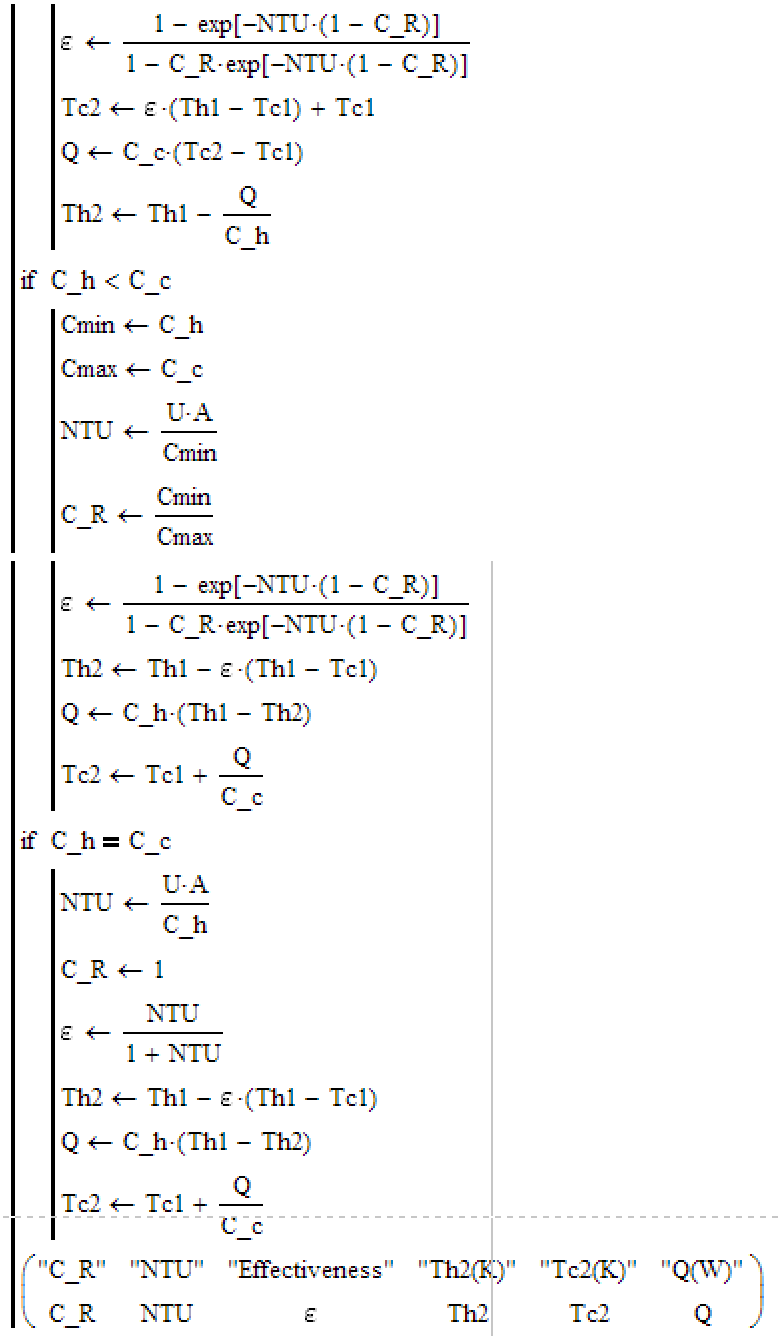


Prob. 3.3.29 Write a Mathcad Function to calculate the NTU, effectiveness, heat transferred etc for a Counter-flow heat exchanger.

Gaseous nitrogen enters the warm end of a counter-flow HX at 20.27 MPa and 295 K at a mass flow rate of 1.25 kg/s. Gaseous nitrogen enters at the cold end of the HX at 80 K. For the warm stream, the sp. heat is 1.296 J/g.K and the sp. heat for the cold stream is 1.08 J/g.K. The mass flow rate of cold stream is 1.125 kg/s. The overall heat transfer coeff is 150 W/m^2.K and the heat transfer area on which U is based is 72.9 m^2. Determine the heat exchanger effectiveness, exit temp of cold stream and the heat transfer rate. [1].

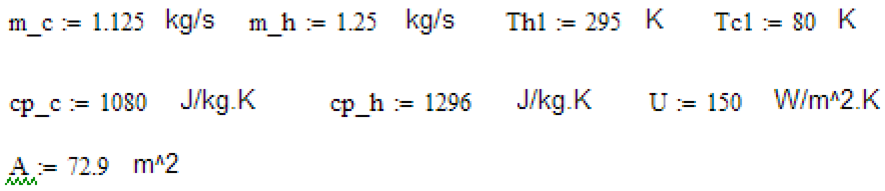
Mathcad Function:

```
CounterflowHX(m_c, m_h, cp_c, cp_h, Th1, Tc1, A, U) :=

    C_h ← m_h·cp_h
    C_c ← m_c·cp_c
    if C_c < C_h
        Cmin ← C_c
        Cmax ← C_h
        NTU ← $\frac{U \cdot A}{Cmin}$
        C_R ← $\frac{Cmin}{Cmax}$
```



```
    $\varepsilon \leftarrow \frac{1 - \exp[-NTU\cdot(1 - C\_R)]}{1 - C\_R\cdot\exp[-NTU\cdot(1 - C\_R)]}$
    $Tc2 \leftarrow \varepsilon\cdot(Th1 - Tc1) + Tc1$
    $Q \leftarrow C\_c\cdot(Tc2 - Tc1)$
    $Th2 \leftarrow Th1 - \frac{Q}{C\_h}$
if $C\_h < C\_c$
    $Cmin \leftarrow C\_h$
    $Cmax \leftarrow C\_c$
    $NTU \leftarrow \frac{U\cdot A}{Cmin}$
    $C\_R \leftarrow \frac{Cmin}{Cmax}$
    $\varepsilon \leftarrow \frac{1 - \exp[-NTU\cdot(1 - C\_R)]}{1 - C\_R\cdot\exp[-NTU\cdot(1 - C\_R)]}$
    $Th2 \leftarrow Th1 - \varepsilon\cdot(Th1 - Tc1)$
    $Q \leftarrow C\_h\cdot(Th1 - Th2)$
    $Tc2 \leftarrow Tc1 + \frac{Q}{C\_c}$
if $C\_h = C\_c$
    $NTU \leftarrow \frac{U\cdot A}{C\_h}$
    $C\_R \leftarrow 1$
    $\varepsilon \leftarrow \frac{NTU}{1 + NTU}$
    $Th2 \leftarrow Th1 - \varepsilon\cdot(Th1 - Tc1)$
    $Q \leftarrow C\_h\cdot(Th1 - Th2)$
    $Tc2 \leftarrow Tc1 + \frac{Q}{C\_c}$
$\begin{pmatrix} "C\_R" & "NTU" & "Effectiveness" & "Th2(K)" & "Tc2(K)" & "Q(W)" \\ C\_R & NTU & \varepsilon & Th2 & Tc2 & Q \end{pmatrix}$
```

Now, solve the problem:



Data:

m_c := 1.125 kg/s m_h := 1.25 kg/s Th1 := 295 K Tc1 := 80 K

cp_c := 1080 J/kg.K cp_h := 1296 J/kg.K U := 150 W/m^2.K

A := 72.9 m^2

And, apply the Function written above:

$$\mathrm{CounterflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U) = \begin{pmatrix} \text{"C\_R"} & \text{"NTU"} & \text{"Effectiveness"} & \text{"Th2(K)"} & \text{"Tc2(K)"} & \text{"Q(W)"} \\ 0.75 & 9 & 0.9714 & 138.3636 & 288.8485 & 2.5375\times 10^{5} \end{pmatrix}$$

i.e.

$$CapRatio := \mathrm{CounterflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,0} \qquad CapRatio = 0.75$$

$$NTU := \mathrm{CounterflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,1} \qquad NTU = 9$$

$$EFF := \mathrm{CounterflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,2} \qquad EFF = 0.9714$$

$$Th2 := \mathrm{CounterflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,3} \qquad Th2 = 138.364 \quad \mathrm{K}$$

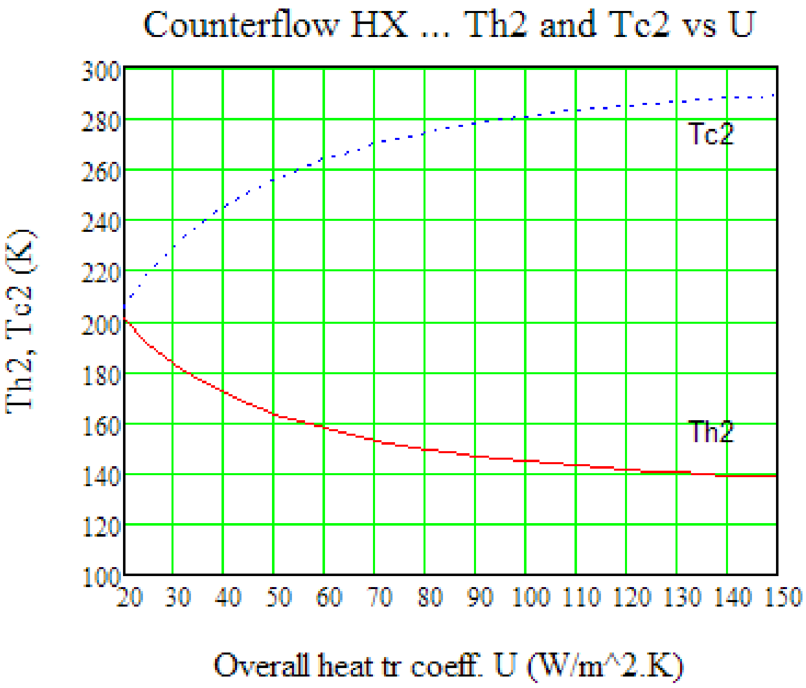
$$Tc2 := \mathrm{CounterflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,4} \qquad Tc2 = 288.849 \quad \mathrm{K}$$

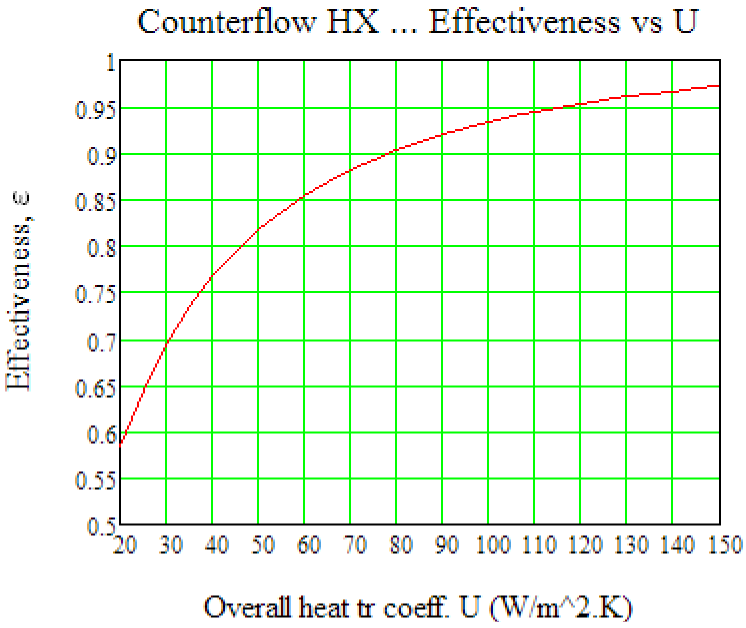
$$Q := \mathrm{CounterflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,5} \qquad Q = 2.5375\times 10^{5} \quad \mathrm{W}$$

……

Plot the exit temps of hot and cold fluids, and effectiveness of HX as the overall heat transfer coeff. U varies from 20 to 150 W/m^2.K:

U := 20, 25 .. 150 ....define a range variable

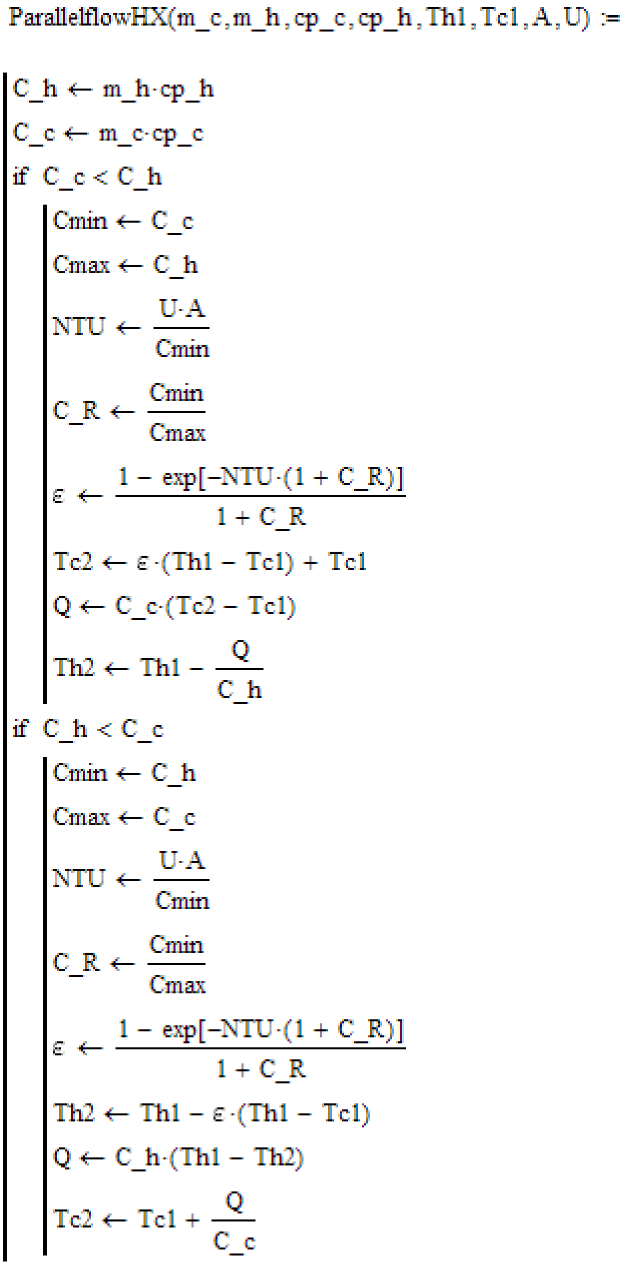




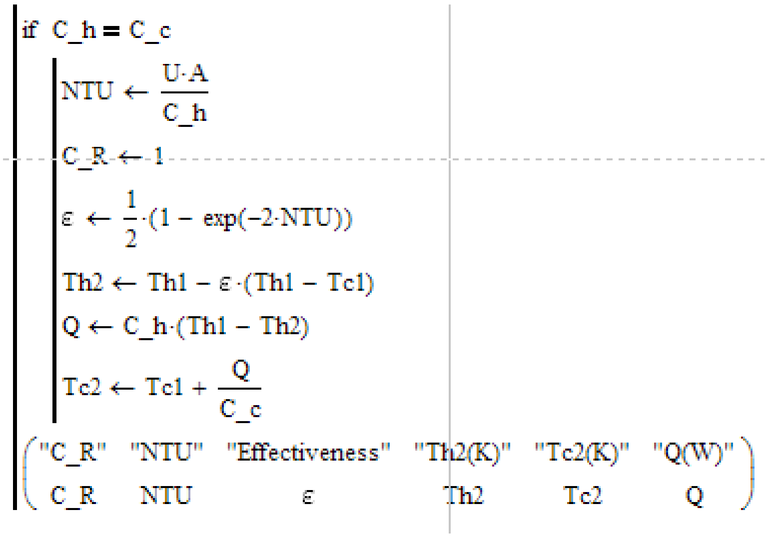
Prob. 3.3.30 (a) Write a Mathcad Function to calculate the NTU, effectiveness, heat transferred etc for a Parallel flow heat exchanger.

