Cryogenic Engineering: Software Solutions Vol II A

Liquefaction systems – Theory + Problems (EES)

Dr. M. Thirumaleshwar

DR. M. THIRUMALESHWAR

CRYOGENIC ENGINEERING: SOFTWARE SOLUTIONS

VOL II A:

LIQUEFACTION SYSTEMS – THEORY+ PROBLEMS (EES)

Cryogenic Engineering: Software Solutions Vol II A
Liquefaction systems – Theory + Problems (EES)
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YOU THINK.
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DEDICATION

This work is lovingly dedicated at the lotus feet of

Bhagavan Sri Sathya Sai Baba

"There is only one religion, the religion of Love.
There is only one caste, the caste of Humanity.
There is only one language, the language of the Heart.
There is only one God, He is Omnipresent."

"Help Ever, Hurt Never!"

– Bhagavan Sri Sathya Sai Baba

PREFACE

This book, viz. **Cryogenic Engineering: Software solutions – Part-II,** is the second in the series on **Cryogenic Engineering: Software Solutions.**

This part is being published in two volumes, viz. Cryogenic Engineering: Software solutions – Part-II A and Part-II B. Part-II A contains the background theory, definitions and formulas and the problems solved with Engineering Equation Solver (EES). Part-II B contains problems solved with Mathcad. So, it is advisable that one refers to both the parts.

As with the Part-I of the series, which dealt with: Introduction and properties of cryogenic fluids and properties of materials at low temperatures, the focus is on the solutions of problems in cryogenic engineering using software such as Mathcad and Engineering Equation Solver (EES). Only the essential theory and summary of equations required for calculations are given at the beginning of the chapter.

Advantages of using computer software to solve problems are reiterated:

i) It helps in solving the problems fast and accurately
ii) Parametric analysis (what-if analysis) and graphical visualization is done very easily. This helps in an in-depth analysis of the problem.
iii) Once a particular type of problem is solved, it can be used as a *template* and solving similar problems later becomes extremely easy.
iv) In addition, one can plot the data, curve fit, write functions for various properties or calculations and re-use them.
v) These possibilities create interest, curiosity and wonder in the minds of students and enthuse them to know more and work more.

This book, viz. **Cryogenic Engineering: Software solutions – Part-II** deals with the liquefaction of gases.

In this book: first, a thermodynamic background for cryogenic liquefaction is given; isenthalpic and isentropic expansion, pay-off functions for a liquefier and thermodynamically ideal liquefaction system are explained.

Next, various cycles for liquefaction of gases are explained and the calculation formulas for the pay-off functions are given. The cycles dealt with are: simple Linde – Hampson (L-H) system, pre-cooled L-H system, Linde dual pressure system, Claude system, Kapitza system, Heylandt system, cycles for liquefaction of Neon, Hydrogen and Helium, Collin's Helium liquefier etc.

Finally, various types of heat exchangers, hear transfer correlations, calculation formulas and graphs for analysis by LMTD and NTU methods are given. And, Second Law analysis of the cryogenic systems is explained briefly.

Some data on commercial liquefiers for Air, Hydrogen and Helium are also given.

Many numerical problems are solved to illustrate the ease of computer calculations using Engineering Equation Solver (EES) and Mathcad software.

Useful data for Nitrogen, Hydrogen, Helium, Argon, Methane, Fluorine and Oxygen are generated from NIST website, i.e. http://webbook.nist.gov/chemistry/fluid/.

EES has built-in functions for properties of several gases, refrigerants and fluids (including cryogenic fluids). Therefore, it is very convenient to use EES in these calculations.

However, Mathcad does not have built-in functions for properties of gases, refrigerants and fluids.

So, Mathcad Functions were written for both saturation properties and superheat gas properties of above mentioned important cryogenic fluids, viz. Nitrogen, Hydrogen, Helium, Argon, Methane, Fluorine and Oxygen. These Functions were used in solving problems, illustrating the ease of using Mathcad in calculations and graphing.

Several Procedures/Functions are written in EES and Mathcad to simplify the standard and most required calculations, which, the students, teachers, researchers and professionals may find very useful.

S.I. Units are used throughout this book. Wide variety of worked examples presented in the book should be useful for those appearing for University, AMIE and Engineering Services examinations.

Acknowledgements: Firstly, I would like to **thank all my students**, who have been an inspiration to me in all my academic efforts.

Sincere thanks are due to **Rev. Fr. Joseph Lobo**, Director, St. Joseph Engineering College, Mangalore, for his kindness, regard and words of encouragement.

I am also thankful to **Dr. Thirumaleshwara Bhat**, Principal, Sri Madhwa Vadiraja Institute of Technology and Management, Bantakal, Udupi, for giving me support in my academic activities.

I gratefully remember my former colleagues at the Cryogenics section of Technical Physics Division, Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology, Indore for their sincere cooperation in a true spirit of team-work in all the projects that we undertook.

I particularly salute and admire the vision and foresight of former Heads of Technical Physics Division, BARC viz. late Mr. C. Ambasankaran, Mr. R.Y. Deshpande, Dr. S.R. Gowariker and late Mr. S.S. Ramamurthy in initiating and guiding many of the ‘first of its kind’ projects for Indian Space Research Organization (ISRO), designed and executed by the Cryogenics section.

I am especially grateful to Prof. R.G.Scurlock, former Director of Institute of Cryogenics, University of Southampton, (U.K.) for writing a message for the Part-I of this series.

It was indeed gracious of my former Professor, under whom I studied for M.Sc. in Cryogenics at the University of Southampton, U.K. during 1970–72, and worked as a Visiting Research Fellow during 1993–94, to honor me by writing this message.

My special thanks to **Bookboon.com** for publishing this free ebook. **Ms Karin Jakobsen and Ms Sophie Tergeist** and their editorial staff have been most patient and helpful.

Finally, I would like to express my sincere thanks and appreciation to my **wife, Kala**, who, as usual, has given me continuous support, help and encouragement in all my academic activities, making many silent sacrifices.

M. Thirumaleshwar
March 2016

ABOUT THE AUTHOR

Dr. M. Thirumaleshwar graduated in Mechanical Engineering from Karnataka Regional Engineering College, Surathkal, Karnataka, India, in the year 1965. He obtained M.Sc (cryogenics) from University of Southampton, U.K. and Ph.D. (cryogenics) from Indian Institute of Science, Bangalore, India.

He is a Fellow of Institution of Engineers (India), Life Member, Indian Society for Technical Education, and a Foundation Fellow of Indian Cryogenics Council.

He has worked in India and abroad on large projects in the areas involving heat transfer, fluid flow, vacuum system design, cryo-pumping etc.

He worked as Head of Cryogenics Dept. in Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology (CAT), Indore, from 1966 to 1992.

He worked as Guest Collaborator with Superconducting Super Collider Laboratory of Universities Research Association, in Dallas, USA from 1990 to 1993.

He also worked at the Institute of Cryogenics, Southampton, U.K. as a Visiting Research Fellow from 1993 to 1994.

He was Head of the Dept. of Mechanical Engineering, Fr. Conceicao Rodrigues Institute of Technology, Vashi, Navi Mumbai, India for eight years.

He also worked as Head of Dept. of Mechanical Engineering and Civil Engineering, and then as Principal, Vivekananda College of Engineering and Technology, Puttur (D.K.), India.

He was Professor and coordinator of Post-graduate program in the Dept. of Mechanical Engineering in St. Joseph Engineering College, Vamanjoor, Mangalore, India.

A book entitled **“Fundamentals of Heat and Mass Transfer”** authored by him and published by M/s Pearson Education, India (2006) **has been adopted as a Text book** for third year engineering students by the Visweswaraya Technological University (V.T.U.), Belgaum, India.

He has authored a free e-book entitled **"Software Solutions to Problems on Heat Transfer"** wherein problems are solved using 4 software viz. Mathcad, EES, FEHT and EXCEL. This book, containing about 2750 pages, is presented in 9 parts and all the 9 parts can be downloaded for free from www.bookboon.com.

He has also authored **free e-books on Thermodynamics** entitled **"Basic Thermodynamics: Software Solutions" and "Applied Thermodynamics: Software Solutions"** wherein problems are solved using 3 software viz. Mathcad, EES, and TEST. Each of these titles is presented in 5 parts and all the books can be downloaded for free from www.bookboon.com

His earlier free ebook, viz. **Cryogenic Engineering: Software Solutions – Part-I** was published by Bookboon about a year ago.

He has also authored **three motivational, free ebooks**, published by www.bookboon.com, entitled as follows:

1. Towards Excellence…How to Study (A Guide book to Students)
2. Towards Excellence…How to teach (A guide book to Teachers)
3. Towards Excellence…Seminars, GD's and Personal Interviews

Dr. M. Thirumaleshwar has attended several National and International conferences and has more than 50 publications to his credit.

ABOUT THE SOFTWARE USED

Following three software are used while solving problems in this book series:

1. Mathcad 7 and Mathcad 15 (Ref: www.ptc.com)
2. Engineering Equation Solver (EES) (Ref: www.fchart.com), and
3. EXCEL

For a brief introduction to Mathcad, EES and EXCEL see the chapter 1 of the following free ebook by the author:

"Software Solutions to Problems on Heat Transfer – CONDUCTION – Part-I": http://bookboon.com/en/software-solutions-to-problems-on-heat-transfer-ebook

3 CRYOGENIC LIQUEFACTION SYSTEMS

Learning objectives:

1. In this chapter, topics on 'cryogenic liquefaction systems' are dealt with.
2. First, a thermodynamic background for cryogenic liquefaction is given; isenthalpic and isentropic expansion, pay-off functions for a liquefier and thermodynamically ideal liquefaction system are explained.
3. Various cycles for liquefaction of gases are explained and the calculation formulas for the pay-off functions are given. The cycles dealt with are: simple Linde-Hampson (L-H) system, pre-cooled L-H system, Linde dual pressure system, Claude system, Kapitza system, Heylandt system, cycles for liquefaction of Neon, Hydrogen and Helium, Collin's Helium liquefier etc.
4. Finally, various types of heat exchangers, hear transfer correlations, calculation formulas and graphs for analysis by LMTD and NTU methods are given. And, Second Law analysis of the cryogenic systems is explained briefly.
5. Some data on commercial liquefiers for Air, Hydrogen and Helium are also given.

6. Many numerical problems are solved to illustrate the ease of computer calculations using Engineering Equation Solver (EES) and Mathcad software.
7. Useful property data for Nitrogen, Hydrogen and Helium from NIST are given.
8. EES has built-in functions for properties of several gases, refrigerants and fluids (including cryogenic fluids). Therefore, it is very convenient to use EES in these calculations.
9. However, Mathcad does not have built-in functions for properties of gases, refrigerants and fluids. **So, Mathcad Functions were written for properties of important cryogenic fluids, viz. Nitrogen, Hydrogen, Helium, Argon, Methane, Fluorine and Oxygen.** These Functions were used in solving problems, illustrating the ease of using Mathcad in calculations and graphing.
10. **Several Procedures/Functions are written in EES and Mathcad to simplify the standard and most required calculations, which, the students, teachers, researchers and professionals may find very useful**.

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3.1 DEFINITIONS, STATEMENTS AND FORMULAS USED [1–9]:

3.1.1 THERMODYNAMIC BACKGROUND: JOULE-THOMSON (J-T) OR ISENTHALPIC EXPANSION:

When a high pressure gas passes through a valve, capillary or restriction, its pressure drops and the temperature also drops, generally.

From the I Law of Thermodynamics, it can be shown that in this process the enthalpy remains constant.

Joule-Thomson coefficient is defined as:

$$\mu_{\mathrm{JT}} = \left(\frac{\partial T}{\partial P}\right)_H$$

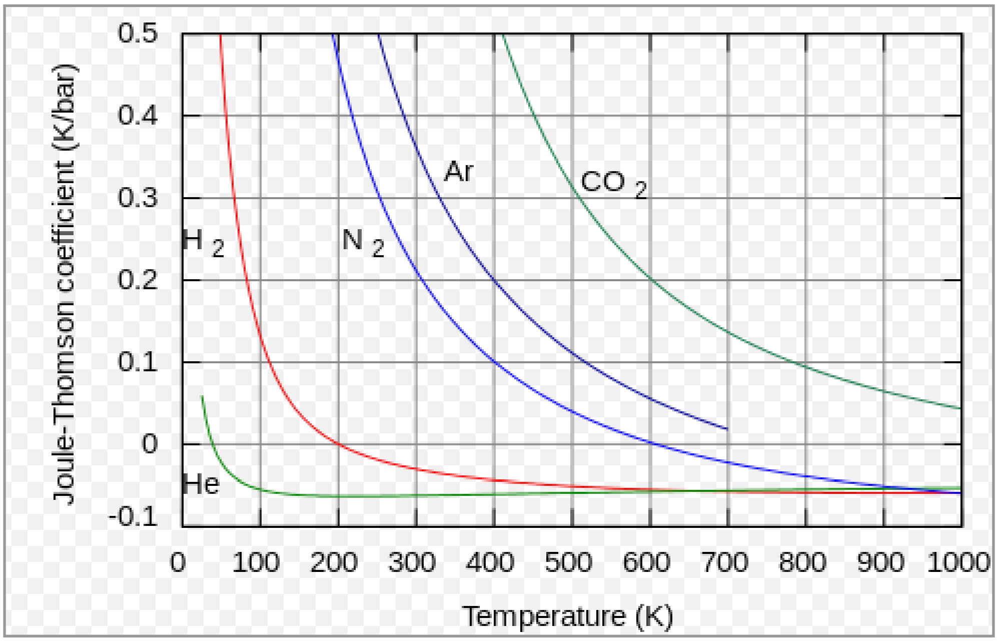
Obviously,

$\mu JT > 0$ to produce a temp drop as the pressure drops,

$\mu JT < 0$ for a temp increase as the pressure drops, and

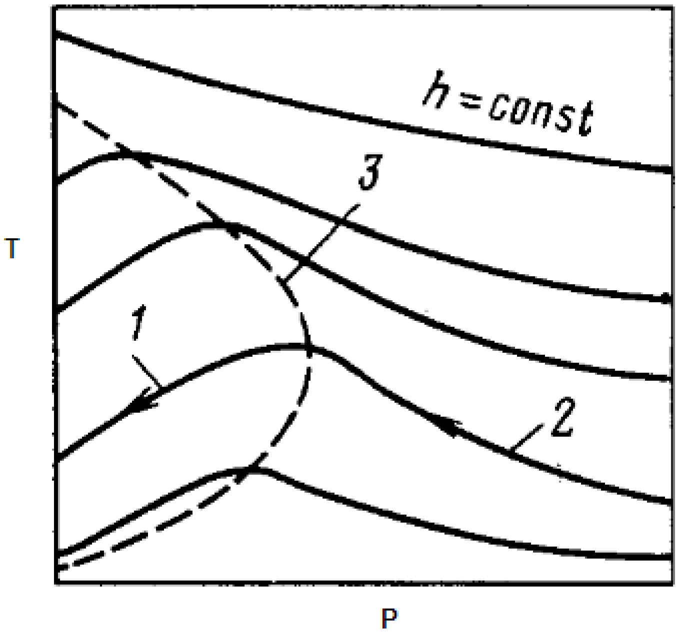
if μJT = zero, then there is no change in temp as the pressure drops through the J-T valve.

Hydrogen, helium, and neon have negative J-T coefficients at ambient temperature; therefore, these gases have to be pre-cooled to temperatures below their respective **'max. inversion temperatures'**, if they have to produce cooling upon expansion in a J-T valve. Following graph illustrates this [7]:



Inversion curve:

Pressure vs Temp curves at constant enthalpies for a gas look as follows [1]:



Note from the above graph that constant enthalpy curves go through a maximum. Locus of these maximums is called **'Inversion line' (3 in fig. above)** and the point where this inversion line cuts the y-axis (i.e $P = 0$) is known as **'Max. inversion temp.'** Note that Inversion line divides the P-T graphs in to two regions, viz.:

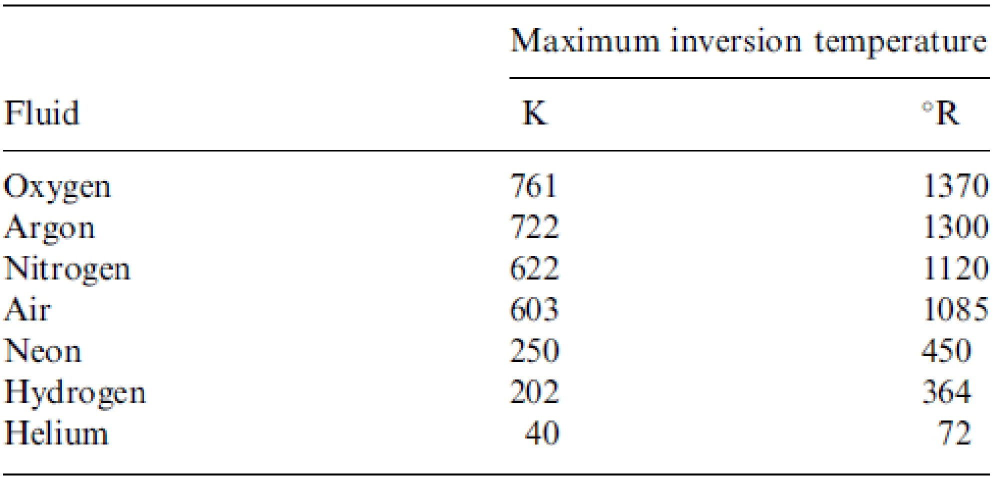
i) Region of cooling upon expansion, i.e. J-T coeff. > 0, is to the left of the Inversion curve (region 1 in fig. above),
ii) Region of heating upon expansion, i.e. J-T coeff. < 0, is to the right of the Inversion curve (region 2 in fig. above).

And, on the Inversion curve, J-T coeff. is zero.

Also note that for cooling to occur during a J-T expansion, ***the initial temp must be below the max. inversion temp.***

Following Table gives max. inversion temperatures for a few gases:[2]

Max. Inversion temps for a few gases

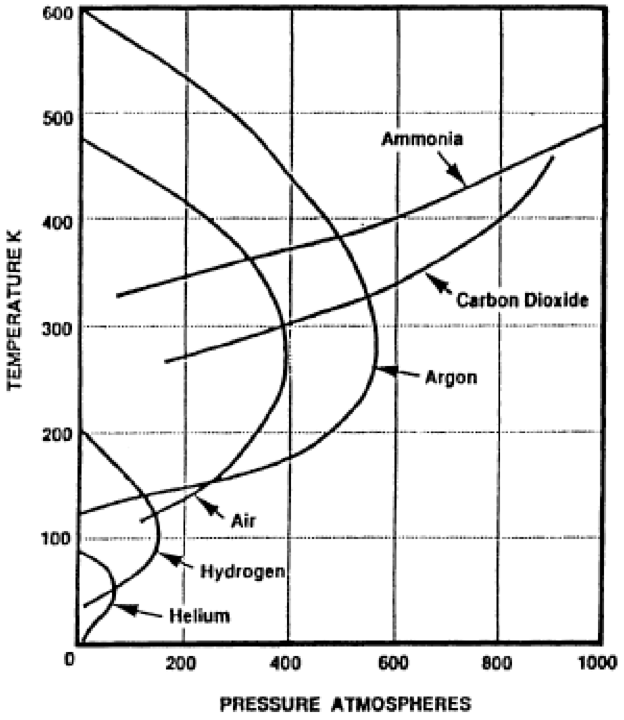


	Maximum inversion temperature	
Fluid	K	°R
Oxygen	761	1370
Argon	722	1300
Nitrogen	622	1120
Air	603	1085
Neon	250	450
Hydrogen	202	364
Helium	40	72

Note from the above Table that max. inversion temps for Neon, Hydrogen and Helium are below the room temp (300 K); therefore, these three gases will have to be pre-cooled to a temperature below their respective max. inversion temperatures to produce cooling upon a J-T expansion.

Joule-Thomson Inversion curves for a few common gases are shown below: [2]

Joule-THomson Inversion curves for some common gases



We have, for J-T coeff. by definition [1]:

$$\mu_{JT} = \left(\frac{\partial T}{\partial p}\right)_h.$$

From Thermodynamics, we can show that:

$$\mu_{JT} = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right].$$

where v is the sp. volume.

Now, for a 'perfect gas', we have:

p. v = R. T,

and, we get, *J-T coeff. for a Perfect gas* as:

$$\mu_{JT} = \frac{1}{c_p}\left[T\left(\frac{v}{T}\right) - v\right] = 0.$$

Note that J-T coeff for a Perfect gas is zero, **i.e. a Perfect gas will undergo no temperature drop during a J-T expansion.**

***For a 'Real gas'* which is represented by the Van der Waal's equation, we have:**

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT,$$

where 'a' is a measure of intermolecular forces and 'b' is a measure of the finite size of the molecules.

Here, a and b are calculated in terms of the critical temp (Tc) and critical pressure (Pc) of a given gas:

$$a = \frac{27}{64} \cdot \frac{R^2 \cdot T_c^{\,2}}{P_c}$$

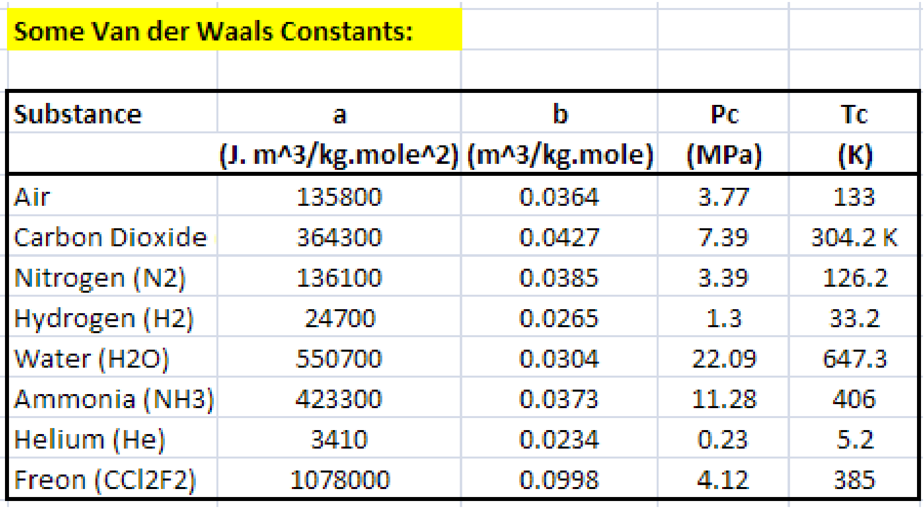
$$b = \frac{1}{8} \cdot \frac{R \cdot T_c}{P_c}$$

It is important to remember proper units while using the Van der Waal's equation.

Units:

p…N/m^2
v…m^3/kg.mol
T…K
R = Universal gas constant = 8314.4 Nm/kg.mol.K
a…Nm^4/(kg.mol)^2
b…m^3/kg.mol

Values of Van der Waal's constants a and b, and pressure and temp at critical point for some gases are given below [8]:



Some Van der Waals Constants:

Substance	a (J. m^3/kg.mole^2)	b (m^3/kg.mole)	Pc (MPa)	Tc (K)
Air	135800	0.0364	3.77	133
Carbon Dioxide	364300	0.0427	7.39	304.2 K
Nitrogen (N2)	136100	0.0385	3.39	126.2
Hydrogen (H2)	24700	0.0265	1.3	33.2
Water (H2O)	550700	0.0304	22.09	647.3
Ammonia (NH3)	423300	0.0373	11.28	406
Helium (He)	3410	0.0234	0.23	5.2
Freon (CCl2F2)	1078000	0.0998	4.12	385

Some Tables give values of a and b in different Units. Then, following conversion factors can be used (remember: 1J = 1Nm):

Units:

1 J·m^3/mol^2 = 1 m^6·Pa/mol^2 = 10 L^2·bar/mol^2

1 L^{2}atm/mol^2 = 101 325 J·m^3/kmol^2 = 101 325 Pa·m^6/kmol^2

1 dm^3/mol = 1 L/mol = 1 m^3/kmol (where kmol is kilomoles = 1000 moles)

J-T coeff for a Van der Waals gas is obtained as[1]:

$$\mu_{JT} = \frac{(2a/RT)(1-b/v)^2 - b}{c_p[1-(2a/(v\,RT))(1-b/v)^2]}.$$

For large values of sp. volume, above eqn can be approximated to:

$$\mu_{JT} = \frac{1}{c_p}\left(\frac{2a}{RT} - b\right),$$

Inversion temp for a Van der Waal's gas is given by:

$$T_i = \frac{2a}{bR}\left(1 - \frac{b}{v}\right)^2$$

And, the max. inversion temp is the temp on the inversion curve at p = 0, and is given by:

$$T_{imax} = 2a/(bR).$$

3.1.2 THERMODYNAMIC BACKGROUND: ISENTROPIC EXPANSION [1]:

Isentropic expansion coeff is defined as:

$$\mu_s = \left(\frac{\partial T}{\partial p}\right)_s.$$

From Thermodynamics, and using Maxwell's equations:

$$\mu_s = \left(\frac{\partial T}{\partial p}\right)_s = -\left(\frac{\partial T}{\partial s}\right)_p \left(\frac{\partial s}{\partial p}\right)_T = +\frac{T}{c_p}\left(\frac{\partial v}{\partial T}\right)_p.$$

The term (∂v/∂T)p in the above eqn is equal to β.v where β is the volumetric coeff of expansion. This is positive for all gases, and T and cp are also positive; so, *isentropic expansion coefficient is always positive*, **i.e. isentropic expansion (say, through an expansion engine) will *always* produce a temp drop.**

For a perfect gas, pv = RT, and we get:

$$\mu_s = v/c_p.$$

For a Van der Waal's gas, one can show that:

$$\mu_s = \frac{v(1-b/v)}{c_p\left[1-(2a/(vRT))(1-b/v)^2\right]}$$

which is positive since v > b.

In a liquefier, both the above methods of cooling are used, i.e. isentropic cooling in an expansion engine in the beginning and isenthalpic expansion in an expansion valve in the final stages.

3.1.3 PAY-OFF FUNCTIONS FOR A LIQUEFIER:

Three pay-off functions are defined[1]:

- Work required per unit mass of gas compressed: -W/m (J/kg)
- Work required per unit mass of gas liquefied: -W/mf (J/kg)
- Fraction liquefied, y = mf/m

Then, obviously, W/m = (W/mf) * y

Note: -ve sign in the above equations only indicates that work is going ***in to*** the system.

Another parameter, i.e. **Figure of Merit (FOM)** is defined to compare the same system of liquefier for different fluids.

FOM is defined as the theoretical minimum work required divided by the actual work required for the system:

FOM = Wi/W = (Wi/m) / (W/mf)
FOM has a value between 0 and 1.

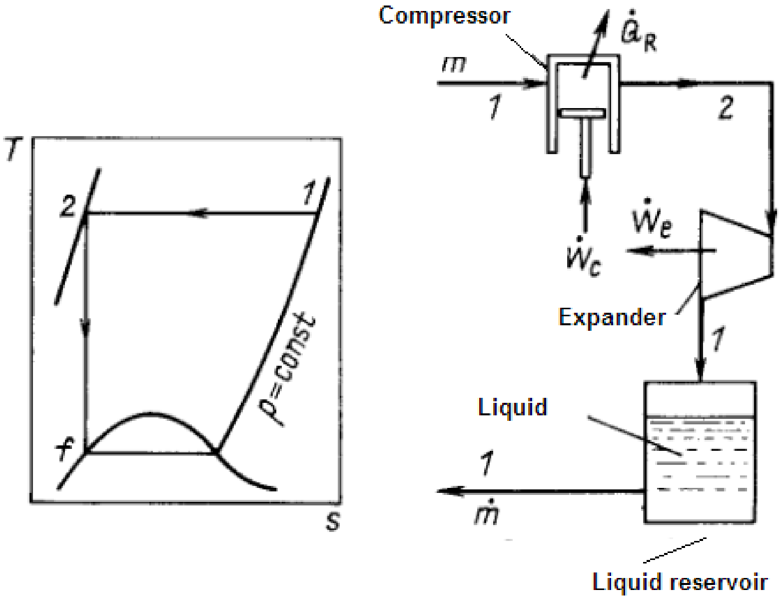
In an actual liquefier/refrigerator system, following performance parameters of components affect the overall performance of the system:

- Compressor and expander adiabatic efficiencies
- Compressor and expander mechanical efficiencies
- Heat exchanger effectiveness
- Pressure drops and
- Heat transfer to/from the system

3.1.4 WORK REQUIRED FOR LIQUEFACTION IN A THERMODYNAMICALLY IDEAL SYSTEM [1]:

A thermodynamically ideal liquefier system is shown below, along with its T-s diagram.
In the ideal system, *all* the gas compressed is liquefied.
1-2: isothermal compression to a required high pressure
2-f: isentropic expansion, such that all the compressed gas is liquefied after expansion
Applying the I Law to the entire system, neglecting changes in P.E. and K.E., we get [1]:

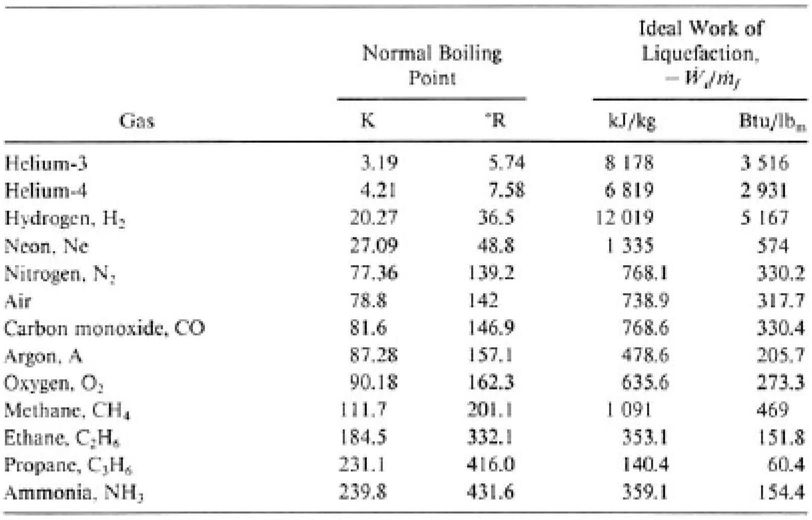
$$-\frac{\dot{W}_i}{\dot{m}} = T_1(s_1 - s_f) - (h_1 - h_f) = -\frac{\dot{W}_i}{\dot{m}_f}.$$



Thermodynamically ideal liquefaction system

Ideal work of liquefaction for a few gases, starting from 300 K, 101.3 kPa are given below [1]:

Ideal work for liquefaction of gases, starting from 300K, 101.3 kPa



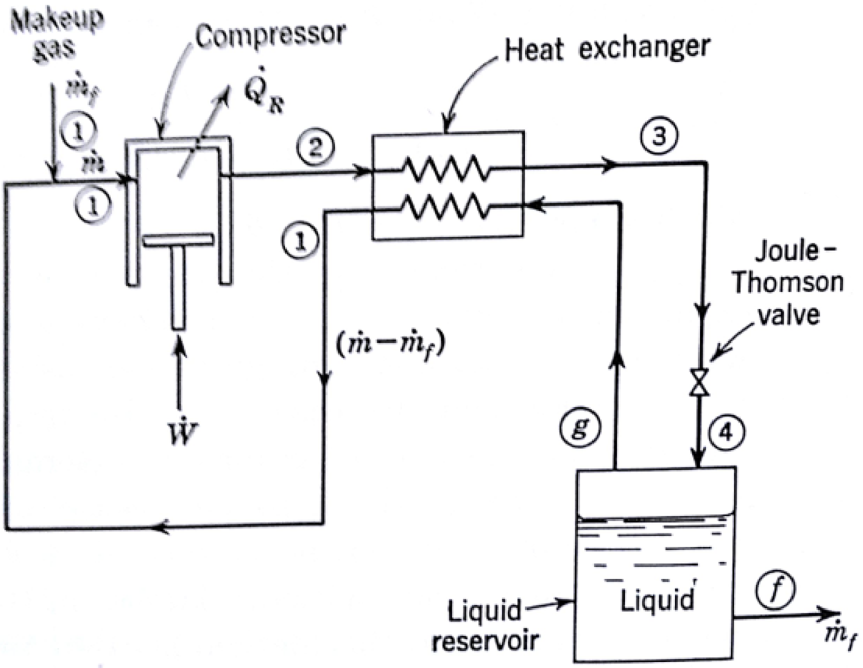
Gas	Normal Boiling Point		Ideal Work of Liquefaction, $-\dot{W}_i/\dot{m}_f$	
	K	°R	kJ/kg	Btu/lb$_m$
Helium-3	3.19	5.74	8 178	3 516
Helium-4	4.21	7.58	6 819	2 931
Hydrogen, H_2	20.27	36.5	12 019	5 167
Neon, Ne	27.09	48.8	1 335	574
Nitrogen, N_2	77.36	139.2	768.1	330.2
Air	78.8	142	738.9	317.7
Carbon monoxide, CO	81.6	146.9	768.6	330.4
Argon, A	87.28	157.1	478.6	205.7
Oxygen, O_2	90.18	162.3	635.6	273.3
Methane, CH_4	111.7	201.1	1 091	469
Ethane, C_2H_6	184.5	332.1	353.1	151.8
Propane, C_3H_6	231.1	416.0	140.4	60.4
Ammonia, NH_3	239.8	431.6	359.1	154.4

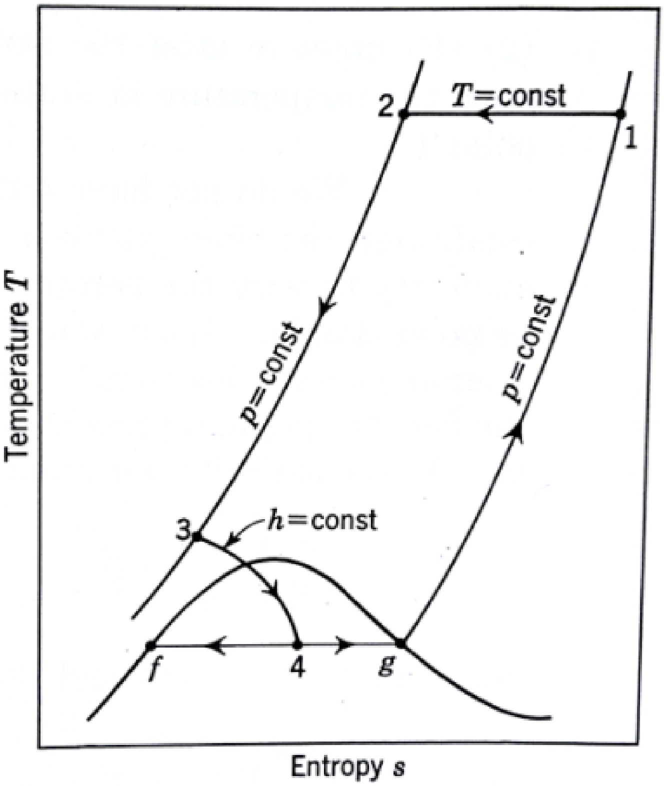
Above mentioned 'Ideal liquefaction system' is not practically possible since:

i) Because of the nature of T-s diagram, the pressure required would be extremely high
ii) The isentropic expansion in an expansion engine to the liquid point is impractical in the presence of two-phase fluid

3.1.5 SIMPLE LINDE-HAMPSON SYSTEM FOR LIQUEFACTION OF GASES OTHER THAN NEON, HYDROGEN AND HELIUM [1, 9]:

Schematic diagram and the T-s diagram for the system are shown below:





In the above:

1-2: Isothermal compression in the compressor
2-3: Cooling to a low temp in the heat exchanger, by heat exchange with the returning cold stream
3-4: J-T expansion in the expansion valve, resulting in liquefaction (4)
g-1: returning cold gas stream in the heat exchanger
Applying the I Law to the whole system (except the compressor):

$$0=(\dot{m}-\dot{m}_f)h_1+\dot{m}_f h_f-\dot{m}h_2$$

Or, the liquid yield is given by:

$$\frac{\dot{m}_f}{\dot{m}}=y=\frac{h_1-h_2}{h_1-h_f}.$$

Note that h_1 and h_f are determined by the ambient conditions; and the variable in our control is h_2.

It can be shown that for maximum yield, the point 2 must lie on the Inversion curve. For air, with the ambient at 294 K, this pressure is about 400 atm, but actual systems use about 200 atm.

Remember that to produce cooling, temp before J-T expansion should be below the Max inversion temp.

Compressor work requirement is obtained by applying the I Law to the compressor:

$$\dot{W}_c - \dot{Q}_r + \dot{m}\left(h_1 - h_2\right) = 0$$

And, heat transferred in compressor for isothermal compression:

$$\dot{Q}_r = \dot{m}T_1\left(s_1 - s_2\right)$$

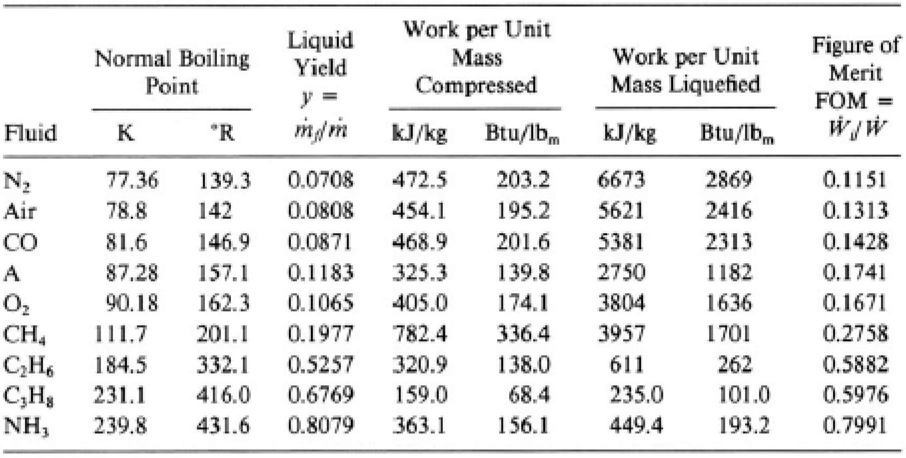
And, the work required:

$$\frac{\dot{W}_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

Then,

$$\frac{\dot{W}_c}{\dot{m}_f} = \frac{\dot{W}_c}{\dot{m}\ \mathrm{y}} = \left[T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)\right]\left(\frac{h_1 - h_f}{h_1 - h_2}\right)$$

Performance of Linde-Hampson system using different fluids, (with p1 = 101.3 kPa (1 atm), T1 = T2 = 300 K (80 F), p2 = 20.265 MPa (200 atm), heat exchanger effectiveness = 100%, compressor overall efficiency = 100%), is given in the following Table [1]:



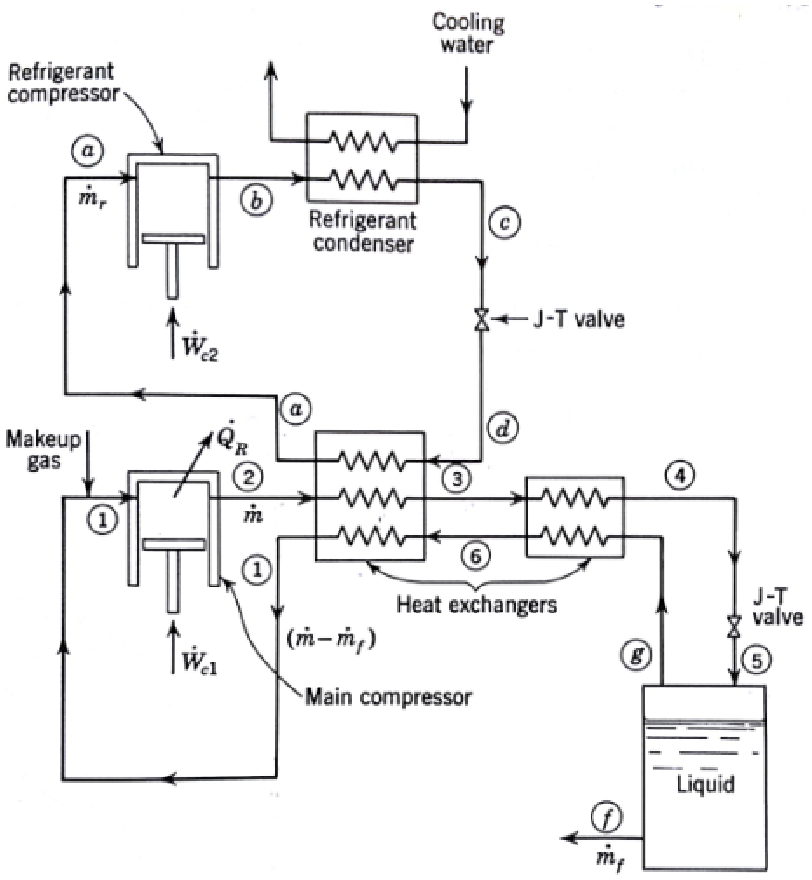
	Normal Boiling Point		Liquid Yield $y =$	Work per Unit Mass Compressed		Work per Unit Mass Liquefied		Figure of Merit FOM =
Fluid	K	°R	$\dot{m}_f/\dot{m}$	kJ/kg	Btu/lb$_m$	kJ/kg	Btu/lb$_m$	$\dot{W}_i/\dot{W}$
N_2	77.36	139.3	0.0708	472.5	203.2	6673	2869	0.1151
Air	78.8	142	0.0808	454.1	195.2	5621	2416	0.1313
CO	81.6	146.9	0.0871	468.9	201.6	5381	2313	0.1428
A	87.28	157.1	0.1183	325.3	139.8	2750	1182	0.1741
O_2	90.18	162.3	0.1065	405.0	174.1	3804	1636	0.1671
CH_4	111.7	201.1	0.1977	782.4	336.4	3957	1701	0.2758
C_2H_6	184.5	332.1	0.5257	320.9	138.0	611	262	0.5882
C_3H_8	231.1	416.0	0.6769	159.0	68.4	235.0	101.0	0.5976
NH_3	239.8	431.6	0.8079	363.1	156.1	449.4	193.2	0.7991

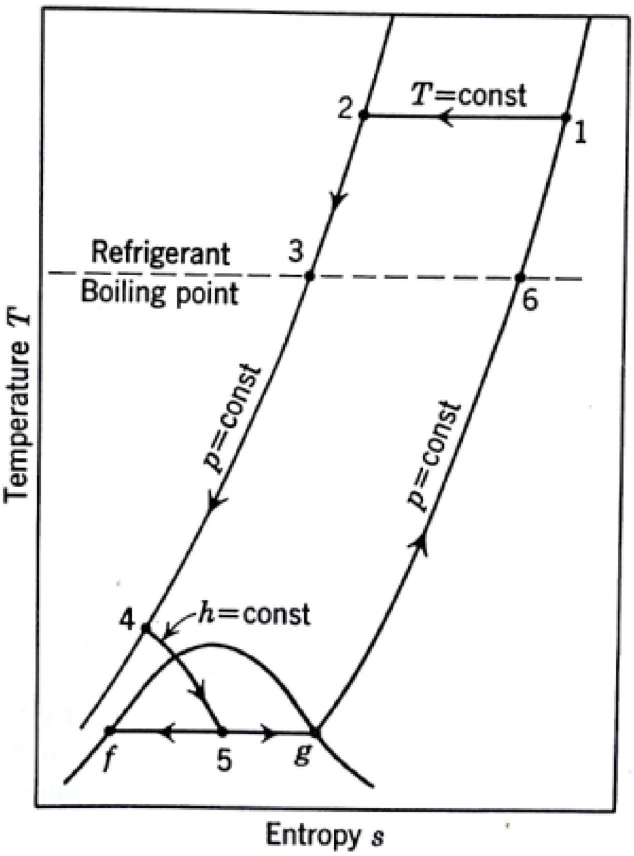
3.1.6 PRE-COOLED LINDE-HAMPSON SYSTEM [1,9]:

Recall that for the simple Linde-Hampson system, the liquid yield is given by:

$$\frac{\dot{m}_f}{\dot{m}} = y = \frac{h_1 - h_2}{h_1 - h_f}.$$

Since h_1, h_f are fixed by ambient conditions, we can increase the liquid yield by decreasing h_2, i.e. by reducing the temp at point 2, i.e at the inlet to the heat exchanger. Such a pre-cooled system and its T-s diagram are shown below:





Here, we have:

1-2: Isothermal compression
2-3: Cooling in the first heat exchanger by the auxiliary refrigeration system
3-4: Cooling in the J-T heat exchanger
4-5: Expansion in the J-T valve
g-1: return path of gas

Applying the I Law to the system containing the two heat exchangers, expansion valve, liquid receiver and the auxiliary refrigerant expansion valve, we get:

$$0=(\dot{m}-\dot{m}_f)h_1+\dot{m}_r h_a+\dot{m}_f h_f-\dot{m}_r h_d.$$

Defining the *refrigerant flow-rate ratio* r as:

$$r=\frac{\dot{m}_r}{\dot{m}},$$

where mr is the mass flow rate of auxiliary refrigerant, solving **for liquid yield** y, we get:

$$y = \frac{h_1 - h_2}{h_1 - h_f} + r\,\frac{h_a - h_c}{h_1 - h_f}$$

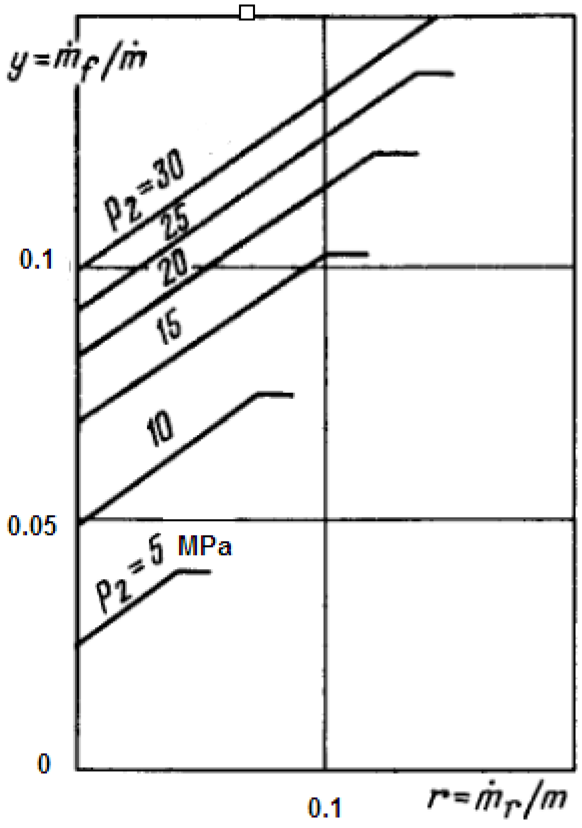
The second term in the above eqn represents *the improvement* in the liquid yield because of pre-cooling.

And, the *max. liquid yield* is:

$$y_{max} = \frac{h_6 - h_3}{h_6 - h_f};$$

where h_3 and h_6 are at the point d at the boiling point of the refrigerant.