(b) Determine the heat exchanger effectiveness, exit temp of cold stream and the heat transfer rate for the Problem 3.3.29 if the HX is parallel flow type. [1].

Mathcad Function for Parallel flow HX:

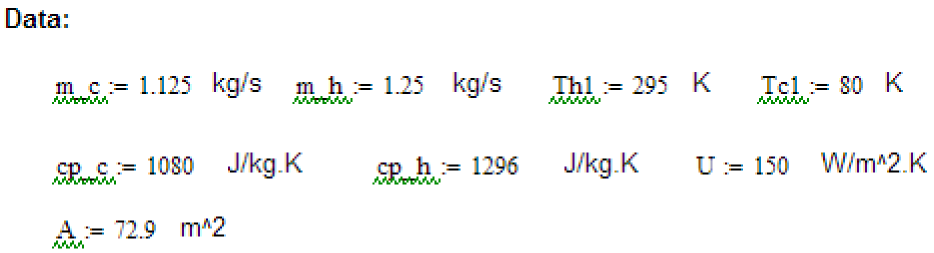


```
ParallelflowHX(m_c,m_h,cp_c,cp_h,Th1,Tc1,A,U) :=
  C_h ← m_h·cp_h
  C_c ← m_c·cp_c
  if C_c < C_h
    Cmin ← C_c
    Cmax ← C_h
    NTU ← $\frac{U \cdot A}{Cmin}$
    C_R ← $\frac{Cmin}{Cmax}$
    ε ← $\frac{1 - \exp[-NTU \cdot (1 + C\_R)]}{1 + C\_R}$
    Tc2 ← ε·(Th1 − Tc1) + Tc1
    Q ← C_c·(Tc2 − Tc1)
    Th2 ← $Th1 - \frac{Q}{C\_h}$
  if C_h < C_c
    Cmin ← C_h
    Cmax ← C_c
    NTU ← $\frac{U \cdot A}{Cmin}$
    C_R ← $\frac{Cmin}{Cmax}$
    ε ← $\frac{1 - \exp[-NTU \cdot (1 + C\_R)]}{1 + C\_R}$
    Th2 ← Th1 − ε·(Th1 − Tc1)
    Q ← C_h·(Th1 − Th2)
    Tc2 ← $Tc1 + \frac{Q}{C\_c}$
```



```
if C_h = C_c
    NTU ← $\frac{U \cdot A}{C\_h}$
    C_R ← 1
    ε ← $\frac{1}{2} \cdot (1 - \exp(-2 \cdot NTU))$
    Th2 ← Th1 − ε·(Th1 − Tc1)
    Q ← C_h·(Th1 − Th2)
    Tc2 ← Tc1 + $\frac{Q}{C\_c}$
$\begin{pmatrix} "C\_R" & "NTU" & "Effectiveness" & "Th2(K)" & "Tc2(K)" & "Q(W)" \\ C\_R & NTU & \varepsilon & Th2 & Tc2 & Q \end{pmatrix}$
```

Now, solve the problem:



Data:

m_c := 1.125 kg/s m_h := 1.25 kg/s Th1 := 295 K Tc1 := 80 K

cp_c := 1080 J/kg.K cp_h := 1296 J/kg.K U := 150 W/m^2.K

A := 72.9 m^2

And, using the Function written above:

$$\text{ParallelflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U) = \begin{pmatrix} \text{"C\_R"} & \text{"NTU"} & \text{"Effectiveness"} & \text{"Th2(K)"} & \text{"Tc2(K)"} & \text{"Q(W)"} \\ 0.75 & 9 & 0.5714 & 202.8572 & 202.8571 & 1.4927 \times 10^{5} \end{pmatrix}$$

i.e.

$CapRatio := \text{ParallelflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,0}$ $\quad CapRatio = 0.75$

$NTU := \text{ParallelflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,1}$ $\quad NTU = 9$

$EFF(U) := \text{ParallelflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,2}$ $\quad EFF(U) = 0.5714$

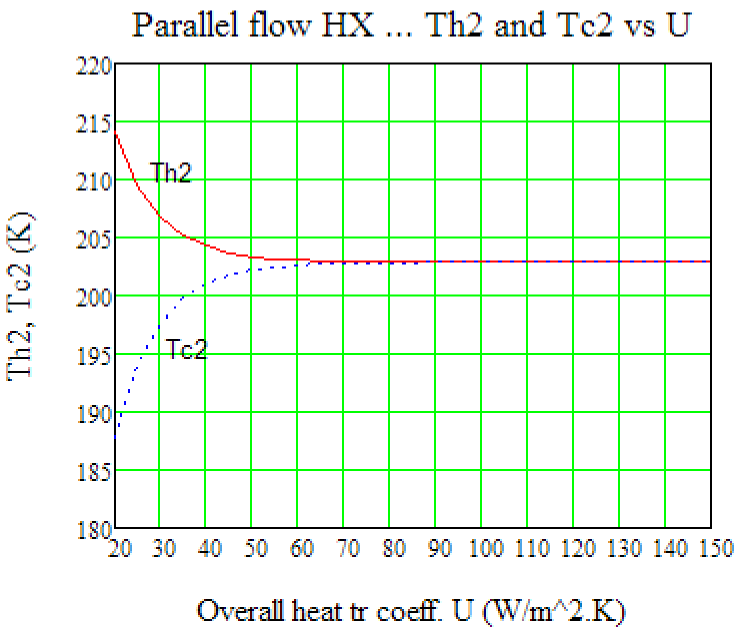
$Th2(U) := \text{ParallelflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,3}$ $\quad Th2(U) = 202.857 \ K$

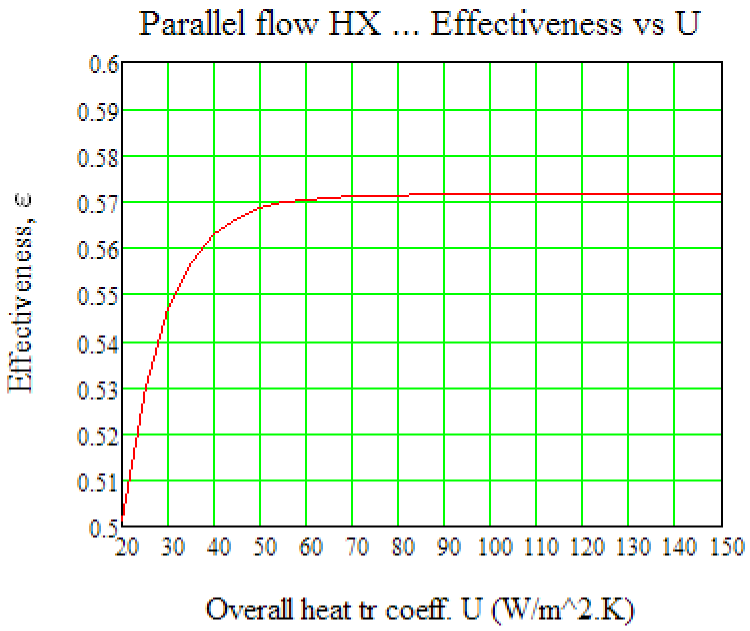
$Tc2(U) := \text{ParallelflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,4}$ $\quad Tc2(U) = 202.857 \ K$

$Q := \text{ParallelflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,5}$ $\quad Q = 1.4927 \times 10^{5} \ W$

Plot the exit temps of hot and cold fluids, and effectiveness as the overall heat transfer coeff varies from 20 to 150 W/m^2.K:

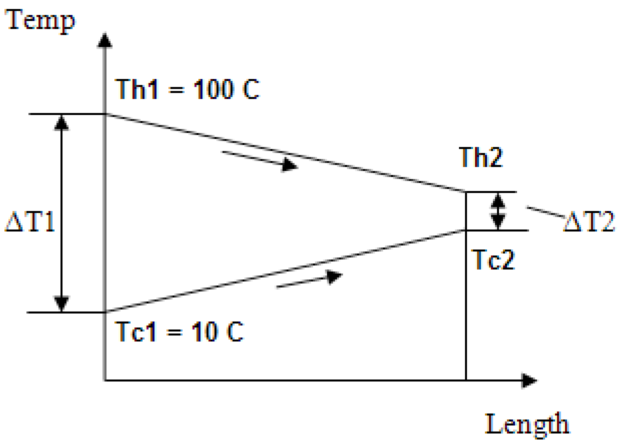
$U := 20, 25 .. 150$ ....define a range variable



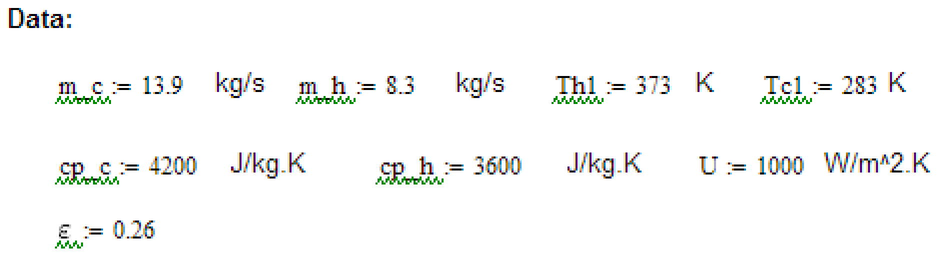


Note: Effectiveness and heat transfer are more for a Counter-flow HX.

Prob. 3.3.31 Oil at 100 C (cp = 3.6 kJ/kg.K) flows at a rate of 8.3 kg/s and enters in to a parallel flow HX. Cooling water (cp = 4.2 kJ/kg.K) enters the HX at 10 C at a rate of 13.9 kg/s. The HX has an effectiveness of 0.26. If the overall heat transfer coeff is 1000 W/m^2.K, find the area required for the HX. Also, find the outlet temperatures of both the fluids.



Use the Mathcad Function for Parallel flow HX, already written above, with the 'Solve block' of Mathcad to easily determine the Area required:



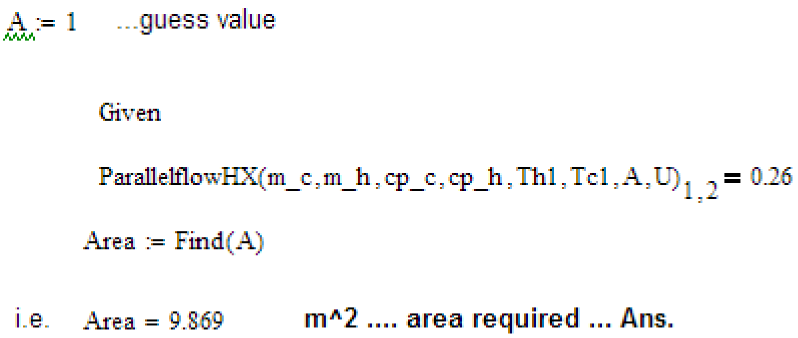
Data:

m_c := 13.9 kg/s   m_h := 8.3 kg/s   Th1 := 373 K   Tc1 := 283 K

cp_c := 4200 J/kg.K   cp_h := 3600 J/kg.K   U := 1000 W/m^2.K

ε := 0.26

Solve Block:



A := 1   ...guess value

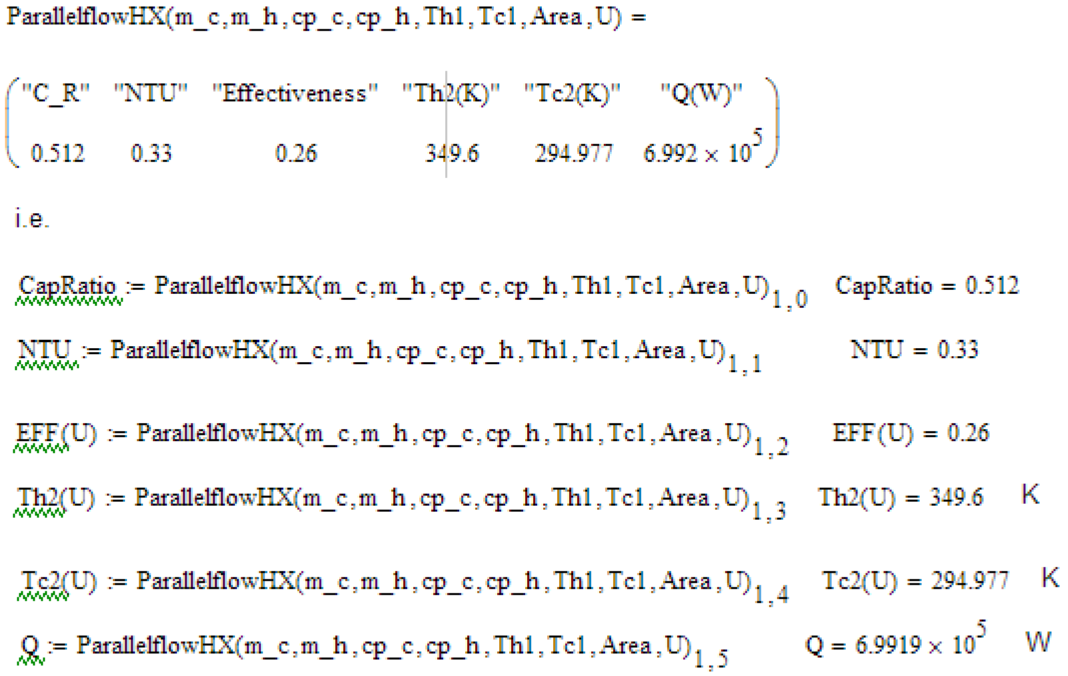
Given

ParallelflowHX(m_c, m_h, cp_c, cp_h, Th1, Tc1, A, U)$_{1,2}$ = 0.26

Area := Find(A)

i.e.   Area = 9.869   **m^2 .... area required ... Ans.**

Now, with this Area entered, apply the Mathcad Function for parallel flow HX, to get exit temps of fluids etc.:



ParallelflowHX(m_c, m_h, cp_c, cp_h, Th1, Tc1, Area, U) =

$$\begin{pmatrix} \text{"C\_R"} & \text{"NTU"} & \text{"Effectiveness"} & \text{"Th2(K)"} & \text{"Tc2(K)"} & \text{"Q(W)"} \\ 0.512 & 0.33 & 0.26 & 349.6 & 294.977 & 6.992 \times 10^{5} \end{pmatrix}$$

i.e.