Fig. below shows variation of y with r for different p_2, for Nitrogen with p_1 = 0.1 MPa and T_1 = 293K [1]:



Compressor Work:

Assuming the main compressor to be reversible and isothermal, and the auxiliary compressor to be reversible and adiabatic, work required per unit mass of gas compressed is:

$$-\dot{W}/\dot{m}=T_1(s_1-s_2)-(h_1-h_2)+r(h_b-h_a).$$

The last term in the above eqn is the *additional work* due to auxiliary compressor (approximately 10% of the total work requirement.)

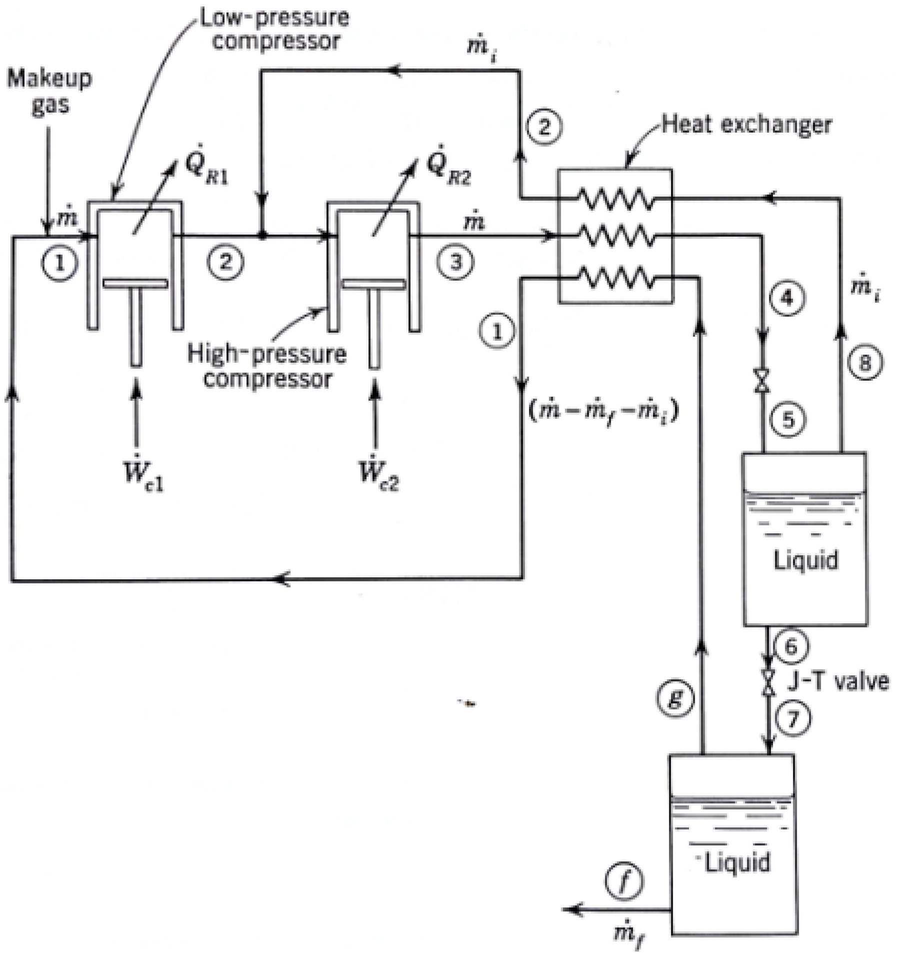
3.1.7 LINDE DUAL PRESSURE SYSTEM [1,9]:

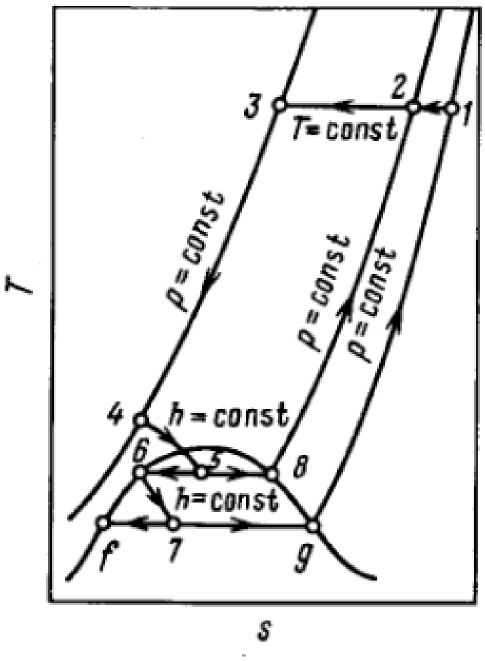
In this system, the gas is compressed in 2 stages. While there is saving in work of compression, liquid yield is reduced slightly.

Since in the simple L-H system, the liquid yield is a small fraction of the gas compressed, not all the gas is expanded to the lowest pressure, but some is expanded to an intermediate pressure. We save the work required since for an isothermal compression, work requirement depends on the pressure ratio, i.e.

$W_c = R\ T_1\ \ln(p_2/p_1)$

Schematic diagram and the T-s diagram for the system are shown below:




T
s
1
2
3
T=const
p=const
p=const
p=const
4
h=const
5
6
8
h=const
f
7
g

We have:

1-2: compression to intermediate pressure
2-3: compression to high pressure, after a return stream is added
3-4: cooling in the heat exchanger
4-5: expansion to an intermediate pressure, where some of the gas is liquefied. Liquid/vapor separated in the liquid receiver, and vapor returned to second compressor
6-7: liquid from the first liquid receiver is expanded through the second J-T valve to the second liquid receiver
g-1: cold vapor returned through the heat exchanger

Applying the I Law to the heat exchanger, two liquid receivers and the two J-T valves, we get the **liquid yield** for this system as:

$$y = \frac{h_1 - h_3}{h_1 - h_f} - i\,\frac{h_1 - h_2}{h_1 - h_f},$$

where, i is the intermediate flow rate ratio, viz.

$$i = \dot{m}_i / \dot{m}$$

where m_i is the mass flow rate of intermediate pressure stream at point 8, and m is the total mass flow rate through high pressure compressor.

In the above eqn for y, the second term represents the *reduction in the liquid yield* as compared to the simple L-H system.

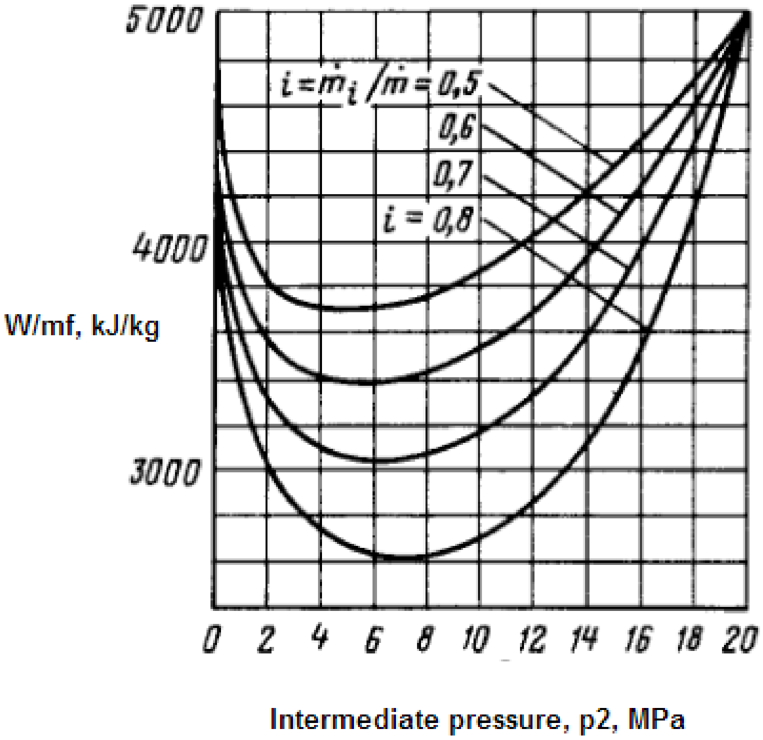
Applying the I Law to the two compressors, we get the **work requirement per unit mass of gas compressed** in the high pressure compressor:

$$-\dot{W}/\dot{m} = [T_1(s_1 - s_3) - (h_1 - h_3)] - i[T_1(s_1 - s_2) - (h_1 - h_2)].$$

From the above eqn, we see that the work required is *reduced* by the second bracketed term, as compared the simple L-H system.

In practical systems, i is of the order of 0.8; so, the reduction in work requirement more than offsets the reduction in liquid yield.

Work required to liquefy unit mass of Air in the Linde dual pressure system, with p_1 = 0.1 MPa, T_1 = 300 K, p_3 = 20 MPa, is shown below [1]:

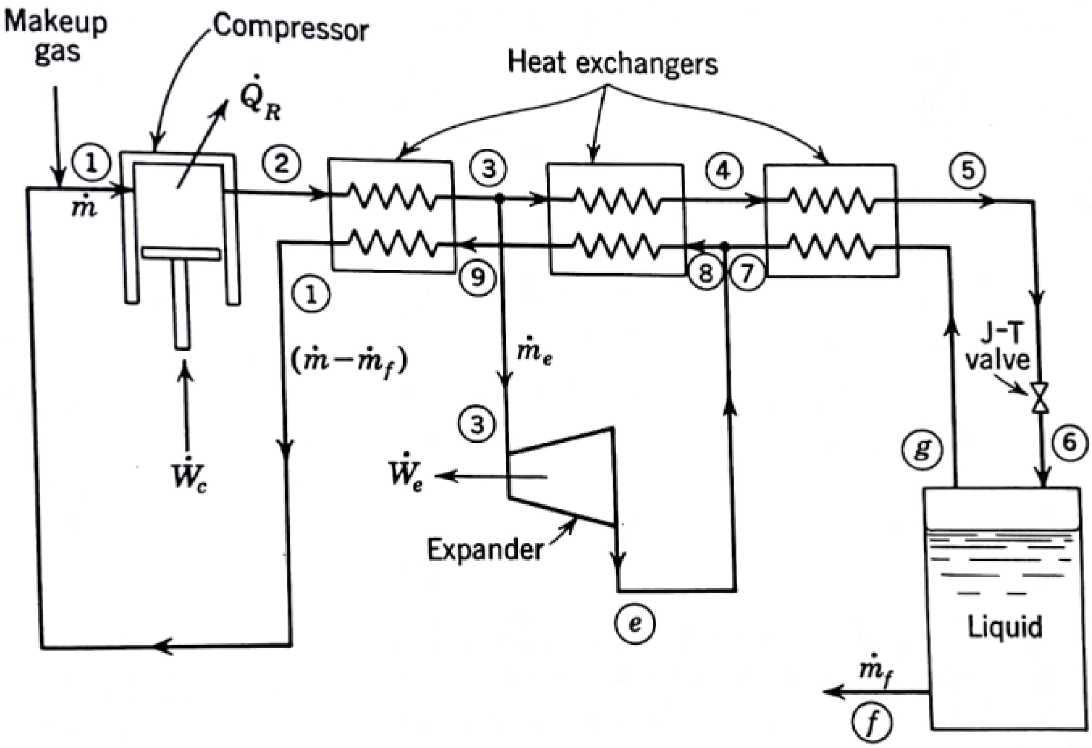


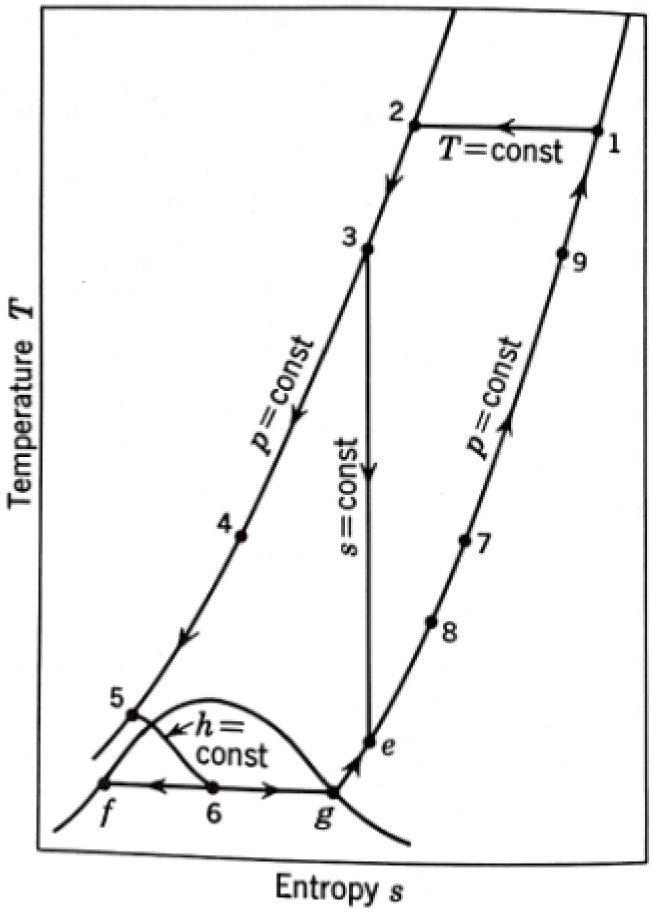
Note: In both the above modifications of the simple L-H system, work requirement per unit mass of *gas liquefied* is ***less*** as compared to that for the simple L-H system.

3.1.8 CLAUDE SYSTEM [1, 9]:

Claude cycle uses an expander for cooling by isentropic expansion of the gas, which gives a larger temp drop than for an isenthalpic expansion, for the same pressure ratio. However, in the final liquefaction stage, a J-T valve has to be used since expander can not have two-phase flow through it.

Following figures show the schematic diagram and the T-s diagram for Claude system:





Here, we have:

1-2: isothermal compression (to about 40 atm)
2-3: cooling in the first heat exchanger
At point 3, approx. 60% to 80% is diverted to be expanded in an expander, and then re-united with the return stream below the second heat exchanger.
3-4: cooling in the second heat exchanger
4-5: cooling in the third heat exchanger
5-6: expansion in J-T valve
g-1:return of the cold vapor through the three heat exchangers

In large Claude systems, the work output of expander is utilized to compress the gas; but, in small systems, expander work is dissipated in a brake. However, whether the expander work is utilized or not, does not affect the liquid yield; it affects only the compressor net work requirement.

Applying the I Law to system consisting of all the components except the compressor, we get:

$$0=(\dot{m}-\dot{m}_f)h_1+\dot{m}_f h_f+\dot{m}_e h_e-\dot{m}h_2-\dot{m}_e h_3.$$

Now, define the expander flow rate ratio, x:

$$x=\dot{m}_e/\dot{m},$$

Then, we get the liquefaction fraction, y as:

$$y=\frac{\dot{m}_f}{\dot{m}}=\frac{h_1-h_2}{h_1-h_f}+x\frac{h_3-h_e}{h_1-h_f}.$$

Second term in the above eqn represents the *improvement* over the simple L-H system.

Compressor work:

If the expander work is not utilized, this is the same as for the simple L-H system. i.e.

$$\frac{\dot{W}_c}{\dot{m}}=T_1(s_1-s_2)-(h_1-h_2)$$

However, if the expander work is used in compressing the gas, then:

$$-\dot{W}/\dot{m}=-\dot{W}_c/\dot{m}-\dot{W}_e/\dot{m}.$$

where, W_c is the compressor work, W_e is the expander work, and W is the net work required.

Applying I Law to the expander, we get the expander work as:

$$\dot{W}_e=\dot{m}_e(h_3-h_e).$$

Then, the net compressor work required is:

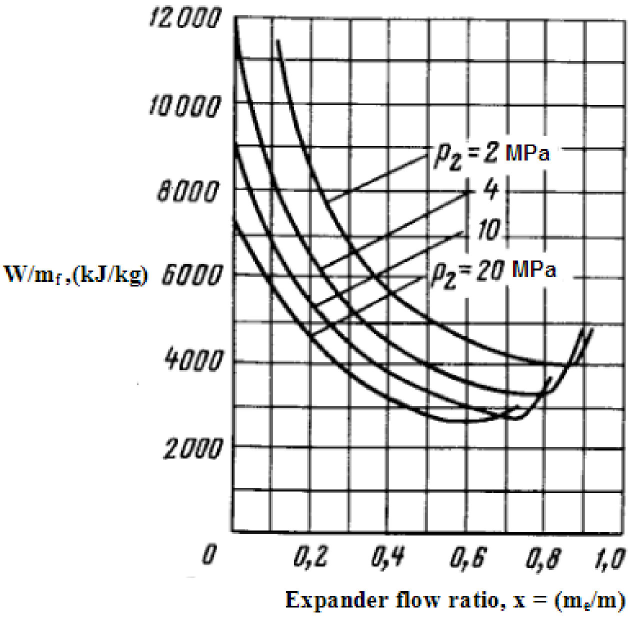
$$-\dot{W}/\dot{m}=[T_1(s_1-s_2)-(h_1-h_2)]-x(h_3-h_e).$$

Obviously, the last term in the above eqn represents the reduction in work requirements due to utilization of expander work.

From the above eqn we see that to calculate W, we need to know h3, i.e. temp T3, just before entering the expander.

For given high pressure p_2 and expander flow rate ratio x, there is an optimum T_3 which will make the work requirement a minimum.

For such a case, work requirement per unit mass liquefied (W/m_f) vs expander flow rate ratio for different p_2, for Air are shown below [1]:



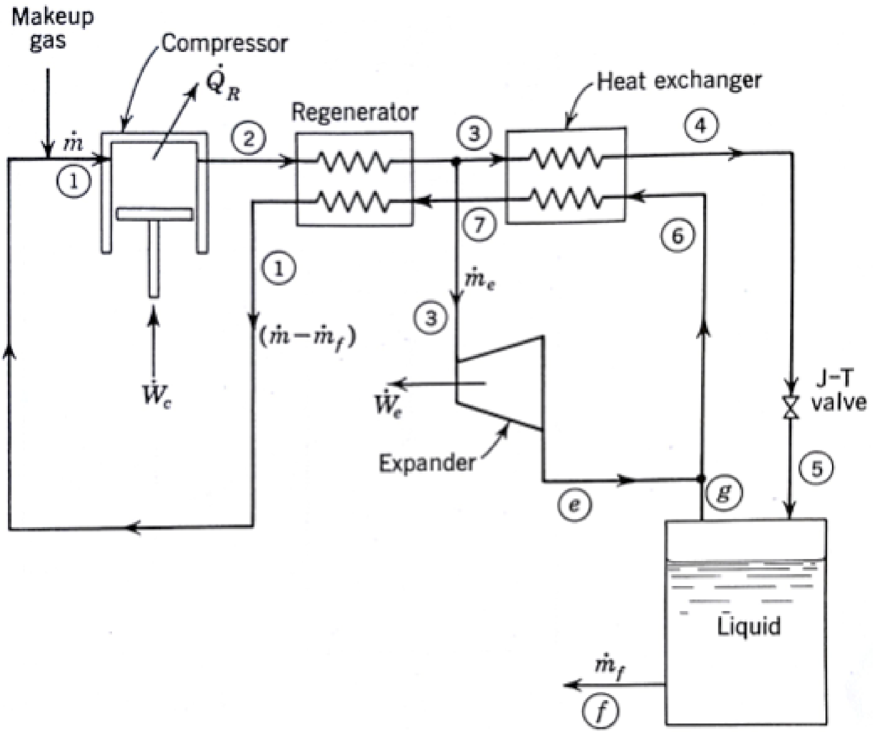
3.1.9 KAPITZA SYSTEM [1, 9]:

The schematic diagram of this system is shown below:

This is a modification of Claude system.

Note the following points:

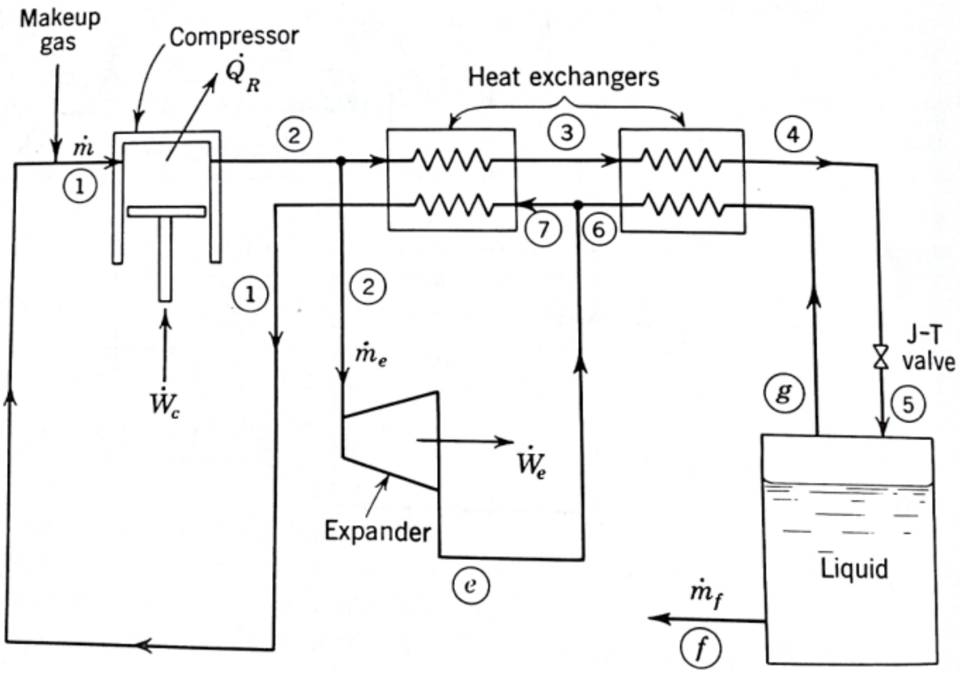
- Third or low temp heat exchanger in the Claude system is eliminated
- *A rotary expansion engine* is used instead of reciprocating one
- First heat exchanger is a set of *two parallel regenerators*, used alternately, which also serve to purify the incoming gas
- Usually operated at *low pressures*, of the order of 7 atm.



3.1.10 HEYLANDT SYSTEM [1, 9]:

This system is mostly used in high pressure liquefaction plants for Air.

Following is the schematic diagram of this system:



Here, the first heat exchanger of Claude system is eliminated.

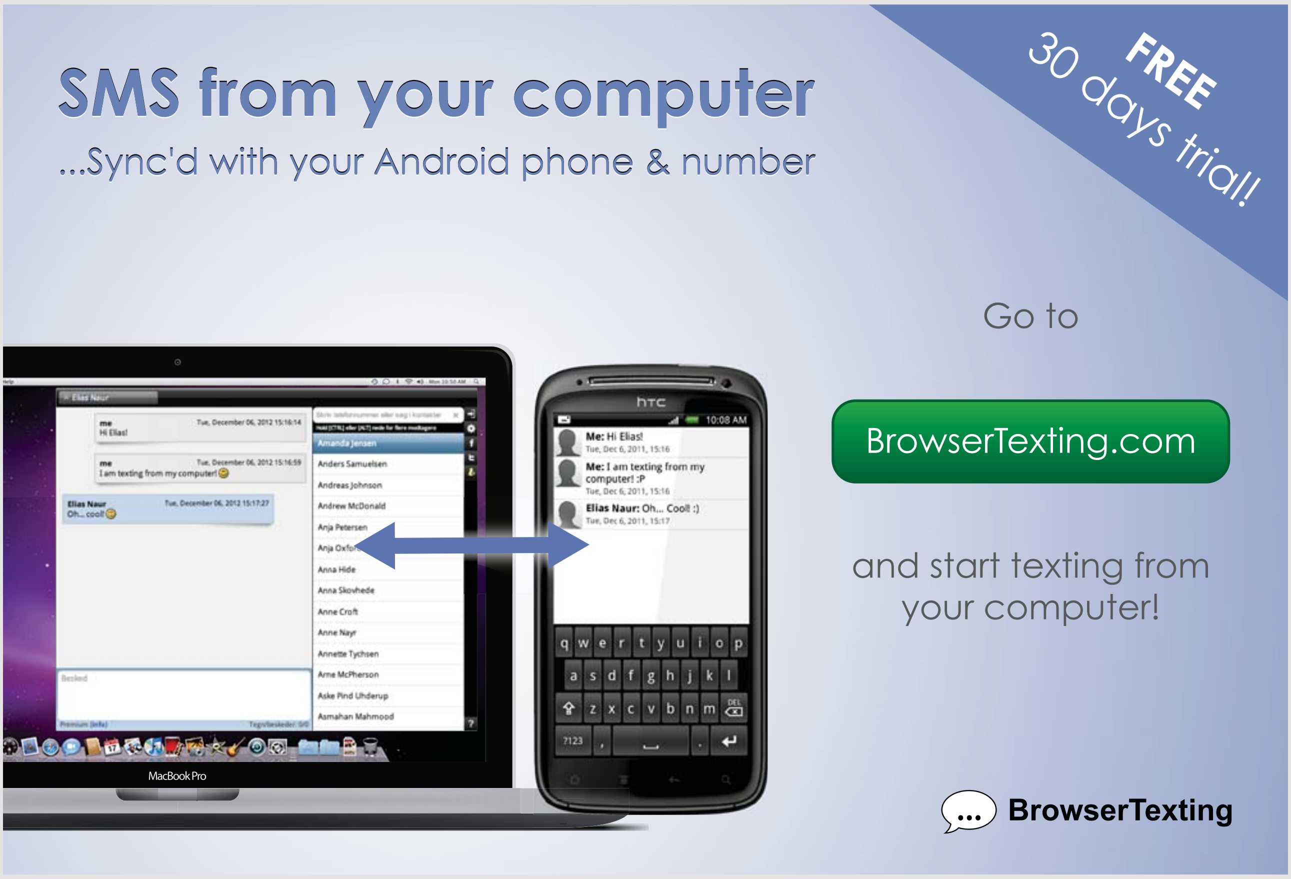
High pressure of the order of 200 atm is used (for Air).

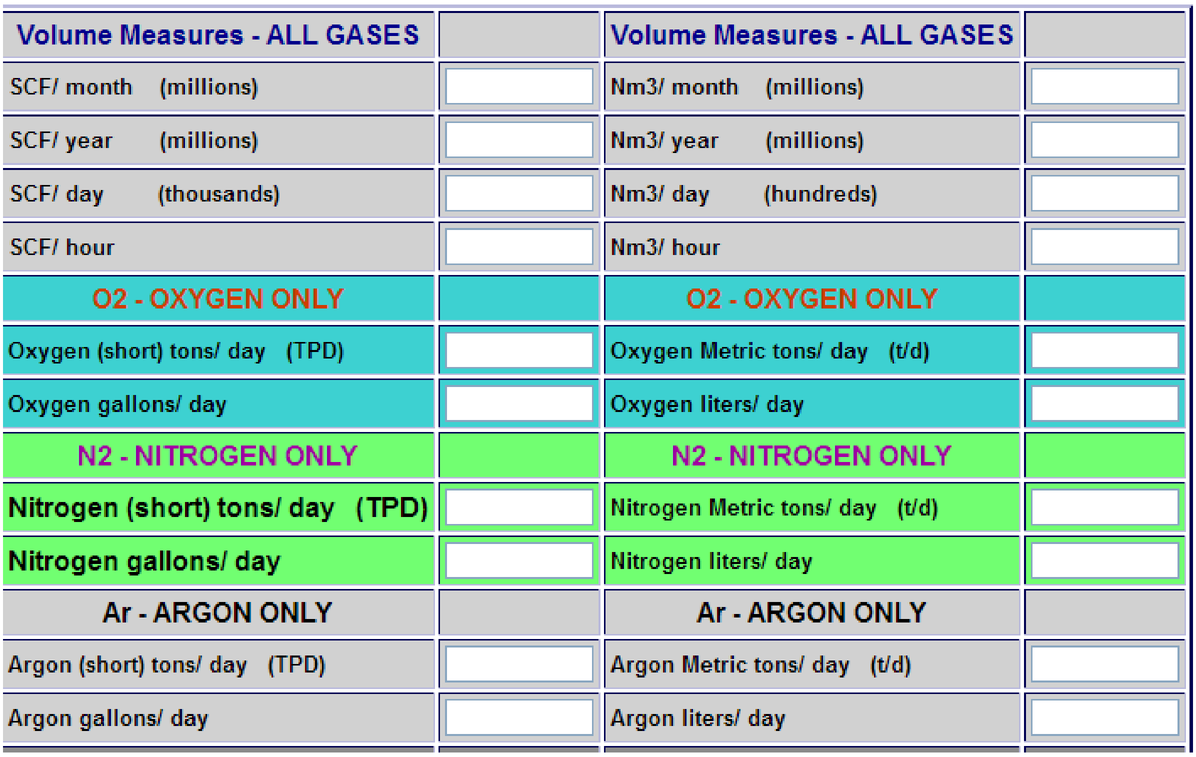
Expansion engine flow rate ratio is approximately 0.6, entry temp to expander being about 21 deg. C, thus reducing the lubrication problems in the expander.

3.1.11 COMMERCIAL AIR LIQUEFACTION SYSTEMS:

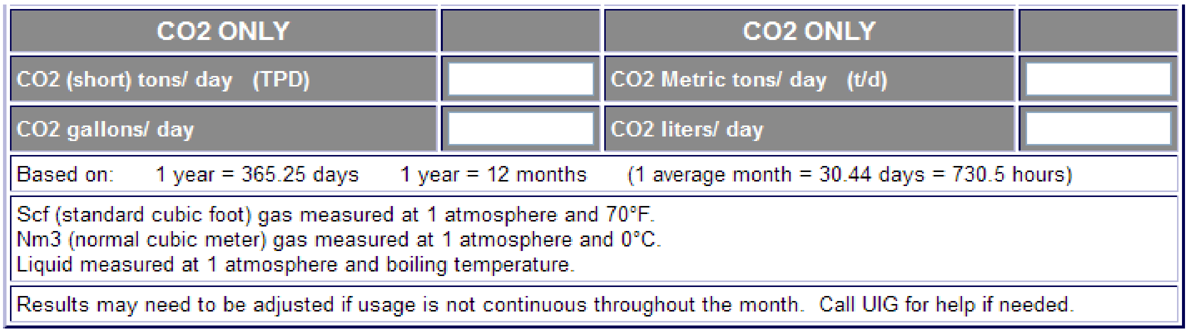
Large Air Separation Units are specified in terms of Tons per day (TPD) of O2 or N2 (or, other gases).

Conversions from TPD to other units of flow rate for various gases are done very conveniently using the following Table / calculator [10]:





Volume Measures - ALL GASES		Volume Measures - ALL GASES	
SCF/ month (millions)		Nm3/ month (millions)	
SCF/ year (millions)		Nm3/ year (millions)	
SCF/ day (thousands)		Nm3/ day (hundreds)	
SCF/ hour		Nm3/ hour	
O2 - OXYGEN ONLY		O2 - OXYGEN ONLY	
Oxygen (short) tons/ day (TPD)		Oxygen Metric tons/ day (t/d)	
Oxygen gallons/ day		Oxygen liters/ day	
N2 - NITROGEN ONLY		N2 - NITROGEN ONLY	
Nitrogen (short) tons/ day (TPD)		Nitrogen Metric tons/ day (t/d)	
Nitrogen gallons/ day		Nitrogen liters/ day	
Ar - ARGON ONLY		Ar - ARGON ONLY	
Argon (short) tons/ day (TPD)		Argon Metric tons/ day (t/d)	
Argon gallons/ day		Argon liters/ day	



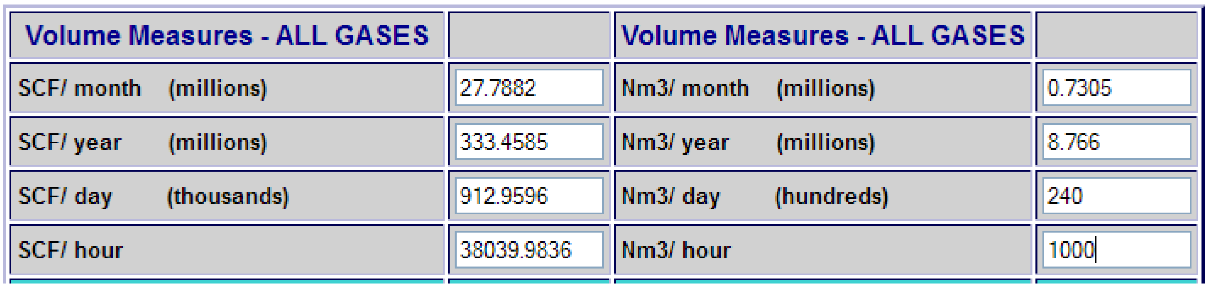
CO2 ONLY		CO2 ONLY	
CO2 (short) tons/ day (TPD)		CO2 Metric tons/ day (t/d)	
CO2 gallons/ day		CO2 liters/ day	

Based on: 1 year = 365.25 days 1 year = 12 months (1 average month = 30.44 days = 730.5 hours)

Scf (standard cubic foot) gas measured at 1 atmosphere and 70°F.
Nm3 (normal cubic meter) gas measured at 1 atmosphere and 0°C.
Liquid measured at 1 atmosphere and boiling temperature.

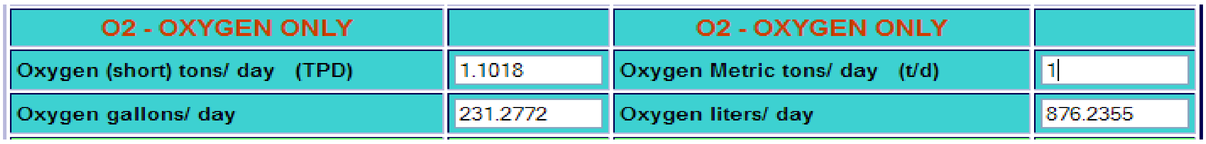
Results may need to be adjusted if usage is not continuous throughout the month. Call UIG for help if needed.

For example, 1000 Nm^3/h is equivalent to:



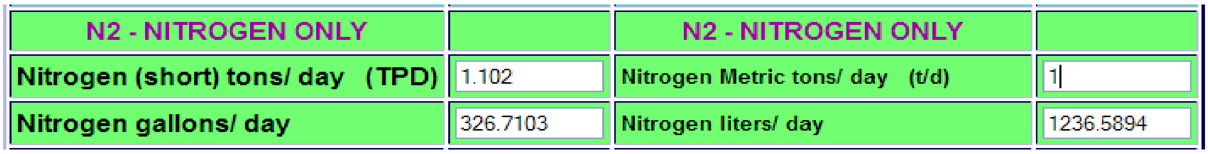
Volume Measures - ALL GASES		Volume Measures - ALL GASES	
SCF/ month (millions)	27.7882	Nm3/ month (millions)	0.7305
SCF/ year (millions)	333.4585	Nm3/ year (millions)	8.766
SCF/ day (thousands)	912.9596	Nm3/ day (hundreds)	240
SCF/ hour	38039.9836	Nm3/ hour	1000

And, to find equivalent of 1 metric Ton per day of Oxygen, enter 1 under 'Oxygen Metric tons/day' and press enter. We immediately get figures in terms of Tons per day, liters per day and gallons per day:

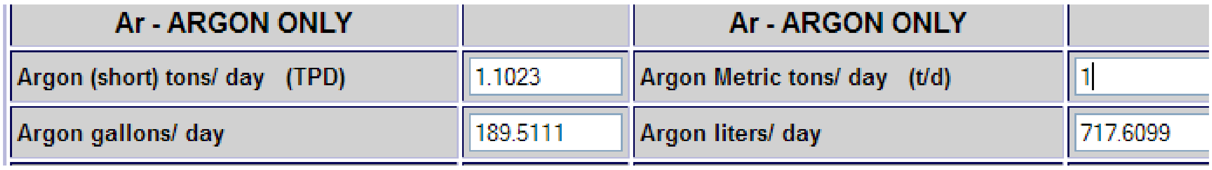


O2 - OXYGEN ONLY		O2 - OXYGEN ONLY	
Oxygen (short) tons/ day (TPD)	1.1018	Oxygen Metric tons/ day (t/d)	1
Oxygen gallons/ day	231.2772	Oxygen liters/ day	876.2355

And, similarly, find equivalent of 1 metric Ton per day of Nitrogen:

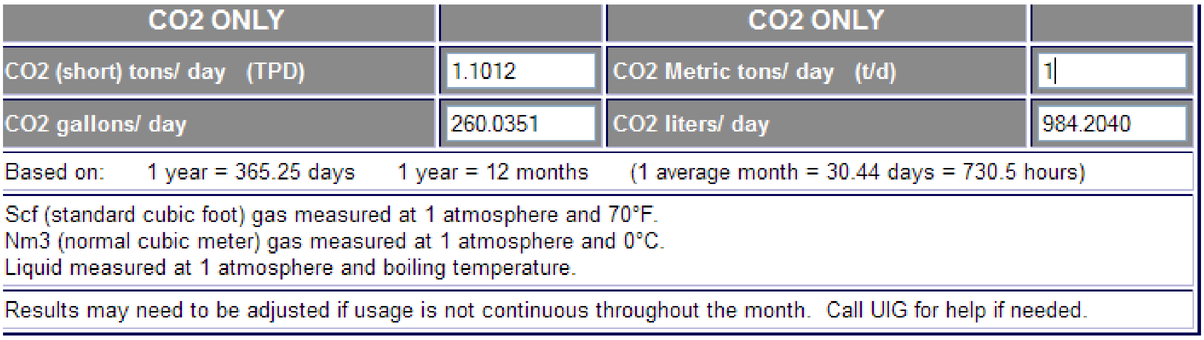


N2 - NITROGEN ONLY		N2 - NITROGEN ONLY	
Nitrogen (short) tons/ day (TPD)	1.102	Nitrogen Metric tons/ day (t/d)	1
Nitrogen gallons/ day	326.7103	Nitrogen liters/ day	1236.5894



Ar - ARGON ONLY		Ar - ARGON ONLY	
Argon (short) tons/ day (TPD)	1.1023	Argon Metric tons/ day (t/d)	1
Argon gallons/ day	189.5111	Argon liters/ day	717.6099

Finally, for CO2:



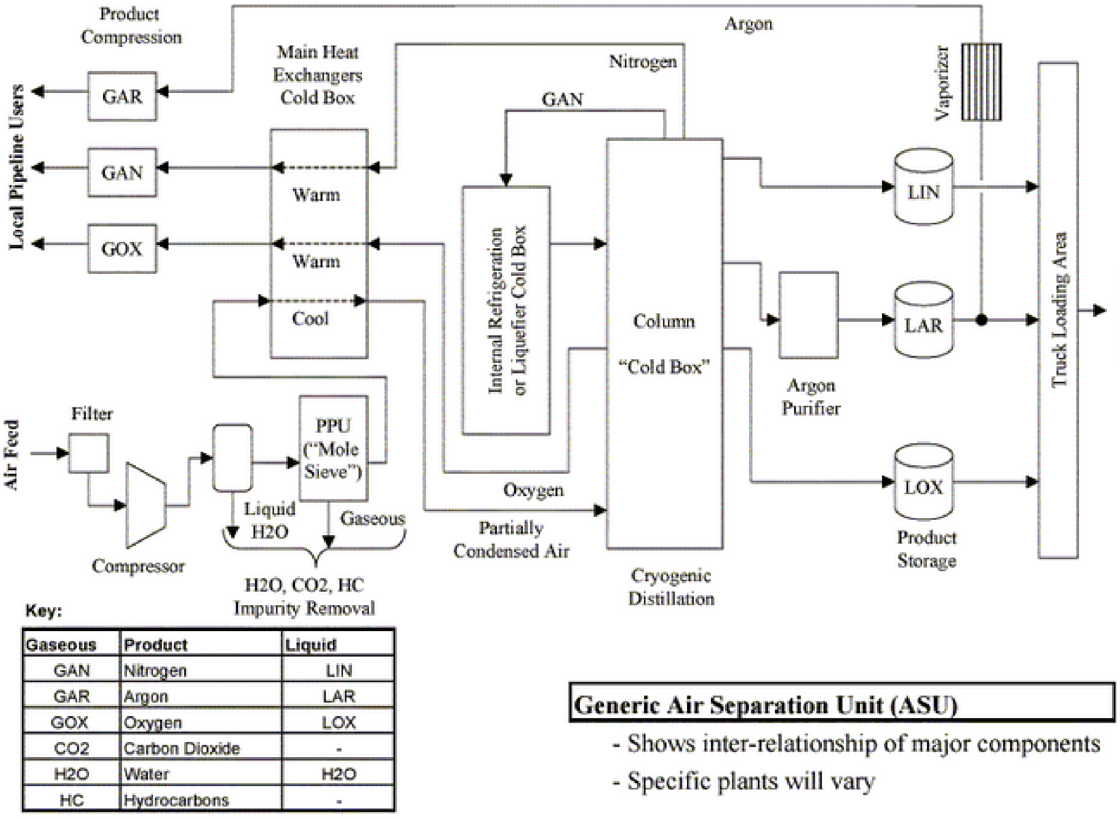
CO2 ONLY		CO2 ONLY	
CO2 (short) tons/ day (TPD)	1.1012	CO2 Metric tons/ day (t/d)	1
CO2 gallons/ day	260.0351	CO2 liters/ day	984.2040

Based on: 1 year = 365.25 days 1 year = 12 months (1 average month = 30.44 days = 730.5 hours)

Scf (standard cubic foot) gas measured at 1 atmosphere and 70°F.
Nm3 (normal cubic meter) gas measured at 1 atmosphere and 0°C.
Liquid measured at 1 atmosphere and boiling temperature.

Results may need to be adjusted if usage is not continuous throughout the month. Call UIG for help if needed.

Generic Air separation flow diagram [10]:

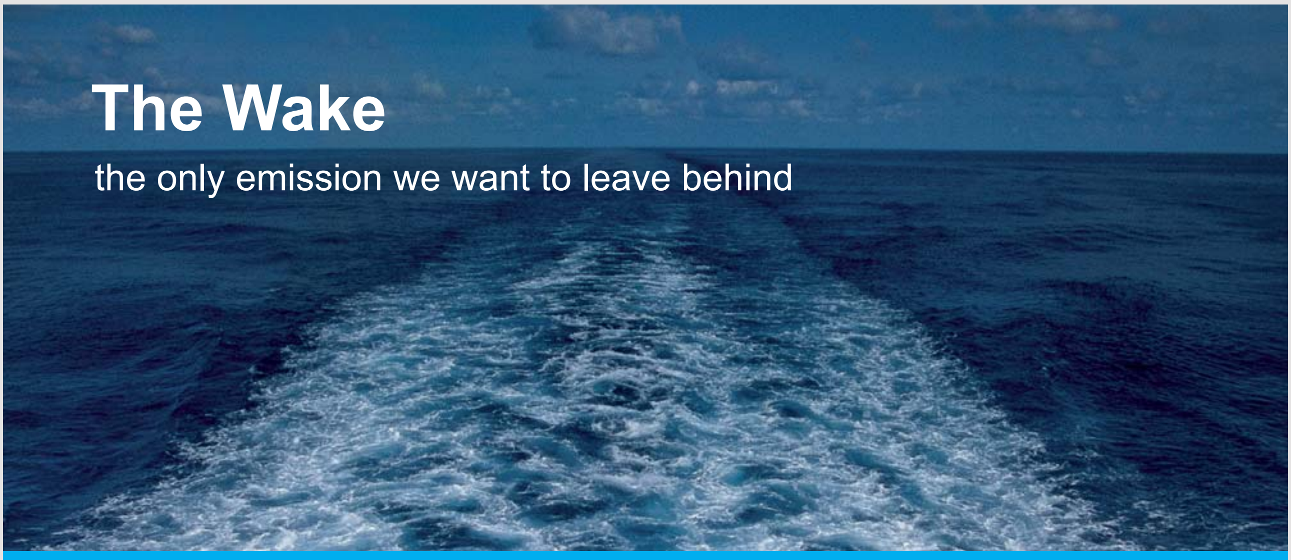


Key:

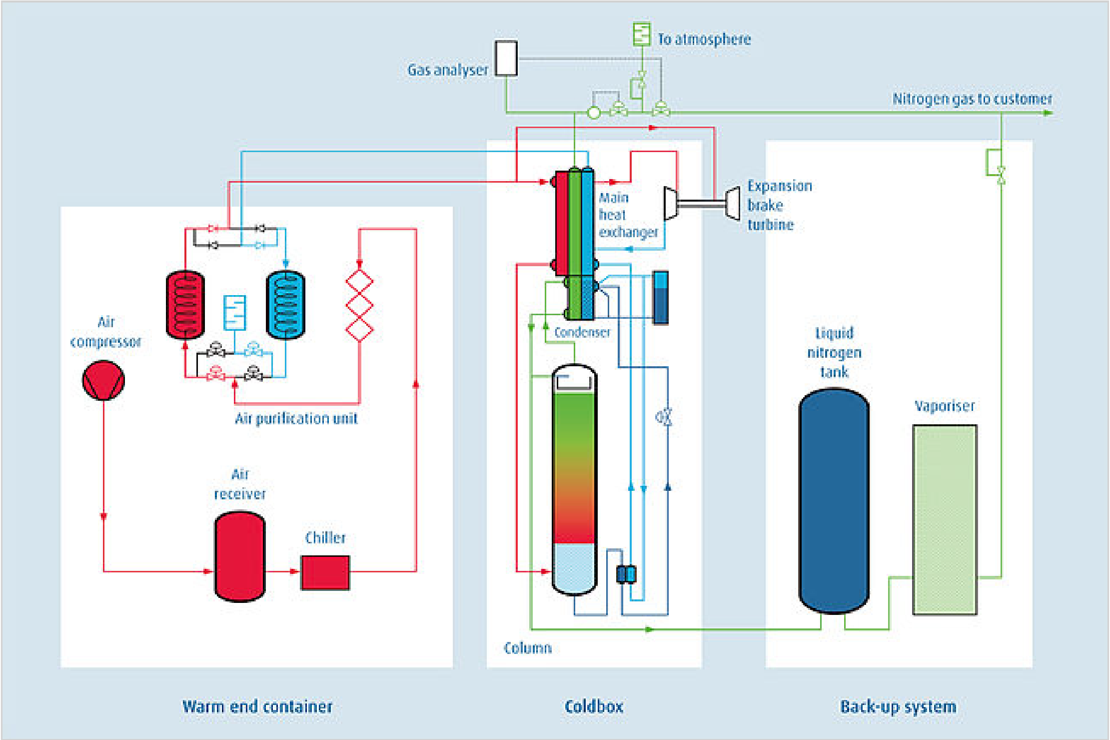
Gaseous	Product	Liquid
GAN	Nitrogen	LIN
GAR	Argon	LAR
GOX	Oxygen	LOX
CO2	Carbon Dioxide	-
H2O	Water	H2O
HC	Hydrocarbons	-

Generic Air Separation Unit (ASU)

- Shows inter-relationship of major components
- Specific plants will vary

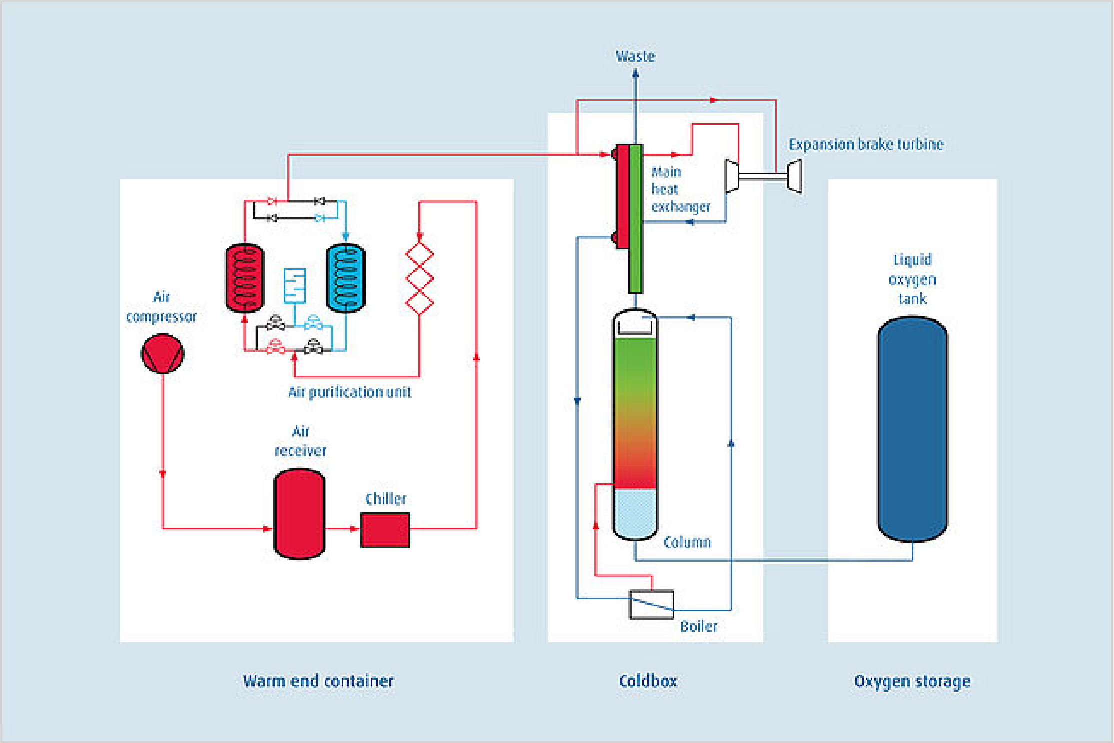


Cryogenic Nitrogen Plant [14]:



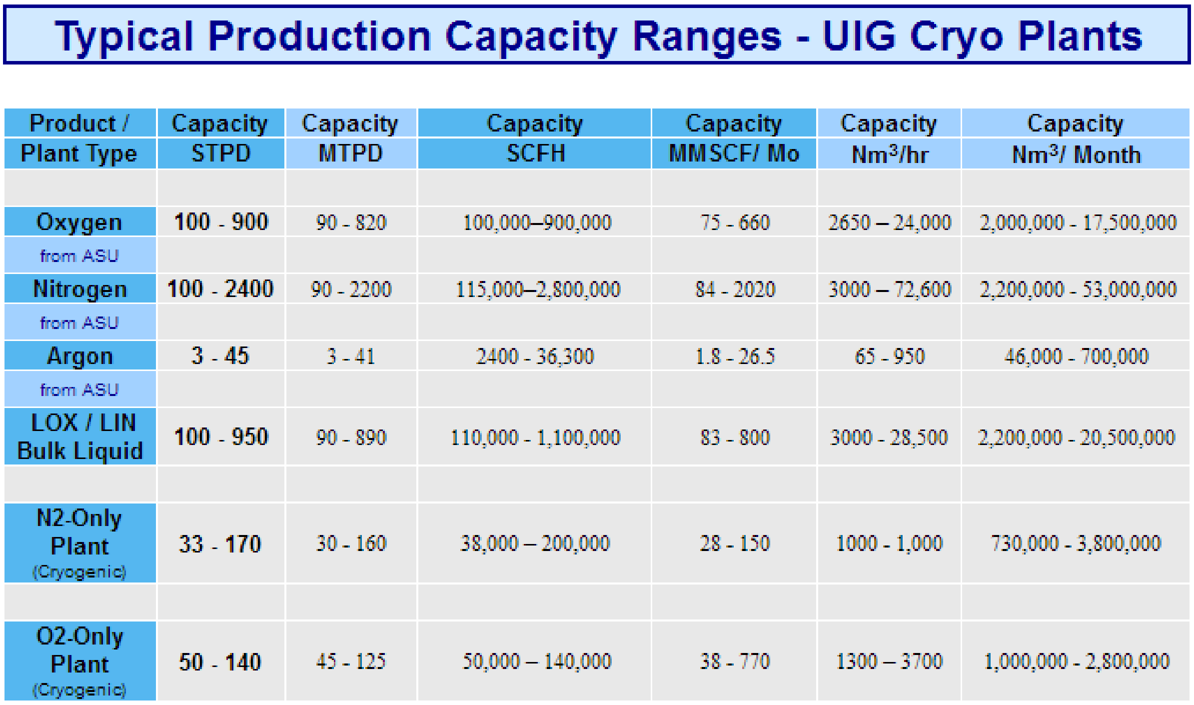
Flowsheet GAN Plant Linde Cryoplants Ltd.

Cryogenic Oxygen Plants [15]:



Flowsheet LOX Plant by Linde Cryoplants Ltd.

Specifications of some typical plants from Universal Industrial Gases, Inc. are shown below [10]:

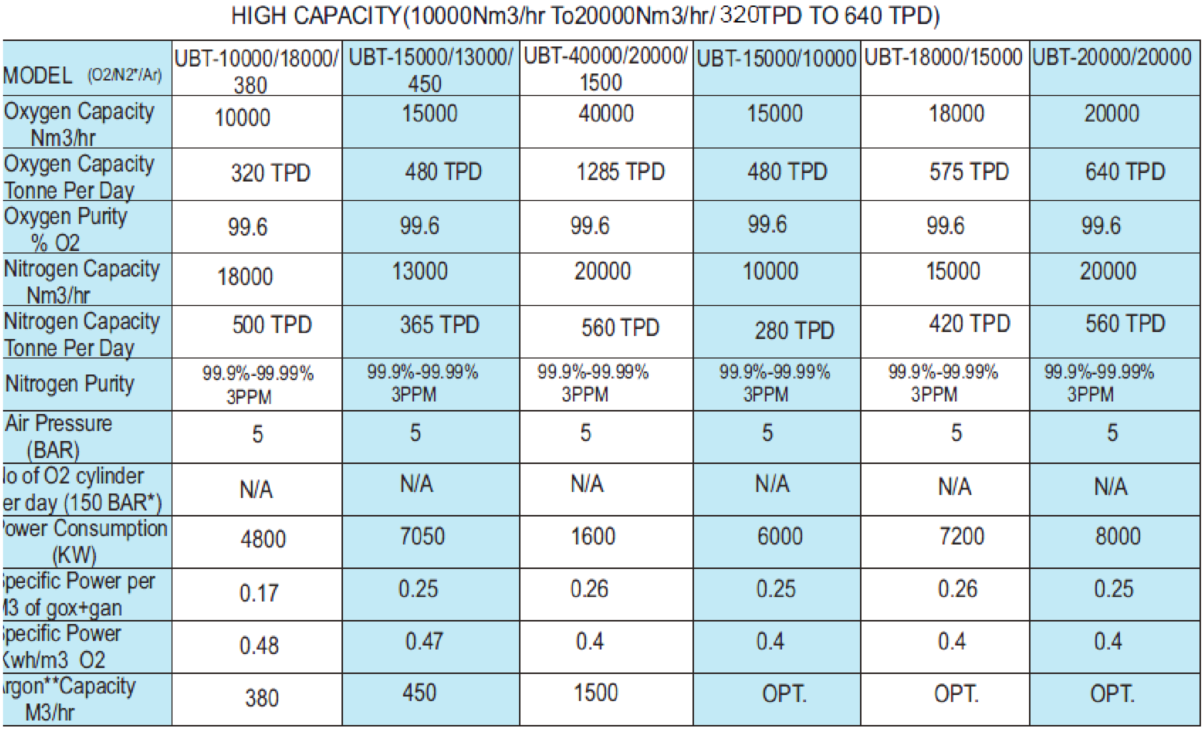


Typical Production Capacity Ranges - UIG Cryo Plants

Product / Plant Type	Capacity STPD	Capacity MTPD	Capacity SCFH	Capacity MMSCF/ Mo	Capacity Nm³/hr	Capacity Nm³/ Month
Oxygen	**100 - 900**	90 - 820	100,000–900,000	75 - 660	2650 – 24,000	2,000,000 - 17,500,000
from ASU						
Nitrogen	**100 - 2400**	90 - 2200	115,000–2,800,000	84 - 2020	3000 – 72,600	2,200,000 - 53,000,000
from ASU						
Argon	**3 - 45**	3 - 41	2400 - 36,300	1.8 - 26.5	65 - 950	46,000 - 700,000
from ASU						
LOX / LIN Bulk Liquid	**100 - 950**	90 - 890	110,000 - 1,100,000	83 - 800	3000 - 28,500	2,200,000 - 20,500,000
N2-Only Plant (Cryogenic)	**33 - 170**	30 - 160	38,000 – 200,000	28 - 150	1000 - 1,000	730,000 - 3,800,000
O2-Only Plant (Cryogenic)	**50 - 140**	45 - 125	50,000 – 140,000	38 - 770	1300 – 3700	1,000,000 - 2,800,000

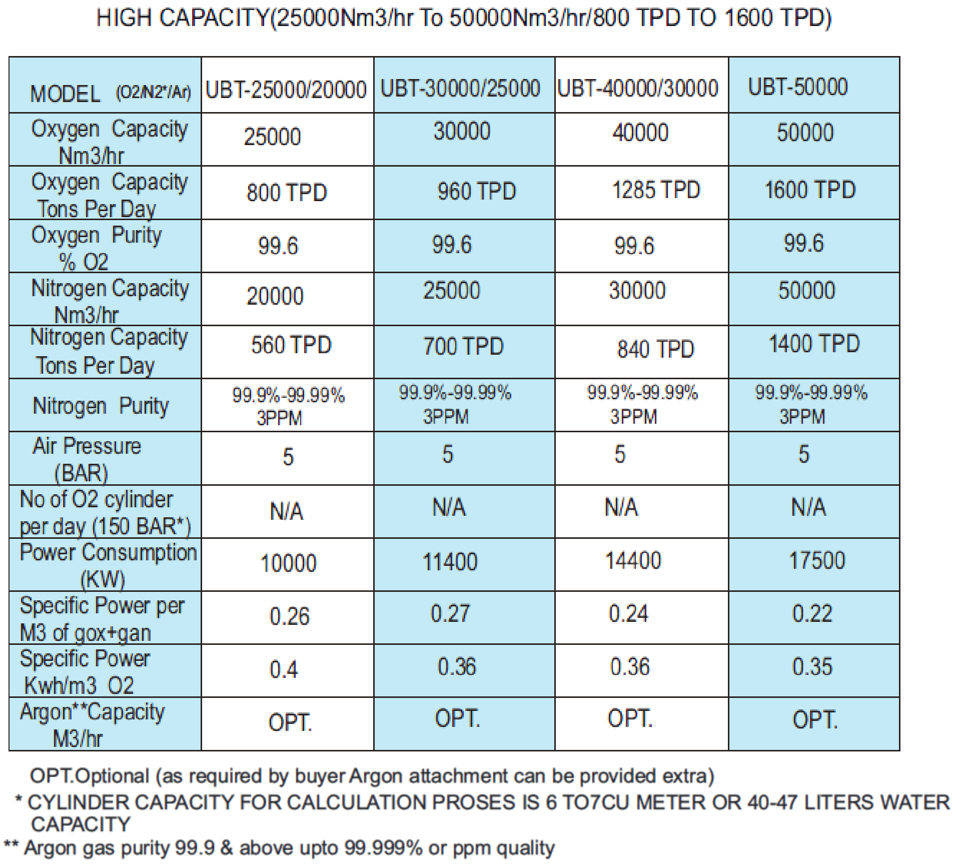
……

Specifications of some low pressure, large air separation plants (10,000 to 50,000 m^3/h of GOX and GAN) are given below [11]:



HIGH CAPACITY(10000Nm3/hr To20000Nm3/hr/ 320TPD TO 640 TPD)

MODEL (O2/N2*/Ar)	UBT-10000/18000/ 380	UBT-15000/13000/ 450	UBT-40000/20000/ 1500	UBT-15000/10000	UBT-18000/15000	UBT-20000/20000
Oxygen Capacity Nm3/hr	10000	15000	40000	15000	18000	20000
Oxygen Capacity Tonne Per Day	320 TPD	480 TPD	1285 TPD	480 TPD	575 TPD	640 TPD
Oxygen Purity % O2	99.6	99.6	99.6	99.6	99.6	99.6
Nitrogen Capacity Nm3/hr	18000	13000	20000	10000	15000	20000
Nitrogen Capacity Tonne Per Day	500 TPD	365 TPD	560 TPD	280 TPD	420 TPD	560 TPD
Nitrogen Purity	99.9%-99.99% 3PPM	99.9%-99.99% 3PPM	99.9%-99.99% 3PPM	99.9%-99.99% 3PPM	99.9%-99.99% 3PPM	99.9%-99.99% 3PPM
Air Pressure (BAR)	5	5	5	5	5	5
lo of O2 cylinder er day (150 BAR*)	N/A	N/A	N/A	N/A	N/A	N/A
'ower Consumption (KW)	4800	7050	1600	6000	7200	8000
;pecific Power per l3 of gox+gan	0.17	0.25	0.26	0.25	0.26	0.25
;pecific Power Kwh/m3 O2	0.48	0.47	0.4	0.4	0.4	0.4
rgon**Capacity M3/hr	380	450	1500	OPT.	OPT.	OPT.



HIGH CAPACITY(25000Nm3/hr To 50000Nm3/hr/800 TPD TO 1600 TPD)

MODEL (O2/N2*/Ar)	UBT-25000/20000	UBT-30000/25000	UBT-40000/30000	UBT-50000
Oxygen Capacity Nm3/hr	25000	30000	40000	50000
Oxygen Capacity Tons Per Day	800 TPD	960 TPD	1285 TPD	1600 TPD
Oxygen Purity % O2	99.6	99.6	99.6	99.6
Nitrogen Capacity Nm3/hr	20000	25000	30000	50000
Nitrogen Capacity Tons Per Day	560 TPD	700 TPD	840 TPD	1400 TPD
Nitrogen Purity	99.9%-99.99% 3PPM	99.9%-99.99% 3PPM	99.9%-99.99% 3PPM	99.9%-99.99% 3PPM
Air Pressure (BAR)	5	5	5	5
No of O2 cylinder per day (150 BAR*)	N/A	N/A	N/A	N/A
Power Consumption (KW)	10000	11400	14400	17500
Specific Power per M3 of gox+gan	0.26	0.27	0.24	0.22
Specific Power Kwh/m3 O2	0.4	0.36	0.36	0.35
Argon**Capacity M3/hr	OPT.	OPT.	OPT.	OPT.

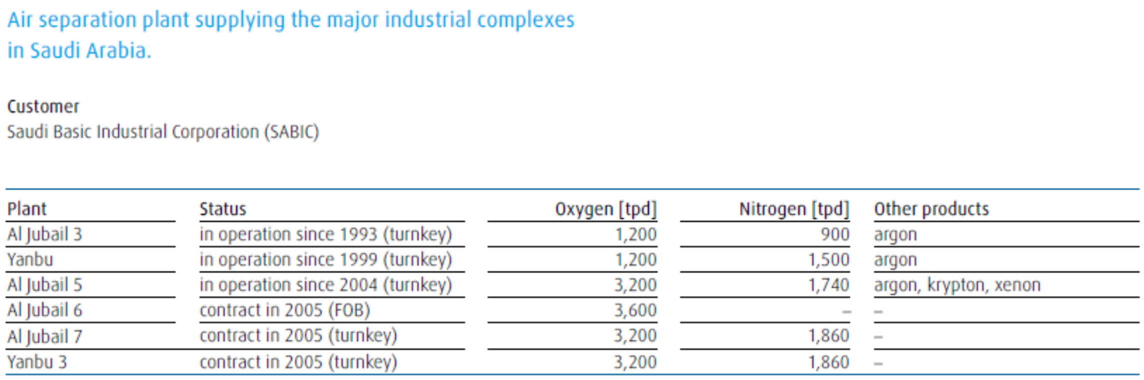
OPT.Optional (as required by buyer Argon attachment can be provided extra)

* CYLINDER CAPACITY FOR CALCULATION PROSES IS 6 TO7CU METER OR 40-47 LITERS WATER CAPACITY

** Argon gas purity 99.9 & above upto 99.999% or ppm quality

Tonnage Air separation plants from Linde Engineering [12]:

These are plants that are individually designed for the specific demands of customers. These plants can produce oxygen, nitrogen, argon, krypton, xenon, helium and neon. **Daily oxygen production capacities are between 450 tons (13,000 Nm³/h) and 7,000 tons (200,000 Nm³/h).**



Air separation plant supplying the major industrial complexes in Saudi Arabia.

Customer
Saudi Basic Industrial Corporation (SABIC)

Plant	Status	Oxygen [tpd]	Nitrogen [tpd]	Other products
Al Jubail 3	in operation since 1993 (turnkey)	1,200	900	argon
Yanbu	in operation since 1999 (turnkey)	1,200	1,500	argon
Al Jubail 5	in operation since 2004 (turnkey)	3,200	1,740	argon, krypton, xenon
Al Jubail 6	contract in 2005 (FOB)	3,600	–	–
Al Jubail 7	contract in 2005 (turnkey)	3,200	1,860	–
Yanbu 3	contract in 2005 (turnkey)	3,200	1,860	–

Largest multi-train separation plant in Mexico:

Specs: Capacity: 63,000 t/d of N2 (17,500 t/d oxygen equivalent), commissioned in 2000 for Pemex:

See the photo below:

For a large steel works in China:

Customer
Wuhan Iron and Steel Company

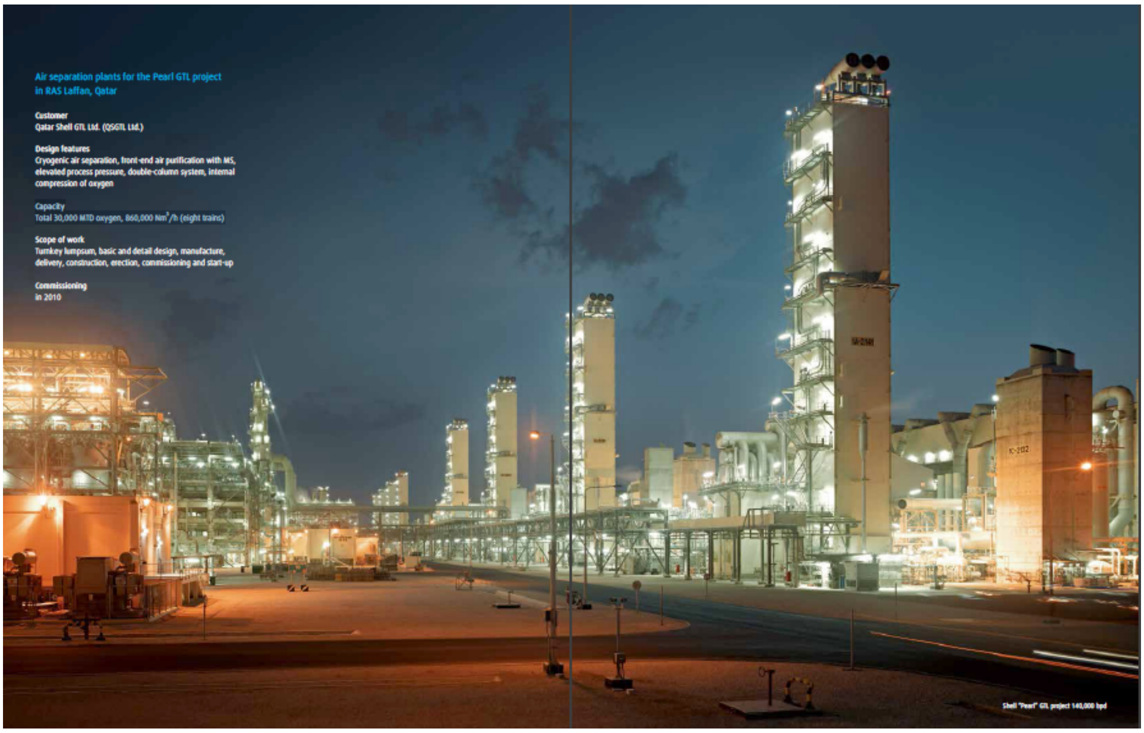
Plant	Status	Oxygen [tpd]	Nitrogen [tpd]	Other products
A/B	commissioned in 1975	690	600	–
C/D	commissioned in 1982	690	605	argon
E/F	commissioned in 1992	2,100	1,860	argon, krypton, xenon
G/H	commissioned in 2004	4,190	4,860	argon, krypton, xenon, helium, neon
I/J	commissioned in 2006	4,190	4,860	argon, krypton, xenon, helium, neon

See the photo below:

For a large steel works for the Pearl GTL project in RAS Laffan, Qatar:

Capacity: Total 30,000 MTD Oxygen, or, 860,000 Nm^3/h (eight trains), commissioned in 2010:

See photo below:



Air separation plants for the Pearl GTL project
in RAS Laffan, Qatar

Customer
Qatar Shell GTL Ltd. (QSGTL Ltd.)

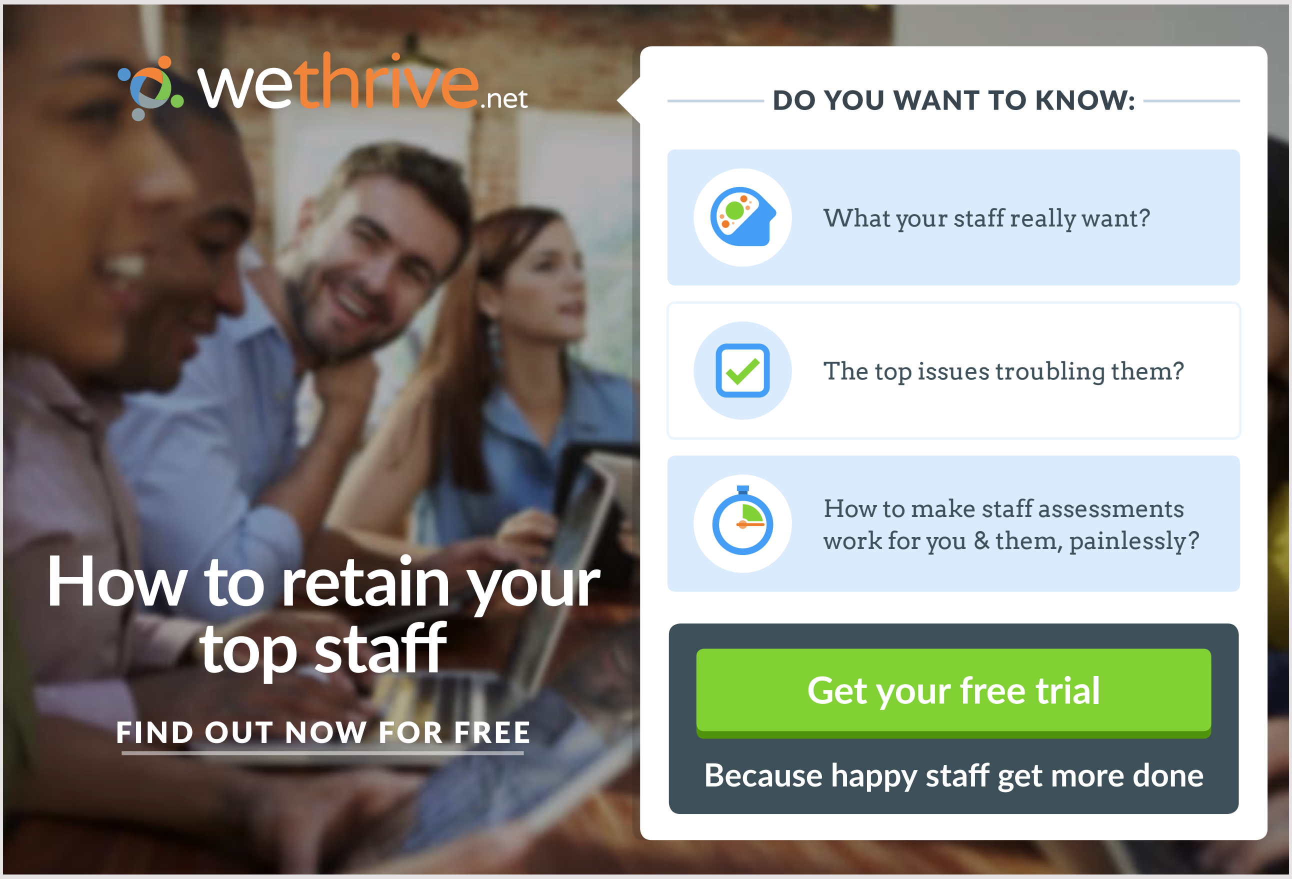
Design features
Cryogenic air separation, front-end air purification with MS, elevated process pressure, double-column system, internal compression of oxygen

Capacity
Total 30,000 MTD oxygen, 860,000 Nm^3/h (eight trains)

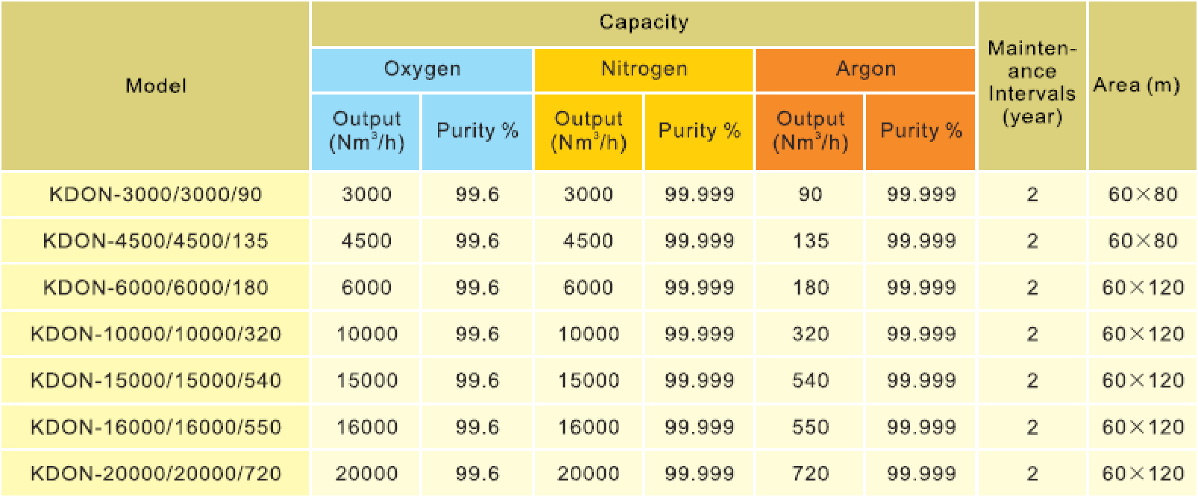
Scope of work
Turnkey lumpsum, basic and detail design, manufacture, delivery, construction, erection, commissioning and start-up

Commissioning
in 2010

Shell "Pearl" GTL project 140,000 bpd



Large and medium sized Air separation plants by Peiyang Chemical Eng. Co. (PCC), China [13]:

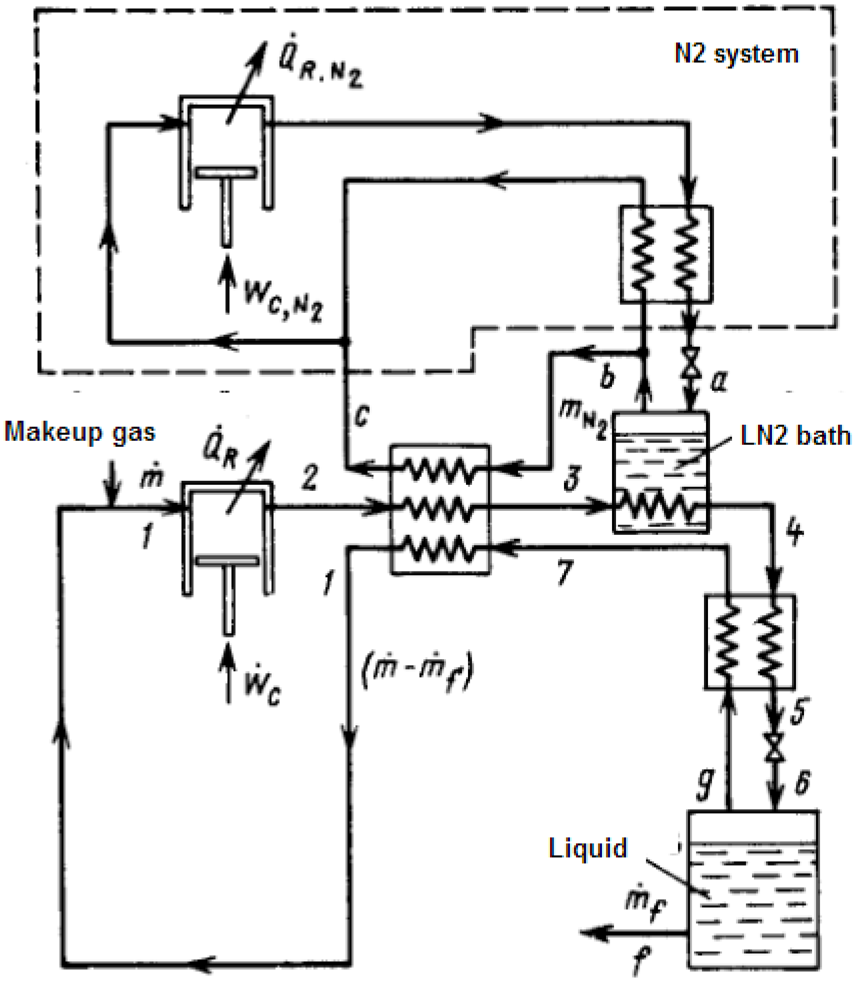


Model	Capacity						Mainten-ance Intervals (year)	Area (m)
	Oxygen		Nitrogen		Argon			
	Output (Nm^3/h)	Purity %	Output (Nm^3/h)	Purity %	Output (Nm^3/h)	Purity %		
KDON-3000/3000/90	3000	99.6	3000	99.999	90	99.999	2	60×80
KDON-4500/4500/135	4500	99.6	4500	99.999	135	99.999	2	60×80
KDON-6000/6000/180	6000	99.6	6000	99.999	180	99.999	2	60×120
KDON-10000/10000/320	10000	99.6	10000	99.999	320	99.999	2	60×120
KDON-15000/15000/540	15000	99.6	15000	99.999	540	99.999	2	60×120
KDON-16000/16000/550	16000	99.6	16000	99.999	550	99.999	2	60×120
KDON-20000/20000/720	20000	99.6	20000	99.999	720	99.999	2	60×120

3.1.12 PRE-COOLED LINDE-HAMPSON (L-H) SYSTEM FOR NEON AND HYDROGEN:

L-H systems are generally suitable for small scale liquefaction plants. Usually pre-cooled with liquid nitrogen to ensure that the gas is cooled much below the max. inversion temperature before it enters the Linde-Hampson part of the liquefier.

Schematic diagram of the system is shown below [1]:



LN2 precooled Linde-Hampson system for Neon or Hydrogen

In small systems, LN2 pre-cooling may be done using LN2 from a separate dewar.

Applying the I Law to the last part of L-H system for Hydrogen (i.e. J-T heat exchanger and the LH2 bath), we get the liquid yield:

$$y = \frac{h_7 - h_4}{h_7 - h_f}.$$

To find the LN2 requirements, apply the I Law to the whole system excepting the two compressors, and we get:

$$0=\dot{m}_{N_2}h_c+(\dot{m}-\dot{m}_f)h_1+\dot{m}_f h_f-\dot{m}_{N_2}h_a-\dot{m}h_2,$$

Now, define the nitrogen boil off rate per unit mass of H2 (or Neon):

$$z=\dot{m}_{N_2}/\dot{m},$$

Then, solving for z from previous two equations:

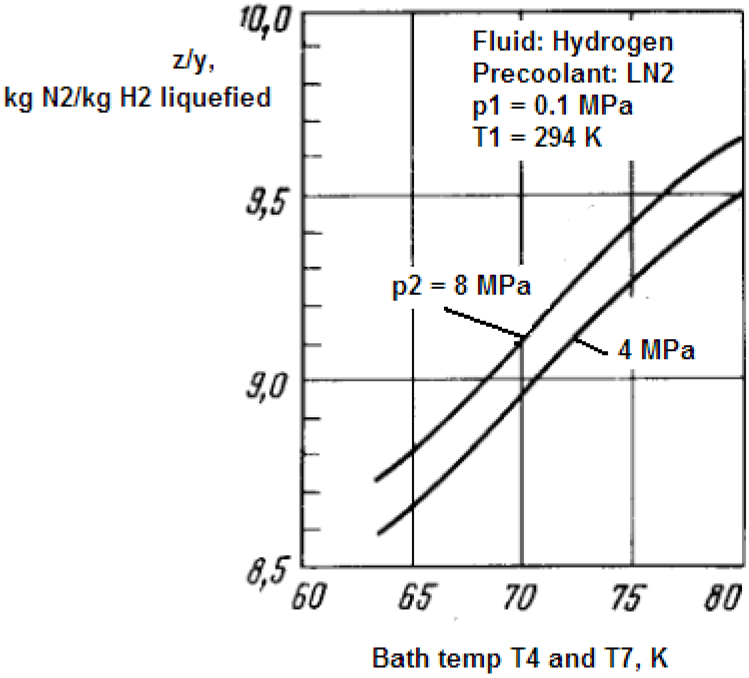
$$z=\frac{h_2-h_1}{h_c-h_a}+y\frac{h_1-h_f}{h_c-h_a}.$$

And, LN2 boiled away per unit mass of H2 (or Neon) is:

$$\frac{\dot{m}_{N_2}}{\dot{m}_f} = \frac{\dot{m}_{N_2}/\dot{m}}{\dot{m}_f/\dot{m}} = \frac{z}{y}.$$

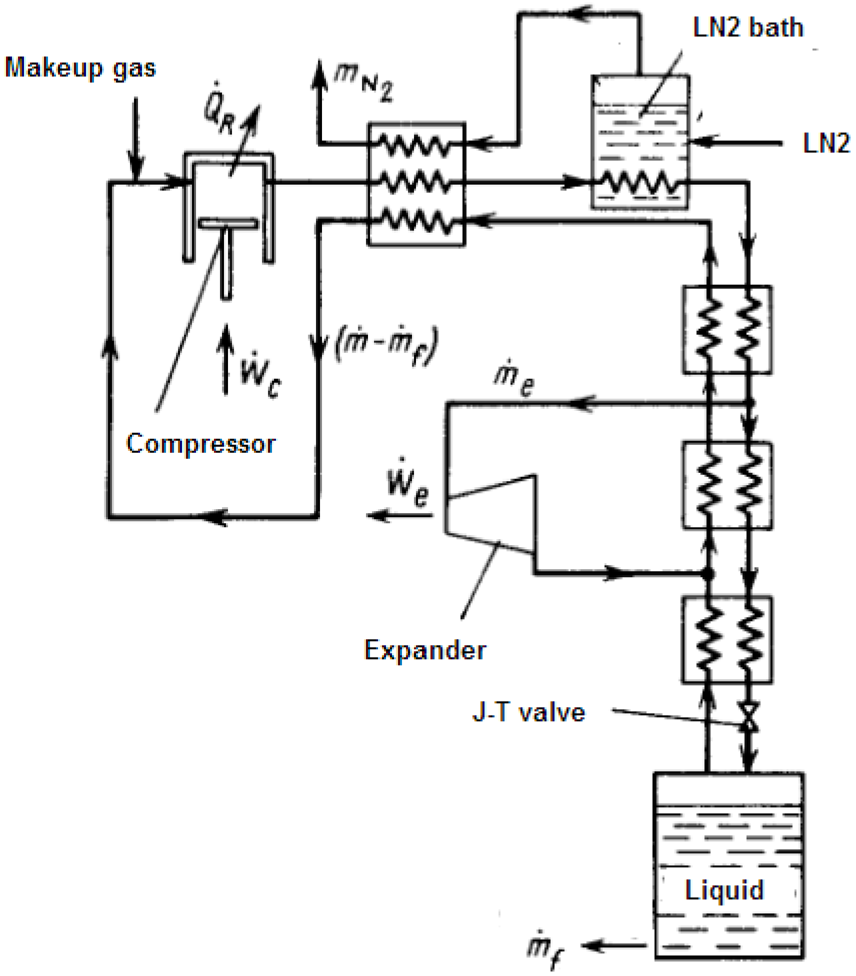
Observe that liquid yield can be improved by lowering temp T4. This is done by lowering the pressure in the LN2 bath. Practical limit to this is the freezing point of LN2 = 63.2 K

Following figure shows the (z/y) plotted against the pre-coolant bath temperature [1]:



3.1.13 CLAUDE SYSTEM FOR NEON AND HYDROGEN [1]:

Following is the schematic diagram of Claude system, with LN2 pre-cooling:

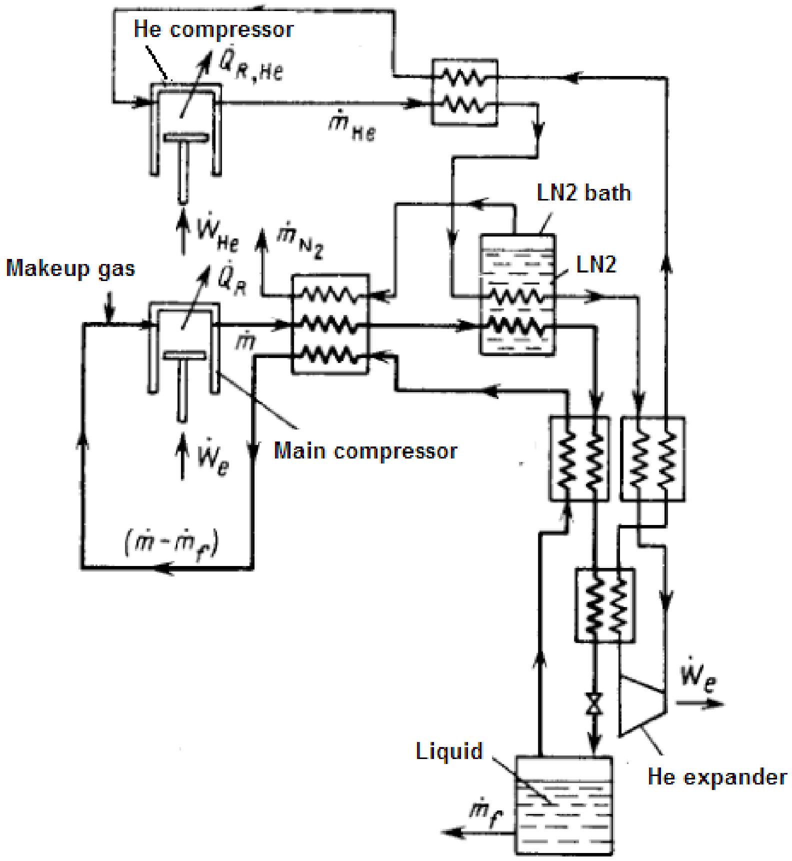


With pre-cooled Claude system for H2, FOM is 50% to 75% higher as compared to pre-cooled L-H system.

3.1.14 HELIUM REFRIGERATED SYSTEM FOR HYDROGEN LIQUEFACTION [1]:

The schematic diagram is shown below.

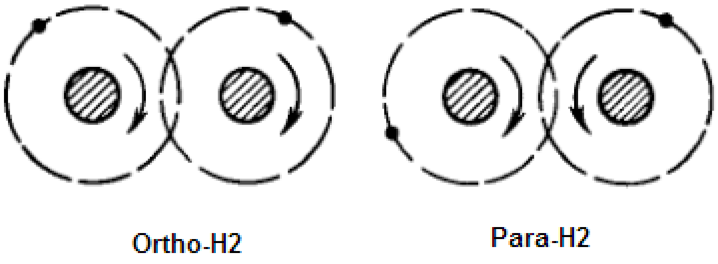
Here, a LN2 bath as well as an auxiliary helium refrigeration system are used for pre-cooling the hydrogen gas. As a result, lower pressures can be used in the hydrogen loop (3 to 8 atm). Helium gas pressure may be about 10 atm. Work requirement is about 60476 kJ/kg liquefied. FOM is about 0.11, including the work required to produce LN2.


He compressor
$\dot{Q}_{R,He}$
$\dot{m}_{He}$
LN2 bath
LN2
$\dot{W}_{He}$
$\dot{m}_{N_2}$
Makeup gas
$\dot{Q}_R$
$\dot{m}$
Main compressor
$\dot{W}_e$
$(\dot{m}-\dot{m}_f)$
$\dot{W}_e$
Liquid
He expander
$\dot{m}_f$

3.1.15 ORTHO-PARA- HYDROGEN CONVERSION IN THE LIQUEFIER [1]:

Hydrogen exists in two molecular forms: ortho-H2 and para-H2.

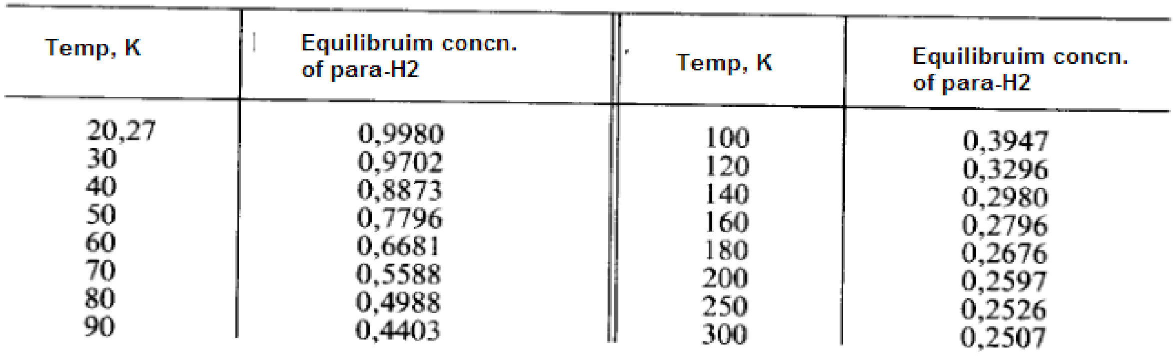
In ortho-H2, the two protons of the H2 molecule possess the spin in the same direction, and in para-H2, the spins are in opposite directions as shown below:



Mixture of o-H2 and p-H2 at high temps is called *normal hydrogen*, and it has 75% o-H2 and 25% p-H2, by volume.

Equilibrium mixture of o-H2 and p-H2 is called *equilibrium hydrogen* (e-H2). Concentration of o-H2 and p-H2 in equilibrium hydrogen depends on temp. At the normal boiling point of hydrogen, e-H2 has a composition of 0.21% o_H2 and 99.79% p-H2, i.e. practically all p-H2.

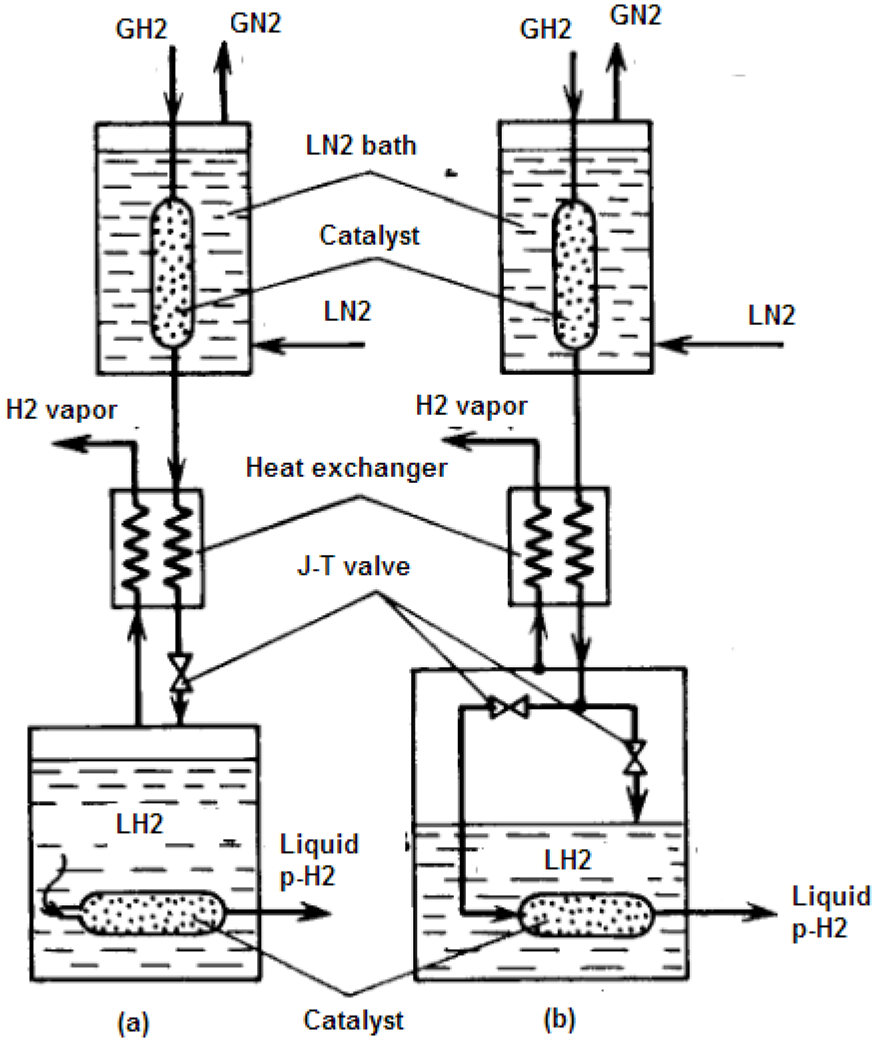
Concentration of p-H2 in equilibrium H2 at different temps is shown in following Table [1]:



Temp, K	Equilibruim concn. of para-H2	Temp, K	Equilibruim concn. of para-H2
20,27	0,9980	100	0,3947
30	0,9702	120	0,3296
40	0,8873	140	0,2980
50	0,7796	160	0,2796
60	0,6681	180	0,2676
70	0,5588	200	0,2597
80	0,4988	250	0,2526
90	0,4403	300	0,2507

Now, during the liquefaction of H2, temp falls from say, 300 K to 20.3 K, and the conversion of ortho-H2 to para-H2 does not occur instantaneously, but occurs in a slow process. However, if this conversion occurs in the LH2 storage dewar, there will be a very high boil-off loss, since the conversion of o-H2 to p-H2 is exothermic reaction. So, the method is to use a *catalyst* to speed up the process of conversion from o-H2 to p-H2 *during the liquefaction* itself.