CapRatio := ParallelflowHX(m_c, m_h, cp_c, cp_h, Th1, Tc1, Area, U)$_{1,0}$ CapRatio = 0.512

NTU := ParallelflowHX(m_c, m_h, cp_c, cp_h, Th1, Tc1, Area, U)$_{1,1}$ NTU = 0.33

EFF(U) := ParallelflowHX(m_c, m_h, cp_c, cp_h, Th1, Tc1, Area, U)$_{1,2}$ EFF(U) = 0.26

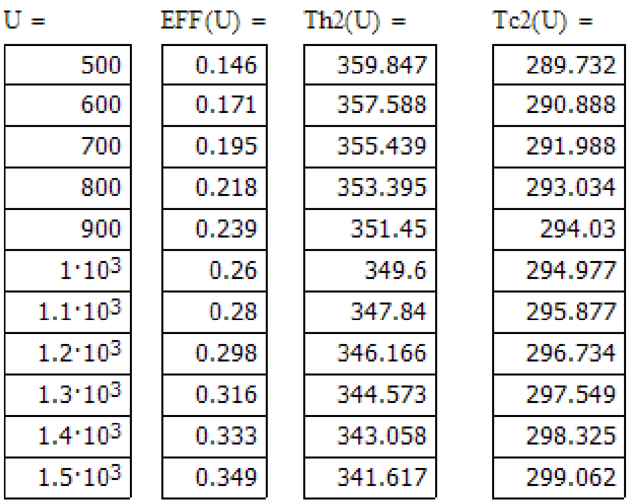
Th2(U) := ParallelflowHX(m_c, m_h, cp_c, cp_h, Th1, Tc1, Area, U)$_{1,3}$ Th2(U) = 349.6 K

Tc2(U) := ParallelflowHX(m_c, m_h, cp_c, cp_h, Th1, Tc1, Area, U)$_{1,4}$ Tc2(U) = 294.977 K

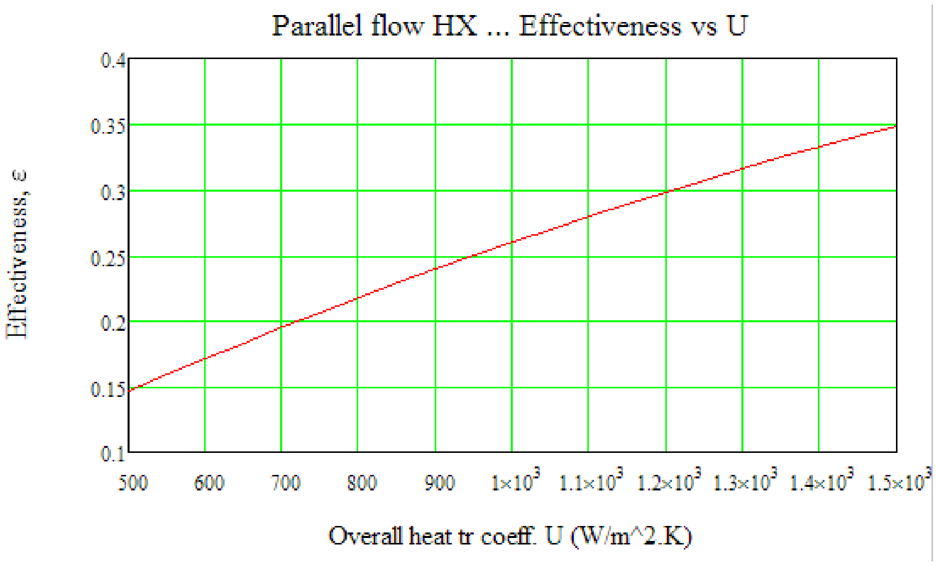
Q := ParallelflowHX(m_c, m_h, cp_c, cp_h, Th1, Tc1, Area, U)$_{1,5}$ $Q = 6.9919 \times 10^{5}$ W

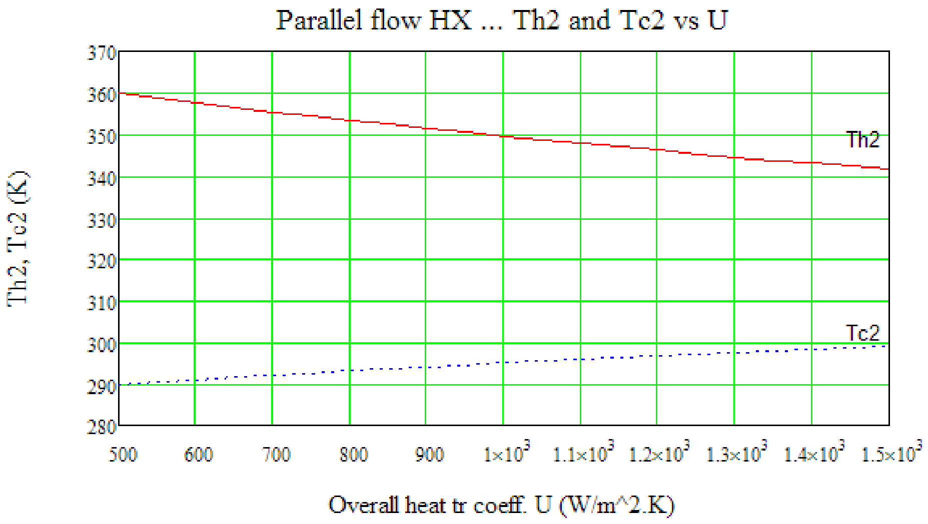
Plot the exit temps of hot and cold fluids, and effectiveness as the overall heat transfer coeff varies from 500 to 1500 W/m^2.K:

U := 500, 550 .. 1500 ....define a range variable



U =	EFF(U) =	Th2(U) =	Tc2(U) =
500	0.146	359.847	289.732
600	0.171	357.588	290.888
700	0.195	355.439	291.988
800	0.218	353.395	293.034
900	0.239	351.45	294.03
$1 \cdot 10^{3}$	0.26	349.6	294.977
$1.1 \cdot 10^{3}$	0.28	347.84	295.877
$1.2 \cdot 10^{3}$	0.298	346.166	296.734
$1.3 \cdot 10^{3}$	0.316	344.573	297.549
$1.4 \cdot 10^{3}$	0.333	343.058	298.325
$1.5 \cdot 10^{3}$	0.349	341.617	299.062

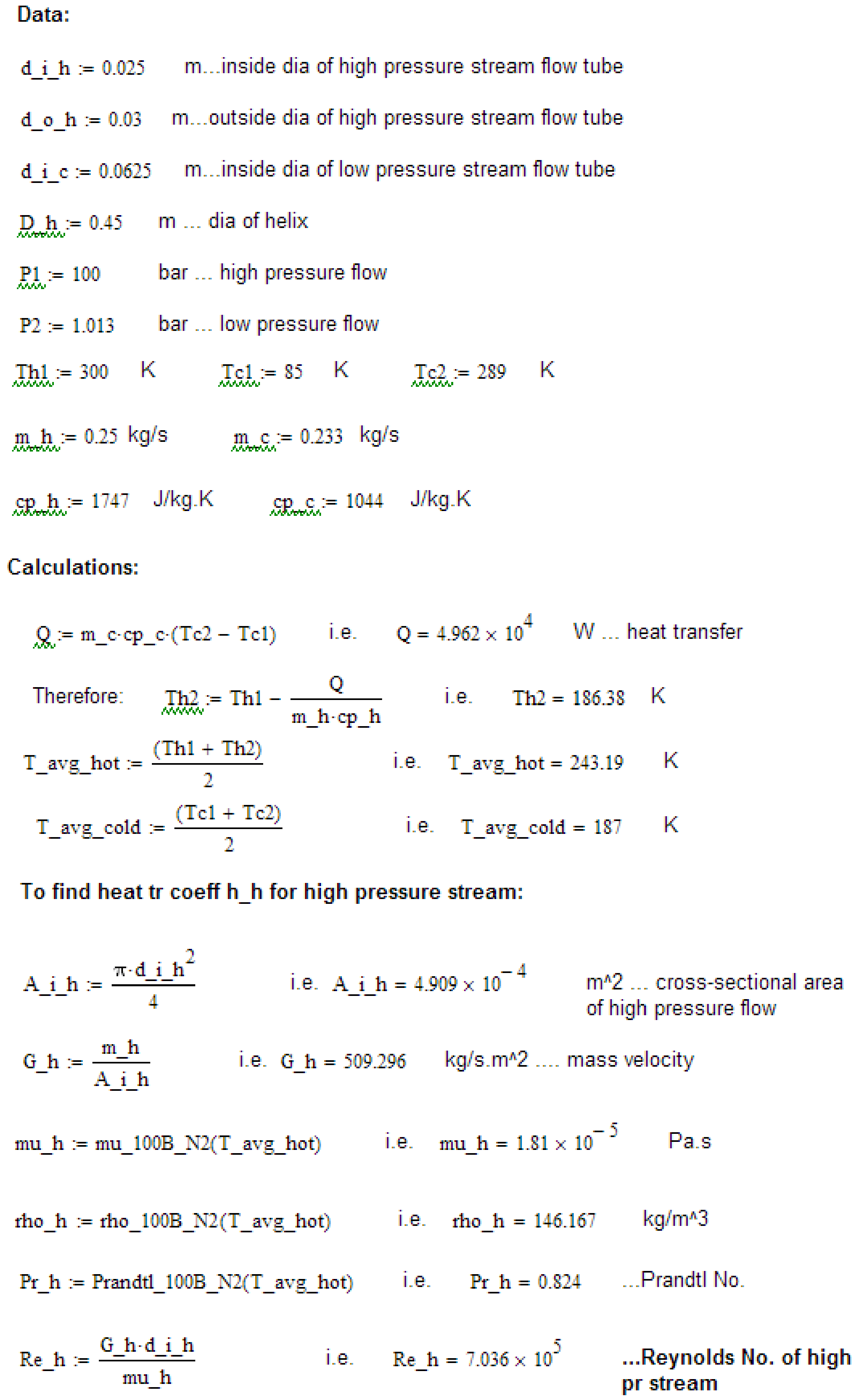




==

Prob. 3.3.32. A Linde concentric tube HX is to be constructed from a 25 mm ID tube having a 2.5 mm wall thickness concentric to a 62.5 mm inside dia tube. Nitrogen at 10.13 MPa and 300 K enters the smaller tube, and air at 101.3 kPa and 85 K enters the larger tube and leaves at 289 K. The mass flow rate of high pressure stream is 0.25 kg/s, and the mass flow rate of low pressure stream is 0.233 kg/s. The mean sp. heat of high pressure stream is 1.747 kJ/kg.K, and the mean sp. heat of low pressure stream is 1.044 kJ/kg.K. If the concentric tube is wound in a 450 mm dia helix and heavily insulated, determine the length of concentric tubing required for this heat exchanger.

Mathcad Solution:



Data:

$d\_i\_h := 0.025$ m...inside dia of high pressure stream flow tube

$d\_o\_h := 0.03$ m...outside dia of high pressure stream flow tube

$d\_i\_c := 0.0625$ m...inside dia of low pressure stream flow tube

$D\_h := 0.45$ m ... dia of helix

$P1 := 100$ bar ... high pressure flow

$P2 := 1.013$ bar ... low pressure flow

$Th1 := 300$ K $Tc1 := 85$ K $Tc2 := 289$ K

$m\_h := 0.25$ kg/s $m\_c := 0.233$ kg/s

$cp\_h := 1747$ J/kg.K $cp\_c := 1044$ J/kg.K

Calculations:

$Q := m\_c \cdot cp\_c \cdot (Tc2 - Tc1)$ i.e. $Q = 4.962 \times 10^{4}$ W ... heat transfer

Therefore: $Th2 := Th1 - \dfrac{Q}{m\_h \cdot cp\_h}$ i.e. $Th2 = 186.38$ K

$T\_avg\_hot := \dfrac{(Th1 + Th2)}{2}$ i.e. $T\_avg\_hot = 243.19$ K

$T\_avg\_cold := \dfrac{(Tc1 + Tc2)}{2}$ i.e. $T\_avg\_cold = 187$ K

To find heat tr coeff h_h for high pressure stream:

$A\_i\_h := \dfrac{\pi \cdot d\_i\_h^{2}}{4}$ i.e. $A\_i\_h = 4.909 \times 10^{-4}$ m^2 ... cross-sectional area of high pressure flow

$G\_h := \dfrac{m\_h}{A\_i\_h}$ i.e. $G\_h = 509.296$ kg/s.m^2 .... mass velocity

$mu\_h := mu\_100B\_N2(T\_avg\_hot)$ i.e. $mu\_h = 1.81 \times 10^{-5}$ Pa.s

$rho\_h := rho\_100B\_N2(T\_avg\_hot)$ i.e. $rho\_h = 146.167$ kg/m^3

$Pr\_h := Prandtl\_100B\_N2(T\_avg\_hot)$ i.e. $Pr\_h = 0.824$ ...Prandtl No.

$Re\_h := \dfrac{G\_h \cdot d\_i\_h}{mu\_h}$ i.e. $Re\_h = 7.036 \times 10^{5}$ **...Reynolds No. of high pr stream**

Since Re_h > 3000, we have:

$$jH\_h := 0.023 \cdot Re\_h^{-0.2} \cdot \left(1 + \frac{3.5 \cdot d\_i\_h}{D\_h}\right)$$

i.e. $jH\_h = 1.86 \times 10^{-3}$ **...Colburn j factor**

Now, we have:

$$jH\_h = \left(\frac{h\_h}{cp\_h \cdot G\_h}\right) \cdot Pr\_h^{\frac{2}{3}}$$ ...by definition

Therefore: heat tr coeff h_h on high pressure side:

$$h\_h := \frac{jH\_h}{Pr\_h^{\frac{2}{3}}} \cdot cp\_h \cdot G\_h$$ i.e. $h\_h = 1.882 \times 10^{3}$ **W/m^2.K.... heat tr. coeff. of high pressure stream**

To find heat tr coeff h_c for low pressure stream:

$$A\_i\_c := \frac{\pi}{4} \cdot \left(d\_i\_c^2 - d\_o\_h^2\right)$$ i.e. $A\_i\_c = 2.361 \times 10^{-3}$ m^2 ... cross-sectional area of low pr. flow

$$G\_c := \frac{m\_c}{A\_i\_c}$$ i.e. $G\_c = 98.683$ kg/s.m^2 ... mass velocity

$mu\_c := mu\_100B\_N2(T\_avg\_cold)$ i.e. $mu\_c = 1.801 \times 10^{-5}$ Pa.s

$rho\_c := rho\_100B\_N2(T\_avg\_cold)$ i.e. $rho\_c = 229.042$ kg/m^3

$Pr\_c := Prandtl\_100B\_N2(T\_avg\_cold)$ i.e. $Pr\_c = 1.076$

$$Re\_c := \frac{G\_c \cdot (d\_i\_c - d\_o\_h)}{mu\_c}$$ i.e. $Re\_c = 1.781 \times 10^{5}$ **...Reynolds No. of low prrssure stream**

Since Re_c > 3000, we have:

$$jH\_c := 0.023 \cdot Re\_c^{-0.2} \cdot \left[1 + \frac{3.5 \cdot (d\_i\_c - d\_o\_h)}{D\_h}\right]$$

i.e. $jH\_c = 2.567 \times 10^{-3}$ **...Colburn j factor**

Therefore: heat tr coeff h_c on low pressure side:

$$h\_c := \frac{jH\_c}{Pr\_c^{\frac{2}{3}}} \cdot cp\_c \cdot G\_c$$ i.e. $h\_c = 251.875$ **W/m^2.K.... heat tr. coeff. of high pressure stream**

Therefore, Overall heat tr coeff, U, based on area on high pressure side:

$$U := \left(\frac{1}{h\_h} + \frac{d\_i\_h}{d\_o\_h \cdot h\_c}\right)^{-1}$$

i.e. $U = 260.424$ **W/m^2.K ... Overall heat tr coeff.**

To find the area and length of HX: Use the Function for Counterflow HX, already written, along with the 'Solve block' of Mathcad:

$C\_c := m\_c \cdot cp\_c$ i.e. $C\_c = 243.252$

$C\_h := m\_h \cdot cp\_h$ i.e. $C\_h = 436.75$

Therefore: cold fluid is the min. fluid.