Following is the schematic of the two possible arrangements [1]:

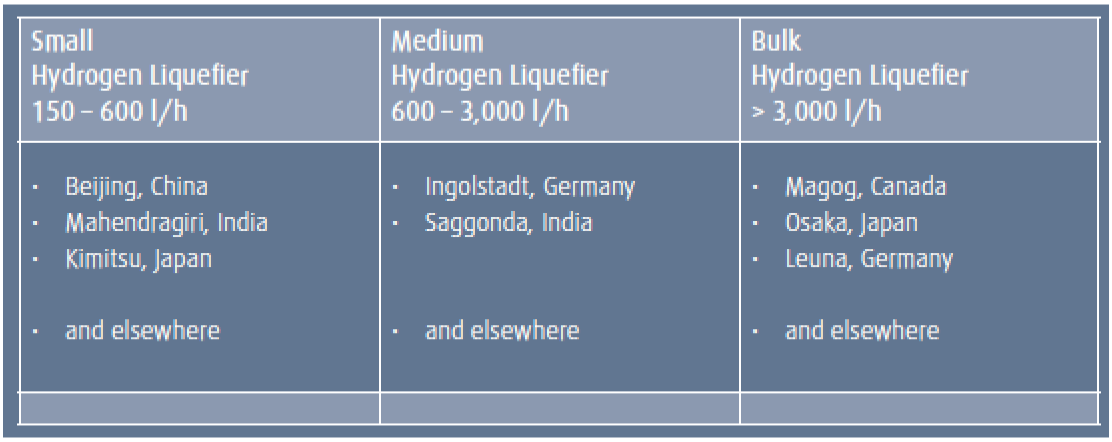


In the above, (a) is the single expansion valve arrangement and (b) is the double expansion valve arrangement. The second arrangement gives approximately 20% higher liquid hydrogen yields.

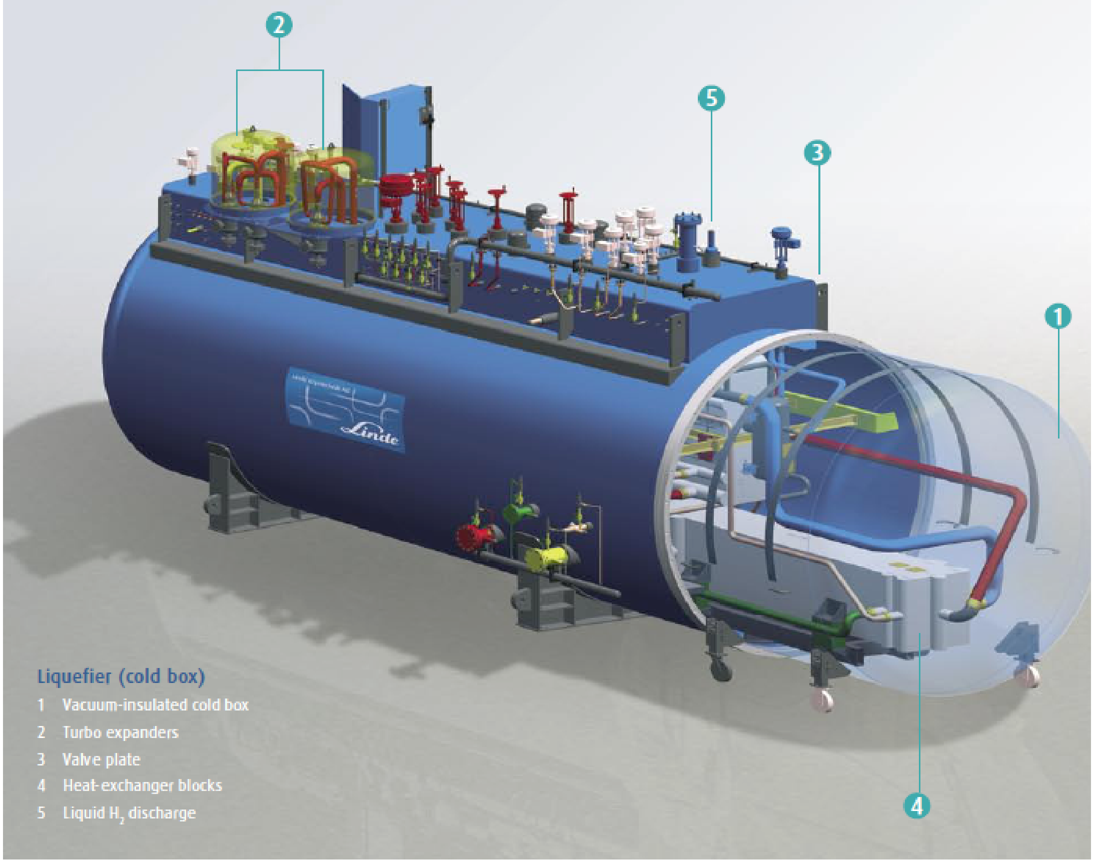
Catalysts used are: hydrous ferric oxide, chromic oxide on alumina particles, charcoal and silica gel, or nickel based catalysts. Of these, hydrous ferric oxide is the most effective.

3.1.16 EXAMPLE OF COMMERCIAL HYDROGEN LIQUEFIER [16]:

LH2 plants from Linde kryotechnik: Capacities of Linde Kryotechnik hydrogen liquefiers range from 150 l/h more than 20,000 l/h. See the Table below:



Small Hydrogen Liquefier 150 – 600 l/h	Medium Hydrogen Liquefier 600 – 3,000 l/h	Bulk Hydrogen Liquefier > 3,000 l/h
· Beijing, China · Mahendragiri, India · Kimitsu, Japan · and elsewhere	· Ingolstadt, Germany · Saggonda, India · and elsewhere	· Magog, Canada · Osaka, Japan · Leuna, Germany · and elsewhere



Liquefier (cold box)

1 Vacuum-insulated cold box
2 Turbo expanders
3 Valve plate
4 Heat-exchanger blocks
5 Liquid H_2 discharge

Linde pamphlet states: “For liquefaction capacities over 1000 l/h, the necessary refrigeration for cooling and ortho-para conversion of the hydrogen is attained by means of a hydrogen Claude process, i.e. at the cold end of the process the H2 feed gas can be combined with the H2 of the refrigeration process.”

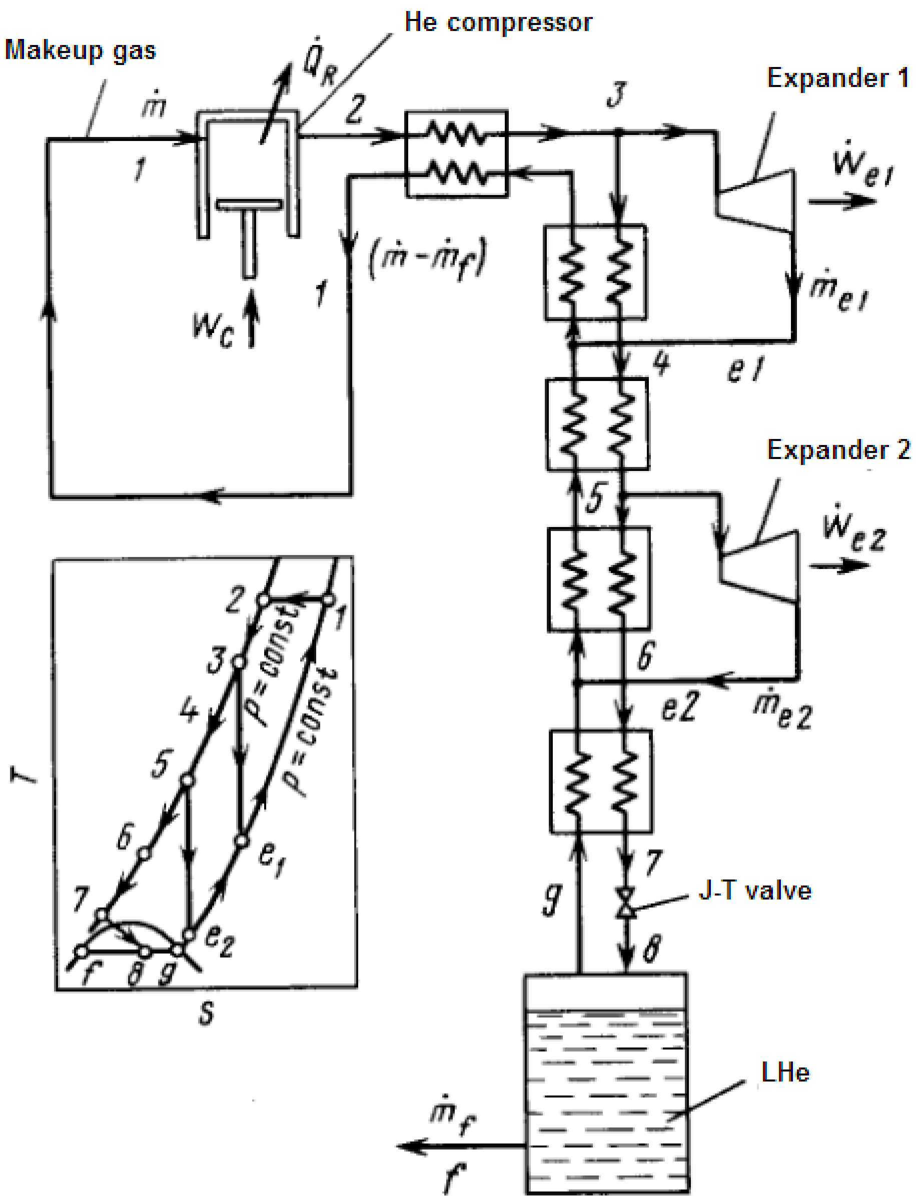
A photo of a 300 m^3 LH2 storage tank (from Linde) is shown below:

300 m^3 storage tank for LH_2

3.1.17 COLLINS SYSTEM FOR HELIUM LIQUEFACTION [1]:

This is a modification of Claude system. In Claude system, one expansion engine is used whereas in *Collins system two or more expansion engines are used.*

Following is the schematic diagram with two expansion engines:



Note: Above system does not use LN2 pre-cooling. With LN2 pre-cooling, LHe production will be increased.

Applying the I Law to all the components except the helium compressor, we get:

$$y=\frac{h_1-h_2}{h_1-h_f}+x_1\frac{\Delta h_{ei}}{h_1-h_f}+x_2\frac{\Delta h_{e2}}{h_1-h_f};$$

where,

$$y=\dot{m}_f/\dot{m}$$

$$x_1=\dot{m}_{e1}/\dot{m};\quad x_2=\dot{m}_{e2}/\dot{m};$$

Δhe1, Δh_e2, are the enthalpy drops in expanders 1 and 2 respectively.

$\dot{m}_{e1}$, $\dot{m}_{e2}$ are the mass flow rates of fluid through expanders 1 and 2 respectively.

Typical values are: p2 = p3 = 14 atm,

For expander 1: x1 = 0.25, inlet temp = 60 K, exit temp = 28.9 K,

For expander 2: x2 = 0.5, inlet temp = 15 K, exit temp = 7.8 K, and

Liquid yield = y = 0.12

With the use of LN2 pre-cooling, liquid yield can be almost tripled.

3.1.18 COMMERCIAL LIQUID HELIUM PLANTS [1]:

As examples, plants produced by Linde Kryotechnik are given below:

Their standard L series covers liquefiers up to a capacity of 290 l/h (equivalent to a refrigeration capacity of 900 W at 4.4 K), and higher capacity liquefiers are custom made for research.

For the L series, following are the specifications:

Standard Scope of Supply:

The standard helium liquefier/refrigerator comprises of:

- Vacuum insulated cold box, either with integrated automatic purifier (L280) or transfer line connection to/from the cryostat (LR280)
- Control cabinet with operator panel, removable from the cold box
- Aluminium plate-fin heat exchangers with LN2 pre-cooling facility
- Two TED dynamic gas bearing turbo expanders
- Oil injected recycle compressor, air or water cooled
- Oil removal system/gas management panel
- Coaxial transfer line from liquefier to dewar (only L280)

Options:

- Pure helium gas buffer
- Line drier
- LHe storage dewar and decant line
- Standard installation kit
- Recovery system
- Spare parts
- Maintenance contract

L280 Liquefaction Performance at ≤ 4.4 K

without LN_2 pre-cooling	with LN_2 pre-cooling	Compressor/Power rating
100 l/h	200 l/h	ESD301/160 kW
112 l/h	225 l/h	ESD351/200 kW
145 l/h	290 l/h	ESD441/250 kW

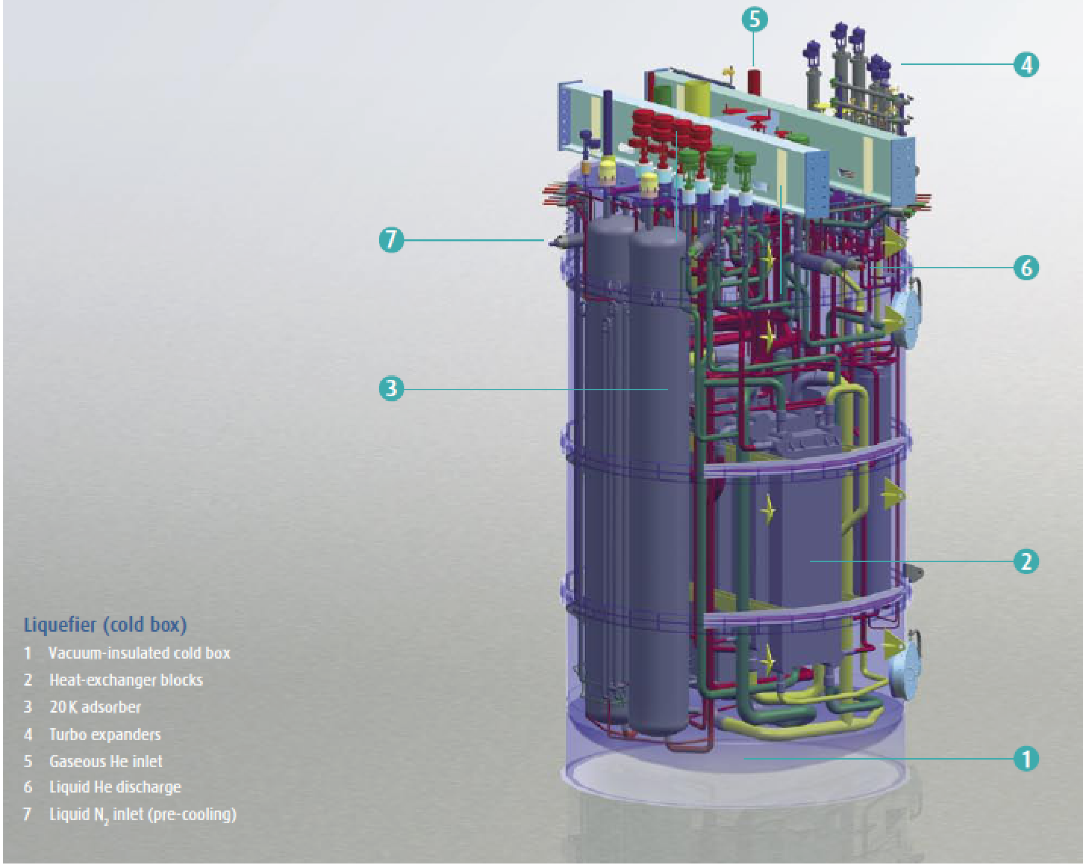
LR280 Refrigeration Performance at ≤ 4.4 K

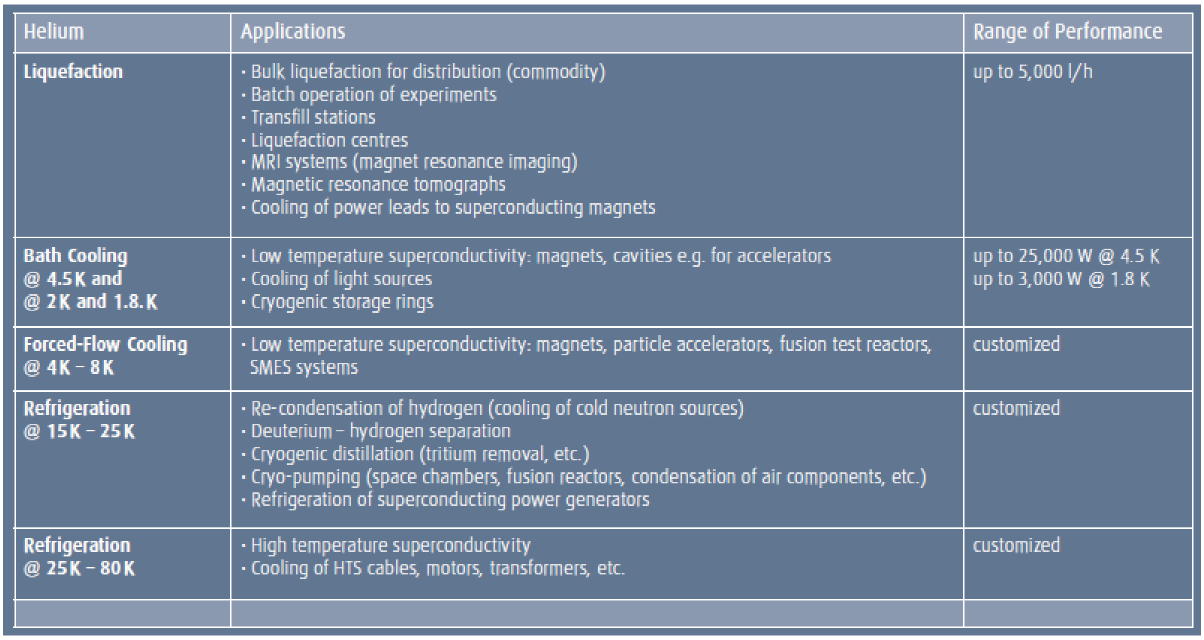
without LN_2 pre-cooling	with LN_2 pre-cooling	Compressor/Power rating
445 Watt	560 Watt	ESD301/160 kW
510 Watt	640 Watt	ESD351/200 kW
640 Watt	900 Watt	ESD441/250 kW

L280/LR280 Main Dimensions

Description	L x W x H [m]	Weight [kg]
Cold box	2.0 x 1.6 x 2.6	2500
Control cabinet	0.8 x 0.4 x 1.9	110
Compressor ESD type	2.7 x 2.2 x 2.2	5400
Oil removal system & Gas management panel	1.6 x 1.3 x 2.4	860 (including filling)

For Research, Science and Industry, Linde Kryotechnik supply the following He liquefiers/ Refrigerators:





Helium	Applications	Range of Performance
Liquefaction	· Bulk liquefaction for distribution (commodity) · Batch operation of experiments · Transfill stations · Liquefaction centres · MRI systems (magnet resonance imaging) · Magnetic resonance tomographs · Cooling of power leads to superconducting magnets	up to 5,000 l/h
Bath Cooling @ 4.5 K and @ 2 K and 1.8 K	· Low temperature superconductivity: magnets, cavities e.g. for accelerators · Cooling of light sources · Cryogenic storage rings	up to 25,000 W @ 4.5 K up to 3,000 W @ 1.8 K
Forced-Flow Cooling @ 4 K – 8 K	· Low temperature superconductivity: magnets, particle accelerators, fusion test reactors, SMES systems	customized
Refrigeration @ 15 K – 25 K	· Re-condensation of hydrogen (cooling of cold neutron sources) · Deuterium – hydrogen separation · Cryogenic distillation (tritium removal, etc.) · Cryo-pumping (space chambers, fusion reactors, condensation of air components, etc.) · Refrigeration of superconducting power generators	customized
Refrigeration @ 25 K – 80 K	· High temperature superconductivity · Cooling of HTS cables, motors, transformers, etc.	customized

3.1.19 HEAT EXCHANGERS AND HEAT TRANSFER CORRELATIONS [1]:

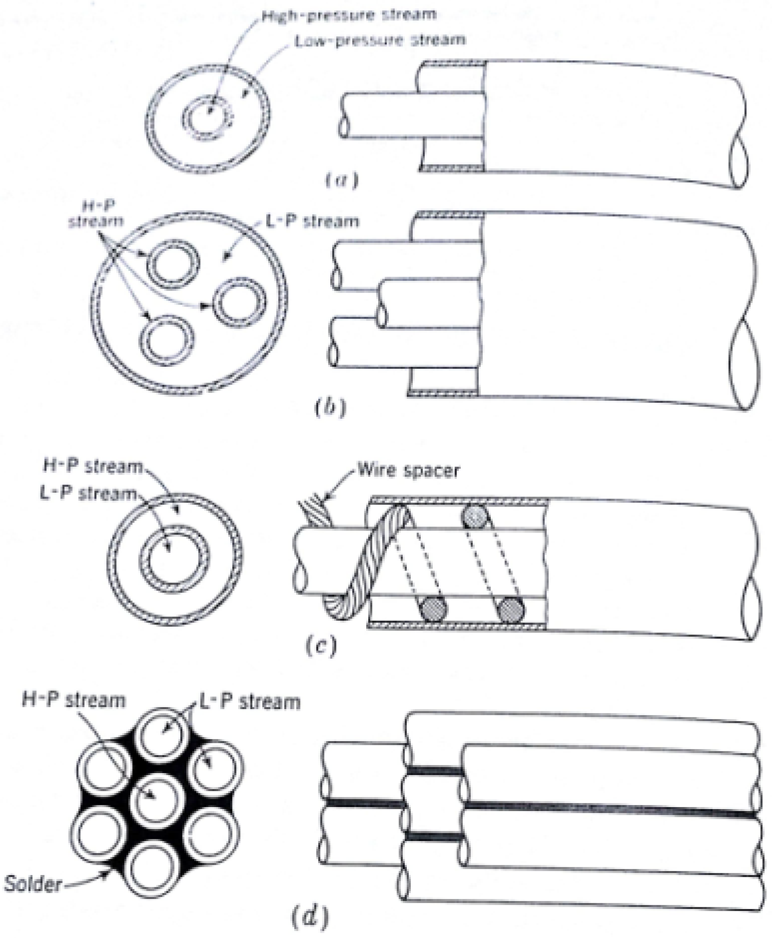


Fig. Simple heat-exchanger configurations. (*a*) Linde concentric-tube exchanger; (*b*) Linde multiple-tube exchanger; (*c*) Linde concentric-tube exchanger with a wire spacer; (*d*) bundle heat exchanger.

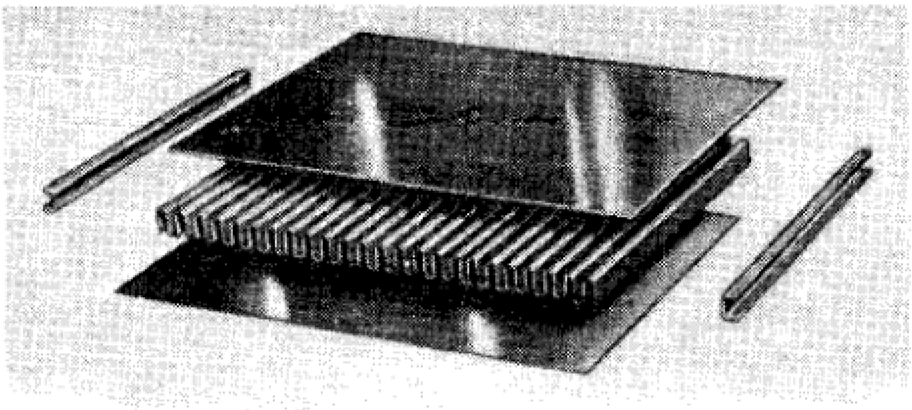


Fig. Plate-fin exchanger

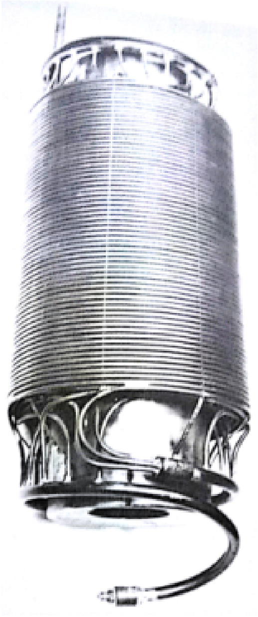


Fig. Giaque-Hampson heat exchanger,
with the outer shell and insulation removed.

Collin's heat exchanger:

Here, the exchanger consists of several concentric copper tubeswith an edge-wound copper helix wrapped in the annular spaces of the tubes. The helix is soft soldered to both sides of the annular space. The helix acts as a fin to extend the heat transfer surface of the annulus. The Collins HX shown below has four concentric tubes with the copper helix wound around three of the tubes. High pressure stream flows in the inner passages while the return, low pressure stream flows in the outer passages.

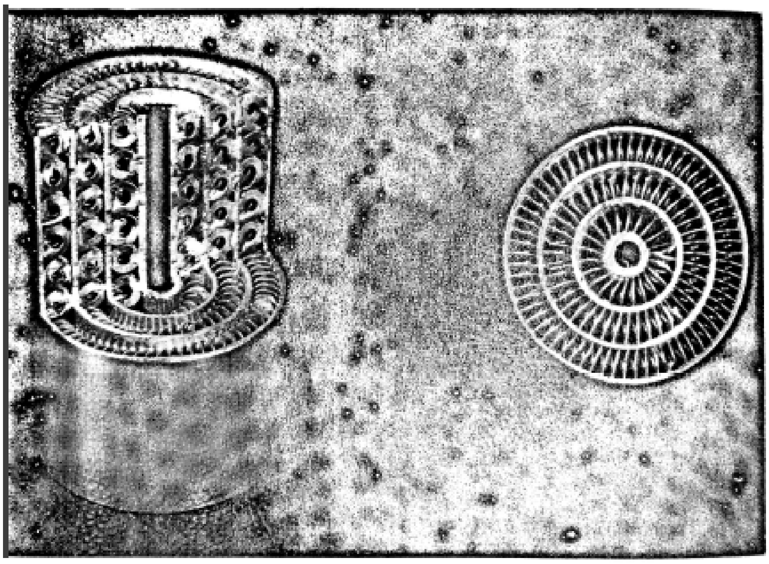


Fig. Collins heat exchanger. (*Joy Manufacturing Company.*)

Important dimensionless numbers:

Prandtl Number:

$$N_{\mathrm{Pr}} = \mu c_p / k_t$$

where μ is fluid viscosity, cp is fluid sp. heat, and k_t is the fluid thermal conductivity.

Colburn J factor:

$$j_H = (h_c / G c_p) N_{\mathrm{Pr}}^{2/3}$$

where h_c is the film heat transfer coeff, G is the mass velocity, given by mass flow rate per unit cross-sectional flow area, i.e.

$$G = \dot{m} / A_{ff}$$

where $A_{ff} = \pi * D^2 / 4$ for flow in a circular tube.

Nusselt Number:

$$N_{\mathrm{Nu}}=h_c D_e/k_t$$

where D_e is the equivalent diameter of flow passage = inside diameter D for a circular tube.

Reynolds Number:

$$N_{\mathrm{Re}}=D_e G/\mu$$

Friction factor:

$$f=(\Delta p/L)(G^2/(2g_c\rho D_e))$$

where ρ is fluid density, g_c conversion factor in Newton's second law of motion.

Correlations for flow inside circular tubes:

For Laminar flow, NRe < 2300:

$$N_{\mathrm{Nu}}=3{,}658+\frac{0{,}0668\,(D_e/L)\,N_{\mathrm{Re}}N_{\mathrm{Pr}}}{1+0{,}04\,[(D_e/L)\,N_{\mathrm{Re}}N_{\mathrm{Pr}}]^{2/3}};$$

where D_e = D, and fluid properties evaluated at bulk or mixed-mean temp.

For Turbulent flow, NRe > 3000:

$$j_H=0{,}023N_{\mathrm{Re}}^{0{,}2}\,[1+3{,}5D_e/D_h];$$

where D_h is the diameter of the helix for tubes wound as in Giauque-Hampson exchanger (for straight tubes, D_h = ∞), and fluid properties evaluated at mean film temp $T_m = (T_b + T_w) / 2$ where T_b is the bulk fluid temp and T_w is the wall temp.

Friction factor correlations:

For flow inside smooth tubes:

For Laminar flow, NRe < 2300:

$$f=64/N_{\mathbf{Re}}$$

For Turbulent flow, 2300 < NRe < 5000:

$$f=0{,}316N_{\mathbf{Re}}^{-0{,}25}$$

For Turbulent flow, N_{Re} > 5000:

$$f=0{,}184\mathrm{N}_{\mathbf{Re}}^{-0{,}20}.$$

For flow inside tubes of non-circular sections:

Above equations of circular sections can be used, but with D = D_e, the equivalent dia.

For heat transfer, D_e is given by:

$$D_e=4A_{ff}L/A.$$

where A_{ff} is the area of cross-section, A is area through which heat is being transferred.

For ex. for a square tube heated on all the 4 sides:

$$D_e=4a^2L/(4aL)=a$$

where a is the length of one side.

For friction factor correlations:

$$D_e=4A_{ff}L/A_W$$

where Aw is the wall area wetted by the fluid.

Ex. for an annulus:

$$D_e = 4(1/(4\pi))(D_2^2 - D_1^2)L/\pi(D_2 + D_1)L = D_2 - D_1$$

where D_1 and D_2 are the inner and outer diameters of the annulus respectively.

For flow normal to the bank of tubes (as in the low pressure stream of Giauque-Hampson HX), following correlations apply:

For banks of staggered tubes, 2000 < N_{Re} < 3.2 * 10^4:

$$j_H = 0{,}33 N_{Re}^{-0,4}$$

with the Reynolds No. defined as:

$$N_{Re} = D_0 G_{max} / \mu$$

Where D_0 is the outside diameter of the tubes, G_{max} = mdot / A_{min}; A_{min} is the minimum flow area between the tubes. Fluid properties are evaluated at the mean film temp, $T_m = (T_b + T_w) / 2$.

For banks of tubes in line (as in the low pressure stream of Giauque-Hampson HX), 2000 < N_{Re} < 3.2 * 10^4:

$$j_H = 0{,}26 N_{Re}^{-0{,}4}$$

with the Reynolds No. defined as for the previous eqn.

For flow across tubes, friction factor is defined as follows:
For flow outside tubes:

$$f' = \frac{\Delta p / N}{G_{max}^2 / (2 g_c \rho)},$$

where N is the total no. of tubes in line across which the fluid flows.

For staggered tubes, N_{Re} = De. Gmax /μ > 1000:

$$f' = \left[1 + 0{,}470 (X_T - 1)^{-1{,}08}\right] N_{Re}^{-0{,}16}$$

where X_T is the transverse pitch/tube outside dia.

For inline tubes, N_{Re} = De. G_{max}/μ > 1000:

$$f' = \left[0{,}176 + 0{,}32 (X_L - 1)^{-n}\right] N_{Re}^{-0{,}15}$$

where X_L is the longitudinal pitch/tube outside dia. and,

$$n = 0{,}43 + 1{,}13 / X_L.$$

Definition of longitudinal and transverse pitch is illustrated in the following figure:

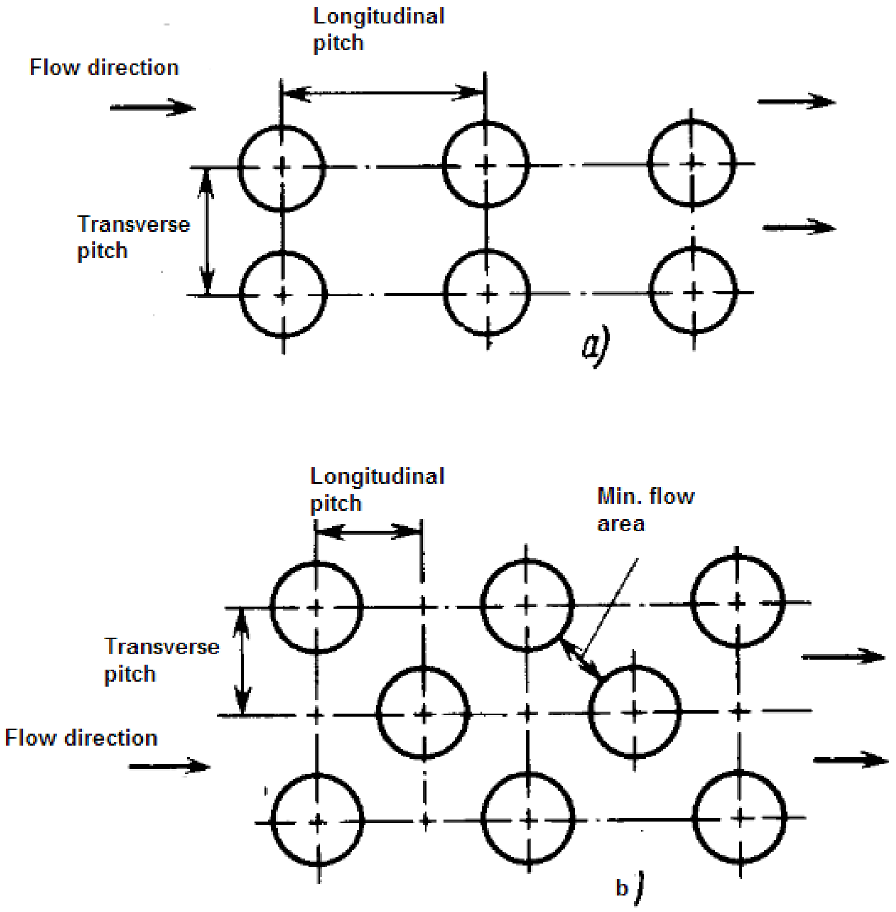


Fig. Definition of longitudinal and transverse pitch (*a*) for tubes in line and (*b*) for staggered-tube arrangement.

For Collins Heat Exchanger surface:

For heat transfer:

For $400 < N_{Re} < 10^4$:

$$j_H = 0{,}118 N_{Re}^{-0{,}3}$$

where equivalent dia, $D_e = 4 *$ (free volume) / (wetted area).

Free volume is the volume of the annulus with the volume of fins excluded, and the wetted area is the entire internal area of the annulus, including the surface area of the fins.

For friction factor:

For 400 < N_{Re} < 10^4:

$$f = 1{,}904 N_{Re}^{-0,2}$$

where equivalent dia is the same as for heat transfer correlation.

Compact Heat exchangers [24]:

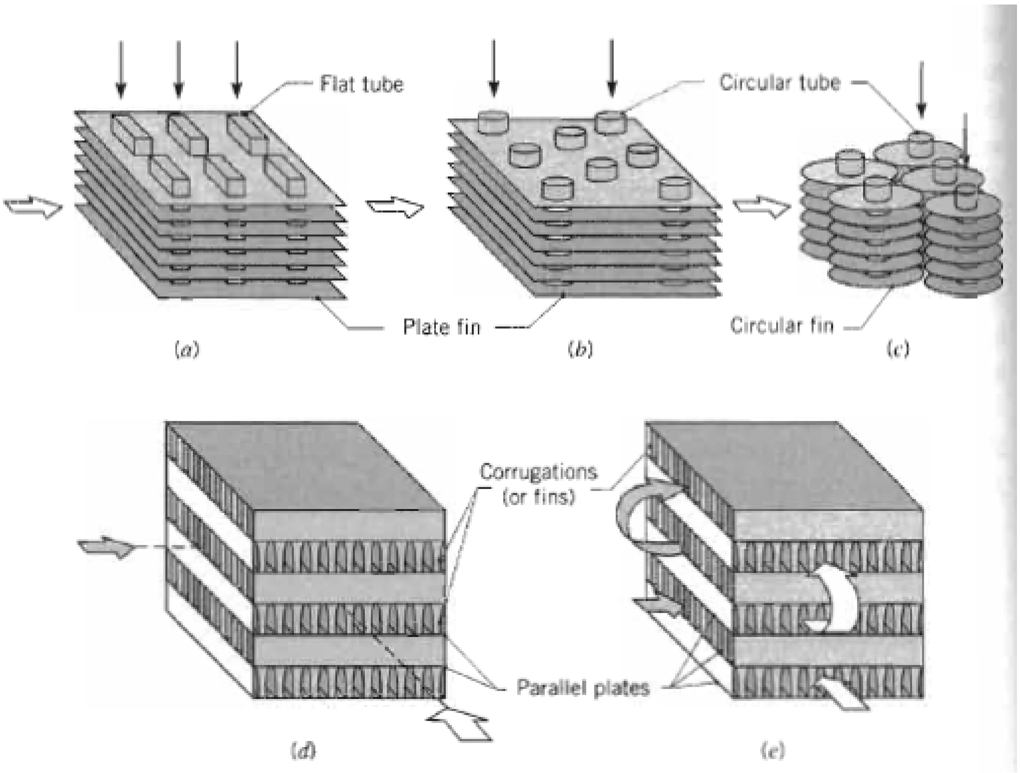


FIG · Compact heat exchanger cores. (*a*) Fin–tube (flat tubes, continuous plate fins). (*b*) Fin–tube (circular tubes, continuous plate fins). (*c*) Fin–tube (circular tubes, circular fins). (*d*) Plate–fin (single pass). (*e*) Plate–fin (multipass).

Plate-fin exchangers:

Heat transfer and friction characteristics are specific to a given plate-fin heat exchanger, and are determined experimentally.

Data for a large number of compact heat exchangers (such as: plate-fin surfaces, tube-fin surfaces, dimpled, flattened tube banks etc.) are presented by Kays and London [22].

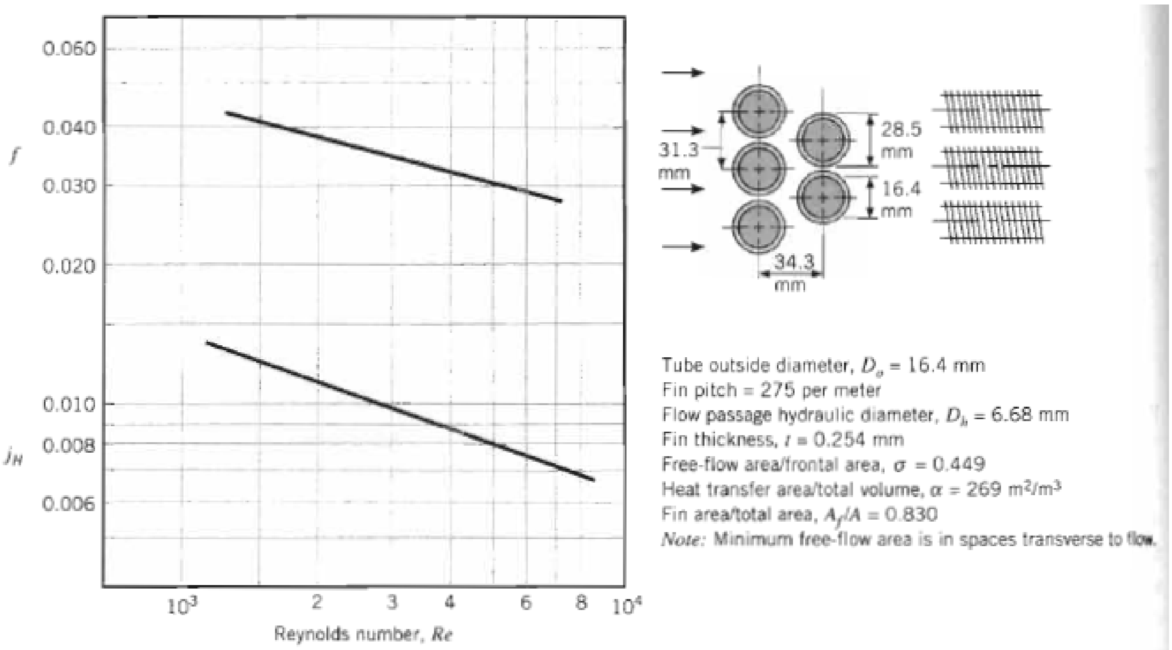
As an example, typical correlations for heat transfer and friction calculations for plate fin surfaces with straight fins, 0.31 in high, 0.006 in thick, 12.5 fins per inch, and

for 500 < N_{Re} < 10^4, are given below:

$$j_H = 0{,}0291 N_{Re}^{-0{,}24};$$
$$f = 0{,}0198 + 8{,}16 N_{Re}^{-1{,}033}$$

j_H and f are presented in the form of plots against N_{Re}.

Typical plots for two types of compact heat exchanger surfaces are shown below:



Tube outside diameter, D_o = 16.4 mm
Fin pitch = 275 per meter
Flow passage hydraulic diameter, D_h = 6.68 mm
Fin thickness, t = 0.254 mm
Free-flow area/frontal area, σ = 0.449
Heat transfer area/total volume, α = 269 m^2/m^3
Fin area/total area, A_f/A = 0.830
Note: Minimum free-flow area is in spaces transverse to flow.

FIG. · Heat transfer and friction factor for a circular tube–circular fin heat exchanger, surface CF-7.0-5/8J from Kays and London

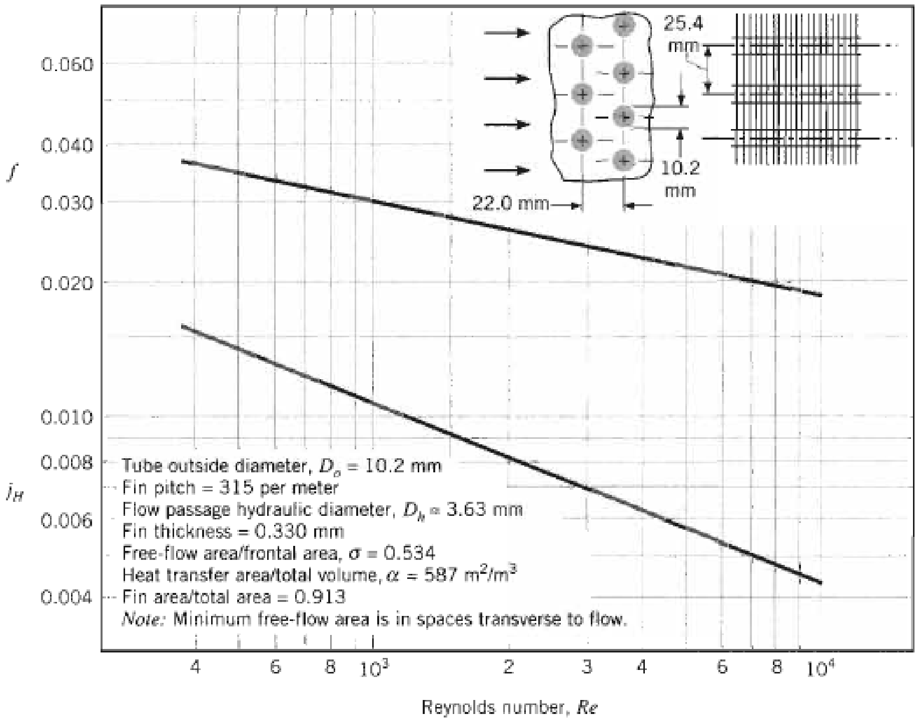


FIG · Heat transfer and friction factor for a circular tube–continuous fin heat exchanger, surface 8.0-3/8T from Kays and London .

Fin effectiveness, surface effectiveness and overall coeff of heat transfer:

Fins are used to extend the area of heat transfer; however, 100% of fin area is *not* effective since there is some temperature gradient along the fin due to its finite thermal conductivity.

Fin-area effectiveness **for plate fins and straight fins** is given by:

$$\eta_f = \frac{\tanh(ML_f)}{ML_f}$$

where

$$M = (h_c / k_f \delta)^{1/2}$$

In the above,

k_f = thermal conductivity of the fin material,

$\delta = V_f / A_f$ = Volume of fin/Surface area of fin,

L_f = Length of fin if one end of fin is free, or one-half the fin length if the fin is attached at both ends.

***Surface effectiveness*:**

If the total heat transfer area is A_0, then:

$A_0 = A_f + A_p$, where A_f = fin surface area, and A_p = bare (or prime) surface area.

Prime surface area is 100% effective.

Then, if the surface effectiveness is η_0, we can write:

$$\eta_0 A_0 = (1) A_p + \eta_f A_f = A_0 - A_f + \eta_f A_f$$

Then, surface effectiveness is given by:

$$\eta_0 = 1 - (A_f / A_0)(1 - \eta_f).$$

Note that effective heat transfer area is: $\eta_0 . A_0$

***Overall coeff of heat transfer*:**

Denoting the warmer stream by subscript 'h' and cooler stream by subscript 'c', the ***overall coeff of heat transfer*** *based on the warm-side heat transfer area* $A_{0,h}$ is given by:

$$\frac{1}{U_h} = \frac{1}{\eta_{0,h} h_{c,h}} + \frac{A_{0,h}/A_{0,c}}{\eta_{0,c} h_{c,c}}$$

In the above eqn, we have neglected the thermal resistance of separating surface. Also, if no fins are used on a surface, its surface effectiveness is unity.

3.1.20 HEAT EXCHANGER ANALYSIS:

There are two approaches:

Effectiveness-NTU method, and Log-mean-temp-difference approach.

Temperature variations which can occur in a heat exchanger are shown below:

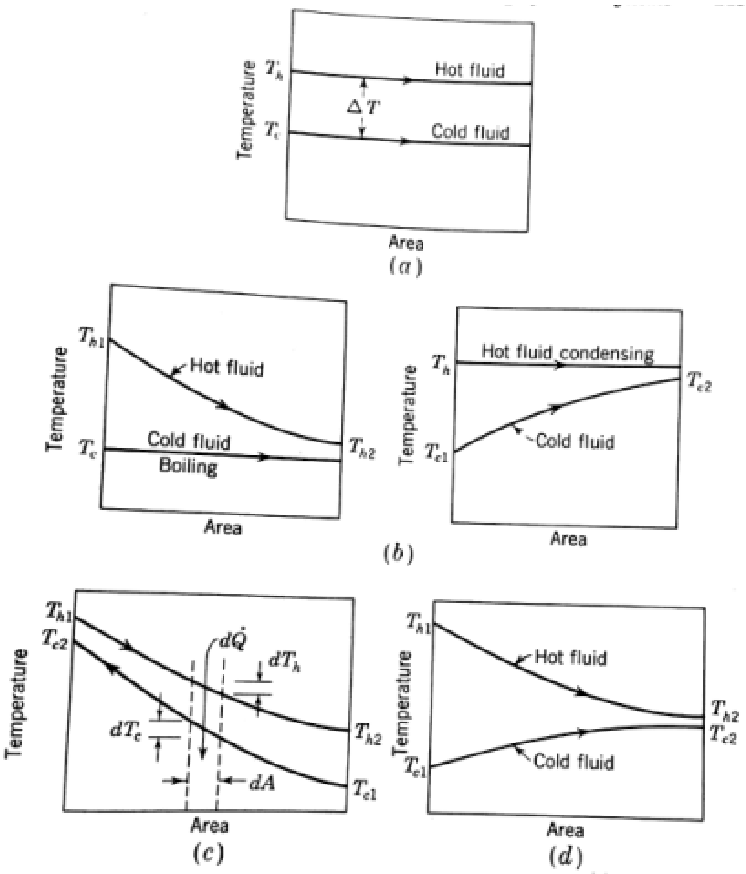


Fig. **Temperature variations which can occur in heat exchangers. (*a*) Condenser-reboiler; (*b*) one fluid condensing or boiling; (*c*) counterflow; (*d*) parallel flow.**

Summary of calculation equations for Effectiveness-NTU method are given below:

By definition,

Effectiveness, ε = actual energy transfer / max. possible energy transfer = Q / Q_{max}, i.e.

$$\varepsilon = \frac{Q}{Q_{max}}$$

And,

$$Q_{max} = C_{min}\left(T_h - T_{c1}\right).$$

No. of heat transfer Units, NTU = U. A / C_{min}

where U is the overall heat transfer coeff, A is the area of heat transfer and Cmin is the minimum of capacity rates of the two fluids. **Capacity rate** is mass flow rate of the fluid multiplied by sp. heat of that fluid.(= mdot * cp).

For a counter-flow heat exchanger (see fig. c above):

$$\varepsilon = \frac{1-\exp\left[-N_{tu}(1-C_R)\right]}{1-C_R\exp\left[-N_{tu}(1-C_R)\right]}.$$

where $C_R = C_{min} / C_{max}$

Two special cases of counter-flow exchanger are:

balanced operation (i.e. $C_{min} = C_{max,}$ or $C_R = 1$), and

phase change on one side (i.e. $C_{max} = \infty$, or $C_R = 0$).

For these two cases, we have:

$$\varepsilon = \frac{N_{tu}}{1+N_{tu}} \quad \text{for} \quad C_R = 1.$$

$$\varepsilon = 1-\exp(-N_{tu}) \quad \text{for} \quad C_R = 0.$$

For a parallel-flow heat exchanger (see fig. d above):

$$\varepsilon = \frac{1-\exp\left[-N_{tu}(1+C_R)\right]}{1+C_R}$$

For *balanced operation*, in parallel flow:

$$\varepsilon = \frac{1}{2}\left[1-\exp(-2N_{tu})\right] \quad \text{for} \quad C_R = 1.$$

For condensation or boiling on one side:

$$\varepsilon = 1-\exp(-N_{tu}) \quad \text{for} \quad C_R = 0.$$

i.e. when there is a phase change on one side, it does not matter if the flow arrangement is counter-flow or parallel flow.

Cross-flow heat exchanger:

Here, fluids flow perpendicular to each other.

Typical cross-flow arrangements are shown in the figure below. Here, note that a fluid is said to be *'mixed'* when it is *not* confined to flow in a definite channel, such as a tube. A fluid is *'unmixed'* when it is confined to flow within a definite channel.

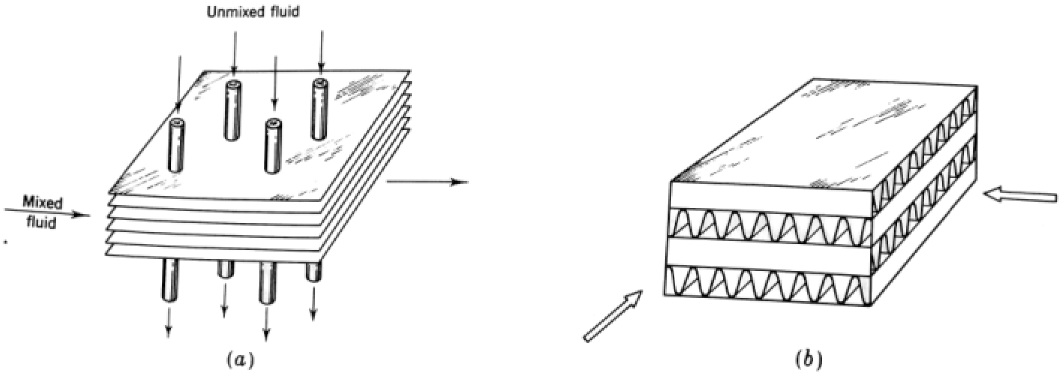
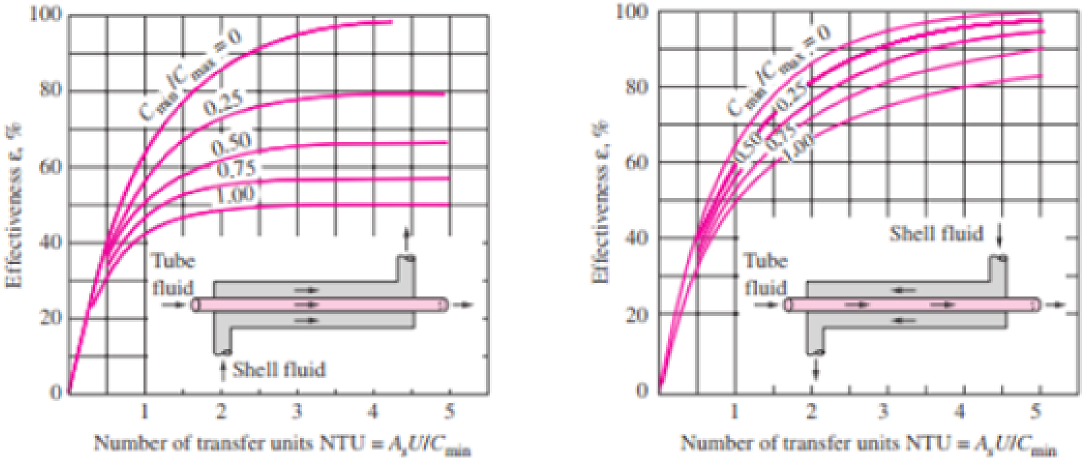


Fig. Crossflow heat exchangers. (a) One fluid mixed, one fluid unmixed (b) both fluids unmixed

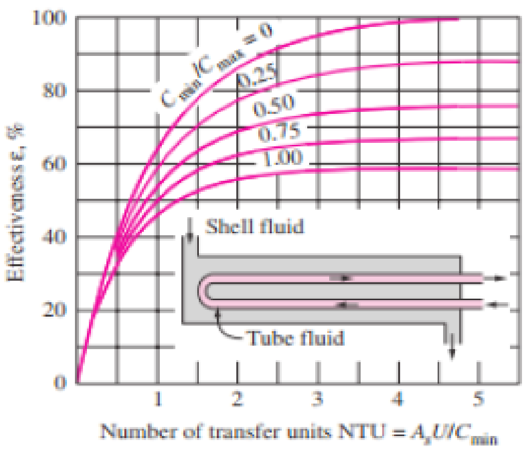
Effectiveness-NTU graphs for some heat exchangers are shown below [23]:

NTU-Effectiveness graphs:

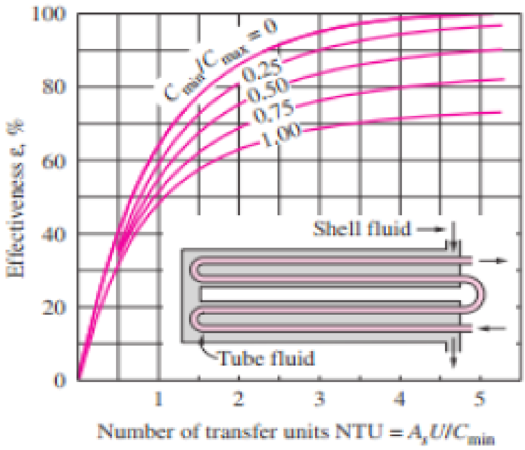


(*a*) Parallel-flow

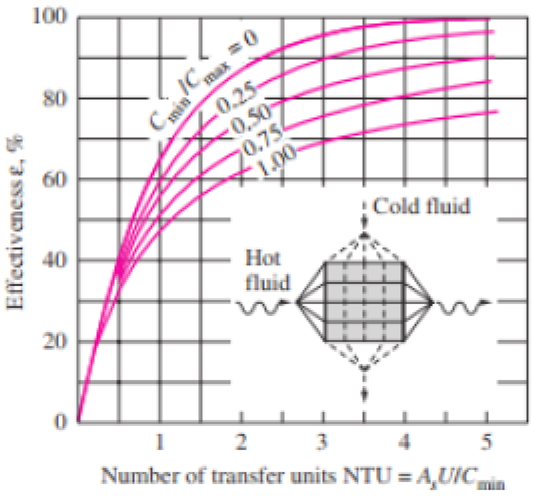
(*b*) Counter-flow



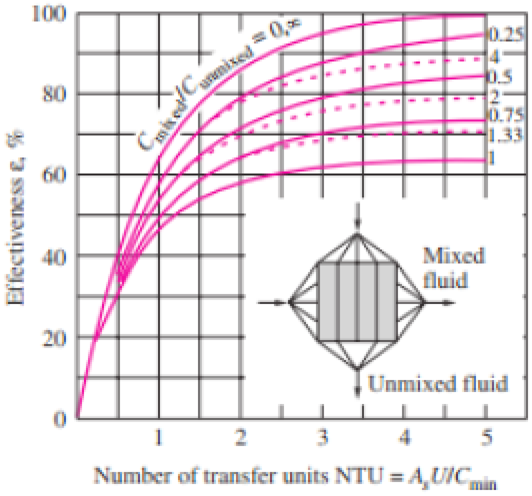
(*c*) One-shell pass and 2, 4, 6, … tube passes



(*d*) Two-shell passes and 4, 8, 12, … tube passes



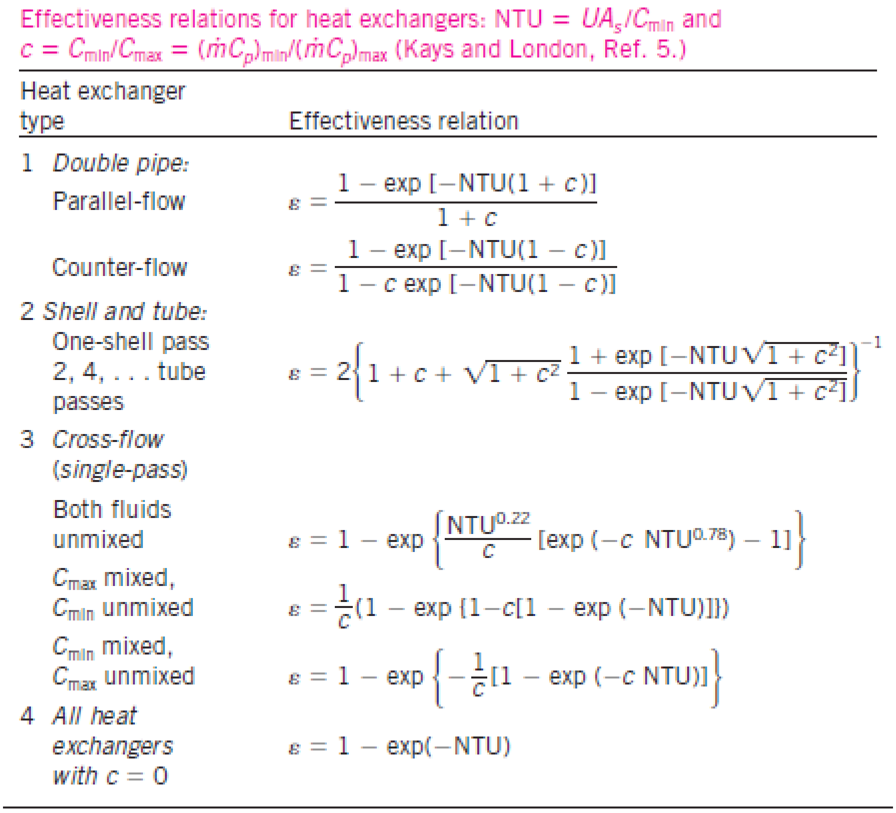
(*e*) Cross-flow with both fluids unmixed



(*f*) Cross-flow with one fluid mixed and the other unmixed

It is shown later that these graphs are reproduced very easily with either EES (see Prob.3.2.39 to 3.2.42), or Mathcad (see Prob.3.3.20 to 3.3.35).

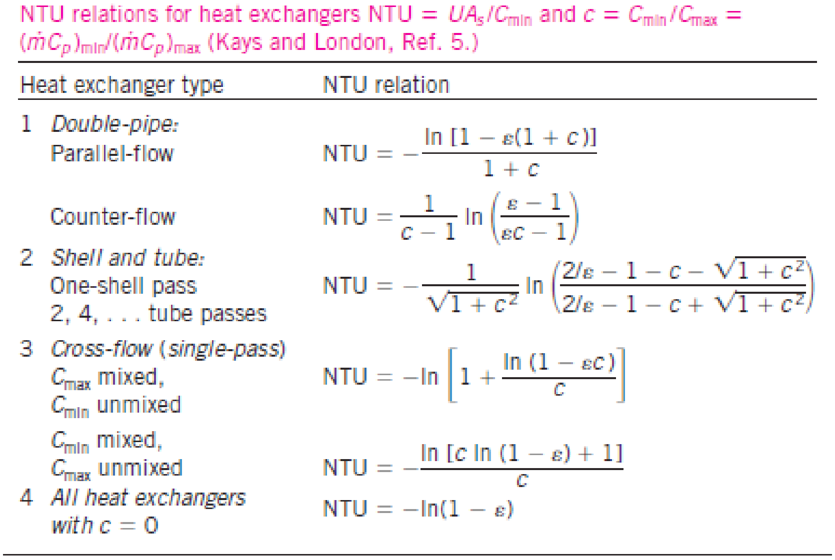
Summary of Effectiveness relations [23]:



Effectiveness relations for heat exchangers: NTU = UA_s/C_{min} and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 + c)]}{1 + c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 - c)]}{1 - c\exp[-\text{NTU}(1 - c)]}$
2 *Shell and tube:*	
One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1 + c + \sqrt{1 + c^2}\,\dfrac{1 + \exp[-\text{NTU}\sqrt{1 + c^2}]}{1 - \exp[-\text{NTU}\sqrt{1 + c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)*	
Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}[\exp(-c\ \text{NTU}^{0.78}) - 1]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\{1 - c[1 - \exp(-\text{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp(-c\ \text{NTU})]\right\}$
4 *All heat exchangers with c = 0*	$\varepsilon = 1 - \exp(-\text{NTU})$

Log-mean-temp-difference (LMTD) approach [23]:



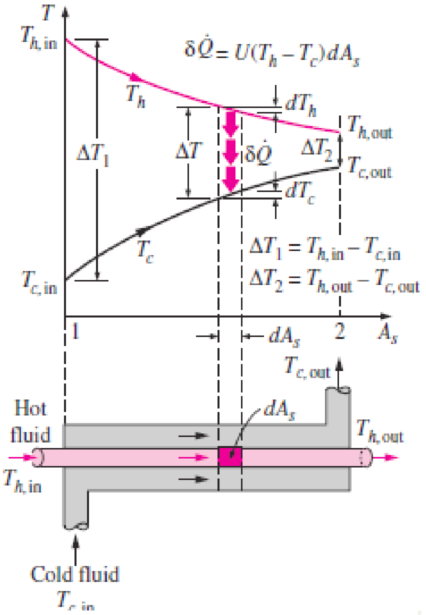
NTU relations for heat exchangers NTU = UA_s/C_{min} and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	NTU relation
1 *Double-pipe:* Parallel-flow	$NTU = -\frac{\ln[1-\varepsilon(1+c)]}{1+c}$
Counter-flow	$NTU = \frac{1}{c-1}\ln\left(\frac{\varepsilon-1}{\varepsilon c-1}\right)$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$NTU = -\frac{1}{\sqrt{1+c^2}}\ln\left(\frac{2/\varepsilon-1-c-\sqrt{1+c^2}}{2/\varepsilon-1-c+\sqrt{1+c^2}}\right)$
3 *Cross-flow (single-pass)* C_{max} mixed, C_{min} unmixed	$NTU = -\ln\left[1+\frac{\ln(1-\varepsilon c)}{c}\right]$
C_{min} mixed, C_{max} unmixed	$NTU = -\frac{\ln[c\ln(1-\varepsilon)+1]}{c}$
4 *All heat exchangers with* $c = 0$	$NTU = -\ln(1-\varepsilon)$

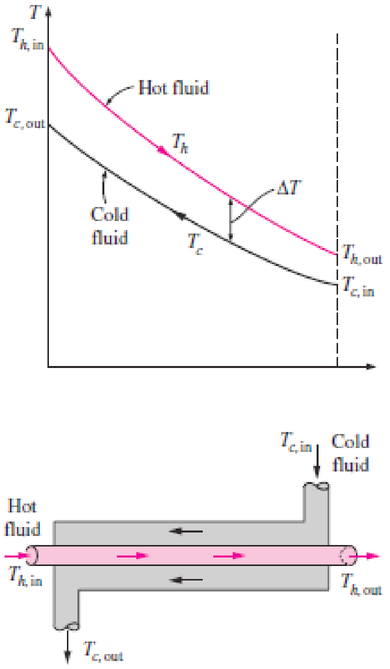
Recollect that heat transfer between the two fluid streams in a heat exchanger can be written as:

Q = U A ΔT_m, where U is the overall heat transfer coeff, A is the area of heat transfer and ΔTm is the *mean temp difference* between the two fluid streams.

ΔT_m has to be used since, generally, the temp difference between the streams is not constant along the length of the heat exchanger. See the fig. below:



Variation of the fluid temperatures in a parallel-flow double-pipe heat exchanger.



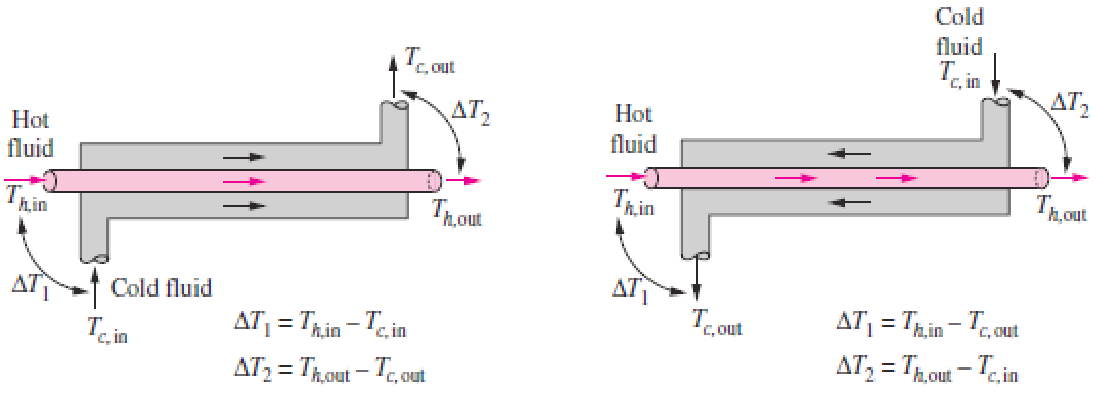
The variation of the fluid temperatures in a counter-flow double-pipe heat exchanger.

It is found that the mean temp difference is a logarithmic expression, and therefore, it is also known as **Logarithmic Mean Temp Difference (LMTD).**

For parallel flow and counter-flow heat exchangers, we have, for LMTD:

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln (\Delta T_1/\Delta T_2)}$$

Remember that ΔT1 and ΔT2 are the max and min temp differences in the heat exchangers, as shown below:

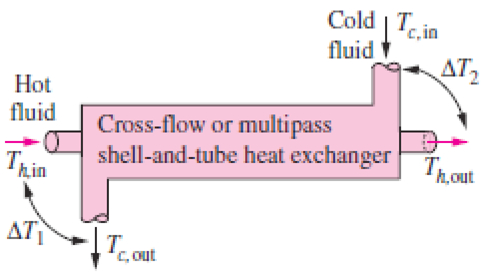


(*a*) Parallel-flow heat exchangers

(*b*) Counter-flow heat exchangers

Note: When ΔT1 = ΔT2 = ΔT, say, then the LMTD is given as: **LMTD = ΔT**

For cross-flow and multi-pass Shell and Tube heat exchangers:



Here, the LMTD is calculated as follows:

First find out the LMTD as if the flows were *counter-flow*.

Then, apply a **correction factor F**, determined from the following graphs, represented as functions of two temperature factors P and R, defined as follows:

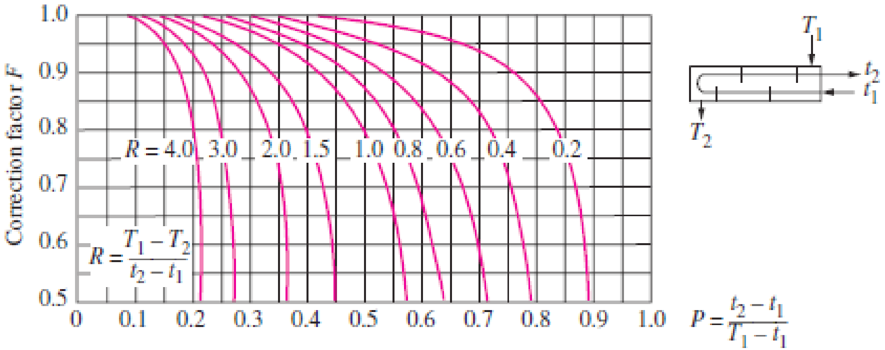
$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}C_p)_{\text{tube side}}}{(\dot{m}C_p)_{\text{shell side}}}$$

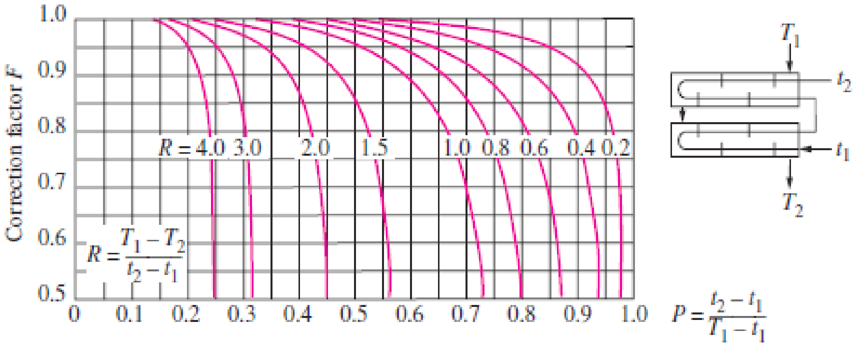
And, actual LMTD is:

$$\Delta T_{\text{lm}} = F\,\Delta T_{\text{lm},\,CF}$$

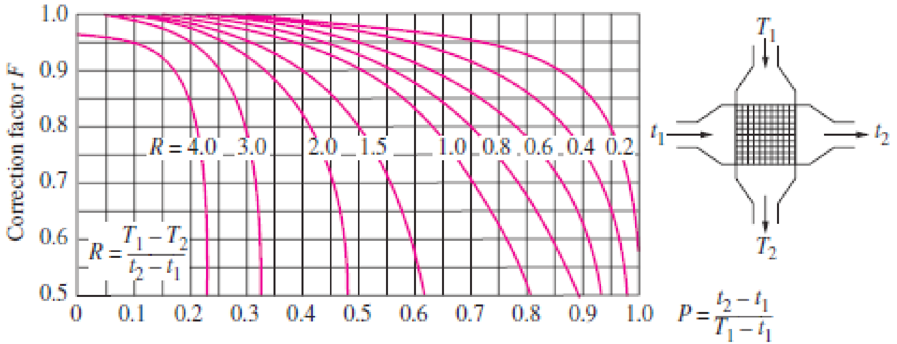
where F is determined from following graphs:



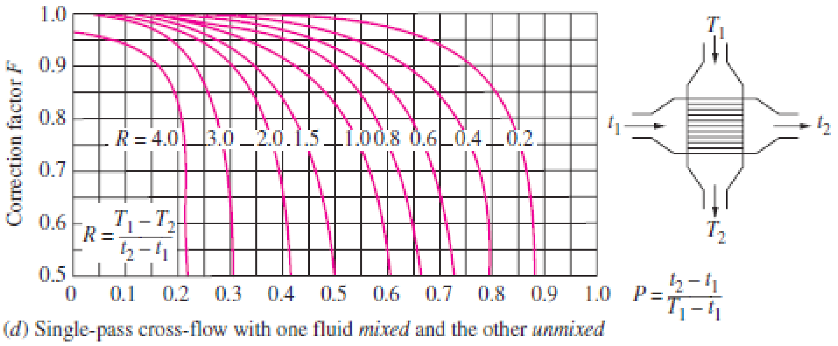
(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes



(*c*) Single-pass cross-flow with both fluids *unmixed*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

3.1.21 SECOND LAW ANALYSIS (OR, EXERGY ANALYSIS) [25, 26, 27]:

Summary of definitions, statements and formulas used, essential for problem solving only are given below:

'Availability' or 'Available energy' or 'Exergy':

For a heat engine, we know from II Law that all the heat available can not be converted to work, and some amount of heat is necessarily to be rejected to the sink. So, the amount of heat rejected is the *'unavailable part'* of the energy and the W_{max} obtained is the *'available part'* of the energy.

Thus, by definition: 'A system delivers the max. possible work as it undergoes a reversible process from the specified initial state to the state if its environment, that is, the *'dead state'*. This is the **'useful work potential'** of the system at the specified state and is called **'Exergy'.**

Exergy of heat:

For a heat engine, if heat Q is supplied at a constant source temp of T_H and the environment is at T_0, the exergy or the max. work output is:

$W_{max} = Q.(1- T_0/T_H)$

i.e. $W_{max} = Q – T_{0.}\ \Delta s$, where Δs is the change in entropy in the process.

Also, the unavailable energy (or 'anergy') = T0 .Δs

For a refrigerator: if heat Q is absorbed at a constant source temp of T_z and the environment is at T_0, the exergy or the *min. work required* is:

$W_{min} = Q\ /\ [T_z\ /\ (T_0 – T_z)] = Q.\ [(T_0\ /\ T_z) – 1)]$

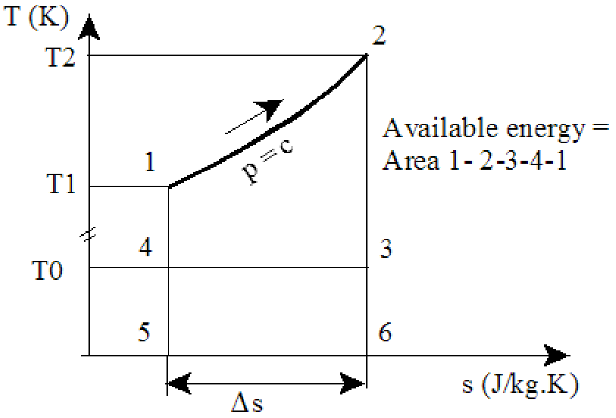
i.e. W_{min} = T0. Δs – Q, where Δs is the change in entropy in the process.

Exergy of Work: Exergy of work is work itself, since there is no thermodynamic restriction on its availability.

Exergy of Kinetic Energy and Potential Energy: Again, the exergies of K.E. and P.E. are the respective energies themselves.

If heat is supplied at varying temperatures, i.e. at constant pressure:

Then, Available energy for a heat engine is easily calculated as follows:



Note that:

Heat supplied = cp. (T2 – T1) …J/kg = area 1-2-6-5-1

Unavailable energy = area 4-3-6-5-4 = T0. Δs = T0. cp .ln(T2/T1)…J/kg

Available energy = area 1-2-3-4-1

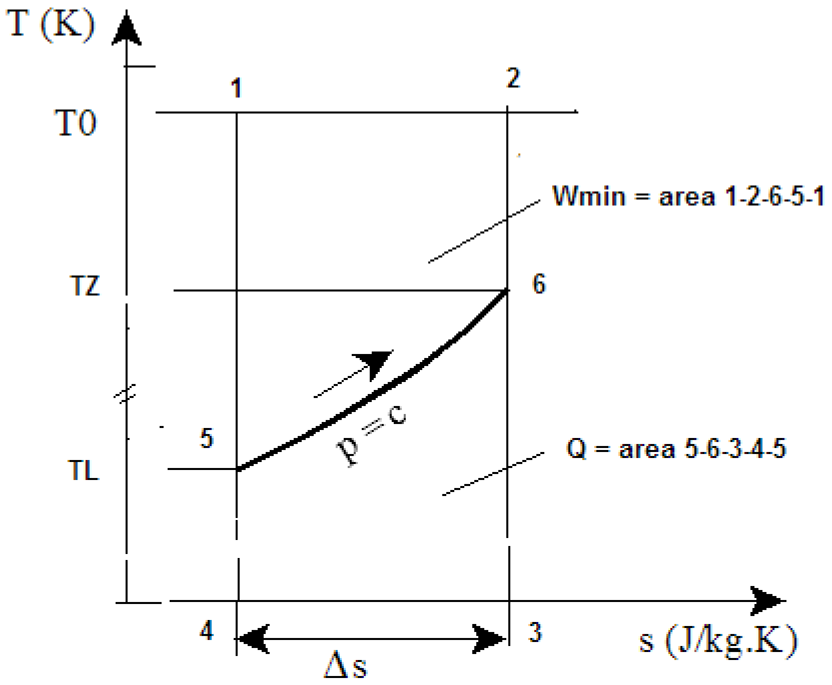
i.e. Available energy = area 1-2-6-5-1 – area 4-3-6-5-4

i.e. Available energy = cp. (T2 – T1) – T0. cp. ln(T2/T1)…J/kg

Note that in the above equations, temp should be in Kelvin.

If heat is supplied at varying temperatures, i.e. at constant pressure:

Available energy for a refrigerator is easily calculated as follows:



COP = Q / Wmin

Exergy of cold produced $eq_L = Q. (T_z - T_0) / T_0$…when refrigerator temp is T_z

This exergy is *negative*, indicating that work has to be supplied.

3.1.22 AVAILABILITY (OR EXERGY) IN STEADY FLOW SYSTEMS [25]:

Let the initial conditions of the flow system be p1, T1, Z1. Let this state be reduced to atmospheric or ‘dead state’ denoted by p0, T0, Z0 (= 0), through an ideal process. Then,

$$W_{max} = \left(h_1 + \frac{C1^2}{2} + Z1\,g\right) - h_0 - T_0 \cdot (s_1 - s_0)$$

In many systems, changes in K.E and P.E. can be considered as negligible. Then,

$$W_{max} = (h_1 - T_0 \cdot s_1) - (h_0 - T_0 \cdot s_0)$$

This is also known as **exergy of mass flow**.

i.e.

$$ef_1 = (h_1 - T_0 \cdot s_1) - (h_0 - T_0 \cdot s_0) \quad \text{J/kg or kJ/kg}$$

Irreversibility:

***Irreversibility* is defined as:**

$$I = W_{max} - W$$

Irreversibility is also known as **‘degradation’** or **‘dissipation’**.

Note: Expression for Irreversibility is the same for both the flow and non-flow processes.

We have, *per unit mass*:

$$I = T_0 \cdot \left(\Delta s_{sys} + \Delta s_{surr} \right) = T_0 \cdot \Delta s_{gen}$$

Second Law efficiency, η_{II}[27]: Thermal efficiency for heat engines and coefficient of performance for refrigerators are based on the first law of thermodynamics and referred to as *the first-law efficiencies.*

Second-law efficiency is defined based on II Law:

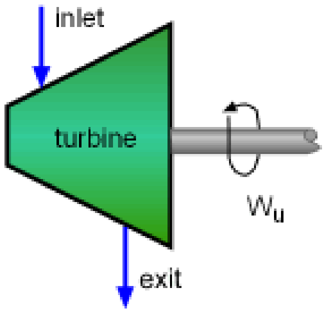
The second-law efficiency *for a heat engine* is defined as the ratio of the useful work output to the maximum possible work output (for work-producing device, such as turbine), or the ratio of the minimum work input to the actual useful work input (for work-consuming device, such as compressor).

For refrigerators or heat Pumps, it is defined as the ratio of the actual COP to the COP of reversible process. For mixing chambers, the second-law efficiency is defined as the ratio of the exergy recovered to the exergy supplied.

Summarizing:

Device	**Second Law efficiency, η_{II}**
Heat engine	η_{th} / η_{rev}
Work producing device (ex: Turbine)	W_u / W_{rev}
Work consuming device (ex: Compressor)	W_{rev} / W_u
Refrigerators/Heat pumps	COP / COP_{rev}
Mixing chambers/heat exchangers	Exergy recovered / Exergy supplied

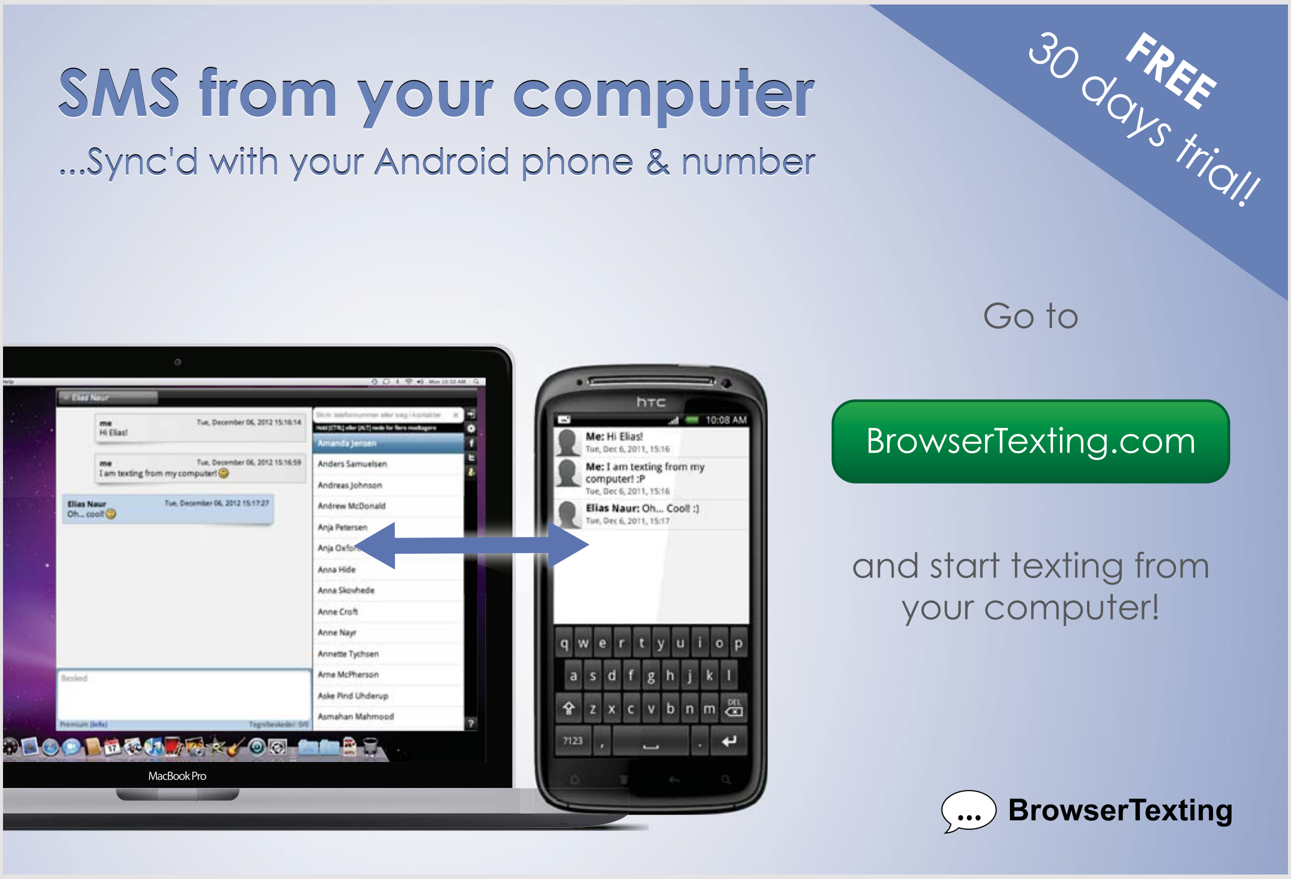
For an adiabatic turbine:



For a turbine the second-law efficiency is defined as:

$$\eta_{II} = \frac{W_u}{W_{rev}}$$

where W_u is the actual useful work and W_{rev} is the reversible work.



From an energy balance, neglecting kinetic and potential energy differences compared to the enthalpy change of the fluid, we get:

$$W_u = m \cdot \left(h_i - h_e\right)$$

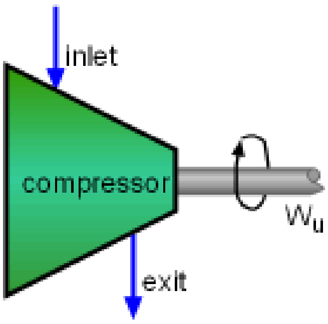
The reversible work for adiabatic turbine, the reversible work equals the difference of the flow exergies at the inlet and the exit:

$$W_{rev} = m \cdot \left(ef_i - ef_e\right)$$

Then, *second-law efficiency of an adiabatic turbine* is given by:

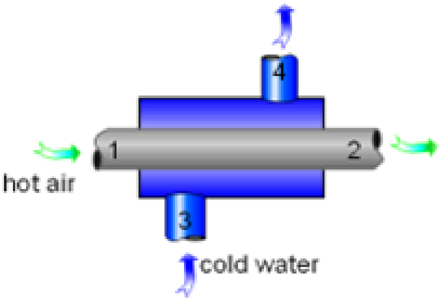
$$\eta_{II} = \frac{h_i - h_e}{ef_i - ef_e}$$

Similarly, *second-law efficiency of a compressor* is:



For heat exchangers and mixing chambers, their second-law efficiencies are given as the ratio of exergy recovered to exergy supplied.

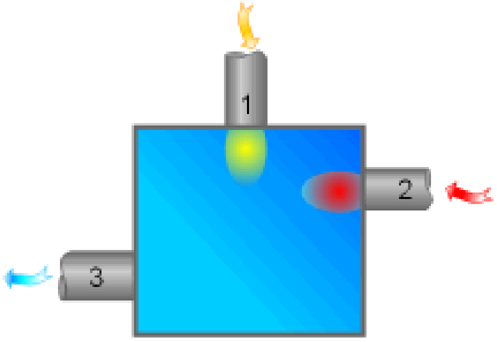
For a heat exchanger: the second-law efficiency is:



$$\eta_{II} = \frac{m_{cold} \cdot (ef_4 - ef_3)}{m_{hot} \cdot (ef_1 - ef_2)}$$

where m_{cold} and m_{hot} are mass flow rates of cold and hot fluids respectively. And, ef stands for exergy of flow.

Similarly, *second-law efficiency of a mixing chamber* is:



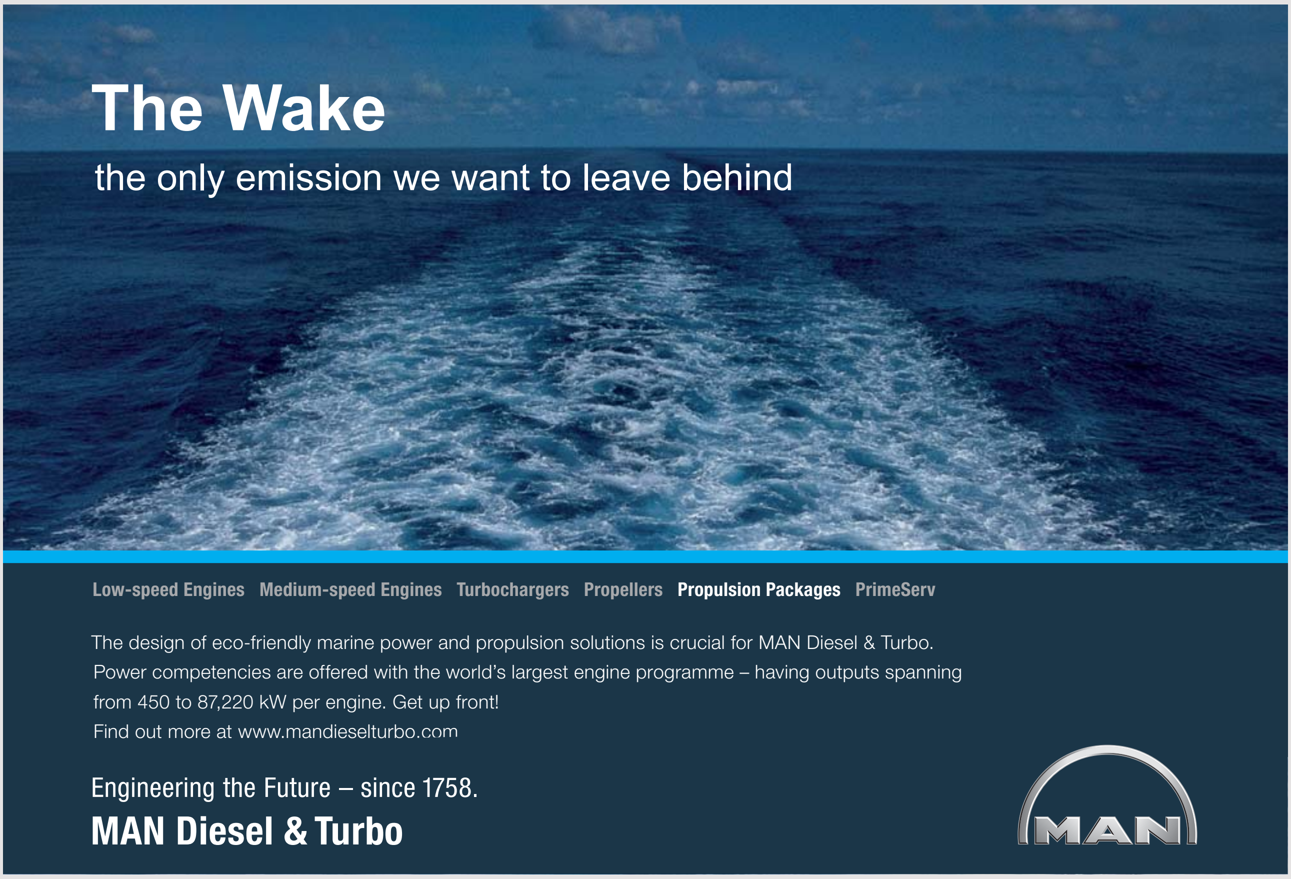
Adiabatic Mixing Chamber

Here, m_1 and m_2 are mass flow rates of hot and cold fluids respectively. And, ef stands for exergy of flow, as usual. Then, second Law efficiency is:

$$\eta_{II} = \frac{m_2 \cdot (ef_3 - ef_2)}{m_1 \cdot (ef_1 - ef_3)}$$

Summarizing:

Device	**Second Law efficiency, η_{II}**
Adiabatic turbine	$\eta_{II}=\frac{h_i-h_e}{ef_i-ef_e}$
Adiabatic compressor	$\eta_{II}=\frac{ef_i-ef_e}{h_i-h_e}$
Heat Exchanger (non-mixing)	$\eta_{II}=\frac{m_{cold}\cdot\left(ef_4-ef_3\right)}{m_{hot}\cdot\left(ef_1-ef_2\right)}$
Adiabatic Mixing chamber	$\eta_{II}=\frac{m_2\cdot\left(ef_3-ef_2\right)}{m_1\cdot\left(ef_1-ef_3\right)}$



Exergy balance:

Exergy balance is written for each component of a system to find out the relative magnitudes of **'exergy losses'** in those components so that corrective action can be taken to reduce the losses.

Writing the exergy balance for components of a steady flow system, such as compressors, turbines, throttle valves, heat exchangers etc is as follows [6]:

Exergy of heat:

$$e_q = q \cdot \frac{(T - T_0)}{T} = q \cdot \left(1 - \frac{T_0}{T}\right)$$

Exergy of work:

Exergy of work is that itself since there is no thermodynamic restriction on its availability.

Exergy of flow of mass flux:

$$e_{f1} = (h_1 - h_0) - T_0 \cdot (s_1 - s_0) \qquad \text{kJ/kg...per unit mass}$$

Exergy balance is written as:

$$e_{f1} + e_{q1} + w_1 = e_{f2} + e_{q2} + w_2 + \Delta e$$

where, 1 represents inlets and 2 represents exits, and Δe is the exergy loss.

*As an example, for a **compressor** we can write:.*

$$e_{f1} + w = e_{f2} + \Delta e$$

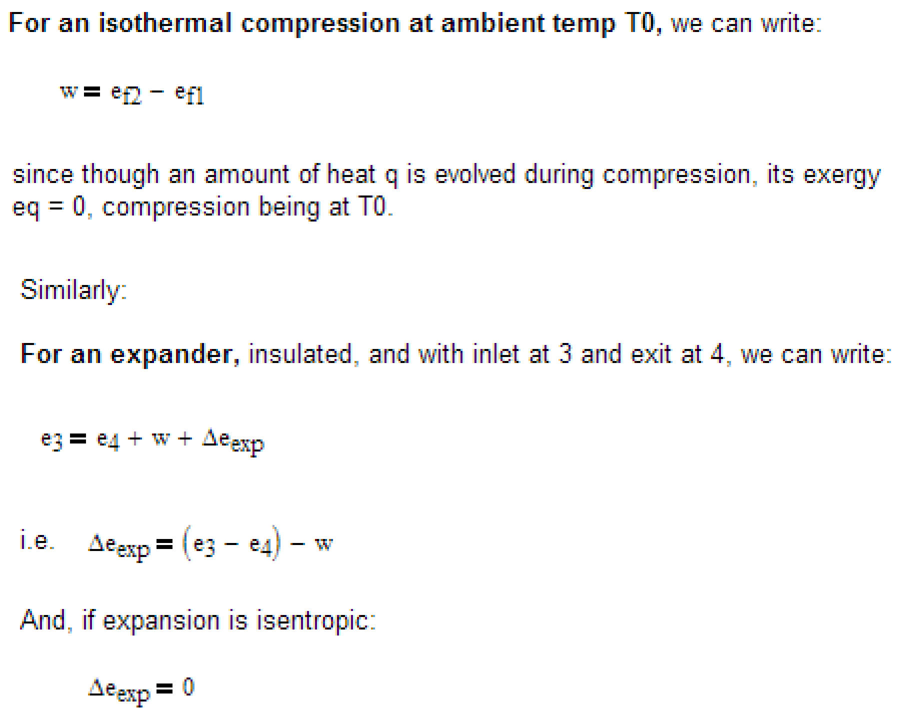
If the compression is adiabatic: q = 0, and eq =0; and **if it is reversible**, $\Delta e = 0$

Therefore:

$$w = e_{f2} - e_{f1}$$

If the compression is adiabatic, but irreversible, then:

$$w = (e_{f2} - e_{f1}) + \Delta e$$



For an isothermal compression at ambient temp T0, we can write:

$$w = e_{f2} - e_{f1}$$

since though an amount of heat q is evolved during compression, its exergy eq = 0, compression being at T0.

Similarly:

For an expander, insulated, and with inlet at 3 and exit at 4, we can write:

$$e_3 = e_4 + w + \Delta e_{exp}$$

i.e. $\Delta e_{exp} = \left(e_3 - e_4\right) - w$

And, if expansion is isentropic:

$$\Delta e_{exp} = 0$$

3.2 PROBLEMS SOLVED WITH EES:

"**Prob. 3.2.1** Calculate the isentropic temp drop for different gases, expanding from 20 atm, 300 K to 1 atm."