Therefore, effectivenesss:

$$\varepsilon := \frac{Tc2 - Tc1}{Th1 - Tc1} \qquad \text{i.e.} \qquad \varepsilon = 0.949$$

Now, use the 'Solve block':

$A := 1$ ...guess value

Given

$$\text{CounterflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, A, U)_{1,2} = 0.949$$

$Area := \text{Find}(A)$

i.e. $Area = 4.689$ **m^2 .... heat transfer area of HX**

Therefore:

$$L := \frac{Area}{\pi \cdot d\_i\_h} \qquad \text{m...length of tube}$$

i.e. $L = 59.7$ **m...length of tube ... Ans.**

With this Area entered, apply the Mathcad Function for Counterflow HX again:

$$\text{CounterflowHX}(m\_c, m\_h, cp\_c, cp\_h, Th1, Tc1, Area, U) = \begin{pmatrix} \text{"C\_R"} & \text{"NTU"} & \text{"Effectiveness"} & \text{"Th2(K)"} & \text{"Tc2(K)"} & \text{"Q(W)"} \\ 0.557 & 5.0198 & 0.949 & 186.3608 & 289.035 & 4.9632 \times 10^{4} \end{pmatrix}$$

Thus:

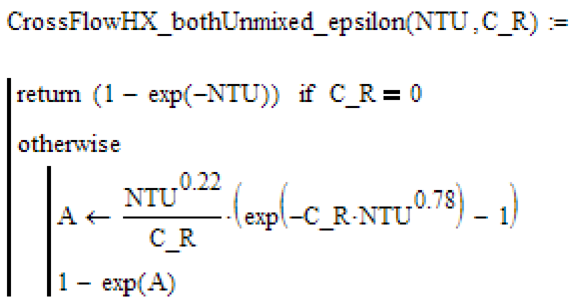
Overall heat transfer coeff. = U = 260.424 W/m^2.K…Ans.

Length of tube in HX = L = 59.7 m…Ans.

Heat transfer in HX = Q = 49632 W…Ans.

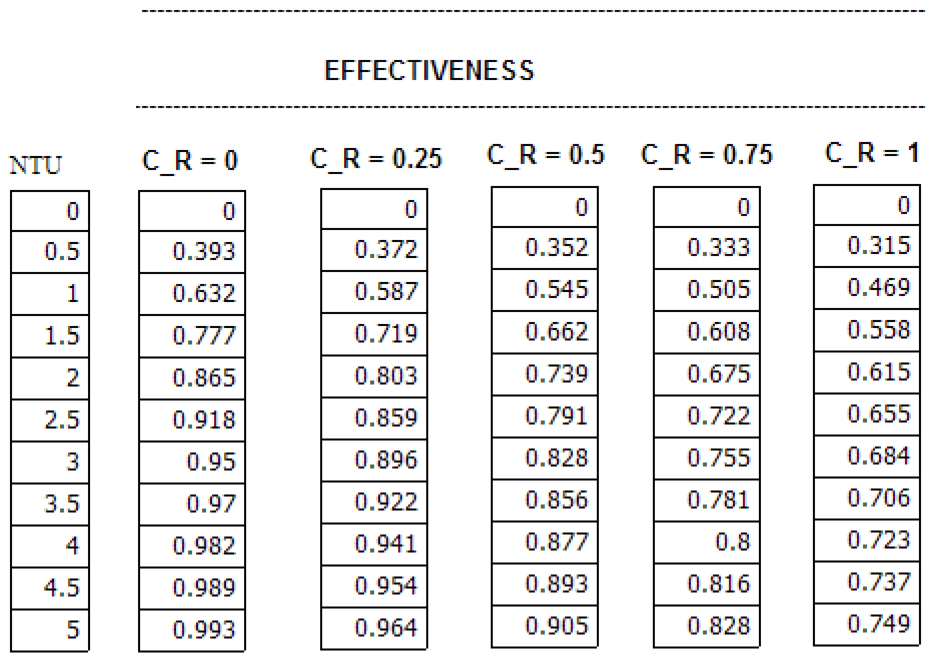
Prob. 3.3.33. Write an EES Function to find the effectiveness of a Cross-flow HX, with both the fluids un-mixed, as a function of NTU and Capacity Ratio, C_R. Then, plot the Effectiveness vs NTU graphs for different values of Capacity ratios.

Mathcad Function:



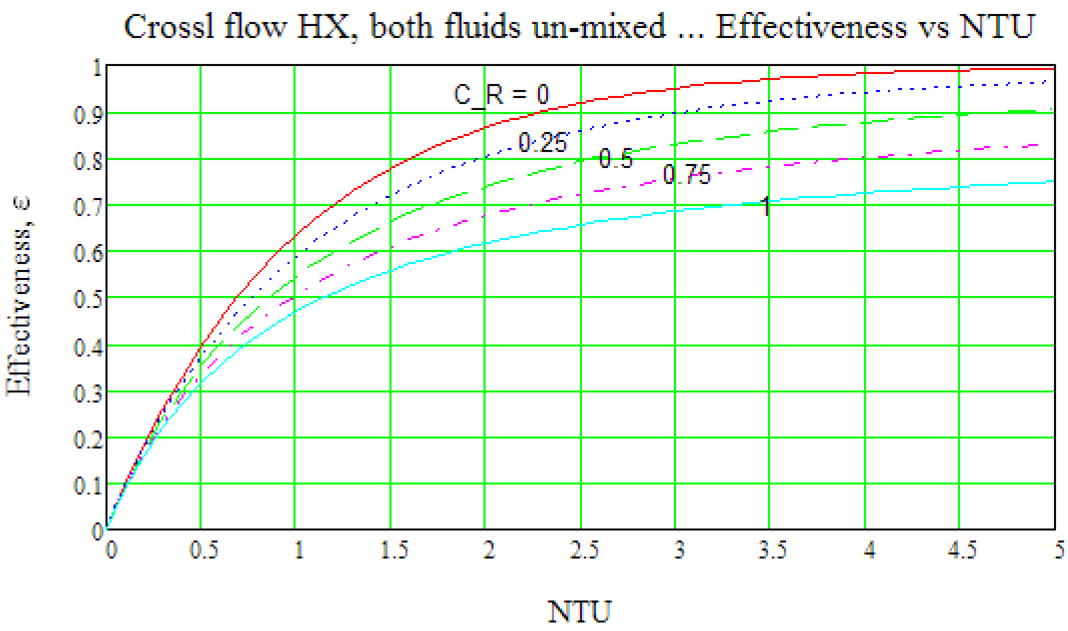
```
CrossFlowHX_bothUnmixed_epsilon(NTU, C_R) :=
    return (1 − exp(−NTU))  if  C_R = 0
    otherwise
        $A \leftarrow \frac{NTU^{0.22}}{C\_R} \cdot \left(\exp\left(-C\_R \cdot NTU^{0.78}\right) - 1\right)$
        $1 - \exp(A)$
```

To plot the epsilon – NTU graphs:



EFFECTIVENESS

NTU	C_R = 0	C_R = 0.25	C_R = 0.5	C_R = 0.75	C_R = 1
0	0	0	0	0	0
0.5	0.393	0.372	0.352	0.333	0.315
1	0.632	0.587	0.545	0.505	0.469
1.5	0.777	0.719	0.662	0.608	0.558
2	0.865	0.803	0.739	0.675	0.615
2.5	0.918	0.859	0.791	0.722	0.655
3	0.95	0.896	0.828	0.755	0.684
3.5	0.97	0.922	0.856	0.781	0.706
4	0.982	0.941	0.877	0.8	0.723
4.5	0.989	0.954	0.893	0.816	0.737
5	0.993	0.964	0.905	0.828	0.749



Prob. 3.3.34. Hot exhaust gases which enter a finned tube, cross flow HX at 300 C and leave at 100 C, are used to heat pressurized water at a flow rate of 1 kg/s from 35 C to 125 C. The exhaust gas sp. heat is approx. 100 J/kg.K and the overall heat transfer coeff based on the gas side surface area is U = 100 W/m^2.K. Determine the required gas side surface area A using the NTU method. (Ref: Incropera)

Solution:

Use the Mathcad Function for a cross flow HX with both fluids un-mixed, written above:

Data:

$m\_c := 1$ kg/s $cp\_c := 4197$ J/kg.K $cp\_h := 1000$ J/kg.K $U := 100$ W/m^2.K

$Tc1 := 35$ C $Tc2 := 125$ C $Th1 := 300$ C $Th2 := 100$ C

Therefore:

$Q := m\_c \cdot cp\_c \cdot (Tc2 - Tc1)$ i.e. $Q = 3.777 \times 10^{5}$ **W... heat transfer rate**

And: $m\_h := \dfrac{Q}{cp\_h \cdot (Th1 - Th2)}$ i.e. $m\_h = 1.889$ **kg/s... mass flow rate of hot fluid**

Then:

$C\_c := m\_c \cdot cp\_c$ i.e. $C\_c = 4.197 \times 10^{3}$ W/K.... capacity rate of cold fluid

$C\_h := m\_h \cdot cp\_h$ i.e. $C\_h = 1.889 \times 10^{3}$ W/K.... capacity rate of hot fluid

Therefore, hot fluid is the 'minimum' fluid.

i.e. $C\_min := C\_h$ $C\_max := C\_c$

i.e. $$C\_R := \frac{C\_min}{C\_max}$$ i.e. $C\_R = 0.45$ **....capacity ratio**

Since hot fluid is minimum fluid, we can find the effectiveness as:

$$\varepsilon := \frac{Th1 - Th2}{Th1 - Tc1}$$ i.e. $\varepsilon = 0.755$ **...effectiveness of HX**

Now, to find the area required for HX:

First, find the NTU by using the Mathcad Function for cross flow HX with both fluids un-mixed, along with the 'Solve block'. Then, find the area from: NTU = U * A / C_min

$NTU := 1$ .... guess value

Given

$CrossFlowHX\_bothUnmixed\_epsilon(NTU, C\_R) = \varepsilon$

$Find(NTU) = 2.024$

i.e. $NTU := 2.024$

Therefore:

$$A := \frac{NTU \cdot C\_min}{U}$$

i.e. $A = 38.226$ **m^2 .... area of HX ... Ans.**

To plot Tc2 vs Th1 for given U values, other quantities remaining the same:

First, write Tc2 as a function of U and Th1:

$$NTU(U) := \frac{U \cdot A}{C\_min}$$

$\varepsilon(U) := CrossFlowHX\_bothUnmixed\_epsilon(NTU(U), C\_R)$

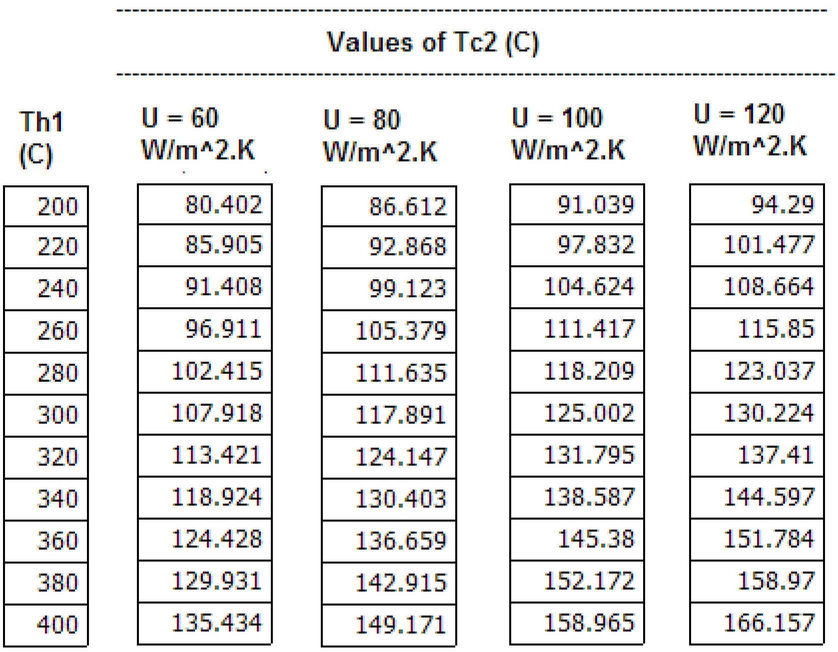
$Th2(U, Th1) := Th1 - \varepsilon(U) \cdot (Th1 - Tc1)$

$m\_c \cdot cp\_c \cdot (Tc2 - Tc1) = m\_h \cdot cp\_h \cdot (Th1 - Th2)$ ...by heat balance

$$Tc2(U, Th1) := \frac{m\_h \cdot cp\_h \cdot (Th1 - Th2(U, Th1))}{m\_c \cdot cp\_c} + Tc1$$ **...Tc2 as a function of U and Th1**

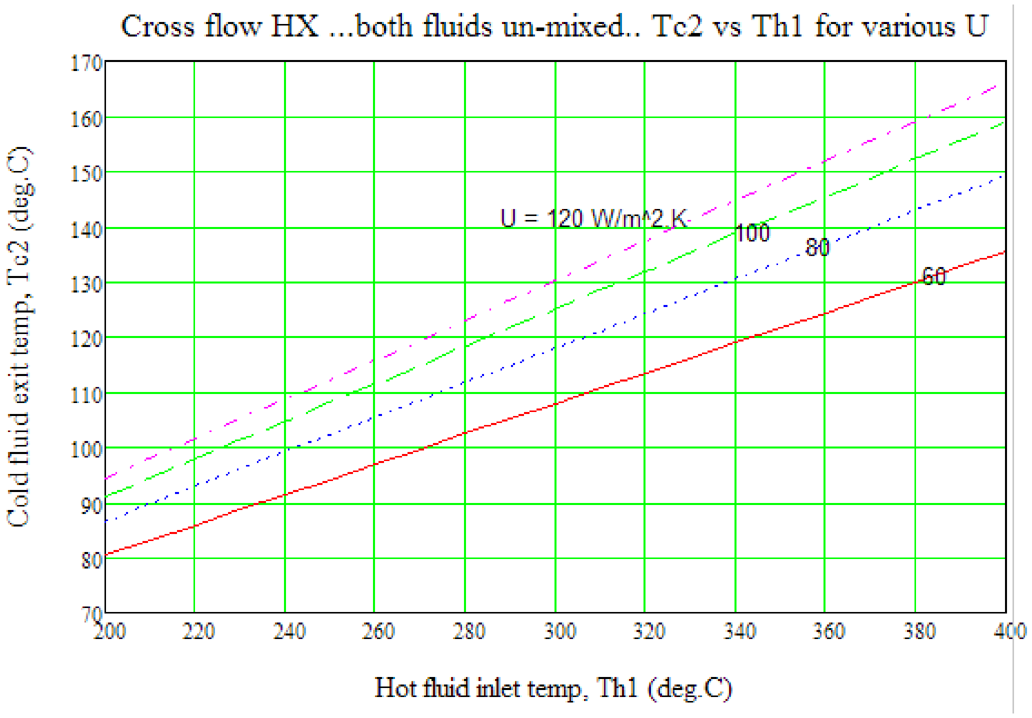
Ex: $Tc2(100, 300) = 125.002$ ....verified with the Tc2 in the stated problem

Now, tabulate the results:



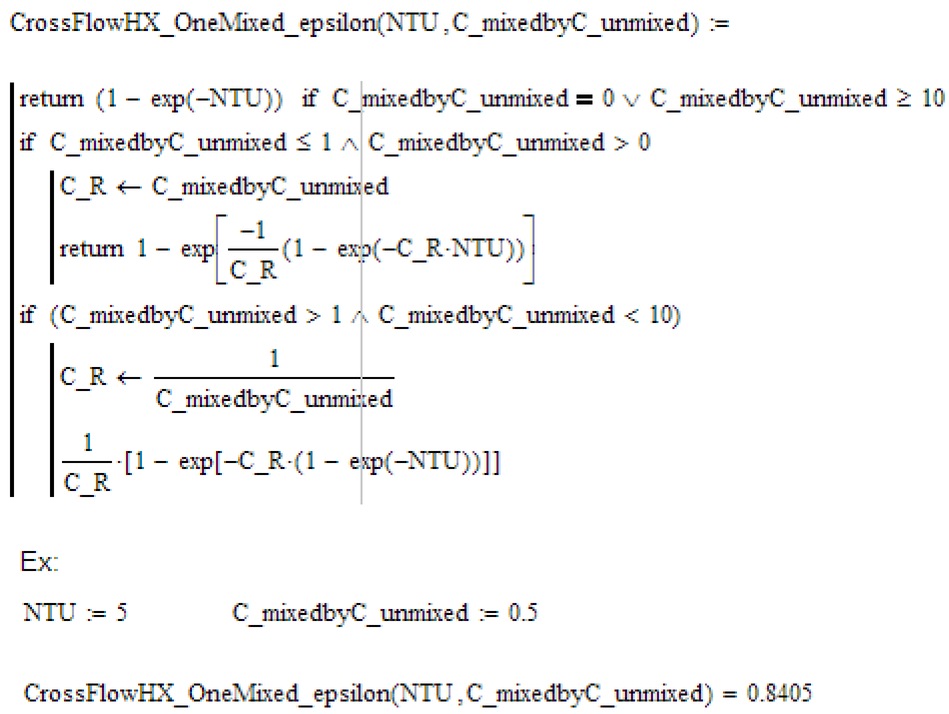
	Values of Tc2 (C)			
Th1 (C)	U = 60 W/m^2.K	U = 80 W/m^2.K	U = 100 W/m^2.K	U = 120 W/m^2.K
200	80.402	86.612	91.039	94.29
220	85.905	92.868	97.832	101.477
240	91.408	99.123	104.624	108.664
260	96.911	105.379	111.417	115.85
280	102.415	111.635	118.209	123.037
300	107.918	117.891	125.002	130.224
320	113.421	124.147	131.795	137.41
340	118.924	130.403	138.587	144.597
360	124.428	136.659	145.38	151.784
380	129.931	142.915	152.172	158.97
400	135.434	149.171	158.965	166.157

And, plot the results:



Prob. 3.3.35. Write a Mathcad Function to find the effectiveness of a Cross-flow HX, with one fluid mixed, and the other un-mixed, as a function of NTU and Capacity Ratio, C_R. Then, plot the Effectiveness vs NTU graphs for different values of Capacity ratios.

Mathcad Function:



```
CrossFlowHX_OneMixed_epsilon(NTU,C_mixedbyC_unmixed) :=

 return (1 − exp(−NTU))  if  C_mixedbyC_unmixed = 0 ∨ C_mixedbyC_unmixed ≥ 10
 if  C_mixedbyC_unmixed ≤ 1 ∧ C_mixedbyC_unmixed > 0
     C_R ← C_mixedbyC_unmixed
     return $1 - \exp\left[\frac{-1}{C\_R}(1 - \exp(-C\_R \cdot NTU))\right]$
 if  (C_mixedbyC_unmixed > 1 ∧ C_mixedbyC_unmixed < 10)
     C_R ← $\frac{1}{C\_mixedbyC\_unmixed}$
     $\frac{1}{C\_R} \cdot [1 - \exp[-C\_R \cdot (1 - \exp(-NTU))]]$
```

Ex:

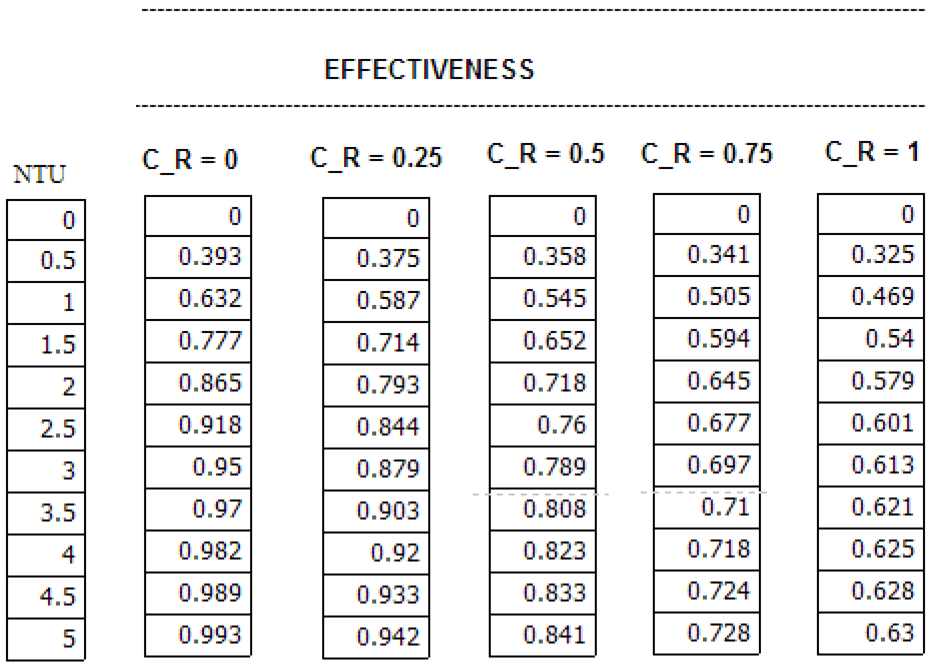
NTU := 5   C_mixedbyC_unmixed := 0.5

CrossFlowHX_OneMixed_epsilon(NTU,C_mixedbyC_unmixed) = 0.8405

“==”

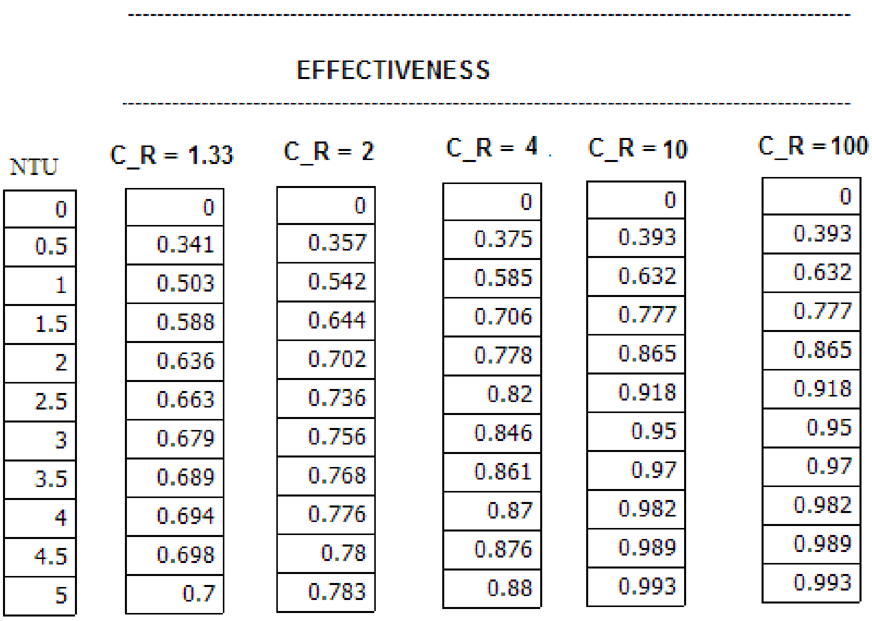
To plot the Effectiveness-NTU graphs:

First, compute the Parametric Table:



EFFECTIVENESS

NTU	C_R = 0	C_R = 0.25	C_R = 0.5	C_R = 0.75	C_R = 1
0	0	0	0	0	0
0.5	0.393	0.375	0.358	0.341	0.325
1	0.632	0.587	0.545	0.505	0.469
1.5	0.777	0.714	0.652	0.594	0.54
2	0.865	0.793	0.718	0.645	0.579
2.5	0.918	0.844	0.76	0.677	0.601
3	0.95	0.879	0.789	0.697	0.613
3.5	0.97	0.903	0.808	0.71	0.621
4	0.982	0.92	0.823	0.718	0.625
4.5	0.989	0.933	0.833	0.724	0.628
5	0.993	0.942	0.841	0.728	0.63

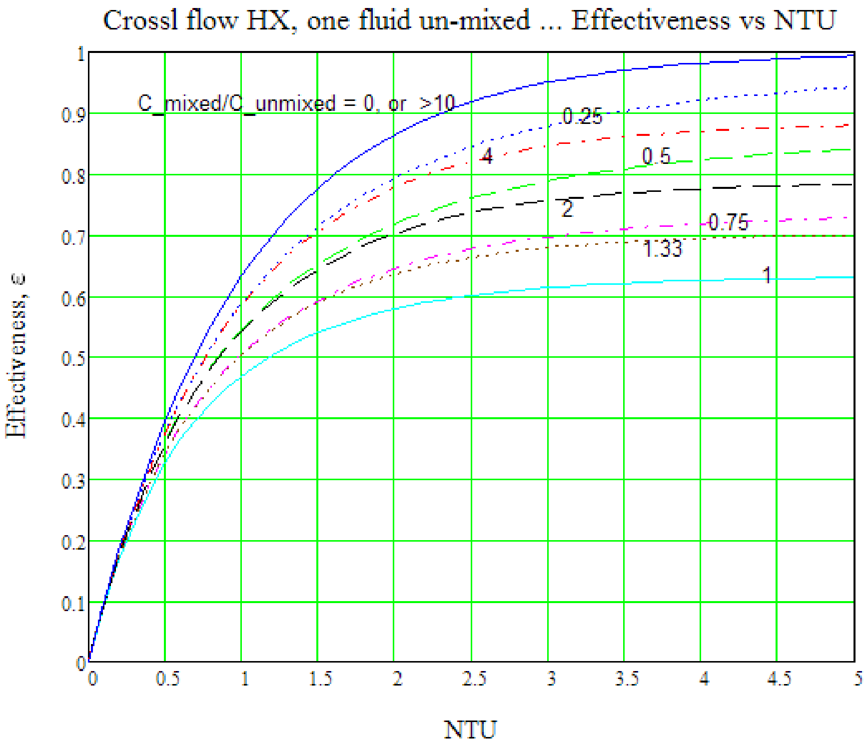


EFFECTIVENESS

NTU	C_R = 1.33	C_R = 2	C_R = 4	C_R = 10	C_R = 100
0	0	0	0	0	0
0.5	0.341	0.357	0.375	0.393	0.393
1	0.503	0.542	0.585	0.632	0.632
1.5	0.588	0.644	0.706	0.777	0.777
2	0.636	0.702	0.778	0.865	0.865
2.5	0.663	0.736	0.82	0.918	0.918
3	0.679	0.756	0.846	0.95	0.95
3.5	0.689	0.768	0.861	0.97	0.97
4	0.694	0.776	0.87	0.982	0.982
4.5	0.698	0.78	0.876	0.989	0.989
5	0.7	0.783	0.88	0.993	0.993

Note that for (C_mixed/C_unmixed) values greater than 10, the effectiveness values remain the same.

Now, plot the results:



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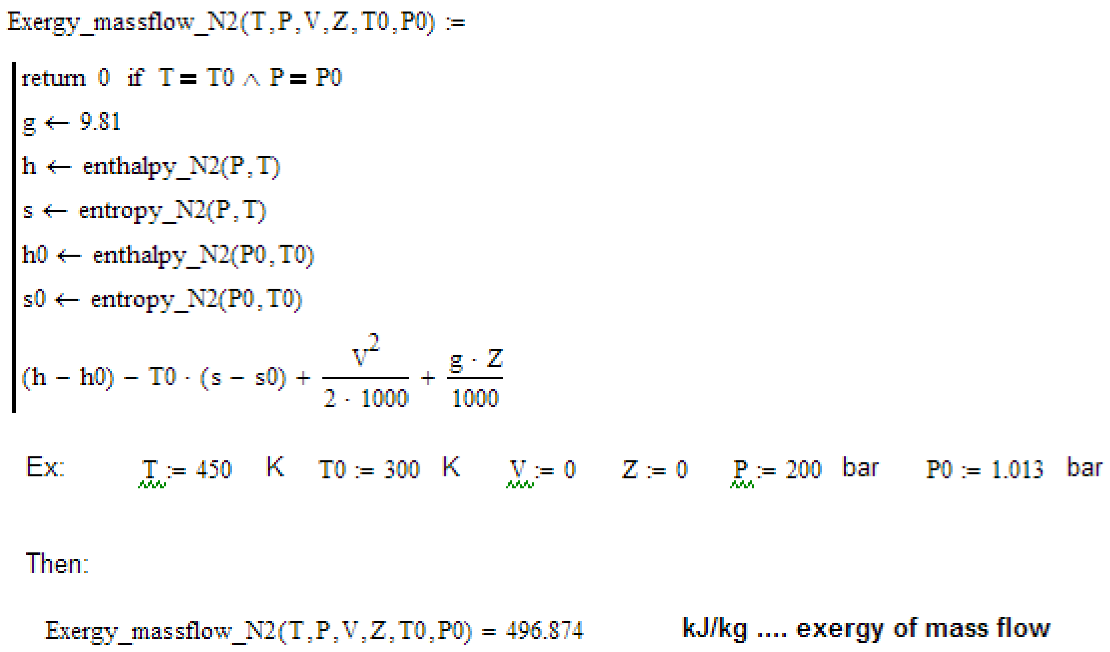
Prob. 3.3.36. Write Mathcad Functions to find the Exergy of mass flow for N2, He and H2.