Solution:

First write an EES Procedure to find out the isentropic temp drop for any fluid, when it expands from initial pressure Pi and temp Ti to a final pressure Pf:

```
$UnitSysystem SI kPa K kg kJ

PROCEDURE Tdrop_isentr(FLUID$, P_i,T_i, P_f:T_f, DELTAT_s)

"Finds the Temp drop for isentropic expansion from Pi, Ti to final pressure Pf."

"Inputs: FLUID$, P_i,T_i, P_f:…Pressures in kPa, Temp in K"

"Outputs: T_f (K) and DELTAT_s (deg.)"

s_i:=Entropy(Fluid$,T=T_i,P=P_i) "[kJ/kg-K]"

s_f:= s_i "[kJ/kg-K]"
```

```
T_f: =Temperature(Fluid$,s=s_f,P=P_f) "[K]"

DELTAT_s: = T_i – T_f

END

"========================================================================"
```

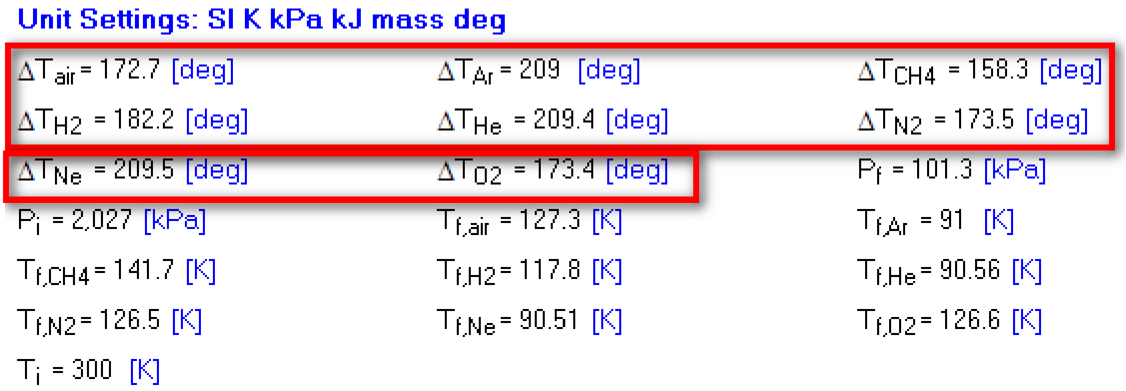
Now, use the above Procedure to find the isentropic temp drop for different fluids:

```
"Data:"
T_i=300 "[K]"

P_i= 20 * 101.325 "[kPa]"

P_f = 101.325 "[kPa]"
```

“Calculations:”

```
CALL Tdrop_isentr(‘Air’, P_i,T_i, P_f:T_f_air, DELTAT_air)

CALL Tdrop_isentr(‘Nitrogen’, P_i,T_i, P_f:T_f_N2, DELTAT_N2)

CALL Tdrop_isentr(‘Oxygen’, P_i,T_i, P_f:T_f_O2, DELTAT_O2)

CALL Tdrop_isentr(‘methane’, P_i,T_i, P_f:T_f_CH4, DELTAT_CH4)

CALL Tdrop_isentr(‘Helium’, P_i,T_i, P_f:T_f_He, DELTAT_He)

CALL Tdrop_isentr(‘Hydrogen’, P_i,T_i, P_f:T_f_H2, DELTAT_H2)

CALL Tdrop_isentr(‘Neon’, P_i,T_i, P_f:T_f_Ne, DELTAT_Ne)

CALL Tdrop_isentr(‘Argon’, P_i,T_i, P_f:T_f_Ar, DELTAT_Ar)
```

Results:



Unit Settings: SI K kPa kJ mass deg

ΔT_{air} = 172.7 [deg]
ΔT_{Ar} = 209 [deg]
ΔT_{CH4} = 158.3 [deg]
ΔT_{H2} = 182.2 [deg]
ΔT_{He} = 209.4 [deg]
ΔT_{N2} = 173.5 [deg]
ΔT_{Ne} = 209.5 [deg]
ΔT_{O2} = 173.4 [deg]
P_f = 101.3 [kPa]
P_i = 2,027 [kPa]
$T_{f,air}$ = 127.3 [K]
$T_{f,Ar}$ = 91 [K]
$T_{f,CH4}$ = 141.7 [K]
$T_{f,H2}$ = 117.8 [K]
$T_{f,He}$ = 90.56 [K]
$T_{f,N2}$ = 126.5 [K]
$T_{f,Ne}$ = 90.51 [K]
$T_{f,O2}$ = 126.6 [K]
T_i = 300 [K]

Isentropic temp drops for various fluids such as Air, Nitrogen, Oxygen, Methane, Helium, Hydrogen, Neon and Argon are shown above. Also, the final temp attained after expansion from 20 atm, 300 K to 1 atm is shown for each gas.

“**Prob. 3.2.2** Calculate the isenthalpic temp drop for different gases, expanding from 20 atm, 300 K to 1 atm.”

Solution:

First write an EES Procedure to find out the isenthalpic temp drop for any fluid, when it expands from initial pressure Pi and temp Ti to a final pressure Pf:

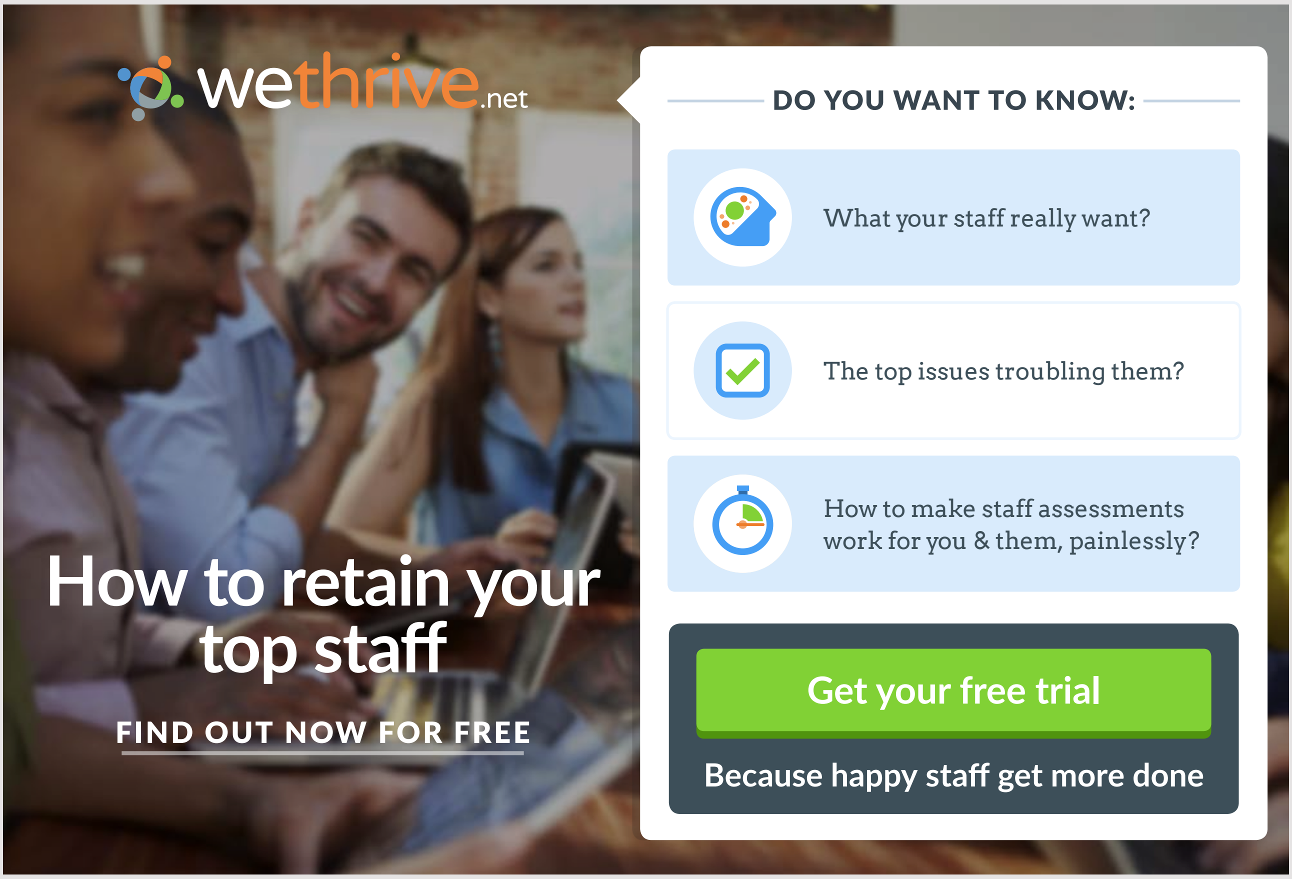
```
$UnitSysystem SI kPa K kg kJ

PROCEDURE Tdrop_isenthalpic(FLUID$, P_i,T_i, P_f:T_f, DELTAT_h)

"Finds Temp drop for isenthalpic expansionfrom Pi, Ti to final pressure Pf."

"Inputs: FLUID$, P_i,T_i, P_f:...Pressures in kPa, Temp in K"

"Outputs: T_f (K) and DELTAT_s (deg.)"

h_i:=Enthalpy(Fluid$,T=T_i,P=P_i) "[kJ/kg]"

h_f:= h_i "[kJ/kg-K]"

T_f: =Temperature(Fluid$,h=h_f,P=P_f) "[K]"

DELTAT_h: = T_i – T_f

END

"=========================================================================="
```

Now, use the above Procedure to find the isentropic temp drop for different fluids:

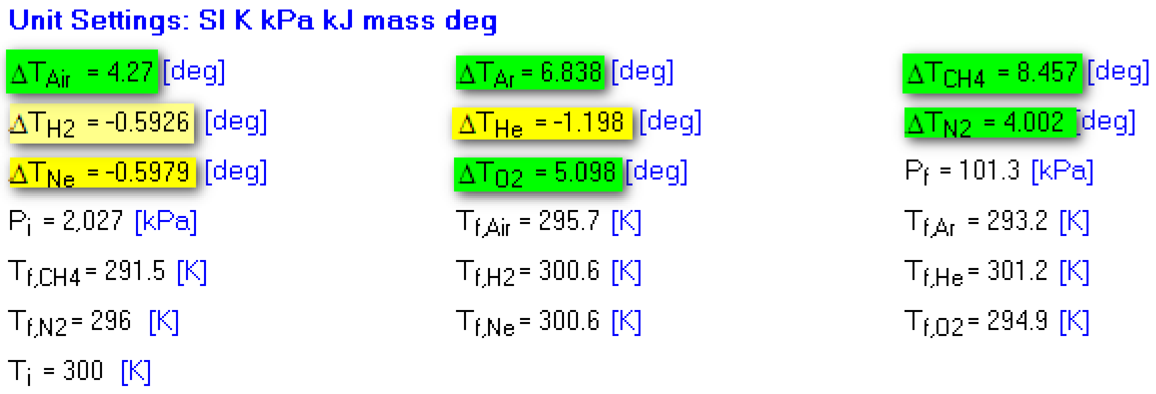
“Data:”

```
T_i=300 "[K]"

P_i= 20 * 101.325 "[kPa]"

P_f = 101.325 "[kPa]"
```

```
"Calculations:"
CALL Tdrop_isenthalpic('Air_ha', P_i,T_i, P_f:T_f_Air, DELTAT_Air)

CALL Tdrop_isenthalpic('Nitrogen', P_i,T_i, P_f:T_f_N2, DELTAT_N2)

CALL Tdrop_isenthalpic('Oxygen', P_i,T_i, P_f:T_f_O2, DELTAT_O2)

CALL Tdrop_isenthalpic('Methane', P_i,T_i, P_f:T_f_CH4, DELTAT_CH4)

CALL Tdrop_isenthalpic('Helium', P_i,T_i, P_f:T_f_He, DELTAT_He)

CALL Tdrop_isenthalpic('Hydrogen', P_i,T_i, P_f:T_f_H2, DELTAT_H2)

CALL Tdrop_isenthalpic('Neon', P_i,T_i, P_f:T_f_Ne, DELTAT_Ne)

CALL Tdrop_isenthalpic('Argon', P_i,T_i, P_f:T_f_Ar, DELTAT_Ar)
```



Results:



Unit Settings: SI K kPa kJ mass deg

ΔT_{Air} = 4.27 [deg]

ΔT_{Ar} = 6.838 [deg]

ΔT_{CH4} = 8.457 [deg]

ΔT_{H2} = -0.5926 [deg]

ΔT_{He} = -1.198 [deg]

ΔT_{N2} = 4.002 [deg]

ΔT_{Ne} = -0.5979 [deg]

ΔT_{O2} = 5.098 [deg]

P_f = 101.3 [kPa]

P_i = 2,027 [kPa]

$T_{f,Air}$ = 295.7 [K]

$T_{f,Ar}$ = 293.2 [K]

$T_{f,CH4}$ = 291.5 [K]

$T_{f,H2}$ = 300.6 [K]

$T_{f,He}$ = 301.2 [K]

$T_{f,N2}$ = 296 [K]

$T_{f,Ne}$ = 300.6 [K]

$T_{f,O2}$ = 294.9 [K]

T_i = 300 [K]

Isenthalpic temp drops for various fluids such as Air, Nitrogen, Oxygen, Methane, Helium, Hydrogen, Neon and Argon are shown above. Also, the final temp attained after expansion from 20 atm, 300 K to 1 atm is shown for each gas.

Note that for Helium, Hydrogen and Neon, final temp Tf is higher than initial temp Ti upon expansion. This is due to the fact that the *max. inversion temp for these fluids is lower than the initial temp before expansion,* Ti = 300 K. See below the max. inversion temp for a few fluids:

Gas	Max. inversion temp. (K)
Carbon dioxide	1500
Oxygen	761
Argon	722
Nitrogen	622
Air	602.78
Neon	250
Hydrogen	202
Helium	40

"**Prob. 3.2.3** Compare the isentropic and isenthalpic temp drops for Nitrogen, expanding from 300 K and various pressures to a final pressure of 1 atm."

Solution:

"Data:"

T_i=300 "[K]"

{P_i= 20 * 101.325 "[kPa]"}

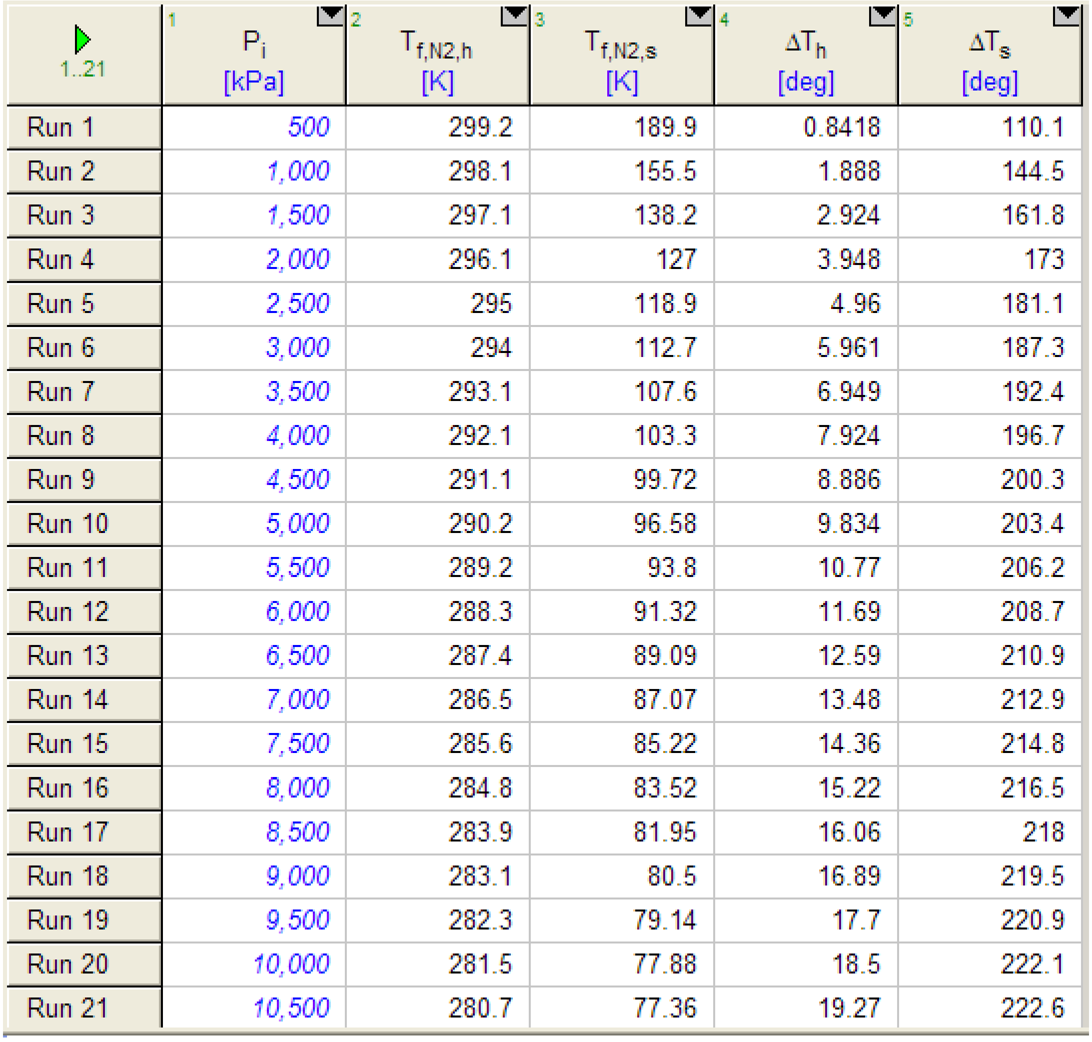
P_f = 101.325 "[kPa]"

"Calculations:"

CALL Tdrop_isentr('Nitrogen', P_i,T_i, P_f:T_f_N2_s, DELTAT_s)

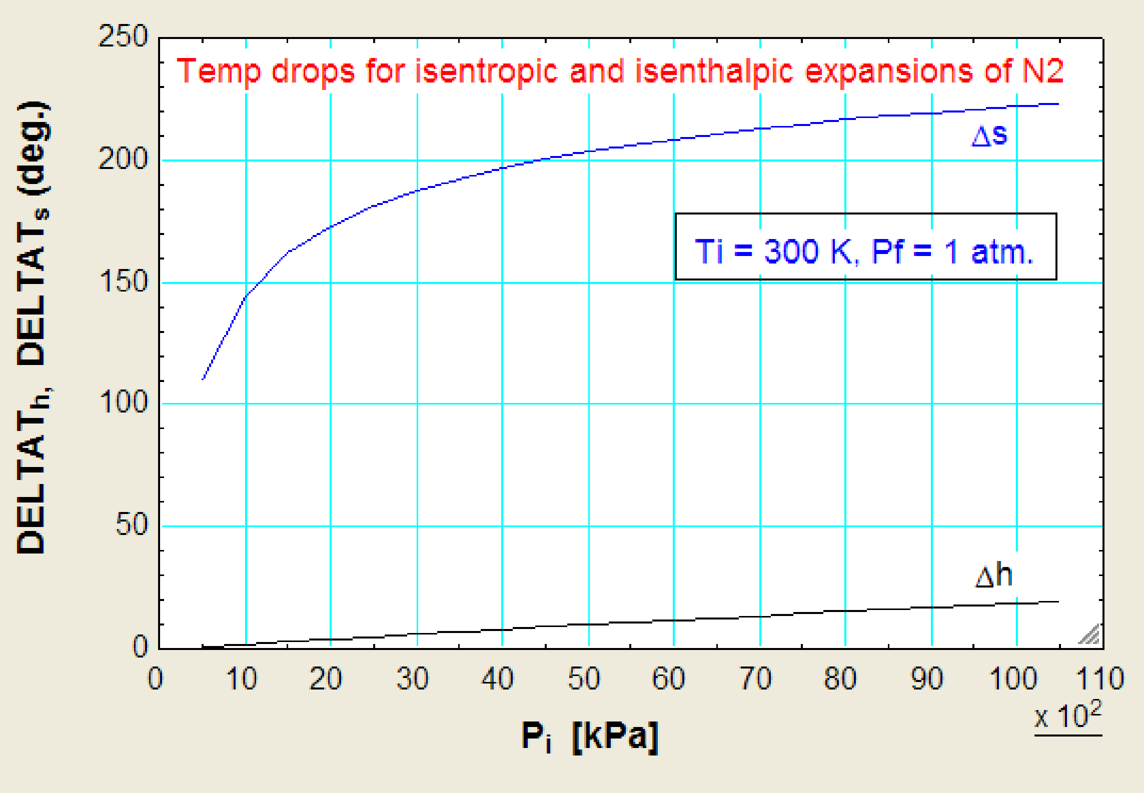
CALL Tdrop_isenthalpic('Nitrogen', P_i,T_i, P_f:T_f_N2_h, DELTAT_h)

Results:



1..21	1 P_i [kPa]	2 $T_{f,N2,h}$ [K]	3 $T_{f,N2,s}$ [K]	4 ΔT_h [deg]	5 ΔT_s [deg]
Run 1	500	299.2	189.9	0.8418	110.1
Run 2	1,000	298.1	155.5	1.888	144.5
Run 3	1,500	297.1	138.2	2.924	161.8
Run 4	2,000	296.1	127	3.948	173
Run 5	2,500	295	118.9	4.96	181.1
Run 6	3,000	294	112.7	5.961	187.3
Run 7	3,500	293.1	107.6	6.949	192.4
Run 8	4,000	292.1	103.3	7.924	196.7
Run 9	4,500	291.1	99.72	8.886	200.3
Run 10	5,000	290.2	96.58	9.834	203.4
Run 11	5,500	289.2	93.8	10.77	206.2
Run 12	6,000	288.3	91.32	11.69	208.7
Run 13	6,500	287.4	89.09	12.59	210.9
Run 14	7,000	286.5	87.07	13.48	212.9
Run 15	7,500	285.6	85.22	14.36	214.8
Run 16	8,000	284.8	83.52	15.22	216.5
Run 17	8,500	283.9	81.95	16.06	218
Run 18	9,000	283.1	80.5	16.89	219.5
Run 19	9,500	282.3	79.14	17.7	220.9
Run 20	10,000	281.5	77.88	18.5	222.1
Run 21	10,500	280.7	77.36	19.27	222.6

Now, draw the graphs:



"**Prob.3.2.4** Nitrogen gas at 200 atm and 300 K is expanded in an expansion valve. Find out the temp drop and the average J-T coeff. for a final pressure of 100 atm. Then plot these quantities as the final pressure varies from 500 kPa to 10000 kPa, other conditions remaining the same."

"Solution:"

"Data:"

T_i=300 "[K]"

P_i= 200 * 101.325 "[kPa]"

P_f = 100 * 101.325 "[kPa]"

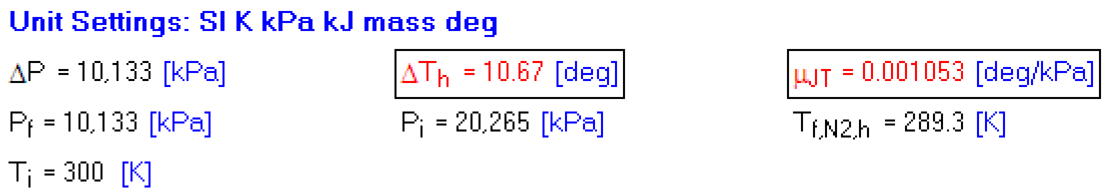
"Calculations:"

CALL Tdrop_isenthalpic('Nitrogen', P_i,T_i, P_f:T_f_N2_h, DELTAT_h)

DELTAP = P_i – P_f "kPa"

mu_JT = DELTAT_h/DELTAP "deg/kPa…J_T coeff."

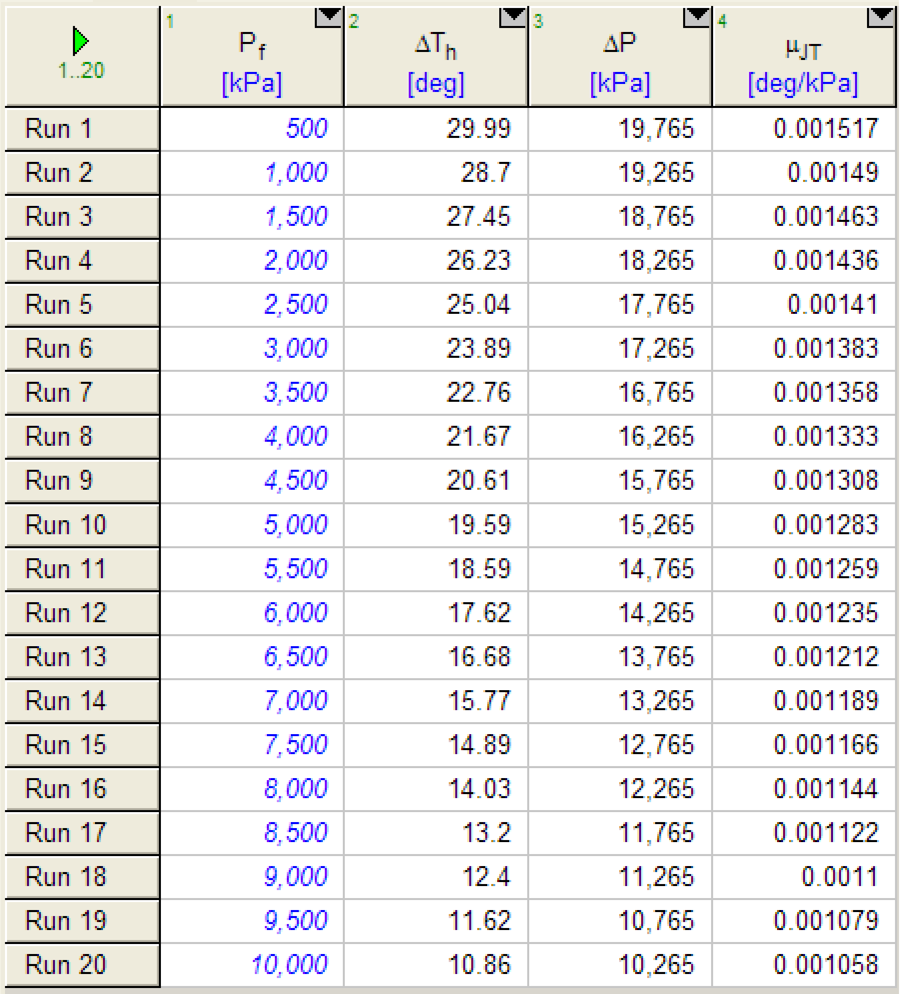
Results:



```
Unit Settings: SI K kPa kJ mass deg
ΔP = 10,133 [kPa]        ΔTh = 10.67 [deg]       μJT = 0.001053 [deg/kPa]
Pf = 10,133 [kPa]        Pi = 20,265 [kPa]       Tf,N2,h = 289.3 [K]
Ti = 300 [K]
```

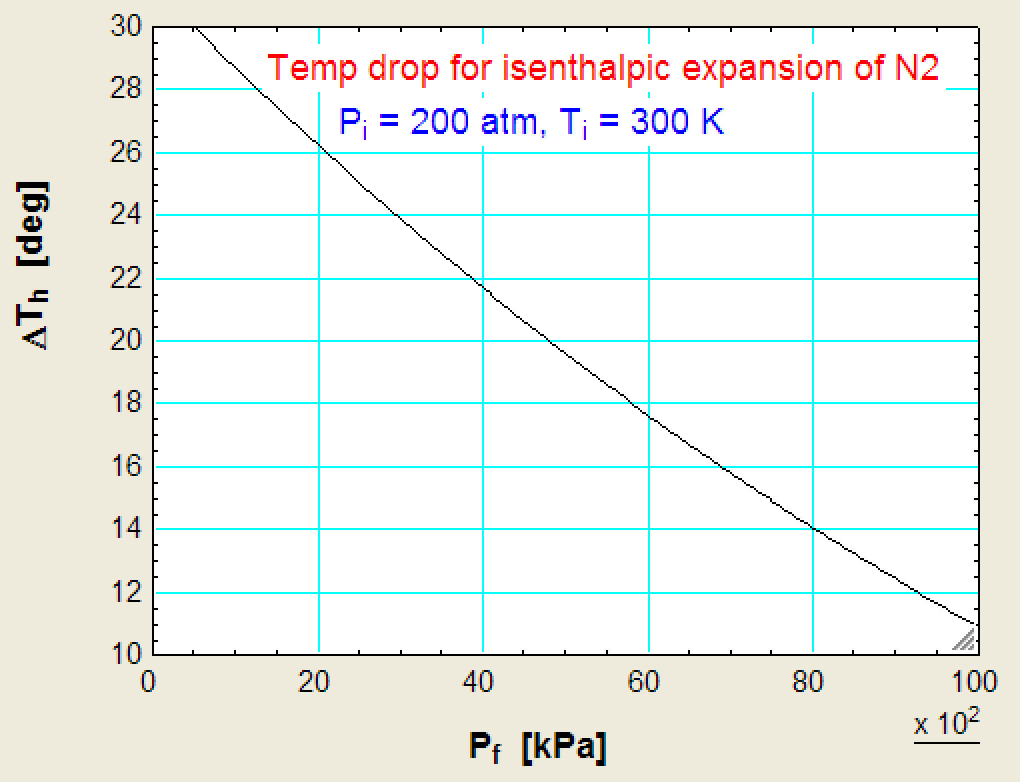
To plot these quantities as the final pressure varies from 500 kPa to 10000 kPa, other conditions remaining the same:

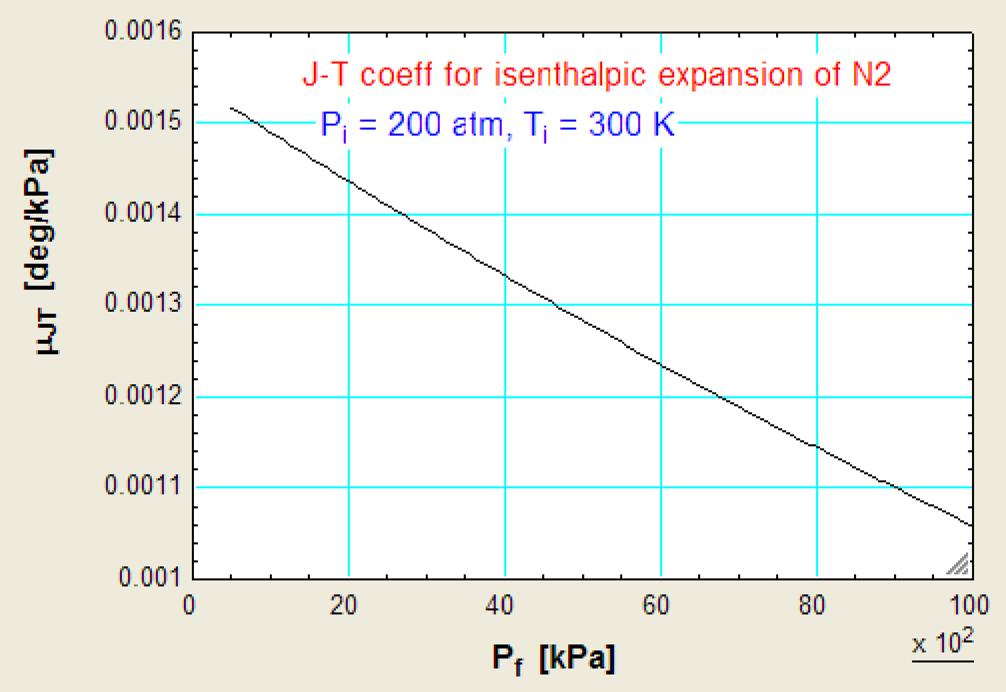
First, compute the Parametric Table:



▶ 1..20	1 P_f [kPa]	2 ΔT_h [deg]	3 ΔP [kPa]	4 μ_{JT} [deg/kPa]
Run 1	500	29.99	19,765	0.001517
Run 2	1,000	28.7	19,265	0.00149
Run 3	1,500	27.45	18,765	0.001463
Run 4	2,000	26.23	18,265	0.001436
Run 5	2,500	25.04	17,765	0.00141
Run 6	3,000	23.89	17,265	0.001383
Run 7	3,500	22.76	16,765	0.001358
Run 8	4,000	21.67	16,265	0.001333
Run 9	4,500	20.61	15,765	0.001308
Run 10	5,000	19.59	15,265	0.001283
Run 11	5,500	18.59	14,765	0.001259
Run 12	6,000	17.62	14,265	0.001235
Run 13	6,500	16.68	13,765	0.001212
Run 14	7,000	15.77	13,265	0.001189
Run 15	7,500	14.89	12,765	0.001166
Run 16	8,000	14.03	12,265	0.001144
Run 17	8,500	13.2	11,765	0.001122
Run 18	9,000	12.4	11,265	0.0011
Run 19	9,500	11.62	10,765	0.001079
Run 20	10,000	10.86	10,265	0.001058

Now, plot the graphs:





"**Prob.3.2.5** Helium gas at 200 atm and 300 K is expanded in an expansion valve. Find out the temp drop and the average J-T coeff. for a final pressure of 100 atm. Then plot these quantities as the final pressure varies from 500 kPa to 10000 kPa, other conditions remaining the same."

"Solution:"

"Data:"

T_i=300 "[K]"

P_i= 200 * 101.325 "[kPa]"

P_f = 100 * 101.325 "[kPa]"

"Calculations:"

CALL Tdrop_isenthalpic('Helium', P_i,T_i, P_f:T_f_He_h, DELTAT_h)

DELTAP = P_i – P_f "kPa"

mu_JT = DELTAT_h/DELTAP "deg/kPa…J_T coeff."

Results:

Unit Settings: SI K kPa kJ mass deg

ΔP = 10,133 [kPa]

μ_{JT} = -0.0006183 [deg/kPa]

P_i = 20,265 [kPa]

T_i = 300 [K]

ΔT_h = -6.265 [deg]

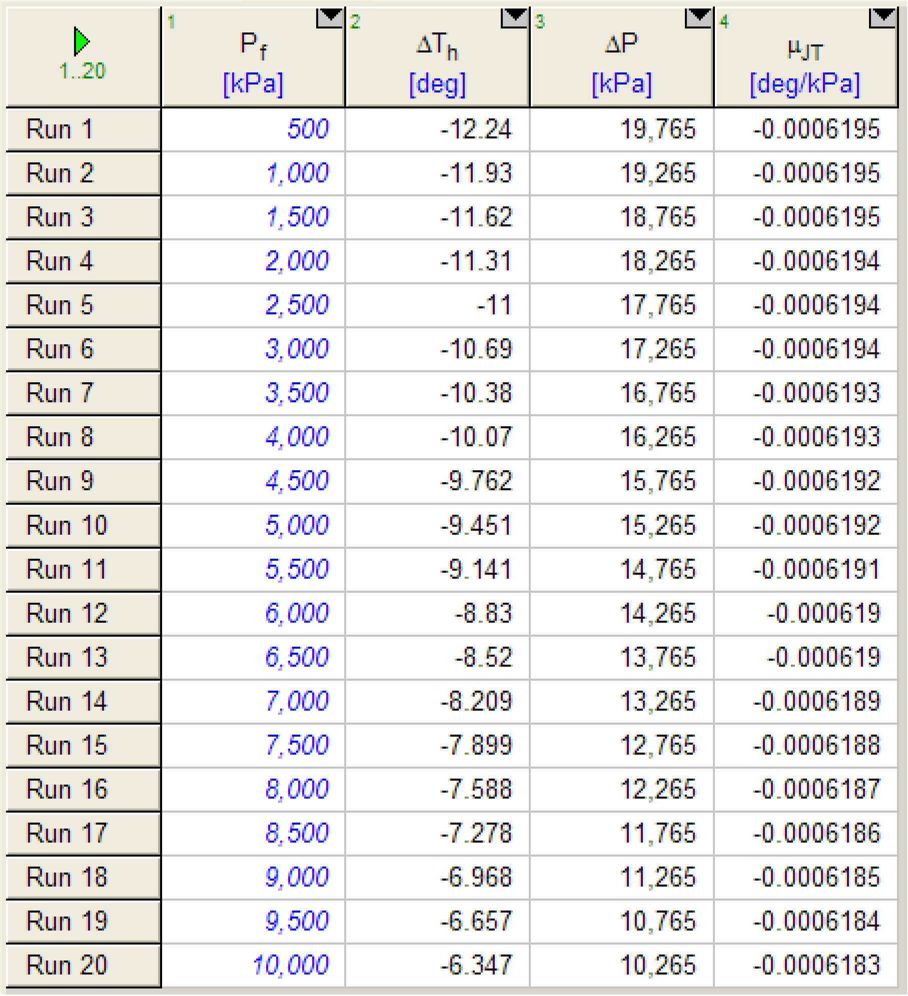
P_f = 10,133 [kPa]

$T_{f,He,h}$ = 306.3 [K]

……

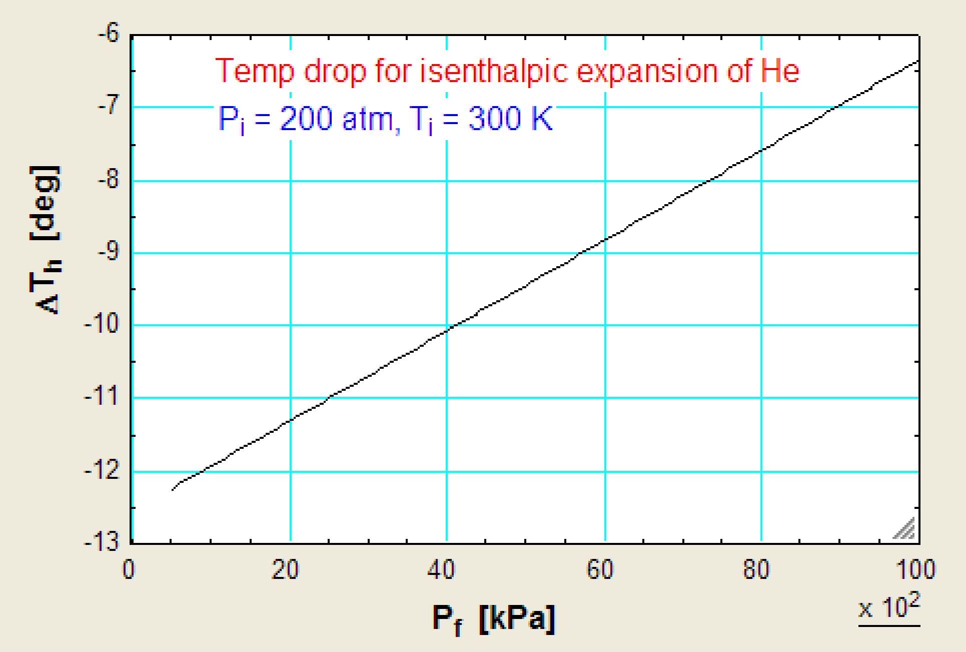
To plot these quantities as the final pressure varies from 500 kPa to 10000 kPa, other conditions remaining the same:

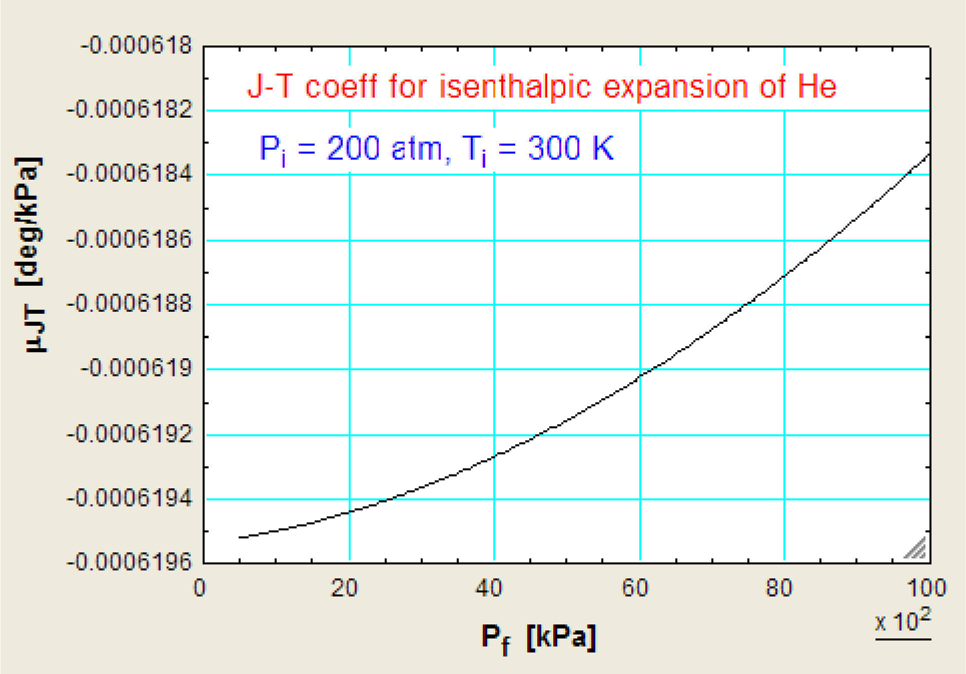
First, compute the Parametric Table:



1..20	1 P_f [kPa]	2 ΔT_h [deg]	3 ΔP [kPa]	4 μ_{JT} [deg/kPa]
Run 1	500	-12.24	19,765	-0.0006195
Run 2	1,000	-11.93	19,265	-0.0006195
Run 3	1,500	-11.62	18,765	-0.0006195
Run 4	2,000	-11.31	18,265	-0.0006194
Run 5	2,500	-11	17,765	-0.0006194
Run 6	3,000	-10.69	17,265	-0.0006194
Run 7	3,500	-10.38	16,765	-0.0006193
Run 8	4,000	-10.07	16,265	-0.0006193
Run 9	4,500	-9.762	15,765	-0.0006192
Run 10	5,000	-9.451	15,265	-0.0006192
Run 11	5,500	-9.141	14,765	-0.0006191
Run 12	6,000	-8.83	14,265	-0.000619
Run 13	6,500	-8.52	13,765	-0.000619
Run 14	7,000	-8.209	13,265	-0.0006189
Run 15	7,500	-7.899	12,765	-0.0006188
Run 16	8,000	-7.588	12,265	-0.0006187
Run 17	8,500	-7.278	11,765	-0.0006186
Run 18	9,000	-6.968	11,265	-0.0006185
Run 19	9,500	-6.657	10,765	-0.0006184
Run 20	10,000	-6.347	10,265	-0.0006183

Now, plot the graphs:





Note that there is *temp increase* during expansion from 200 atm to any lower pressure (i.e. ΔT is negative). This is due to the fact that the initial temp Ti before expansion (i.e. 300 K) is higher than the max. inversion temp of Helium, i.e. 40 K. For the same reason, the J-T coeff is also negative.

"**Prob.3.2.6** Nitrogen gas at 200 atm and 300 K is expanded isentropically in an expansion engine. Find out the temp drop and the average isentropic expansion coeff. for a final pressure of 100 atm. Then plot these quantities as the final pressure varies from 500 kPa to 10000 kPa, other conditions remaining the same."

"Solution:"

"Data:"

T_i=300 "[K]"

P_i= 200 * 101.325 "[kPa]"

P_f = 100 * 101.325 "[kPa]"

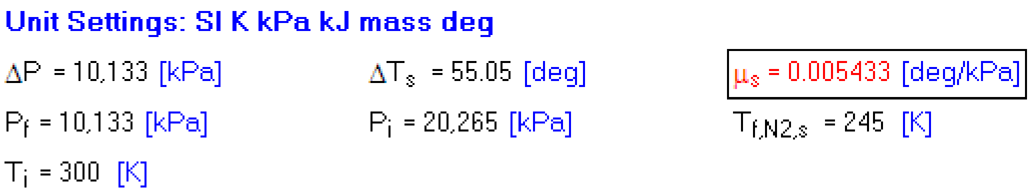
"Calculations:"

CALL Tdrop_isentr('Nitrogen', P_i,T_i, P_f:T_f_N2_s, DELTAT_s)

DELTAP = P_i – P_f "kPa"

mu_s = DELTAT_s/DELTAP "deg/kPa…isentr. expn. coeff."

Results:



Unit Settings: SI K kPa kJ mass deg

ΔP = 10,133 [kPa]

ΔT_s = 55.05 [deg]

μ_s = 0.005433 [deg/kPa]

P_f = 10,133 [kPa]

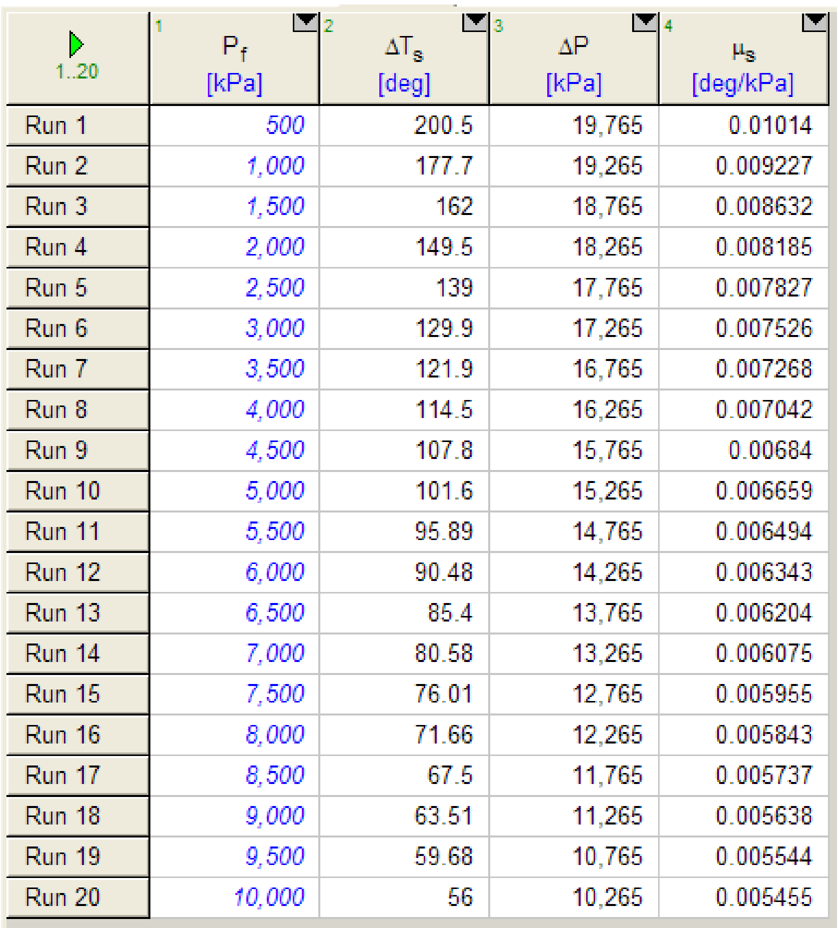
P_i = 20,265 [kPa]

$T_{f,N2,s}$ = 245 [K]

T_i = 300 [K]

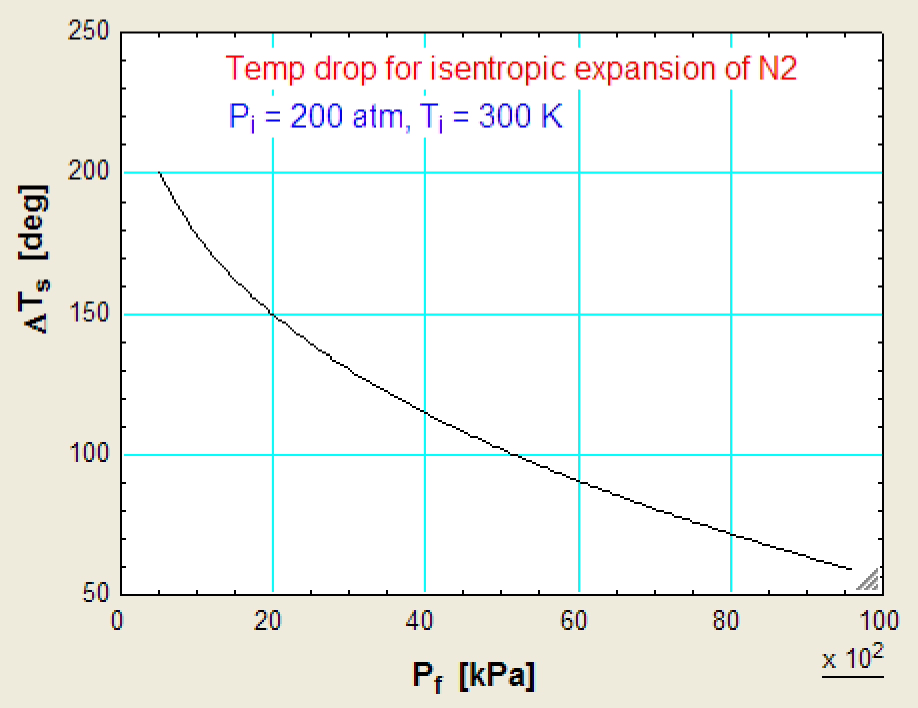
To plot these quantities as the final pressure varies from 500 kPa to 10000 kPa, other conditions remaining the same:

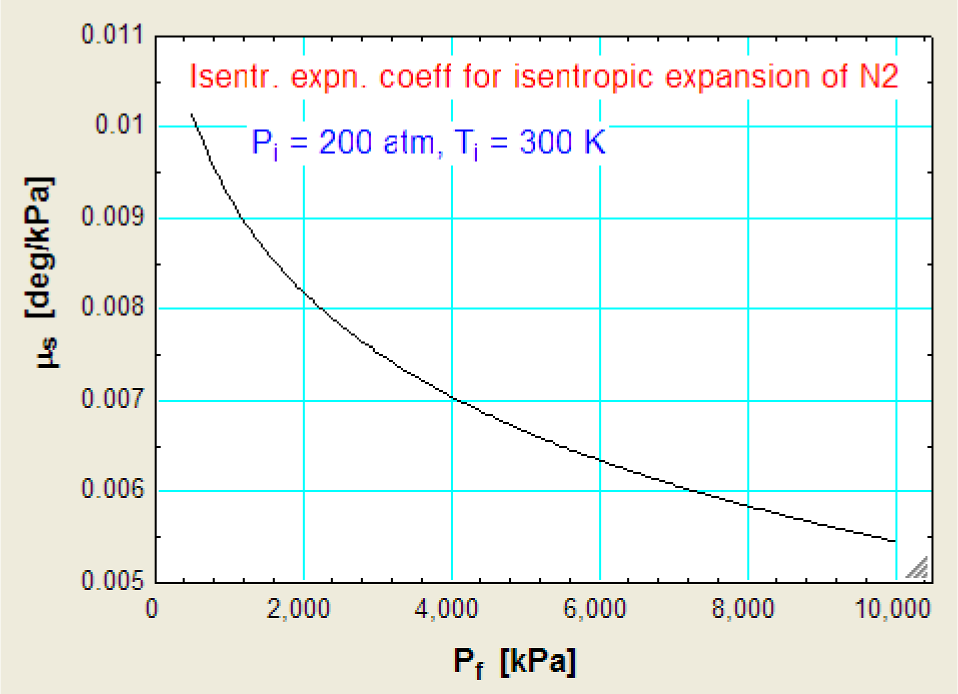
First, compute the Parametric Table:



1..20	1 P_f [kPa]	2 ΔT_s [deg]	3 ΔP [kPa]	4 μ_s [deg/kPa]
Run 1	500	200.5	19,765	0.01014
Run 2	1,000	177.7	19,265	0.009227
Run 3	1,500	162	18,765	0.008632
Run 4	2,000	149.5	18,265	0.008185
Run 5	2,500	139	17,765	0.007827
Run 6	3,000	129.9	17,265	0.007526
Run 7	3,500	121.9	16,765	0.007268
Run 8	4,000	114.5	16,265	0.007042
Run 9	4,500	107.8	15,765	0.00684
Run 10	5,000	101.6	15,265	0.006659
Run 11	5,500	95.89	14,765	0.006494
Run 12	6,000	90.48	14,265	0.006343
Run 13	6,500	85.4	13,765	0.006204
Run 14	7,000	80.58	13,265	0.006075
Run 15	7,500	76.01	12,765	0.005955
Run 16	8,000	71.66	12,265	0.005843
Run 17	8,500	67.5	11,765	0.005737
Run 18	9,000	63.51	11,265	0.005638
Run 19	9,500	59.68	10,765	0.005544
Run 20	10,000	56	10,265	0.005455

Now, plot the graphs:





"**Prob.3.2.7** Helium gas at 200 atm and 300 K is expanded isentropically in an expansion engine. Find out the temp drop and the average isentropic expansion coeff. for a final pressure of 100 atm. Then plot these quantities as the final pressure varies from 500 kPa to 10000 kPa, other conditions remaining the same."

"Solution:"

"Data:"

T_i=300 "[K]"

P_i= 200 * 101.325 "[kPa]"

P_f = 100 * 101.325 "[kPa]"

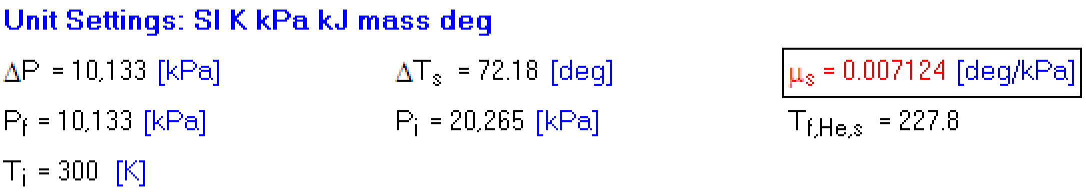
"Calculations:"

CALL Tdrop_isentr('Helium', P_i,T_i, P_f:T_f_He_s, DELTAT_s)

DELTAP = P_i – P_f "kPa"

mu_s = DELTAT_s/DELTAP "deg/kPa…isentr. expn. coeff."

Results:

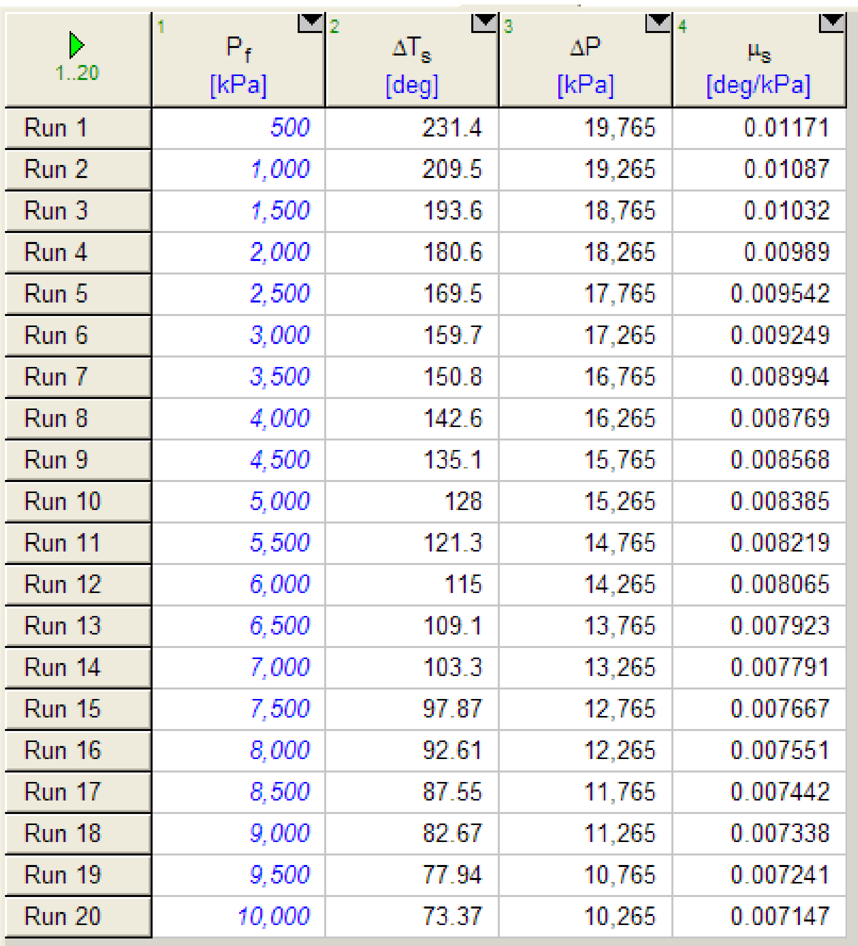


Unit Settings: SI K kPa kJ mass deg

ΔP = 10,133 [kPa]
ΔT_s = 72.18 [deg]
μ_s = 0.007124 [deg/kPa]
P_f = 10,133 [kPa]
P_i = 20,265 [kPa]
$T_{f,He,s}$ = 227.8
T_i = 300 [K]

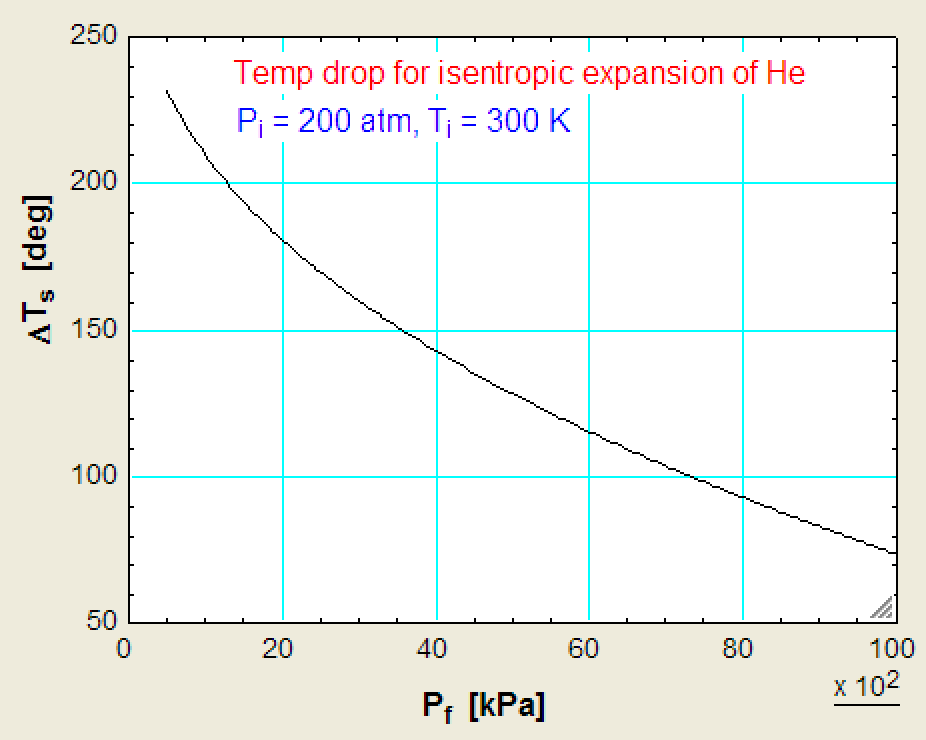
To plot these quantities as the final pressure varies from 500 kPa to 10000 kPa, other conditions remaining the same:

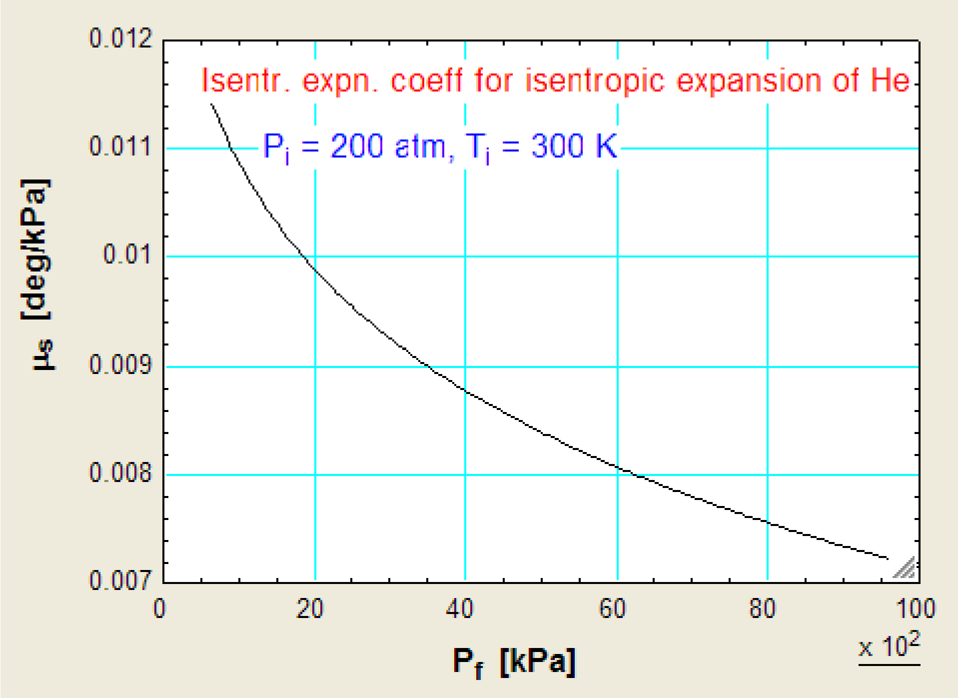
First, compute the Parametric Table:



1..20	1 P_f [kPa]	2 ΔT_s [deg]	3 ΔP [kPa]	4 μ_s [deg/kPa]
Run 1	500	231.4	19,765	0.01171
Run 2	1,000	209.5	19,265	0.01087
Run 3	1,500	193.6	18,765	0.01032
Run 4	2,000	180.6	18,265	0.00989
Run 5	2,500	169.5	17,765	0.009542
Run 6	3,000	159.7	17,265	0.009249
Run 7	3,500	150.8	16,765	0.008994
Run 8	4,000	142.6	16,265	0.008769
Run 9	4,500	135.1	15,765	0.008568
Run 10	5,000	128	15,265	0.008385
Run 11	5,500	121.3	14,765	0.008219
Run 12	6,000	115	14,265	0.008065
Run 13	6,500	109.1	13,765	0.007923
Run 14	7,000	103.3	13,265	0.007791
Run 15	7,500	97.87	12,765	0.007667
Run 16	8,000	92.61	12,265	0.007551
Run 17	8,500	87.55	11,765	0.007442
Run 18	9,000	82.67	11,265	0.007338
Run 19	9,500	77.94	10,765	0.007241
Run 20	10,000	73.37	10,265	0.007147

Now, plot the graphs:





Note that there is always a *temp decrease* during isentropic expansion from 200 atm to any lower, and the isentr. expn. coeff is positive.

"**Prob. 3.2.8** Write an EES Function to calculate the Ideal work required for liquefaction for different gases, starting with an initial pressure P1 (kPa) and temp T1 (K). Using this Function, calculate the ideal work of liquefaction for Air, Argon, Hydrogen, Helium, Oxygen, Methane, Neon and Nitrogen, starting from 1 atm pressure and a temp of 300 K."

Solution:

First, write the EES Function:

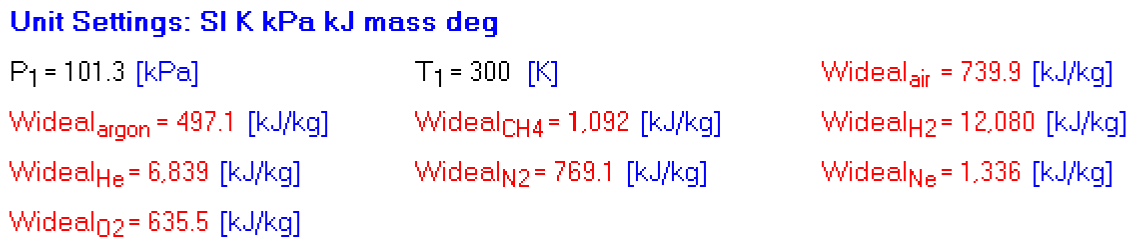
```
$UnitSysystem SI kPa K kg kJ

FUNCTION Ideal_Work(FLUID$, P_1,T_1)

"Inputs: FLUID$, P_1,T_1:…Pressures in kPa, Temp in K"

"Output: W_id (kJ/kg)"

s_1:=Entropy(Fluid$,T=T_1,P=P_1) "[kJ/kg-K]"
```

```
s_f:=Entropy(Fluid$,P=P_1,x=0) “[kJ/kg-K]”

h_1:=Enthalpy(Fluid$,T=T_1,P=P_1) “[kJ/kg]”

h_f:=Enthalpy(Fluid$,P=P_1,x=0) “[kJ/kg]”

Ideal_Work=T_1*(s_1-s_f)-(h_1-h_f) “[kJ/kg]”

END
```

Now, calculate the ideal work required for different gases:

```
“Data:”
T_1=300 “[K]”

P_1=101.325 “[kPa]”

“Calculations:”
Wideal_air = Ideal_Work(‘Air_ha’, P_1,T_1) “kJ/kg”

Wideal_argon = Ideal_Work(‘Argon’, P_1,T_1) “kJ/kg”

Wideal_H2 = Ideal_Work(‘Hydrogen’, P_1,T_1) “kJ/kg”

Wideal_He = Ideal_Work(‘Helium’, P_1,T_1) “kJ/kg”

Wideal_O2 = Ideal_Work(‘Oxygen’, P_1,T_1) “kJ/kg”

Wideal_CH4 = Ideal_Work(‘Methane’, P_1,T_1) “kJ/kg”

Wideal_Ne = Ideal_Work(‘Neon’, P_1,T_1) “kJ/kg”

Wideal_N2 = Ideal_Work(‘Nitrogen’, P_1,T_1) “kJ/kg”
```

Results:



```
Unit Settings: SI K kPa kJ mass deg

P1 = 101.3 [kPa]                  T1 = 300  [K]                  Wideal_air = 739.9 [kJ/kg]
Wideal_argon = 497.1 [kJ/kg]      Wideal_CH4 = 1,092 [kJ/kg]     Wideal_H2 = 12,080 [kJ/kg]
Wideal_He = 6,839 [kJ/kg]         Wideal_N2 = 769.1 [kJ/kg]      Wideal_Ne = 1,336 [kJ/kg]
Wideal_O2 = 635.5 [kJ/kg]
```

Ideal work of liquefaction for different gases, starting with 1 atm and 300 K, are shown above.

"**Prob. 3.2.9** Write an EES Functions to calculate: (i) the Isothermal compressor work, and (ii) the actual compressor work required to compress different gases from an initial pressure P1 (kPa) to a final pressure P2 (kPa) at a temp T1 (K). Take Isothermal efficiency of compressor as eta_isoth. Then, calculate the Isothermal and actual work required to compress Nitrogen from 1 atm to 200 atm at a temp of 300 K, with isoth. efficiency = 80%."

Solution:

First, write the EES Functions:

```
$UnitSysystem SI kPa K kg kJ

FUNCTION Wcomp_isoth(Fluid$, T_1, P_1, P_2)

{Finds Isoth. work for a compressor, Temp in Kelvin, Pressures in kPa

Inlet pressure = P_1, Outlet pressure = P_2, Temp = T_1

 Isoth work in kJ/kg}

s_1:=Entropy(Fluid$,T=T_1,P=P_1) “[kJ/kg-K]”

s_2:=Entropy(Fluid$,P=P_2,T=T_1) “[kJ/kg-K]”

h_1:=Enthalpy(Fluid$,T=T_1,P=P_1) “[kJ/kg]”

h_2:=Enthalpy(Fluid$,P=P_2,T=T_1) “[kJ/kg]”

Wcomp_isoth:= T_1 * (s_1 – s_2) – (h_1 – h_2) “kJ/kg”

END
```

```
$UnitSysystem SI kPa K kg kJ

FUNCTION Wcomp_act(Fluid$, T_1, P_1, P_2,eta_isoth)

{Finds actual work for a compressor, Temp in Kelvin, Pressures in kPa

Inlet pressure = P_1, Outlet pressure = P_2, Temp = T_1, Isoth. effcy = eta_isoth

Actual work in kJ/kg}

Wcomp_act= Wcomp_isoth(Fluid$, T_1, P_1, P_2) / eta_isoth “kJ/kg”

END
```

Now, calculate the isothermal work required for Nitrogen gas:

"Calculations:"

"Data:"

Fluid$ = 'Nitrogen'

T1 = 300 "K"

P1 = 101.325 "kPa"

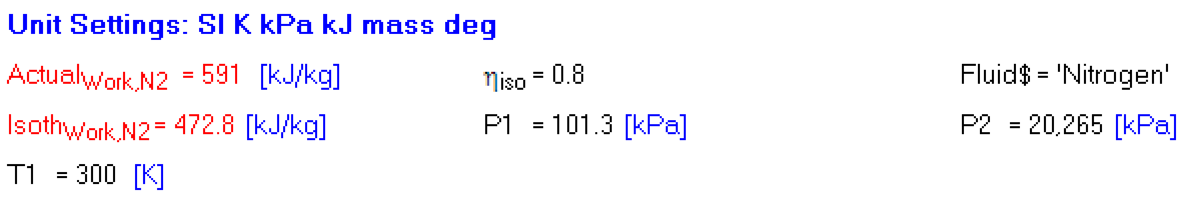
P2 = 200 * 101.325 "kPa"

eta_iso = 0.8 "…Isothermal effcy."

Isoth_Work_N2 = Wcomp_isoth(Fluid$, T1, P1, P2)

Actual_Work_N2 = Wcomp_act(Fluid$, T1, P1, P2, eta_iso)

Results:



Unit Settings: SI K kPa kJ mass deg

$Actual_{Work,N2}$ = 591 [kJ/kg]
η_{iso} = 0.8
Fluid$ = 'Nitrogen'
$Isoth_{Work,N2}$ = 472.8 [kJ/kg]
P1 = 101.3 [kPa]
P2 = 20,265 [kPa]
T1 = 300 [K]

Thus:

Isothermal work required to compress N2 from 1 atm to 200 atm at 300 K = 472.8 kJ/kg…Ans.

Actual work required to compress N2 from 1 atm to 200 atm at 300 K, when isoth. effcy is 80% = 591 kJ/kg…Ans.

"**Prob. 3.2.10** Calculate the Isothermal work required to compress Helium from 1 atm to 25 atm at a temp of 300 K.Then, plot the Isothermal work vs P2, as P2 varies from 200 kPa to 3000 kPa, P1 and T1 remaining the same as earlier."

"Calculations:"

```
Fluid$ = 'Helium'

T1 = 300 "K"

P1 = 101.325 "kPa"

P2 = 25 * 101.325 "kPa"

Isoth_Work_He = Wcomp_isoth(Fluid$, T1, P1, P2)
```

Results:

Unit Settings: SI K kPa kJ mass deg

Fluid$ = 'Helium'

P1 = 101.3 [kPa]

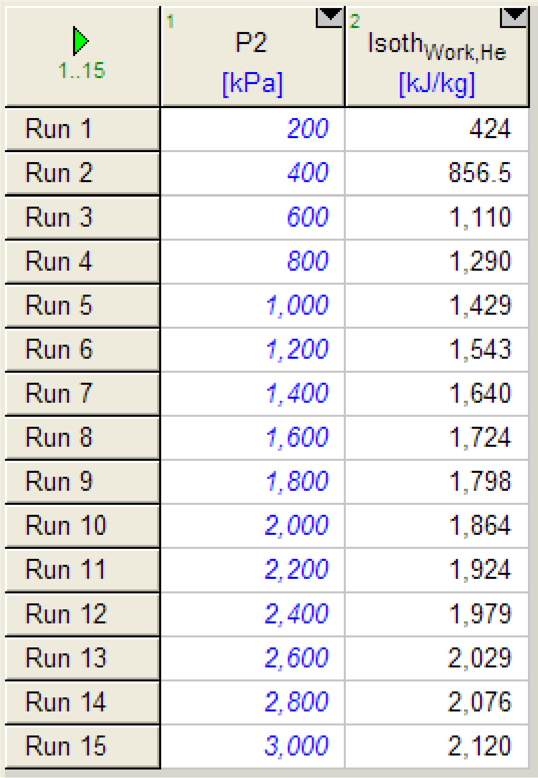
T1 = 300 [K]

$Isoth_{Work,He}$ = 2,013 [kJ/kg]

P2 = 2,533 [kPa]

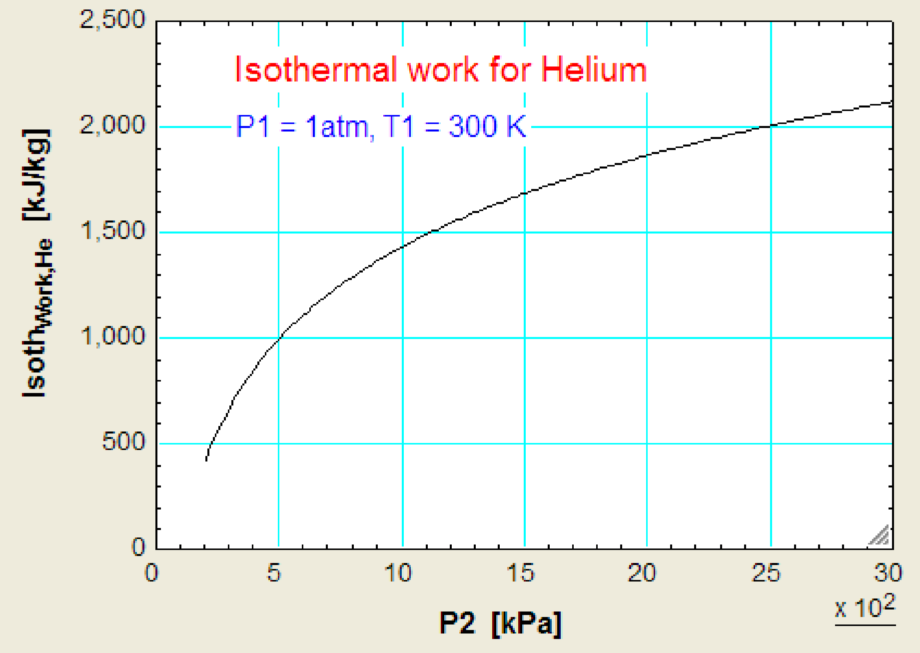
(b) Plot the Isoth. work against P2 as P2 varies from 200 kPa to 3000 kPa, P1 and T1 remaining the same as earlier.

First, compute the Parametric Table:



1..15	1 P2 [kPa]	2 $Isoth_{Work,He}$ [kJ/kg]
Run 1	200	424
Run 2	400	856.5
Run 3	600	1,110
Run 4	800	1,290
Run 5	1,000	1,429
Run 6	1,200	1,543
Run 7	1,400	1,640
Run 8	1,600	1,724
Run 9	1,800	1,798
Run 10	2,000	1,864
Run 11	2,200	1,924
Run 12	2,400	1,979
Run 13	2,600	2,029
Run 14	2,800	2,076
Run 15	3,000	2,120

Now, plot the graph:



“**Prob. 3.2.11** Determine the Ideal work required to liquefy Helium starting from 1 atm, 20 K. Also, find the heat rejected in the ideal isothermal compressor.”

Solution:

“Data:”

Fluid$ = ‘Helium’

T1 = 20 “K”

P1 = 101.325 “kPa”

“Calculations:”

Wideal_He = Ideal_Work(Fluid$, P1,T1) “kJ/kg”

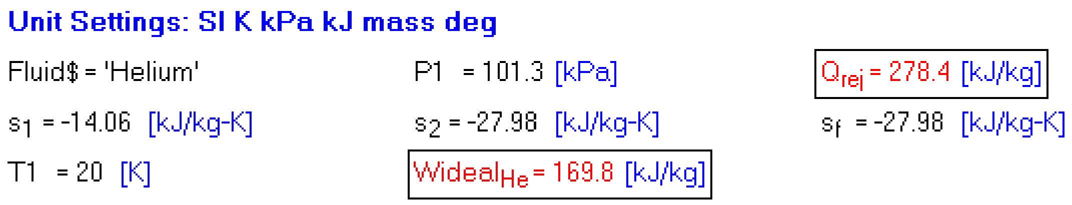
s_1 = Entropy(Fluid$,T=T1,P=P1) “[kJ/kg-K]”

s_f =Entropy(Fluid$,P=P1,x=0) “[kJ/kg-K]”

s_2 = s_f

Q_rej = T1 * (s_1 – s_2) “kJ/kg”

Results:



```
Unit Settings: SI K kPa kJ mass deg
Fluid$ = 'Helium'          P1  = 101.3 [kPa]            Qrej = 278.4 [kJ/kg]
s1 = -14.06  [kJ/kg-K]     s2 = -27.98  [kJ/kg-K]       sf  = -27.98  [kJ/kg-K]
T1  = 20  [K]              WidealHe = 169.8 [kJ/kg]
```

“**Prob. 3.2.12** Write an EES Procedure to calculate, in an Ideal Linde system, the fraction of compressed gas which is liquefied, and the work required per kg of gas compressed, work required per kg of gas liquefied, and the Figure of Merit (FOM), to liquefy different gases from an initial pressure P1 (kPa) and a temp T1 (K) assuming isothermal compression, isentropic expansion and 100% efficiency for the components.”

Solution:

$UnitSysystem SI kPa K kg kJ

```
PROCEDURE LindeSystem(FLUID$, P_1,P_2,T_1:y,Wperkggas,Wperkgliq, W_ideal, FOM)

"Inputs: FLUID$, P_1, P_2, T_1:...Pressures in kPa, Temp in K"

"Outputs: y, fraction liquefied, Wperkggas (kJ/kg of gas compressed), Wperkgliq (kJ/kg of liquid), W_ideal is the min. work of liquefaction (kJ/kg), and FOM is the Figure of Merit."

s_1:=Entropy(Fluid$,T=T_1,P=P_1) "[kJ/kg-K]"

s_2:=Entropy(Fluid$,P=P_2,T=T_1) "[kJ/kg-K]"

h_1:=Enthalpy(Fluid$,T=T_1,P=P_1) "[kJ/kg]"

h_2:=Enthalpy(Fluid$,P=P_2,T=T_1) "[kJ/kg]"

h_f:=Enthalpy(Fluid$,x=0,P=P_1) "[kJ/kg]"

y:= (h_1 – h_2) / (h_1 – h_f) "...fraction liquefied"
```

```
Wperkggas:=T_1 * (s_1 – s_2) – (h_1 – h_2) "[kJ/kg]"

Wperkgliq:=(T_1 * (s_1 – s_2 )- (h_1 – h_2)) / y "[kJ/kg]"

W_ideal:= Ideal_Work(FLUID$, P_1,T_1) "kJ/kg…ideal work of liquefaction"

FOM:= W_ideal / Wperkgliq "…Figure of Merit"

END
```

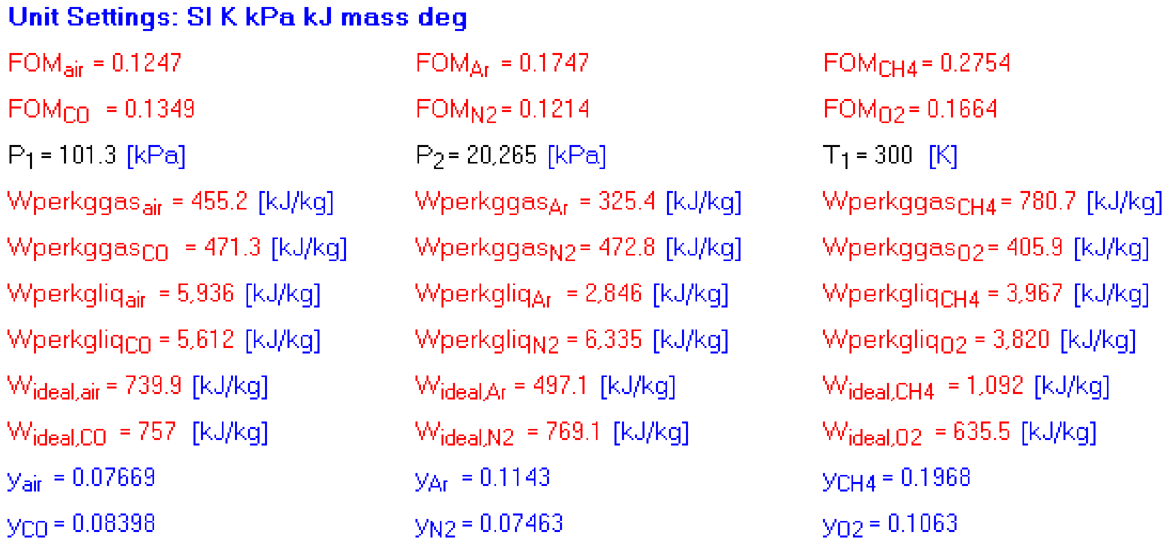
"**Prob. 3.2.13** Using the above EES Procedure, calculate the fraction liquefied etc. for the Ideal Linde cycle, for different gases. Take the initial pressure and temp as P1 = 1 atm and T1 = 300 K, and the final pressure P2 = 200 atm."

Solution:

```
"Data:"

T_1=300 "[K]"

P_1=101.325 "[kPa]"

P_2= 200 * 101.325 "[kPa]"

"Calculations:"

CALL LindeSystem('Nitrogen', P_1,P_2,T_1:y_N2,Wperkggas_N2,Wperkgliq_N2, W_ideal_N2, FOM_N2)

CALL LindeSystem('Air_ha', P_1,P_2,T_1:y_air,Wperkggas_air,Wperkgliq_air, W_ideal_air, FOM_air)

CALL LindeSystem('CarbonMonoxide', P_1,P_2,T_1:y_CO,Wperkggas_CO,Wperkgliq_CO, W_ideal_CO, FOM_CO)

CALL LindeSystem('Argon', P_1,P_2,T_1:y_Ar,Wperkggas_Ar,Wperkgliq_Ar, W_ideal_Ar, FOM_Ar)

CALL LindeSystem('Oxygen', P_1,P_2,T_1:y_O2,Wperkggas_O2,Wperkgliq_O2, W_ideal_O2, FOM_O2)
```

CALL LindeSystem('Methane', P_1,P_2,T_1:y_CH4,Wperkggas_CH4,Wperkgliq_CH4, W_ideal_CH4, FOM_CH4)

Results:



```
Unit Settings: SI K kPa kJ mass deg
FOM_air = 0.1247                   FOM_Ar = 0.1747                   FOM_CH4 = 0.2754
FOM_CO = 0.1349                    FOM_N2 = 0.1214                   FOM_O2 = 0.1664
P_1 = 101.3 [kPa]                  P_2 = 20,265 [kPa]                T_1 = 300 [K]
Wperkggas_air = 455.2 [kJ/kg]      Wperkggas_Ar = 325.4 [kJ/kg]      Wperkggas_CH4 = 780.7 [kJ/kg]
Wperkggas_CO = 471.3 [kJ/kg]       Wperkggas_N2 = 472.8 [kJ/kg]      Wperkggas_O2 = 405.9 [kJ/kg]
Wperkgliq_air = 5,936 [kJ/kg]      Wperkgliq_Ar = 2,846 [kJ/kg]      Wperkgliq_CH4 = 3,967 [kJ/kg]
Wperkgliq_CO = 5,612 [kJ/kg]       Wperkgliq_N2 = 6,335 [kJ/kg]      Wperkgliq_O2 = 3,820 [kJ/kg]
W_ideal,air = 739.9 [kJ/kg]        W_ideal,Ar = 497.1 [kJ/kg]        W_ideal,CH4 = 1,092 [kJ/kg]
W_ideal,CO = 757 [kJ/kg]           W_ideal,N2 = 769.1 [kJ/kg]        W_ideal,O2 = 635.5 [kJ/kg]
y_air = 0.07669                    y_Ar = 0.1143                     y_CH4 = 0.1968
y_CO = 0.08398                     y_N2 = 0.07463                    y_O2 = 0.1063
```

Now, above results are presented conveniently in a tabular form below:

P1 = 1 atm, T1 = 300 K, P2 = 200 atm, T2 = 300 K

Fluid	Liquid yield $y = mf/m$	Work/mass compressed (kJ/kg)	Work/mass liquefied (kJ/kg)	Ideal Work required (kJ/kg)	FOM = Work per mass liq./Ideal work
Nitrogen	0.07463	472.8	6335	769.1	0.1214
Air	0.07669	455.2	5936	739.9	0.1247
CO	0.08398	471.3	5612	757	0.1349
Argon	0.1143	325.4	2846	497.1	0.1747
Oxygen	0.1063	405.9	3820	635.5	0.1664
Methane	0.1968	780.7	3967	1092	0.2754

"**Prob. 3.2.14** Calculate the fraction liquefied etc. for the Ideal Linde cycle, for Nitrogen gas. Take the initial pressure and temp as P1 = 1 atm and T1 = 300 K, and the final pressure P2 = 200 atm. P. Also plot the liquid yield and FOM against the final pressure as the final pressure varies from 100 bar to 200 bar."

Solution:

"Data:"

T_1=300 "[K]"

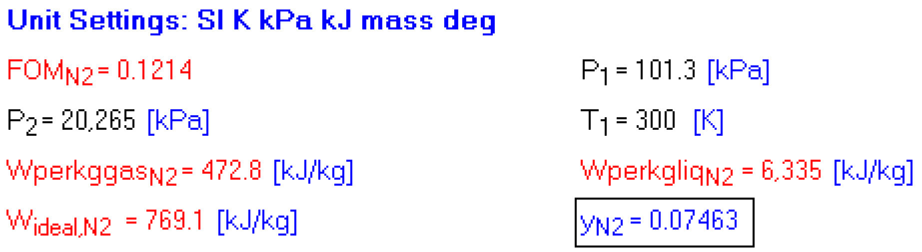
P_1=101.325 "[kPa]"

P_2= 200 * 101.325 "[kPa]"

"Calculations:"

CALL LindeSystem('Nitrogen', P_1,P_2,T_1:y_N2,Wperkggas_N2,Wperkgliq_N2, W_ideal_N2, FOM_N2)

Results:



Unit Settings: SI K kPa kJ mass deg

FOM_{N2} = 0.1214

P_1 = 101.3 [kPa]

P_2 = 20,265 [kPa]

T_1 = 300 [K]

$Wperkggas_{N2}$ = 472.8 [kJ/kg]

$Wperkgliq_{N2}$ = 6,335 [kJ/kg]

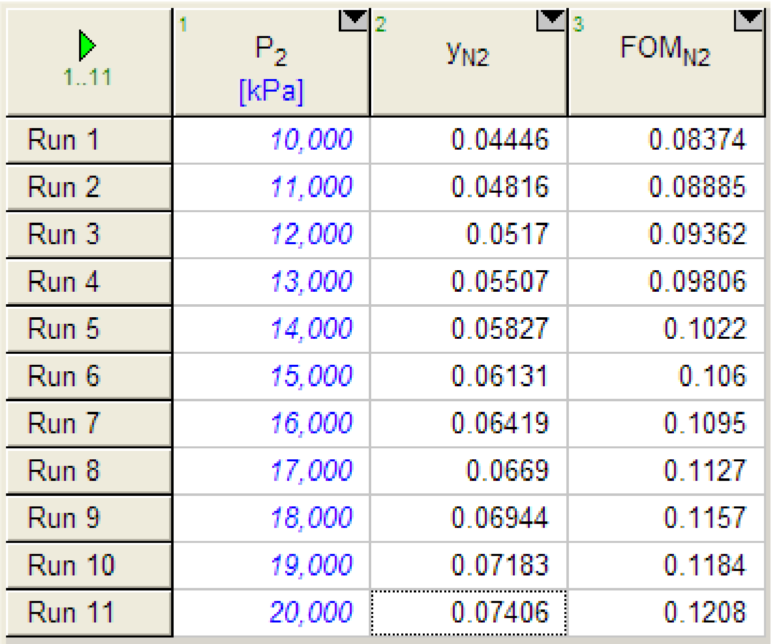
$W_{ideal,N2}$ = 769.1 [kJ/kg]

y_{N2} = 0.07463

Thus: Liquid yield = 0.07463 = 7.463%…Ans.

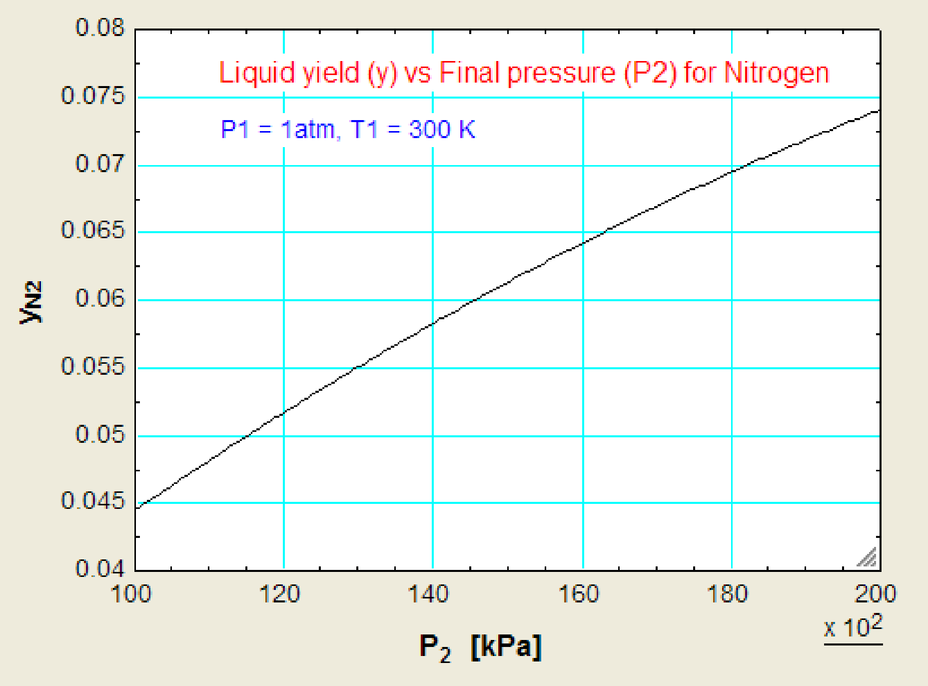
Now, to plot y and FOM against final pressure P2:

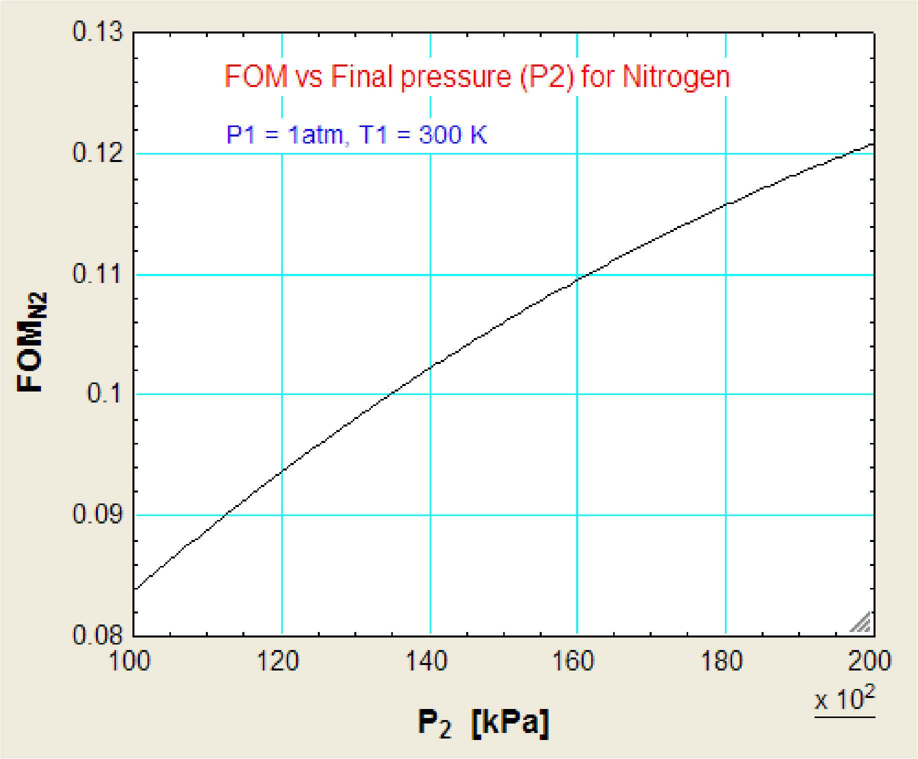
First, compute the Parametric Table:



1..11	1 P_2 [kPa]	2 y_{N2}	3 FOM_{N2}
Run 1	10,000	0.04446	0.08374
Run 2	11,000	0.04816	0.08885
Run 3	12,000	0.0517	0.09362
Run 4	13,000	0.05507	0.09806
Run 5	14,000	0.05827	0.1022
Run 6	15,000	0.06131	0.106
Run 7	16,000	0.06419	0.1095
Run 8	17,000	0.0669	0.1127
Run 9	18,000	0.06944	0.1157
Run 10	19,000	0.07183	0.1184
Run 11	20,000	0.07406	0.1208

Now, plot the results:





Prob. 3.2.15 Determine the liquid yield, the work per unit mass compressed, the work per unit mass liquefied, and the figure of merit for a simple Linde-Hampson system using Argon as the working fluid. The system operates between 101.3 kPa and 293 K at point 1 and 20.67 MPa at point 2. The system may be assumed reversible, except for the expansion through the expansion valve. [Pune Univ. 2013]

Also plot the liquid yield and FOM against the final pressure as the final pressure varies from 100 bar to 210 bar.

Solution:

“Data:”

T_1=293 “[K]”

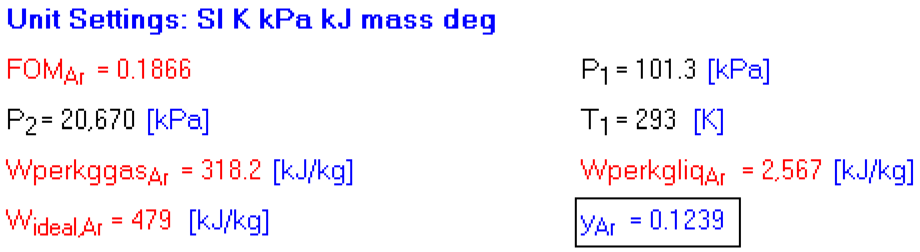
P_1=101.3 “[kPa]”

P_2= 20670 “[kPa]”

"Calculations:"

CALL LindeSystem('Argon', P_1,P_2,T_1:y_Ar,Wperkggas_Ar,Wperkgliq_Ar, W_ideal_Ar, FOM_Ar)

Results:



Unit Settings: SI K kPa kJ mass deg

FOM_{Ar} = 0.1866

P_1 = 101.3 [kPa]

P_2 = 20,670 [kPa]

T_1 = 293 [K]

$Wperkggas_{Ar}$ = 318.2 [kJ/kg]

$Wperkgliq_{Ar}$ = 2,567 [kJ/kg]

$W_{ideal,Ar}$ = 479 [kJ/kg]

y_{Ar} = 0.1239

Thus:

The liquid yield, y = 0.1239 = 12.39%…Ans.