Mathcad Function for N2:

This function returns the specific availability of N2 in kJ/kg as a function of

T [K], P [bar], V [m/sec], Z [m], and 'dead state' P0 (bar), T0 (K)



```
Exergy_massflow_N2(T,P,V,Z,T0,P0) :=
    return 0 if T = T0 ∧ P = P0
    g ← 9.81
    h ← enthalpy_N2(P,T)
    s ← entropy_N2(P,T)
    h0 ← enthalpy_N2(P0,T0)
    s0 ← entropy_N2(P0,T0)
    $(h - h0) - T0 \cdot (s - s0) + \frac{V^2}{2 \cdot 1000} + \frac{g \cdot Z}{1000}$
```

Ex: T := 450 K T0 := 300 K V := 0 Z := 0 P := 200 bar P0 := 1.013 bar

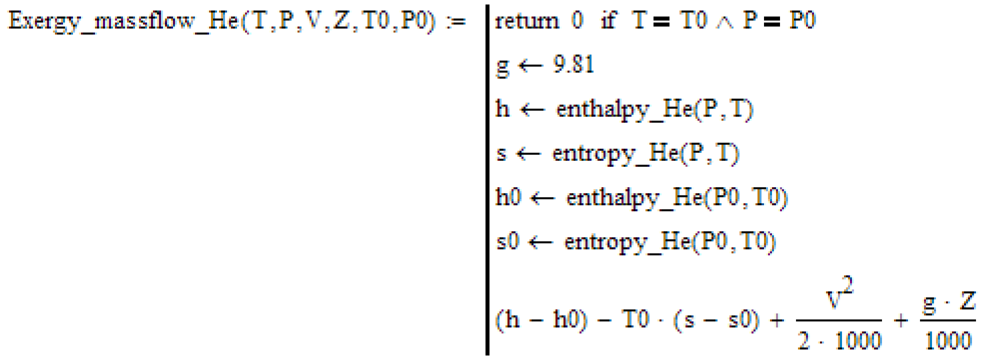
Then:

Exergy_massflow_N2(T,P,V,Z,T0,P0) = 496.874 **kJ/kg .... exergy of mass flow**

Mathcad Function for He:

This function returns the specific availability of He in kJ/kg as a function of

T [K], P [bar], V [m/sec], Z [m], and 'dead state' P0 (bar), T0 (K)



```
Exergy_massflow_He(T,P,V,Z,T0,P0) :=
    return 0 if T = T0 ∧ P = P0
    g ← 9.81
    h ← enthalpy_He(P,T)
    s ← entropy_He(P,T)
    h0 ← enthalpy_He(P0,T0)
    s0 ← entropy_He(P0,T0)
    $(h - h0) - T0 \cdot (s - s0) + \frac{V^2}{2 \cdot 1000} + \frac{g \cdot Z}{1000}$
```

Ex: $T := 350$ K $T0 := 300$ K $V := 0$ $Z := 0$ $P := 200$ bar $P0 := 1.013$ bar

Then:

$\text{Exergy\_massflow\_He}(T,P,V,Z,T0,P0) = 3.375 \times 10^{3}$ **kJ/kg .... exergy of mass flow**

Mathcad Function for H2:

This function returns the specific availability of H2 in kJ/kg as a function of

T [K], P [bar], V [m/sec], Z [m], and 'dead state' P0 (bar), T0 (K)

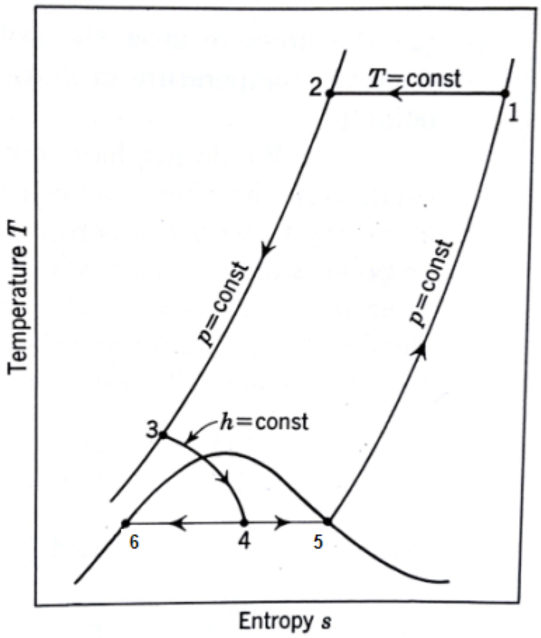
```
Exergy_massflow_H2(T,P,V,Z,T0,P0) := | return 0 if T = T0 ∧ P = P0
                                     | g ← 9.81
                                     | h ← enthalpy_H2(P,T)
                                     | s ← entropy_H2(P,T)
                                     | h0 ← enthalpy_H2(P0,T0)
                                     | s0 ← entropy_H2(P0,T0)
                                     | (h − h0) − T0 · (s − s0) + V^2/(2 · 1000) + g · Z/1000
```

Ex: $T := 350$ K $T0 := 300$ K $V := 0$ $Z := 0$ $P := 200$ bar $P0 := 1.013$ bar

Then:

$\text{Exergy\_massflow\_H2}(T,P,V,Z,T0,P0) = 6.752 \times 10^{3}$ **kJ/kg .... exergy of mass flow**

Prob.3.3.37. Make an exergy analysis for a simple Linde-Hampson system using Nitrogen as the working fluid. The system operates between 101.3 kPa and 300 K at point 1 and 20 MPa at point 2. The system may be assumed reversible, except for the expansion through the expansion valve.

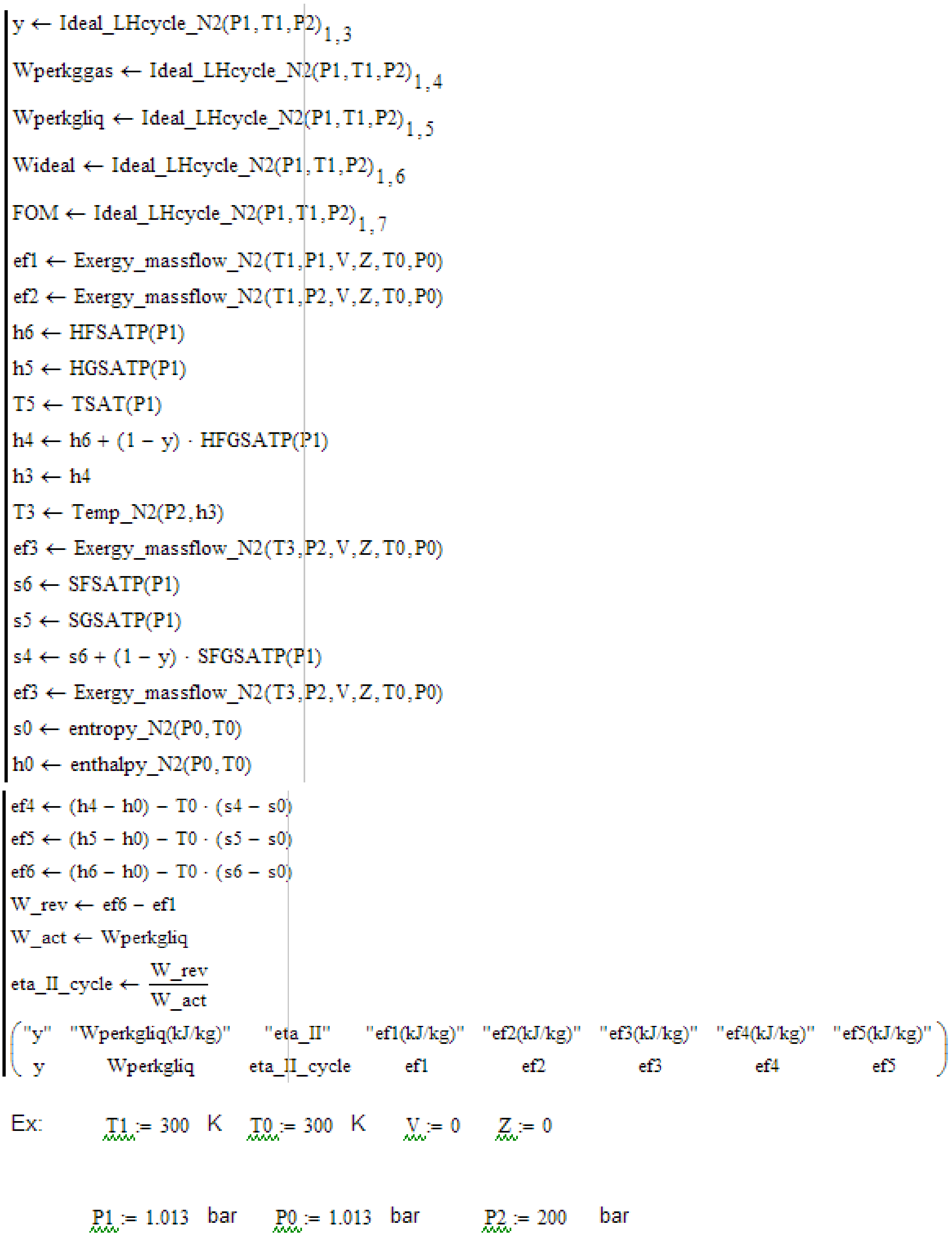


Solution:

To determine the liquid yield, the work per unit mass compressed, the work per unit mass liquefied, and the figure of merit for a simple Linde-Hampson system. Then, determine the exergetic effcy of the cycle. Also, determine the exergy losses in the compressor (assuming the isothermal effcy of compressor as 70%), expansion valve and the heat exchanger.

Mathcad Function:

$\text{ExergyAnalysis\_SimpleLindeHampson\_N2}(P1,T1,P2,P0,T0,V,Z) :=$

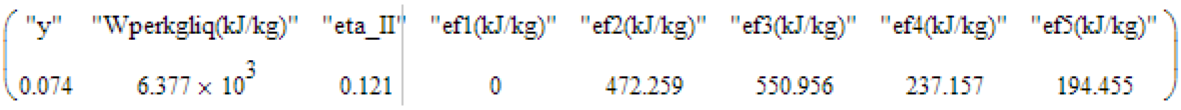


```
y ← Ideal_LHcycle_N2(P1,T1,P2)1,3
Wperkggas ← Ideal_LHcycle_N2(P1,T1,P2)1,4
Wperkgliq ← Ideal_LHcycle_N2(P1,T1,P2)1,5
Wideal ← Ideal_LHcycle_N2(P1,T1,P2)1,6
FOM ← Ideal_LHcycle_N2(P1,T1,P2)1,7
ef1 ← Exergy_massflow_N2(T1,P1,V,Z,T0,P0)
ef2 ← Exergy_massflow_N2(T1,P2,V,Z,T0,P0)
h6 ← HFSATP(P1)
h5 ← HGSATP(P1)
T5 ← TSAT(P1)
h4 ← h6 + (1 − y) · HFGSATP(P1)
h3 ← h4
T3 ← Temp_N2(P2,h3)
ef3 ← Exergy_massflow_N2(T3,P2,V,Z,T0,P0)
s6 ← SFSATP(P1)
s5 ← SGSATP(P1)
s4 ← s6 + (1 − y) · SFGSATP(P1)
ef3 ← Exergy_massflow_N2(T3,P2,V,Z,T0,P0)
s0 ← entropy_N2(P0,T0)
h0 ← enthalpy_N2(P0,T0)
ef4 ← (h4 − h0) − T0 · (s4 − s0)
ef5 ← (h5 − h0) − T0 · (s5 − s0)
ef6 ← (h6 − h0) − T0 · (s6 − s0)
W_rev ← ef6 − ef1
W_act ← Wperkgliq
eta_II_cycle ← W_rev / W_act
⎛ "y"  "Wperkgliq(kJ/kg)"  "eta_II"       "ef1(kJ/kg)"  "ef2(kJ/kg)"  "ef3(kJ/kg)"  "ef4(kJ/kg)"  "ef5(kJ/kg)" ⎞
⎝  y       Wperkgliq       eta_II_cycle      ef1           ef2           ef3           ef4           ef5      ⎠
```

Ex: T1 := 300 K T0 := 300 K V := 0 Z := 0

P1 := 1.013 bar P0 := 1.013 bar P2 := 200 bar

Applying the above written Function, we get:



```
⎛ "y"   "Wperkgliq(kJ/kg)"  "eta_II"  "ef1(kJ/kg)"  "ef2(kJ/kg)"  "ef3(kJ/kg)"  "ef4(kJ/kg)"  "ef5(kJ/kg)" ⎞
⎝ 0.074    6.377 × 10^3      0.121         0          472.259       550.956       237.157       194.455    ⎠
```

i.e.

$\text{LiqFraction\_y}(P2) := \text{ExergyAnalysis\_SimpleLindeHampson\_N2}(P1,T1,P2,P0,T0,V,Z)_{1,0}$ $\text{LiqFraction\_y}(P2) = 0.074$

$\text{Wperkgliq}(P2) := \text{ExergyAnalysis\_SimpleLindeHampson\_N2}(P1,T1,P2,P0,T0,V,Z)_{1,1}$ $\text{Wperkgliq}(P2) = 6.377 \times 10^{3}$

$\text{Wperkggas}(P2) := \text{Wperkgliq}(P2) \cdot \text{LiqFraction\_y}(P2)$ $\text{Wperkggas}(P2) = 472.259$

$\text{eta\_II}(P2) := \text{ExergyAnalysis\_SimpleLindeHampson\_N2}(P1,T1,P2,P0,T0,V,Z)_{1,2}$ $\text{eta\_II}(P2) = 0.121$

$ef1 := \text{ExergyAnalysis\_SimpleLindeHampson\_N2}(P1,T1,P2,P0,T0,V,Z)_{1,3}$ $ef1 = 0$

$ef2 := \text{ExergyAnalysis\_SimpleLindeHampson\_N2}(P1,T1,P2,P0,T0,V,Z)_{1,4}$ $ef2 = 472.259$

$ef3 := \text{ExergyAnalysis\_SimpleLindeHampson\_N2}(P1,T1,P2,P0,T0,V,Z)_{1,5}$ $ef3 = 550.956$

$ef4 := \text{ExergyAnalysis\_SimpleLindeHampson\_N2}(P1,T1,P2,P0,T0,V,Z)_{1,6}$ $ef4 = 237.157$

$ef5 := \text{ExergyAnalysis\_SimpleLindeHampson\_N2}(P1,T1,P2,P0,T0,V,Z)_{1,7}$ $ef5 = 194.455$

Note: Exergetic effcy = eta_II = 0.121…Ans.

Exergy losses in components:

Exergy loss in compressor (of isoth. effcy. = 70%):

$\text{Wperkggas}(P2) := \dfrac{\text{Wperkgliq}(P2) \cdot \text{LiqFraction\_y}(P2)}{0.7}$ i.e. $\text{Wperkggas}(P2) = 674.656$ kJ/kg

Writing an exergy balance for the compressor:

$ef1 + \text{Wperkggas}(P2) = ef2 + \Delta e_{comp}$ where Δe_{comp} is the exergy loss in compressor

Therefore: $\Delta e_{comp} := ef1 - ef2 + \text{Wperkggas}(P2)$

i.e. $\Delta e_{comp} = 202.397$ **kJ/kg .... Ans.**

Exergy loss in Heat Exchanger:

Writing an exergy balance for the HX:

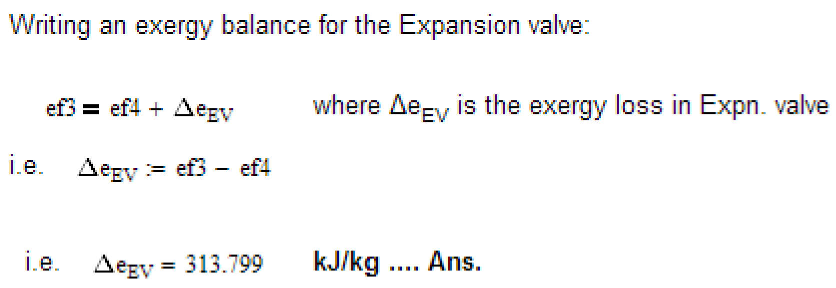
$ef2 + (1 - \text{LiqFraction\_y}(P2)) \cdot ef5 = ef3 + (1 - \text{LiqFraction\_y}(P2)) \cdot ef1 + \Delta e_{HX}$

where Δe_{HX} is the exergy loss in HX

Then: $\Delta e_{HX} := ef2 + (1 - \text{LiqFraction\_y}(P2)) \cdot ef5 - [ef3 + (1 - \text{LiqFraction\_y}(P2)) \cdot ef1]$

i.e. $\Delta e_{HX} = 101.357$ **kJ/kg .... Ans.**

Exergy loss in Expansion valve:



Writing an exergy balance for the Expansion valve:

$ef3 = ef4 + \Delta e_{EV}$ where Δe_{EV} is the exergy loss in Expn. valve

i.e. $\Delta e_{EV} := ef3 - ef4$

i.e. $\Delta e_{EV} = 313.799$ **kJ/kg .... Ans.**

Thus:

Liquid yield, y = 0.074 kg liq. / kg gas compressed…Ans.

Work per kg gas compressed = Wperkggas = 472.26 kJ/kg…Ans.

Work per kg liquid = Wperkgliq = 6377 kJ/kg…Ans.

Exergetic effcy (or, second law effcy) of cycle = ηII = 0.121 (= FOM)…Ans.

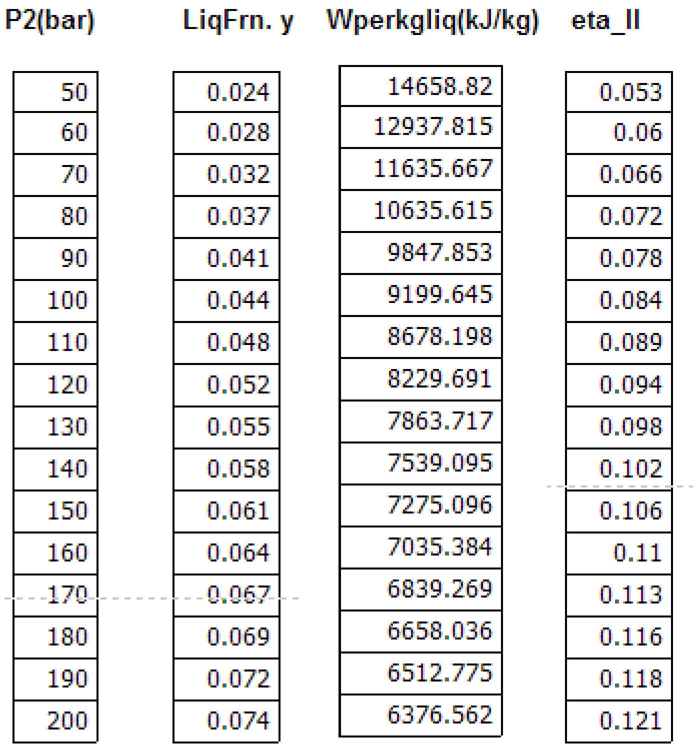
Exergy losses in compressor (assuming its isoth. effcy is 70%) = Δecomp = 202.34 kJ/kg…Ans.

Exergy losses in Heat Exchanger = ΔeHX = 101.36 kJ/kg…Ans.

Exergy losses in Expansion Valve = ΔeEV = 313.8 kJ/kg…Ans.

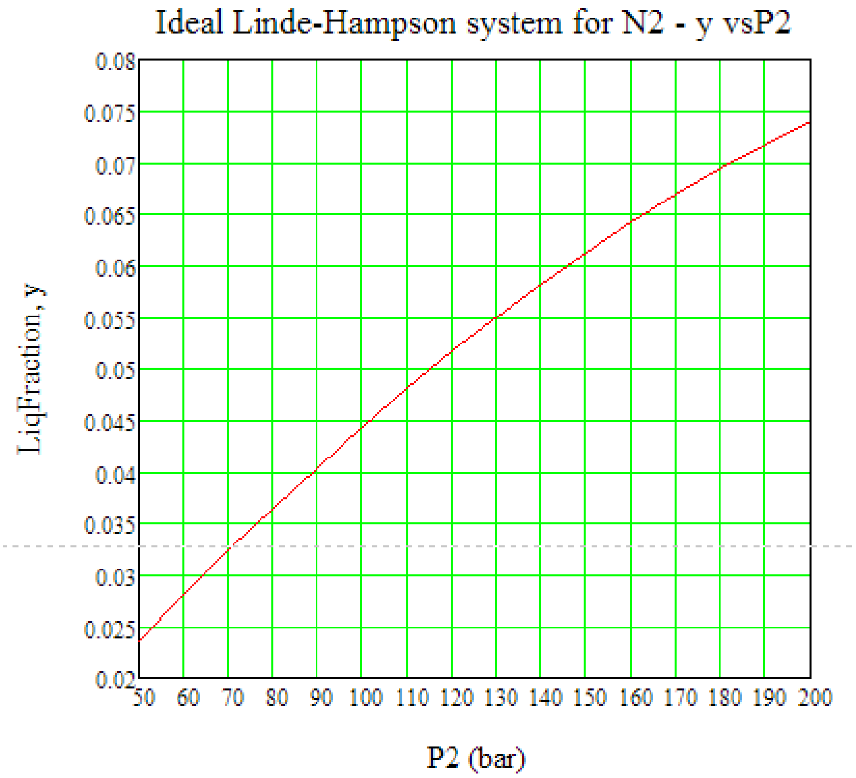
Plot y, Wperkgliq, and η_{II} against P2, other parameters remaining constant:

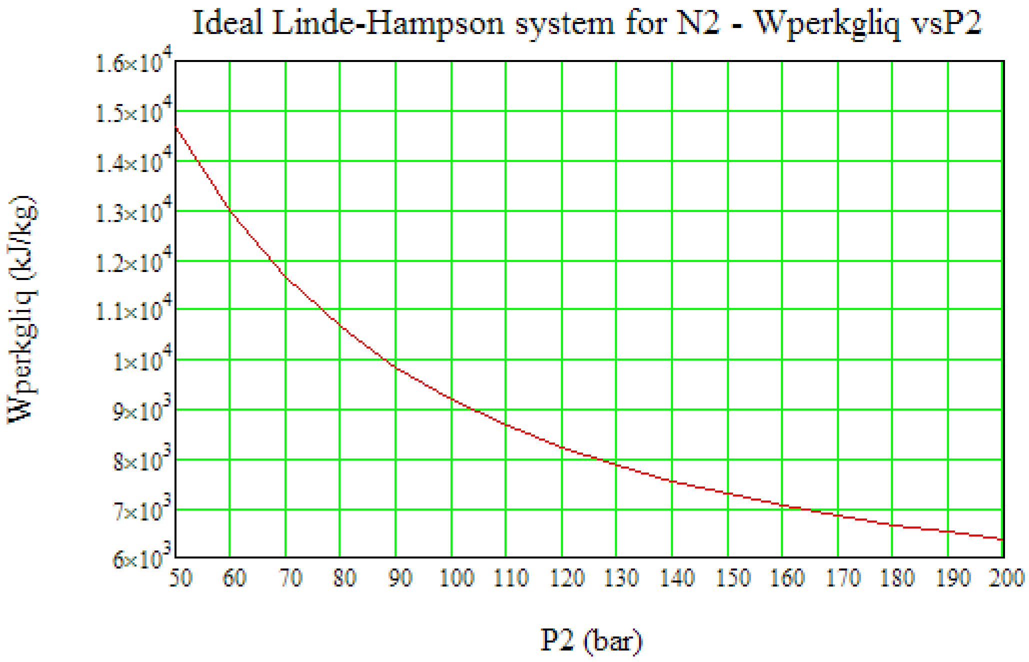
First, compute the Parametric Tables:

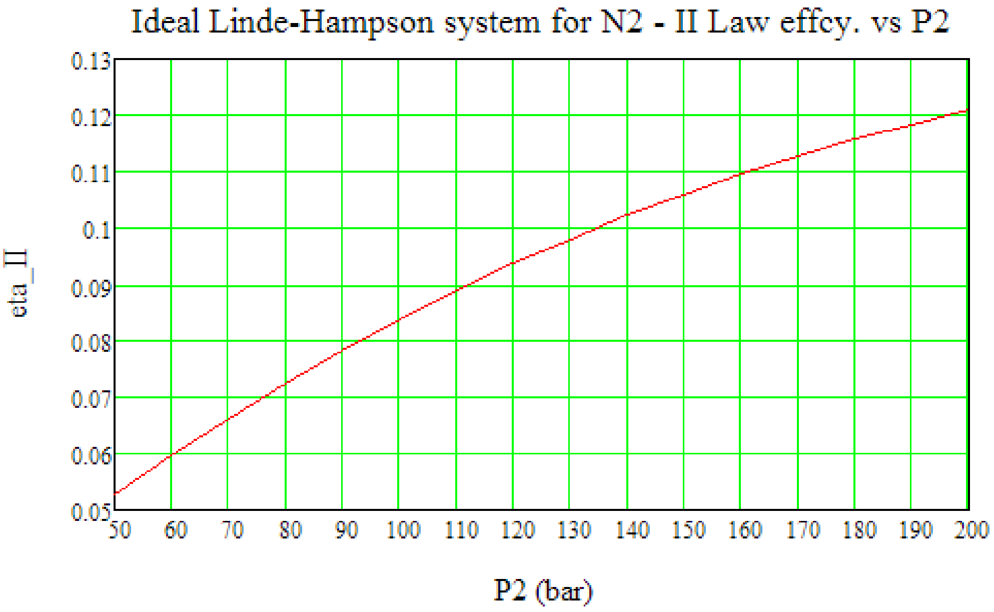


P2(bar)	LiqFrn. y	Wperkgliq(kJ/kg)	eta_II
50	0.024	14658.82	0.053
60	0.028	12937.815	0.06
70	0.032	11635.667	0.066
80	0.037	10635.615	0.072
90	0.041	9847.853	0.078
100	0.044	9199.645	0.084
110	0.048	8678.198	0.089
120	0.052	8229.691	0.094
130	0.055	7863.717	0.098
140	0.058	7539.095	0.102
150	0.061	7275.096	0.106
160	0.064	7035.384	0.11
170	0.067	6839.269	0.113
180	0.069	6658.036	0.116
190	0.072	6512.775	0.118
200	0.074	6376.562	0.121

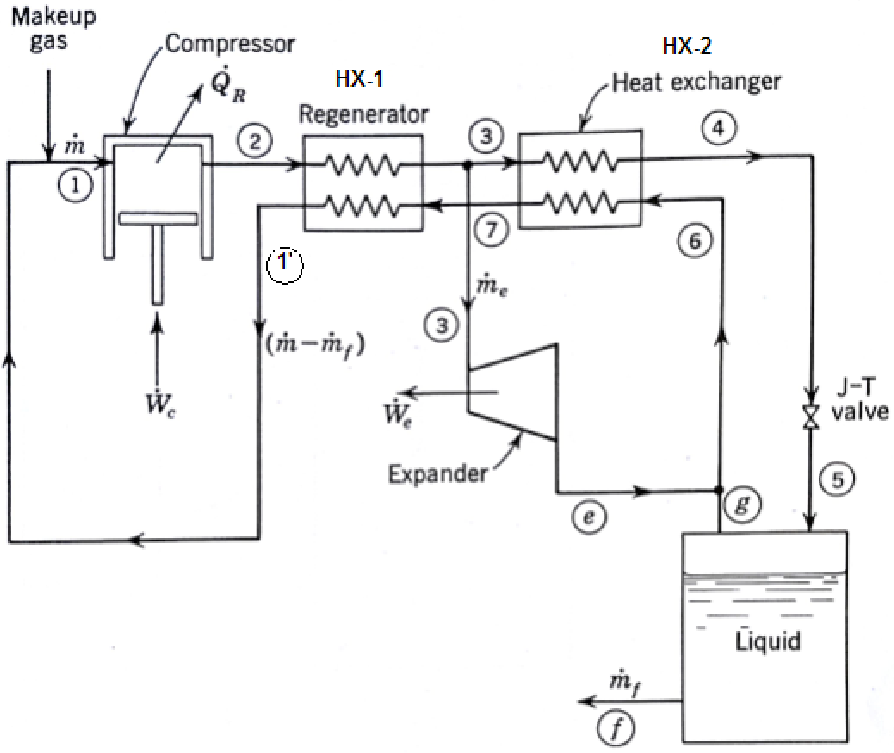
Now, plot the results:







Prob. 3.3.38 In an Ideal Kapitza liquefaction system for Nitrogen, the system operates from 101.3 kPa (1 atm) and 300 K to 5.0 MPa. At 5.0 MPa and 250 K, 50% of the main flow is diverted to the reversible adiabatic expander. Remainder flows through the heat exchanger and expands through the expansion valve to 101.3 kPa. Determine the liquid yield and the work per unit mass liquefied, assuming that the expander work is utilized in the compression process. Make an exergy analysis.



Note: Calculation formulas for y, Wperkggas, Wperkgliq, Fom etc. are the same as for Ideal Claude cycle.

Data:

P1 := 1.013 bar P2 := 50 bar T1 := 300 K T3 := 250 K

FlowRatio_x := 0.5 T0 := 300 K V := 0 Z := 0 P0 := 1.013 bar

Then, using the Function for Ideal Claude cycle, written above:

IdealClaudeSystem_N2(P1, P2, T1, T3, FlowRatio_x) =

$$\begin{pmatrix} \text{"LiqFraction-y"} & \text{"Wexp\_perkggas(kJ/kg)"} & \text{"Wnet\_perkggas(kJ/kg)"} & \text{"Wnet\_perkgliq(kJ/kg)"} & \text{"Wideal(kJ/kg)"} & \text{"FOM"} \\ 0.212 & 81.787 & 263.723 & 1.243 \times 10^{3} & 771.028 & 0.62 \end{pmatrix}$$

i.e.

y := IdealClaudeSystem_N2(P1, P2, T1, T3, FlowRatio_x)$_{1,0}$ y = 0.212

Work of expander per kg gas compressed in compressor =

Wexp_perkggas := IdealClaudeSystem_N2(P1, P2, T1, T3, FlowRatio_x)$_{1,1}$ Wexp_perkggas = 81.787

$\mathrm{Wnet\_perkggas} := \mathrm{IdealClaudeSystem\_N2}(P1, P2, T1, T3, \mathrm{FlowRatio\_x})_{1,2}$ $\mathrm{Wnet\_perkggas} = 263.723$

$\mathrm{Wnet\_perkgliq} := \mathrm{IdealClaudeSystem\_N2}(P1, P2, T1, T3, \mathrm{FlowRatio\_x})_{1,3}$ $\mathrm{Wnet\_perkgliq} = 1.243 \times 10^{3}$

$\mathrm{Wideal} := \mathrm{IdealClaudeSystem\_N2}(P1, P2, T1, T3, \mathrm{FlowRatio\_x})_{1,4}$ $\mathrm{Wideal} = 771.028$

$\mathrm{FOM} := \mathrm{IdealClaudeSystem\_N2}(P1, P2, T1, T3, \mathrm{FlowRatio\_x})_{1,5}$ $\mathrm{FOM} = 0.62$

Further:

$s1 := \mathrm{entropy\_N2}(P1, T1)$ i.e. $s1 = 6.844$ kJ/kg.K

$h1 := \mathrm{enthalpy\_N2}(P1, T1)$ i.e. $h1 = 311.192$ kJ/kg

$s2 := \mathrm{entropy\_N2}(P2, T1)$ i.e. $s2 = 5.658$ kJ/kg.K

$s3 := \mathrm{entropy\_N2}(P2, T3)$ i.e. $s3 = 5.45$ kJ/kg.K

$se := s3$ ...for isentropic expansion

$he := \mathrm{enthalpy\_N2\_Ps}(P1, se)$ i.e. $he = 80.254$ kJ/kg

Alternatively:

$he := \mathrm{enthalpy\_N2}(P2, T3) - \frac{\mathrm{Wexp\_perkggas}}{\mathrm{FlowRatio\_x}}$ i.e. $he = 80.254$ kJ/kg

$Tg := \mathrm{TSAT}(P1)$ i.e. $Tg = 77.166$ K

$T5 := Tg$ $Tf := Tg$

$h5 := \mathrm{HFSATP}(P1) + (1 - y) \cdot \mathrm{HFGSATP}(P1)$ i.e. $h5 = 34.668$ kJ/kg

$h4 := h5$ ...for isenthalpic expansion in the expansion valve

$T4 := \mathrm{Temp\_N2}(P2, h4)$ i.e. $T4 = 130.785$ K

$Te := \mathrm{Temp\_N2}(P1, he)$ i.e. $Te = 80.149$ K

$h2 := \mathrm{enthalpy\_N2}(P1, T1)$ i.e. $h2 = 311.192$ kJ/kg

$hg := \mathrm{HGSATP}(P1)$ i.e. $hg = 76.974$ kJ/kg

$h3 := \mathrm{enthalpy\_N2}(P2, T3)$ i.e. $h3 = 243.827$ kJ/kg

By heat balance at the mixer, where the exit of expander and the sat vapor from the liquid reservoir are mixed:

$$h6 := \frac{(1 - \mathrm{FlowRatio\_x} - y) \cdot hg + \mathrm{FlowRatio\_x} \cdot he}{1 - y}$$

i.e. $h6 = 79.056$ kJ/kg

By heat balance on HX-2:

$(1 - FlowRatio\_x) \cdot (h3 - h4) = (1 - y) \cdot (h7 - h6)$

i.e. we get:

$h7 := h6 + \frac{(1 - FlowRatio\_x) \cdot (h3 - h4)}{1 - y}$ i.e. $h7 = 211.804$ kJ/kg

Then:

$T7 := Temp\_N2(P1, h7)$ i.e. $T7 = 204.607$ K

$T6 := Temp\_N2(P1, h6)$ i.e. $T6 = 79.069$ K

By heat balance on HX-1:

$h2 - h3 = (1 - y) \cdot (h1\_prime - h7)$

i.e. $h1\_prime := h7 + \frac{h2 - h3}{1 - y}$

i.e. $h1\_prime = 297.314$ kJ/kg

Also, we have:

$s0 := entropy\_N2(P0, T0)$ i.e. $s0 = 6.844$ kJ/kg.K

$h0 := enthalpy\_N2(P0, T0)$ i.e. $h0 = 311.192$ kJ/kg

$s5 := SFSATP(P1) + (1 - y) \cdot SFGSATP(P1)$ i.e. $s5 = 4.866$ kJ/kg.K.. in two-phase region

$hg := HGSATP(P1)$ i.e. $hg = 76.974$ kJ/kg

$sg := SGSATP(P1)$ i.e. $sg = 5.415$ kJ/kg.K

Exergies:

$ef1 := Exergy\_massflow\_N2(T1, P1, V, Z, T0, P0)$ i.e. $ef1 = 0$ kJ/kg

$ef1\_prime := Exergy\_massflow\_N2(T1\_prime, P1, V, Z, T0, P0)$ i.e. $ef1\_prime = 0.315$ kJ/kg

$ef2 := Exergy\_massflow\_N2(T1, P2, V, Z, T0, P0)$ i.e. $ef2 = 345.509$ kJ/kg

$ef3 := Exergy\_massflow\_N2(T3, P2, V, Z, T0, P0)$ i.e. $ef3 = 350.952$ kJ/kg

$ef4 := Exergy\_massflow\_N2(T4, P2, V, Z, T0, P0)$ i.e. $ef4 = 507.667$ kJ/kg

$ef5 := (h5 - h0) - T0 \cdot (s5 - s0)$ i.e. $ef5 = 316.803$ kJ/kg

$ef6 := Exergy\_massflow\_N2(T6, P1, V, Z, T0, P0)$ i.e. $ef6 = 190.617$ kJ/kg

$ef7 := Exergy\_massflow\_N2(T7, P1, V, Z, T0, P0)$ i.e. $ef7 = 20.211$ kJ/kg

$ef\_e := \text{Exergy\_massflow\_N2}(Te, P1, V, Z, T0, P0)$ i.e. $ef\_e = 187.378$ kJ/kg

$ef\_g := (hg - h0) - T0 \cdot (sg - s0)$ i.e. $ef\_g = 194.455$ kJ/kg

Exergy losses in components:

Exergy loss in compressor (of isoth. effcy. = 70%):

$Wperkggas := \dfrac{T1 \cdot (s1 - s2) - (h1 - h2)}{0.7}$ i.e. $Wperkggas = 508.184$ kJ/kg

Writing an exergy balance for the compressor:

$ef1 + Wperkggas = ef2 + \Delta e_{comp}$ where Δe_{comp} is the exergy loss in compressor

Therefore: $\Delta e_{comp} := ef1 - ef2 + Wperkggas$

i.e. $\Delta e_{comp} = 162.675$ **kJ/kg .... Ans.**

Exergy loss in Heat Exchanger, HX-1:

Writing an exergy balance for the HX-1:

$ef2 + (1 - y) \cdot ef7 = ef3 + (1 - y) \cdot ef1\_prime + \Delta e_{HX1}$ where Δe_{HX1} is the exergy loss in HX-1

Then: $\Delta e_{HX1} := ef2 + (1 - y) \cdot ef7 - [ef3 + (1 - y) \cdot ef1\_prime]$

i.e. $\Delta e_{HX1} = 10.232$ **kJ/kg .... Ans.**

Exergy loss in Heat Exchanger, HX-2:

Writing an exergy balance for the HX-2:

$(1 - FlowRatio\_x) \cdot ef3 + (1 - y) \cdot ef6 = (1 - FlowRatio\_x) \cdot ef4 + (1 - y) \cdot ef7 + \Delta e_{HX2}$

...where Δe_{HX} is the exergy loss in HX-2

Then: $\Delta e_{HX2} := [(1 - FlowRatio\_x) \cdot ef3 + (1 - y) \cdot ef6] - [(1 - FlowRatio\_x) \cdot ef4 + (1 - y) \cdot ef7]$

i.e. $\Delta e_{HX2} = 55.889$ **kJ/kg .... Ans.**

Exergy loss in Expander:

Writing an exergy balance for the Expander:

$ef3 = ef\_e + \Delta e_{EXP}$ where Δe_{EXP} is the exergy loss in Expander

i.e. $\Delta e_{EXP} := ef3 - ef\_e$

i.e. $\Delta e_{EXP} = 163.574$ **kJ/kg .... Ans.**

Exergy loss in Expansion valve:

Writing an exergy balance for the Expansion valve:

$ef4 = ef5 + \Delta e_{EV}$ where Δe_{EV} is the exergy loss in Expn. valve

i.e. $\Delta e_{EV} := ef4 - ef5$

i.e. $\Delta e_{EV} = 190.864$ **kJ/kg .... Ans.**

Note: Exergy losses in compressor, expander and expansion valve are the major ones.

Plot y, Wperkgliq, eta_II against P2:

Write the required quantities as functions of P2:

$y(P2) := IdealClaudeSystem\_N2(P1, P2, T1, T3, FlowRatio\_x)_{1,0}$ $y(P2) = 0.212$

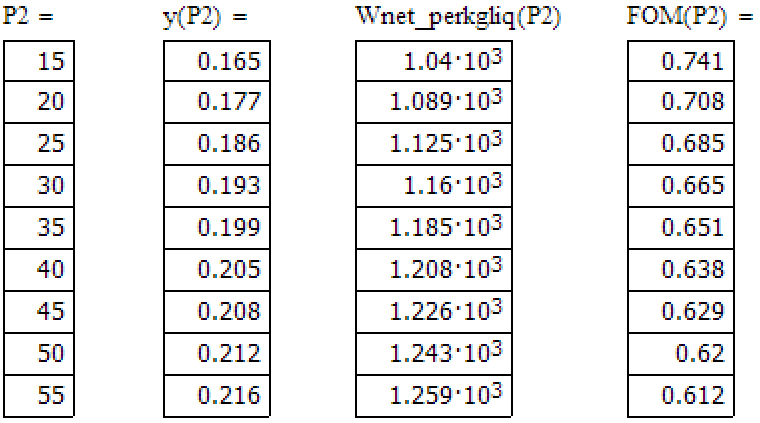
$Wnet\_perkgliq(P2) := IdealClaudeSystem\_N2(P1, P2, T1, T3, FlowRatio\_x)_{1,3}$ $Wnet\_perkgliq(P2) = 1.243 \times 10^{3}$

$Wideal := IdealClaudeSystem\_N2(P1, P2, T1, T3, FlowRatio\_x)_{1,4}$ $Wideal = 771.028$

$FOM(P2) := IdealClaudeSystem\_N2(P1, P2, T1, T3, FlowRatio\_x)_{1,5}$ $FOM(P2) = 0.62$

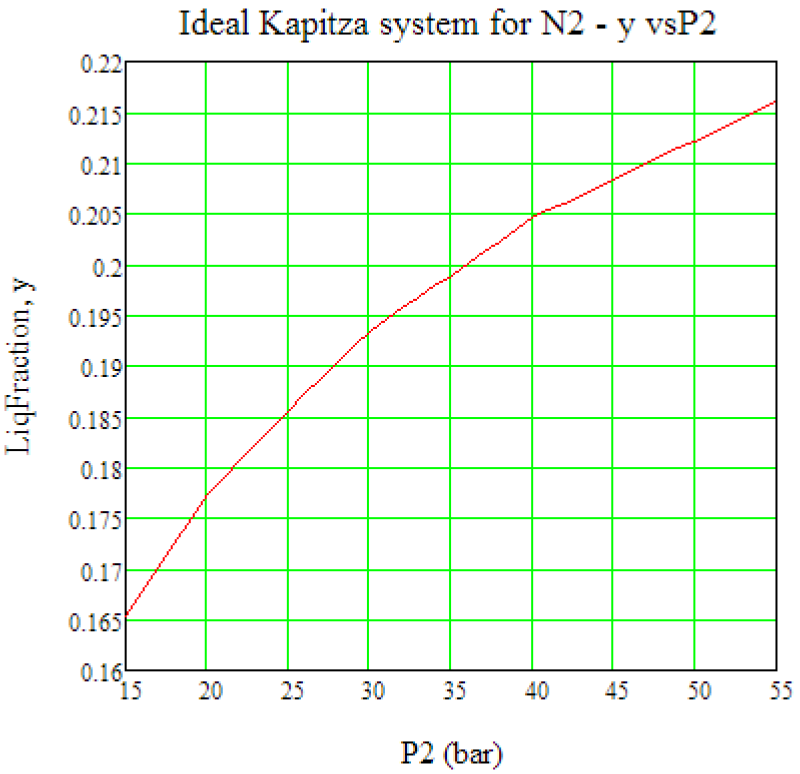
$P2 := 15, 20 .. 55$ ...define a range variable

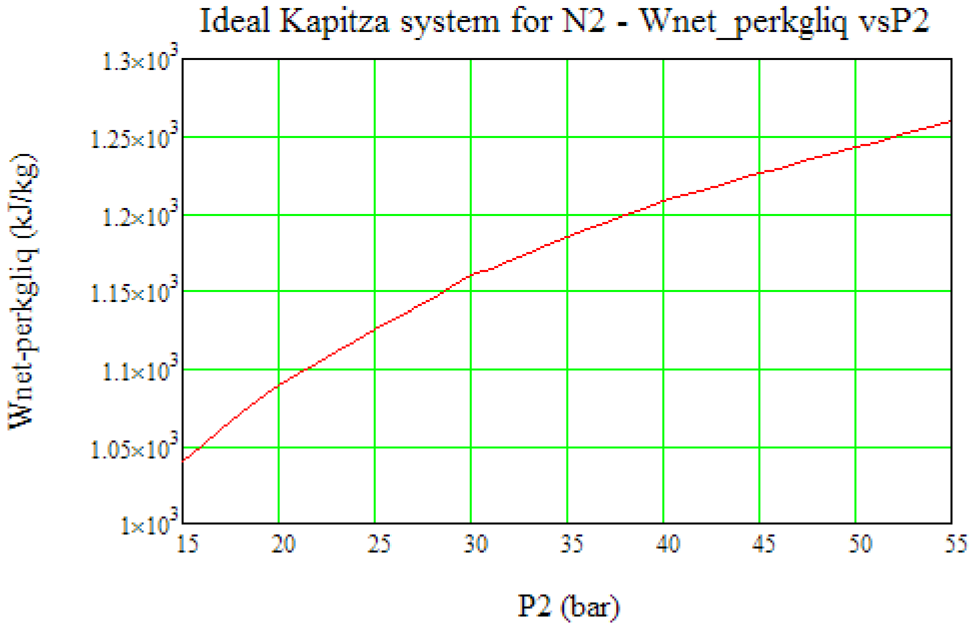
We get:

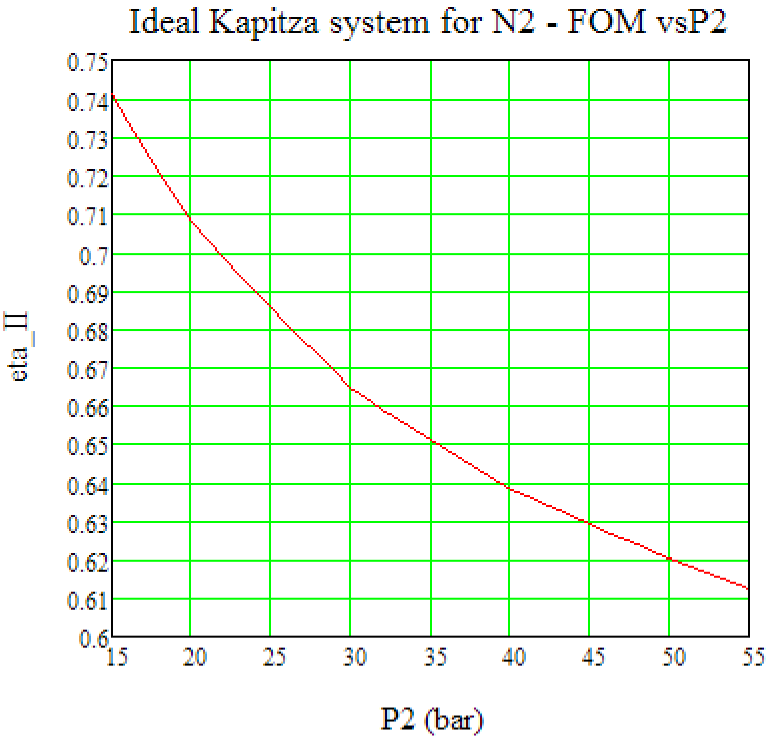


P2 =	y(P2) =	Wnet_perkgliq(P2)	FOM(P2) =
15	0.165	$1.04 \cdot 10^{3}$	0.741
20	0.177	$1.089 \cdot 10^{3}$	0.708
25	0.186	$1.125 \cdot 10^{3}$	0.685
30	0.193	$1.16 \cdot 10^{3}$	0.665
35	0.199	$1.185 \cdot 10^{3}$	0.651
40	0.205	$1.208 \cdot 10^{3}$	0.638
45	0.208	$1.226 \cdot 10^{3}$	0.629
50	0.212	$1.243 \cdot 10^{3}$	0.62
55	0.216	$1.259 \cdot 10^{3}$	0.612

Now, plot the results:







Note: FOM is also equal to eta_II

===

3.5 REFERENCES

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