Work per unit mass compressed, Wperkggas = 318.2 kJ/kg…Ans.

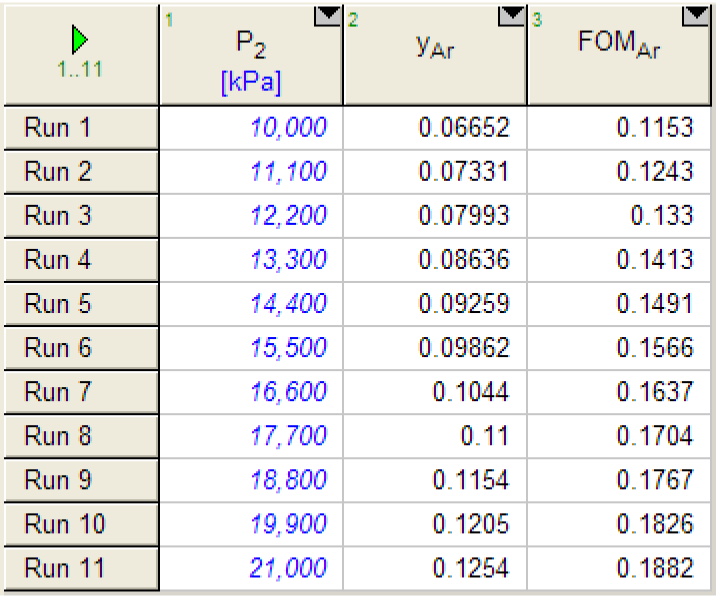
Work per unit mass liquefied, Wperkgliq = 2567 kJ/kg…Ans.

Figure of Merit = FOM = 0.1866…Ans.

……

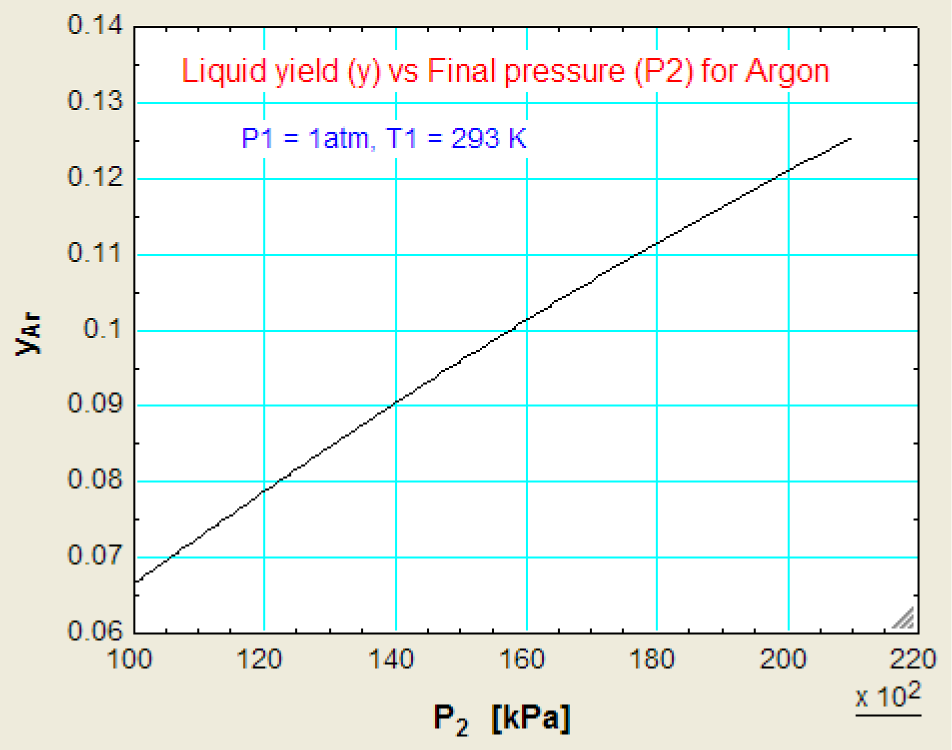
Now, to plot y and FOM against final pressure P2:

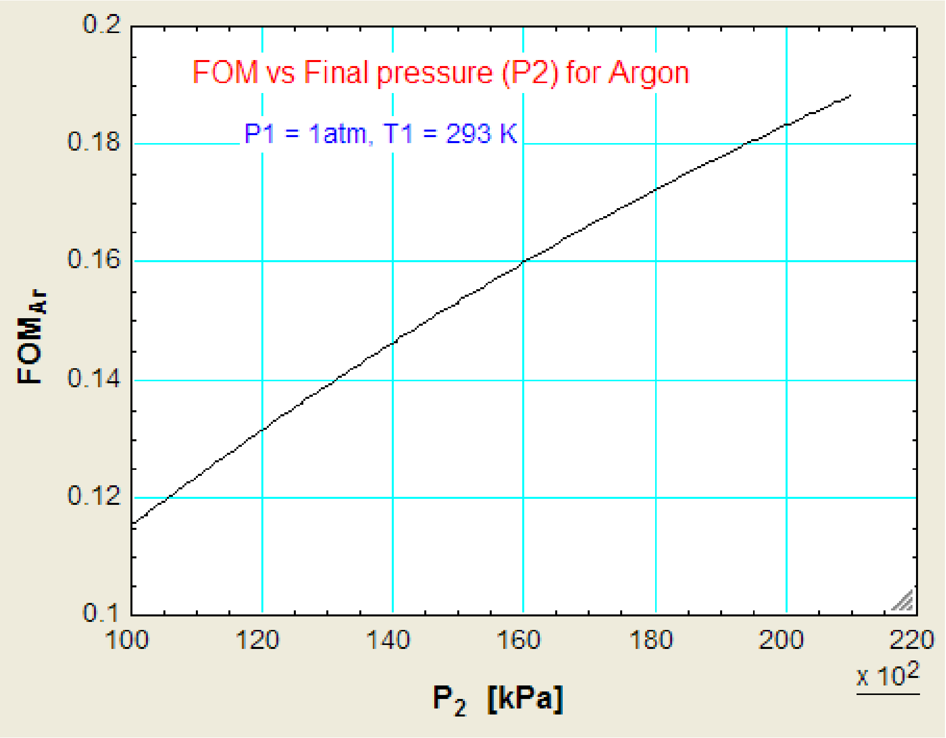
First, compute the Parametric Table:



▶ 1..11	1 P_2 [kPa]	2 y_{Ar}	3 FOM_{Ar}
Run 1	10,000	0.06652	0.1153
Run 2	11,100	0.07331	0.1243
Run 3	12,200	0.07993	0.133
Run 4	13,300	0.08636	0.1413
Run 5	14,400	0.09259	0.1491
Run 6	15,500	0.09862	0.1566
Run 7	16,600	0.1044	0.1637
Run 8	17,700	0.11	0.1704
Run 9	18,800	0.1154	0.1767
Run 10	19,900	0.1205	0.1826
Run 11	21,000	0.1254	0.1882

Now, plot the results:





"**Prob. 3.2.16** Plot liquid yield vs compression temp (T1) for a simple Linde-Hampson system using Nitrogen as the working fluid. The system operates between P1 = 1 atm (i.e. 101.3 kPa) and T1 = 110 K to 330 K at point 1 and P2 = 200, 150, 100 and 50 atm. The system may be assumed reversible, except for the expansion through the expansion valve. And, assume a heat exchanger effectiveness of 100%."

"Solution:"

"Data:"

T1=330 "[K]"

P1=101.3 "[kPa]"

P2= 200 * 101.3 "[kPa]"

"Calculations:"

CALL LindeSystem('Nitrogen', P1,P2,T1:y_N2,Wperkggas_N2,Wperkgliq_N2, W_ideal_N2, FOM_N2)

Results:

Unit Settings: SI K kPa kJ mass deg

FOM_{N2} = 0.0929
P1 = 101.3 [kPa]
P2 = 20,260 [kPa]
T1 = 330 [K]
$Wperkggas_{N2}$ = 522.9 [kJ/kg]
$Wperkgliq_{N2}$ = 9,589 [kJ/kg]
$W_{ideal,N2}$ = 890.8 [kJ/kg]
y_{N2} = 0.05453

Note that liquid yield is 0.05453 = 5.453% when P2 = 200 atm, T1 = T2 = 330 K.

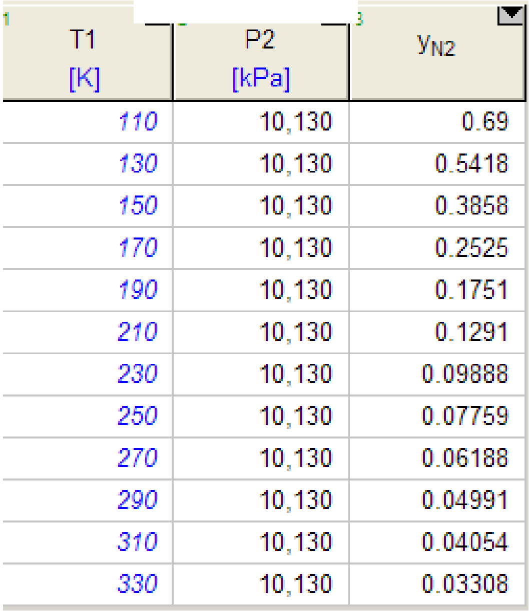
To plot the liquid yield vs T1 (=T2) for different values of P2:

First, compute the Parametric Tables:

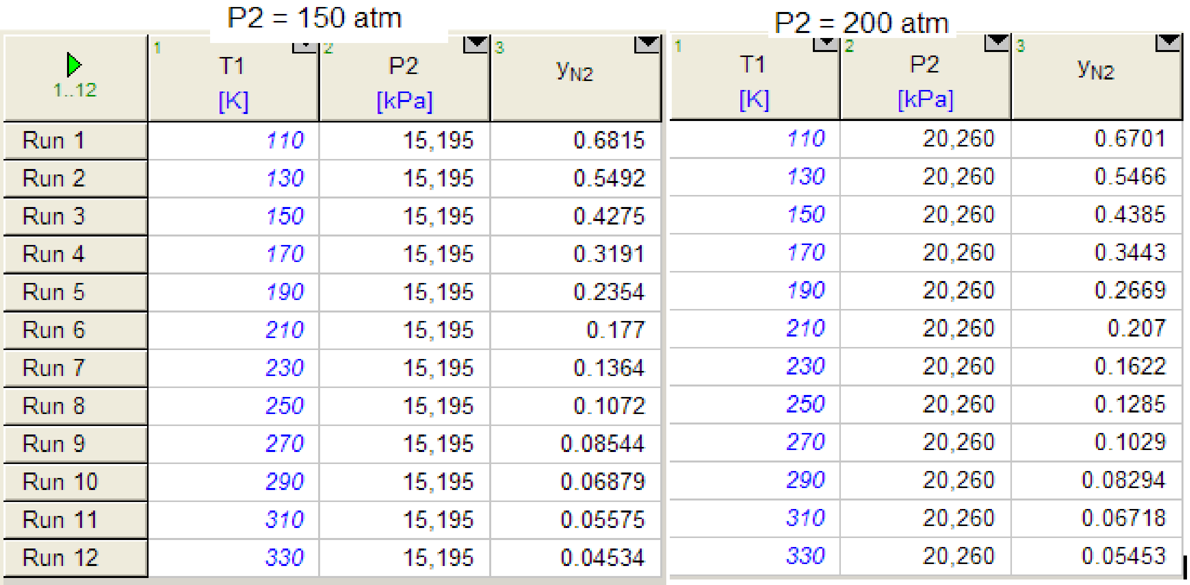
P2 = 50 atm

1..12	T1 [K]	P2 [kPa]	y_{N2}
Run 1	110	5,065	0.6929
Run 2	130	5,065	0.4887
Run 3	150	5,065	0.1938
Run 4	170	5,065	0.1229
Run 5	190	5,065	0.0878
Run 6	210	5,065	0.06612
Run 7	230	5,065	0.05134
Run 8	250	5,065	0.04067
Run 9	270	5,065	0.03267
Run 10	290	5,065	0.0265
Run 11	310	5,065	0.02164
Run 12	330	5,065	0.01775

P2 = 100 atm



T1 [K]	P2 [kPa]	y_{N2}
110	10,130	0.69
130	10,130	0.5418
150	10,130	0.3858
170	10,130	0.2525
190	10,130	0.1751
210	10,130	0.1291
230	10,130	0.09888
250	10,130	0.07759
270	10,130	0.06188
290	10,130	0.04991
310	10,130	0.04054
330	10,130	0.03308



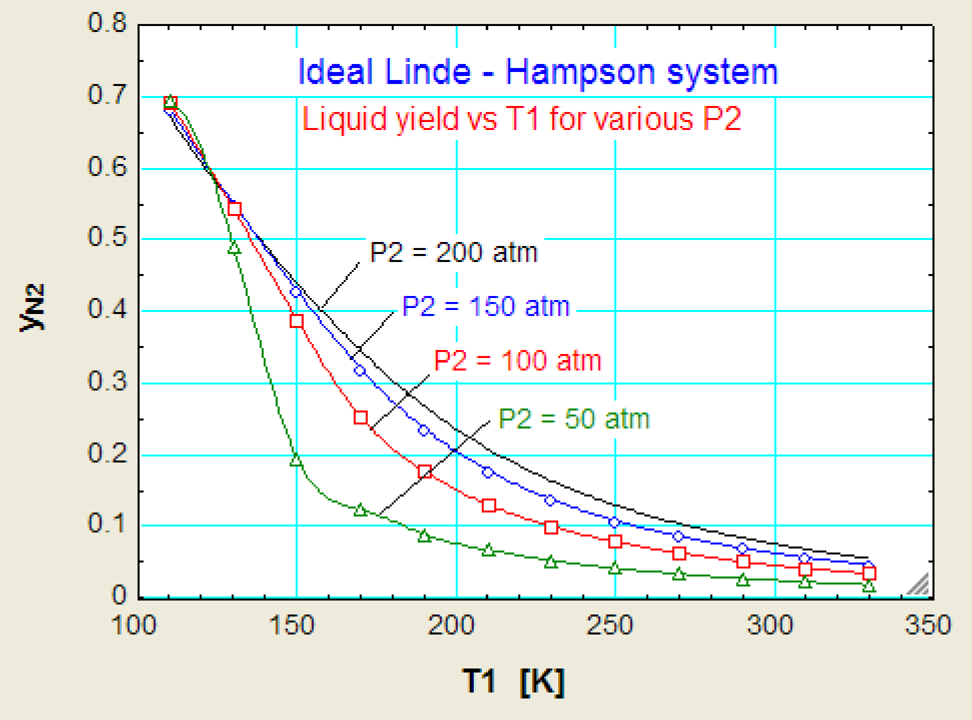
P2 = 150 atm

1..12	T1 [K]	P2 [kPa]	y_{N2}
Run 1	110	15,195	0.6815
Run 2	130	15,195	0.5492
Run 3	150	15,195	0.4275
Run 4	170	15,195	0.3191
Run 5	190	15,195	0.2354
Run 6	210	15,195	0.177
Run 7	230	15,195	0.1364
Run 8	250	15,195	0.1072
Run 9	270	15,195	0.08544
Run 10	290	15,195	0.06879
Run 11	310	15,195	0.05575
Run 12	330	15,195	0.04534

P2 = 200 atm

T1 [K]	P2 [kPa]	y_{N2}
110	20,260	0.6701
130	20,260	0.5466
150	20,260	0.4385
170	20,260	0.3443
190	20,260	0.2669
210	20,260	0.207
230	20,260	0.1622
250	20,260	0.1285
270	20,260	0.1029
290	20,260	0.08294
310	20,260	0.06718
330	20,260	0.05453

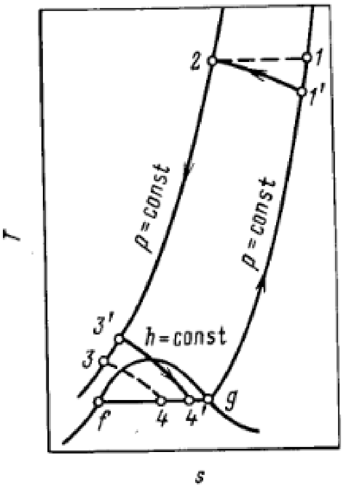
Now, plot the results:



"**Prob.3.2.17** Write an EES Procedure to find the effect of Heat Exchanger effectiveness on the liquid yield, and increase in compressor work per kg of gas compressed, for a simple Linde – Hampson system. Take P1, T1 as initial pressure and temp, and P2 is the final pressure after an isothermal compression. epsilon_HX is the heat exchanger effectiveness.
(b) Then, find the liquid yield and increase in compressor work per kg of mass compressed against heat exchanger effectiveness for Nitrogen. Take P1 = 1 atm, P2 = 200 atm, T1 = 300 K and epsilon_HX = 0.9. Also plot the liquid yield and increase in compressor work against HX effectiveness."

Solution:

Temp – entropy diagram for Linde-Hampson cycle with a HX which is less than 100% effective:



Summary of equations for this case:

HX effectiveness:

$$\varepsilon = \frac{C_c(T'_1 - T_g)}{C_{min}(T_2 - T_g)} = \frac{h'_1 - h_g}{h_1 - h_g},$$

Liquid yield:

$$y = \frac{h'_1 - h_2}{h'_1 - h_f}.$$

Or,

$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) \quad (1 - \varepsilon)(h_1 - h_g)}.$$

Additional compressor work required:

$$\Delta \dot{W} = \dot{m}\ (h_1 - h'_1).$$

Or,

$$\Delta\dot{W} = \dot{m}\,(1-\varepsilon)(h_1 - h_g).$$

First, write the EES Procedure:

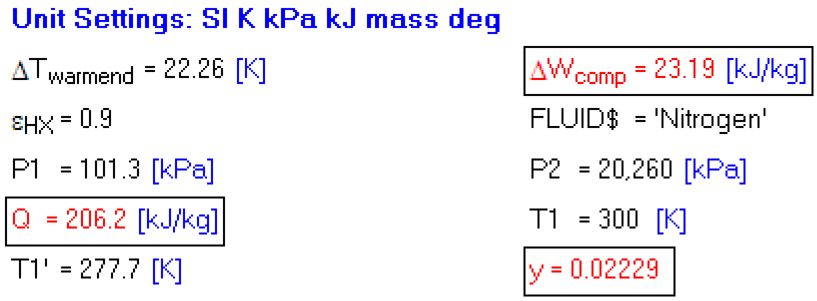
```
PROCEDURE HX_LindeSystem(FLUID$, P1,P2,T1, epsilon_HX: y, DELTAT_warmend, T1_prime,Q,DELTAW_comp)

"Inputs: FLUID$, P1,P2, T1:…Pressures in kPa, Temp in K"
"epsilon_HX = effectiveness of HX, y = liquef. fraction (mf/m)"
"y = liquefn. fraction = mf/m where mf = mass liquefied, m = mass of gas compressed"

"Outputs: DELTAT_warmend = temp diff at warm end of HX, T1_prime = exit temp of low temp stream at the warm end of HX (K)"
"Q = heat transfer in HX (kJ/kg of gas compressed)"
"DELTAW_comp = Additional compressor work required due to HX ineffectiveness (kJ/kg of gas compressed)"

Tsat: = Temperature(FLUID$, P = P1, x = 0))"sat. temp at P1, (K)"

h_f:=Enthalpy(Fluid$,x=0,P=P1) "[kJ/kg]"

h_g:=Enthalpy(Fluid$,x=1,P=P1) "[kJ/kg]"

DELTAT_warmend: = (1 – epsilon_HX) * (T1 – Tsat) "temp drop at warm end of HX (K)"

T1_prime: = T1 – DELTAT_warmend "exit temp of low pressure stream (K)"

h_1_prime:=Enthalpy(Fluid$,T=T1_prime,P=P1) "[kJ/kg]"

h_1: = Enthalpy(Fluid$,T=T1,P=P1) "[kJ/kg]"

h_2: = Enthalpy(Fluid$,T=T1,P=P2) "[kJ/kg]"

y: = (h_1_prime – h_2) / (h_1_prime – h_f) "…liquefn. fraction"

Q: = (1 – y) * (h_1_prime – h_g) "kJ/kg of mass compressed…heat transfer in HX"
```

```
DELTAW_comp: = (h_1 – h_1_prime) "…additional compr. work required due to HX ineffcy…(kJ/kg of mass compressed)"

END

"========================================================================="
```

(b) Now, solve the problem:

```
"Data:"
FLUID$ = 'Nitrogen'
P1 = 101.3 "kPa"
T1 = 300 "K"
P2=200 * 101.3 "kPa"
epsilon_HX = 0.9

"Calculations:"
CALL HX_LindeSystem(FLUID$, P1,P2,T1, epsilon_HX: y, DELTAT_warmend, T1_prime,Q,DELTAW_comp)
```

Results:



Unit Settings: SI K kPa kJ mass deg

$\Delta T_{warmend}$ = 22.26 [K]

ΔW_{comp} = 23.19 [kJ/kg]

ε_{HX} = 0.9

FLUID$ = 'Nitrogen'

P1 = 101.3 [kPa]

P2 = 20,260 [kPa]

Q = 206.2 [kJ/kg]

T1 = 300 [K]

T1' = 277.7 [K]

y = 0.02229

Note that for epsilon_HX = 0.9:

Liquid yield = y = 0.02229 kg/kg gas compressed…Ans.

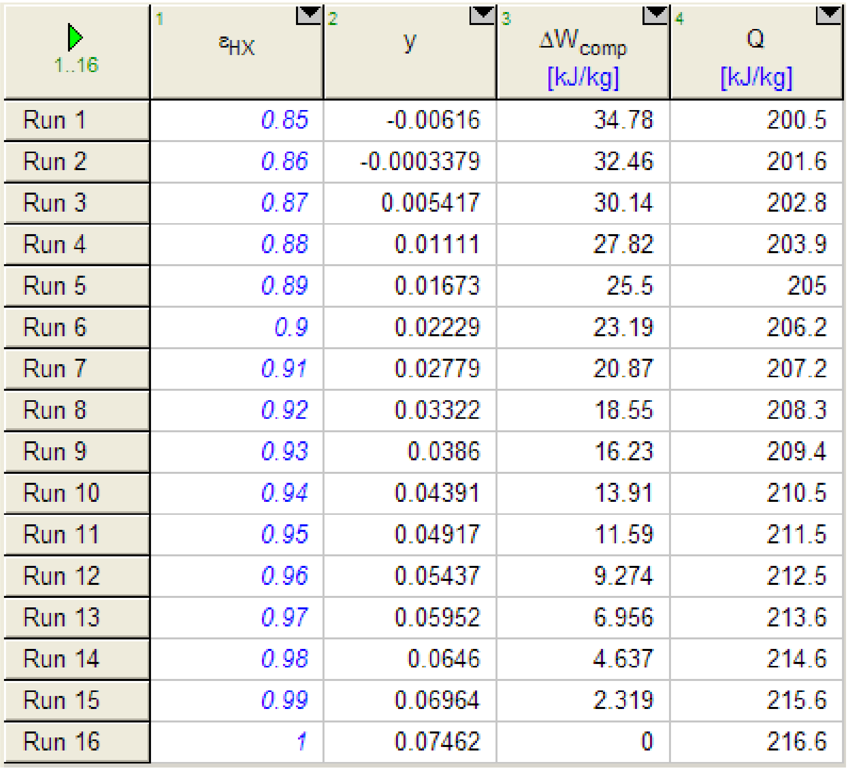
Increase in compressor work = DELTAW_comp = 23.19 kJ/kg of gas compressed…Ans.

Heat transfer in the HX = Q = 206.2 kJ/kg…Ans.

……

To plot y and ΔW_comp against epsilon_HX:

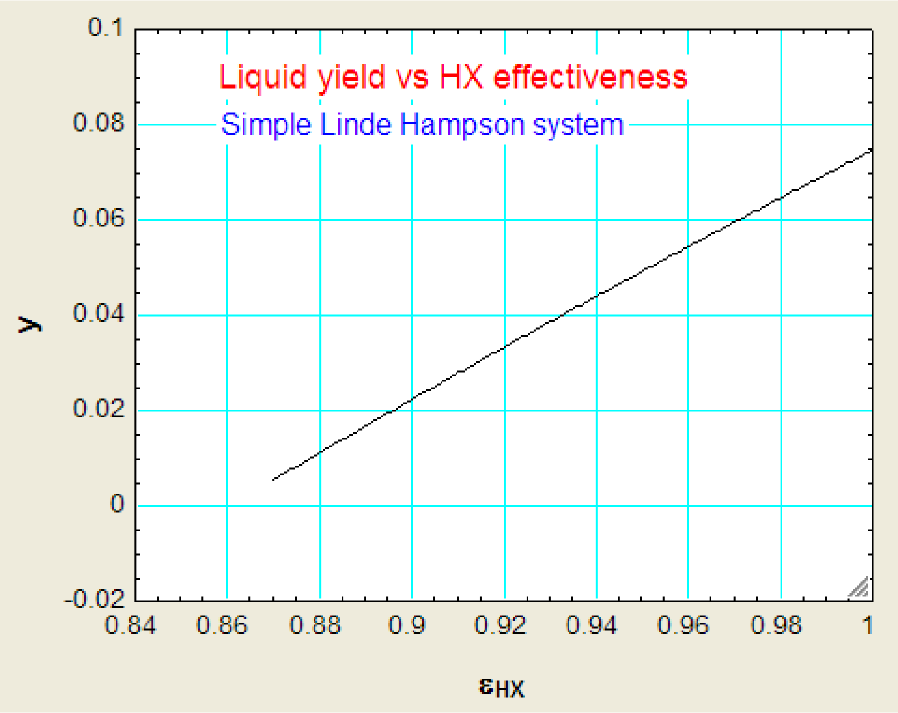
First, compute the Parametric Table:

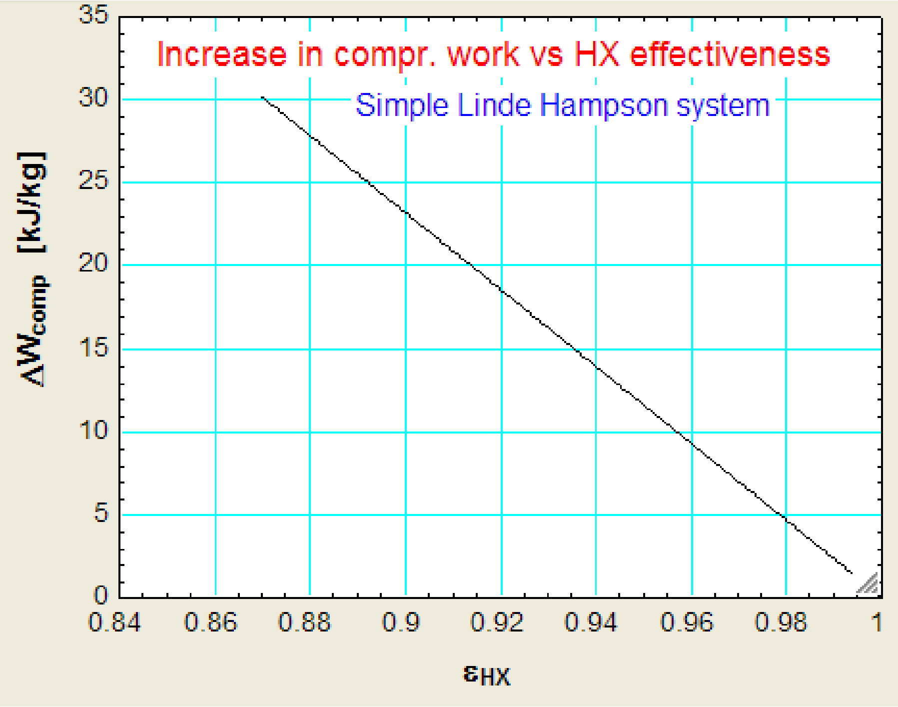


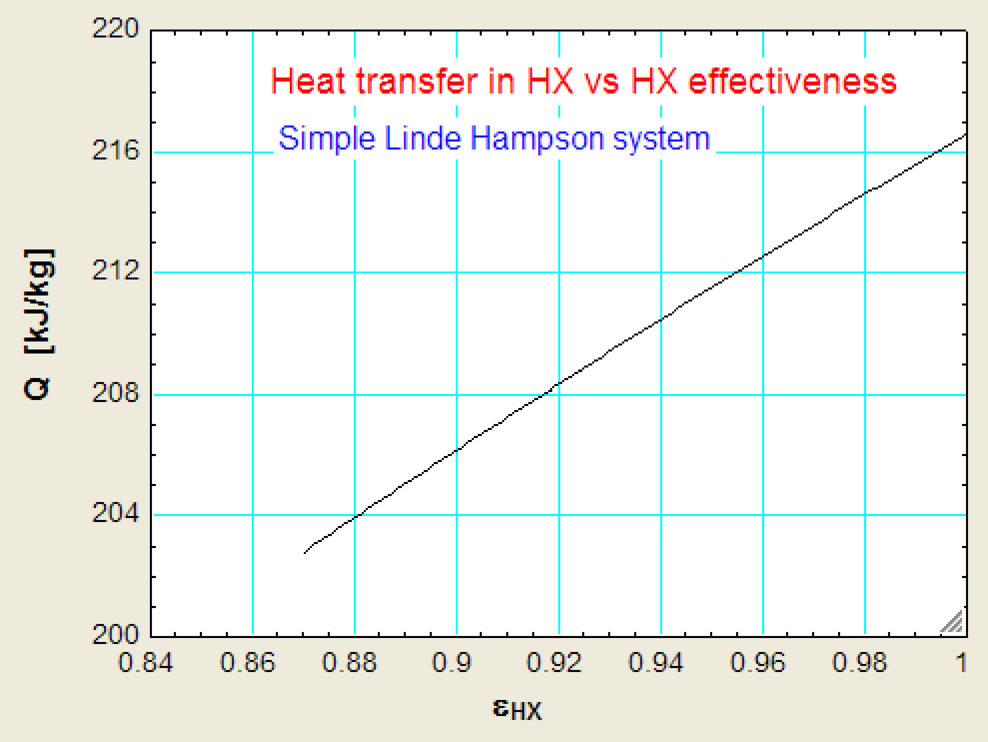
1..16	1 ε_{HX}	2 y	3 ΔW_{comp} [kJ/kg]	4 Q [kJ/kg]
Run 1	0.85	-0.00616	34.78	200.5
Run 2	0.86	-0.0003379	32.46	201.6
Run 3	0.87	0.005417	30.14	202.8
Run 4	0.88	0.01111	27.82	203.9
Run 5	0.89	0.01673	25.5	205
Run 6	0.9	0.02229	23.19	206.2
Run 7	0.91	0.02779	20.87	207.2
Run 8	0.92	0.03322	18.55	208.3
Run 9	0.93	0.0386	16.23	209.4
Run 10	0.94	0.04391	13.91	210.5
Run 11	0.95	0.04917	11.59	211.5
Run 12	0.96	0.05437	9.274	212.5
Run 13	0.97	0.05952	6.956	213.6
Run 14	0.98	0.0646	4.637	214.6
Run 15	0.99	0.06964	2.319	215.6
Run 16	1	0.07462	0	216.6

Note from the above that for HX-effectiveness less than 0.86, *the liquid yield is nil.*

Now, plot the results:






Heat transfer in HX vs HX effectiveness
Simple Linde Hampson system
Q [kJ/kg]
220
216
212
208
204
200
0.84
0.86
0.88
0.9
0.92
0.94
0.96
0.98
1
ε_{HX}

……………………………………………………………………………………………

"**Prob. 3.2.18** Find the liquid yield, work required per unit mass of gas compressed and FOM for a simple Linde-Hampson system using Nitrogen as the working fluid. The system operates between P1 = 1 atm, T1 = 294 K at point 1 and P2 = 200 atm. And, assume heat exchanger effectiveness of 100%,
(b) If the heat exchanger effectiveness is 95%, calculate the liquid yield, compressor work requirement and the FOM."

"Solution:"

"Data:"

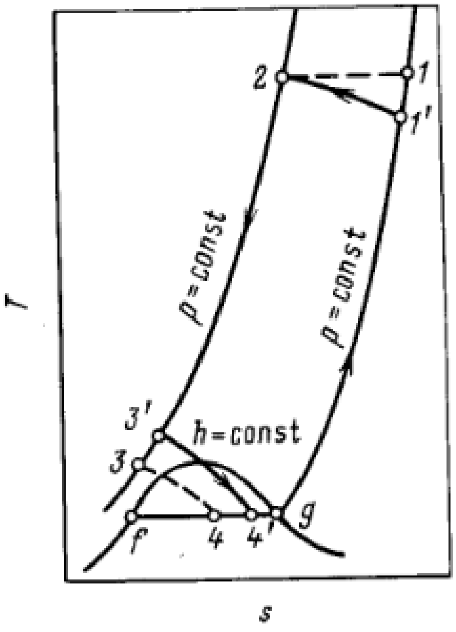
FLUID$ = 'Nitrogen'
P1 = 101.3 "kPa"
T1 = 294 "K"
P2=200 * 101.3 "kPa"
epsilon_HX = 0.95 "…effeciveness of HX"

"Case (i). When the HX is 100% effective:"

CALL LindeSystem('Nitrogen', P1,P2,T1:y_1,Wperkggas_1,Wperkgliq_1, W_ideal_1, FOM_1)

"Case (ii). When the HX is 95% effective:"

Temp – entropy diagram for Linde-Hampson cycle with a HX which is less than 100% effective:



CALL HX_LindeSystem(FLUID$, P1,P2,T1, epsilon_HX: y_2, DELTAT_warmend, T1_prime,Q,DELTAW_comp)

"Work reqd. per kg of gas compressed, for case(ii):"

Wperkggas_2 = Wperkggas_1 + DELTAW_comp "kJ/kg of gas compressed"

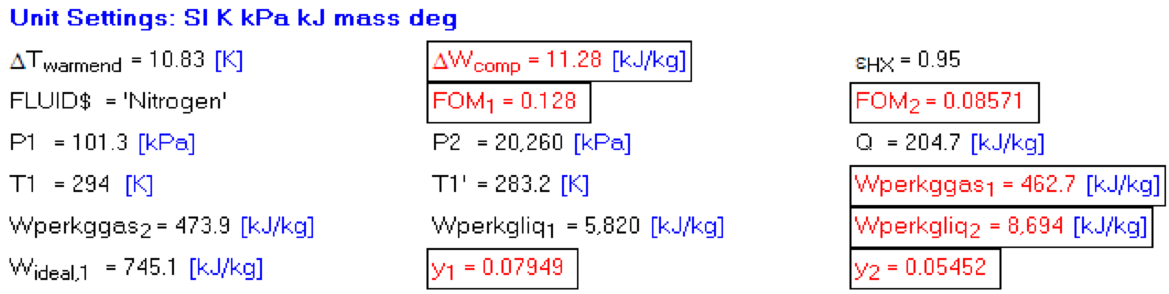
"Work reqd. per kg of gas liquefied, for case(ii):"

Wperkgliq_2 = Wperkggas_2 / y_2

"Then, FOM is given by:"

FOM_2 = W_ideal_1 / Wperkgliq_2

Results:



Unit Settings: SI K kPa kJ mass deg

$\Delta T_{warmend}$ = 10.83 [K]
ΔW_{comp} = 11.28 [kJ/kg]
ε_{HX} = 0.95
FLUID$ = 'Nitrogen'
FOM_1 = 0.128
FOM_2 = 0.08571
P1 = 101.3 [kPa]
P2 = 20,260 [kPa]
Q = 204.7 [kJ/kg]
T1 = 294 [K]
T1' = 283.2 [K]
$Wperkggas_1$ = 462.7 [kJ/kg]
$Wperkggas_2$ = 473.9 [kJ/kg]
$Wperkgliq_1$ = 5,820 [kJ/kg]
$Wperkgliq_2$ = 8,694 [kJ/kg]
$W_{ideal,1}$ = 745.1 [kJ/kg]
y_1 = 0.07949
y_2 = 0.05452

Thus:

For HX effectiveness of 100%:

Liquid yield = y1 = 0.07949 = 7.949%…Ans.

Work/unit mass compressed = Wperkggas1 = 462.7 kJ/kg…Ans.

FOM = FOM1 = 0.128…Ans.

For HX effectiveness of 95%:

Liquid yield = y2 = 0.05452 = 5.452%…Ans.

Work/unit mass compressed = Wperkggas2 = 473.9 kJ/kg…Ans.

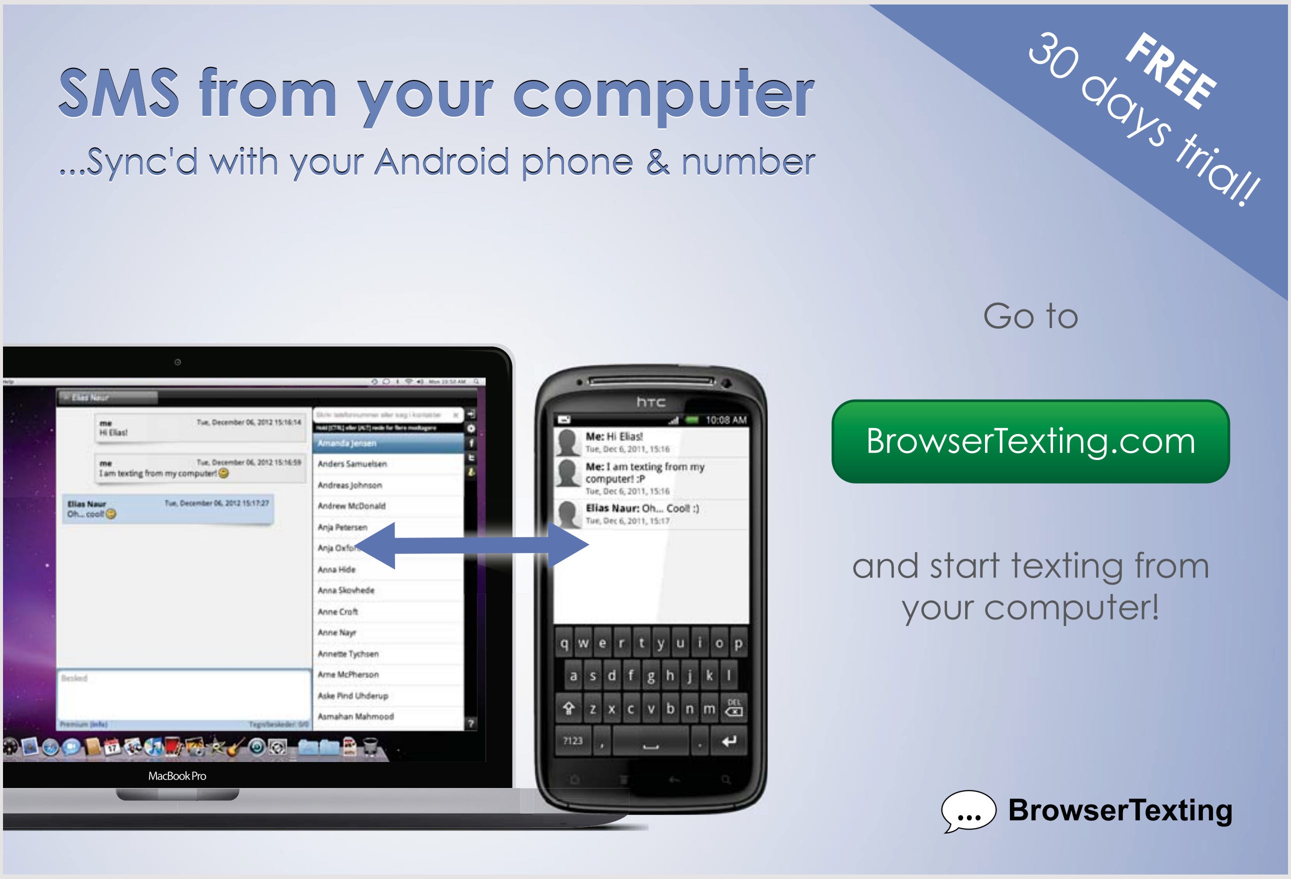
FOM = FOM2 = 0.08571…Ans.

"**Prob.3.2.19** Write an EES Function to find the liquid yield as a function of pressure (P3, kPa) and temp (T3, K) just before the expansion valve in a Linde – Hampson system. And, plot the liquid yield, y against P3 for various values of T3."

Solution:

First, write an EES Function to find out liquid yield as a function of P3, T3 for any fluid:

```
FUNCTION LindeSystem_Liquid_yield_y(FLUID$, P1,P3,T3)

"Inputs: FLUID$, P1, P3, T3:…Pressures in kPa, Temp in K"
"P3, T3…pressure and temp just before the J-T valve"

"Output: y, fraction liquefied"

h3:=Enthalpy(Fluid$,T=T3,P=P3) "[kJ/kg]…enthalpy of fluid approaching J-T valve"

hg:=Enthalpy(Fluid$,P=P1,x=1) "[kJ/kg]…enthalpy of gas exiting the liquid vessel, sat. vap."

hf:=Enthalpy(Fluid$,x=0,P=P1) "[kJ/kg]…enthalpy of liquid exiting the liquid vessel, sat. liq."
```

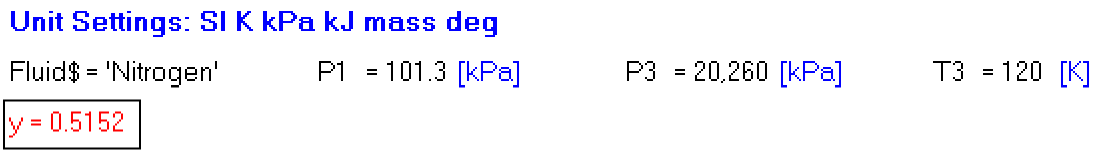


```
LindeSystem_Liquid_yield_y:= (h3 – hg) / (hf – hg) "…fraction liquefied"

END

"=================================================================================="
```

Now, solve the problem:

```
"Data:"

Fluid$ = 'Nitrogen'

P1 = 101.3 "kPa"

P3 = 200 * 101.3 "kPa"

T3 = 120 "K"

y = LindeSystem_Liquid_yield_y(FLUID$, P1,P3,T3)
```

Results:

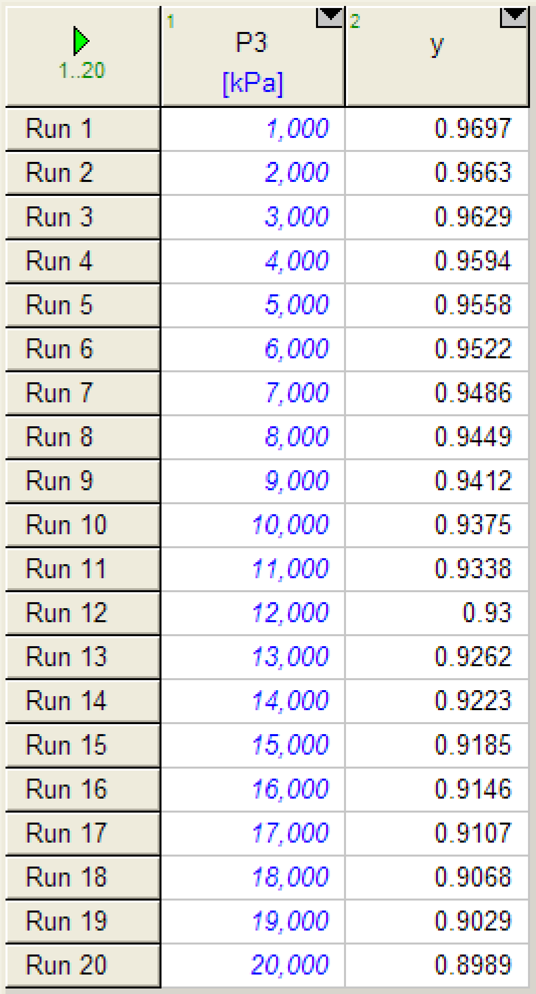


```
Unit Settings: SI K kPa kJ mass deg
Fluid$ = 'Nitrogen'          P1  = 101.3 [kPa]          P3  = 20,260 [kPa]          T3  = 120  [K]
y = 0.5152
```

Now, to plot the liquid yield, y against P3 for various values of T3:

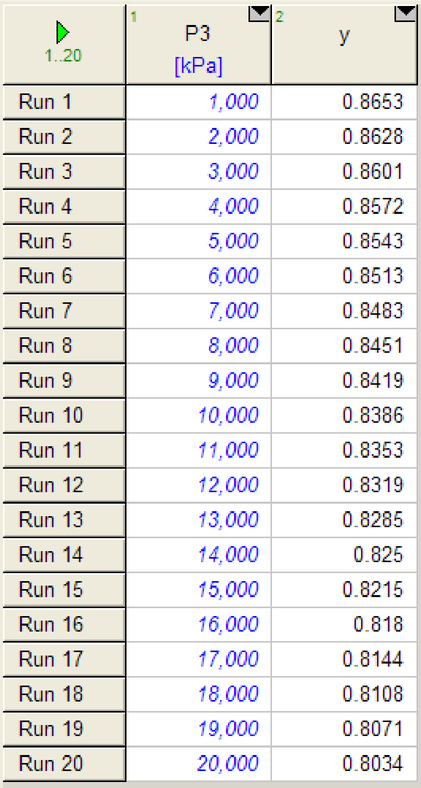
First, compute the Parametric Tables:

T3 = 80 K



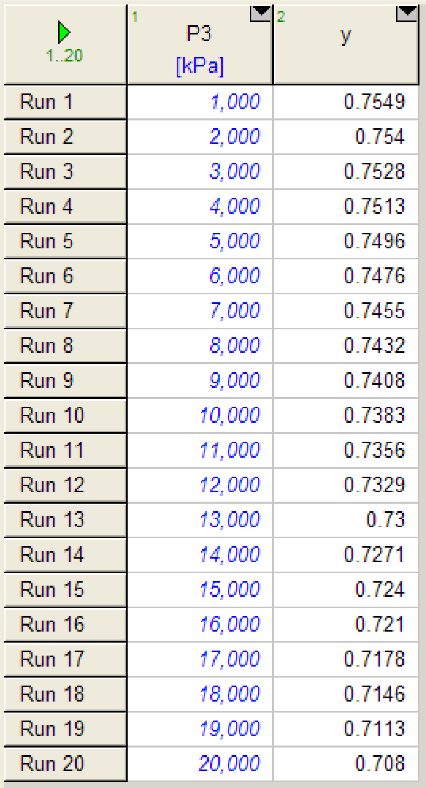
1..20	P3 [kPa]	y
Run 1	1,000	0.9697
Run 2	2,000	0.9663
Run 3	3,000	0.9629
Run 4	4,000	0.9594
Run 5	5,000	0.9558
Run 6	6,000	0.9522
Run 7	7,000	0.9486
Run 8	8,000	0.9449
Run 9	9,000	0.9412
Run 10	10,000	0.9375
Run 11	11,000	0.9338
Run 12	12,000	0.93
Run 13	13,000	0.9262
Run 14	14,000	0.9223
Run 15	15,000	0.9185
Run 16	16,000	0.9146
Run 17	17,000	0.9107
Run 18	18,000	0.9068
Run 19	19,000	0.9029
Run 20	20,000	0.8989

T3 = 90 K

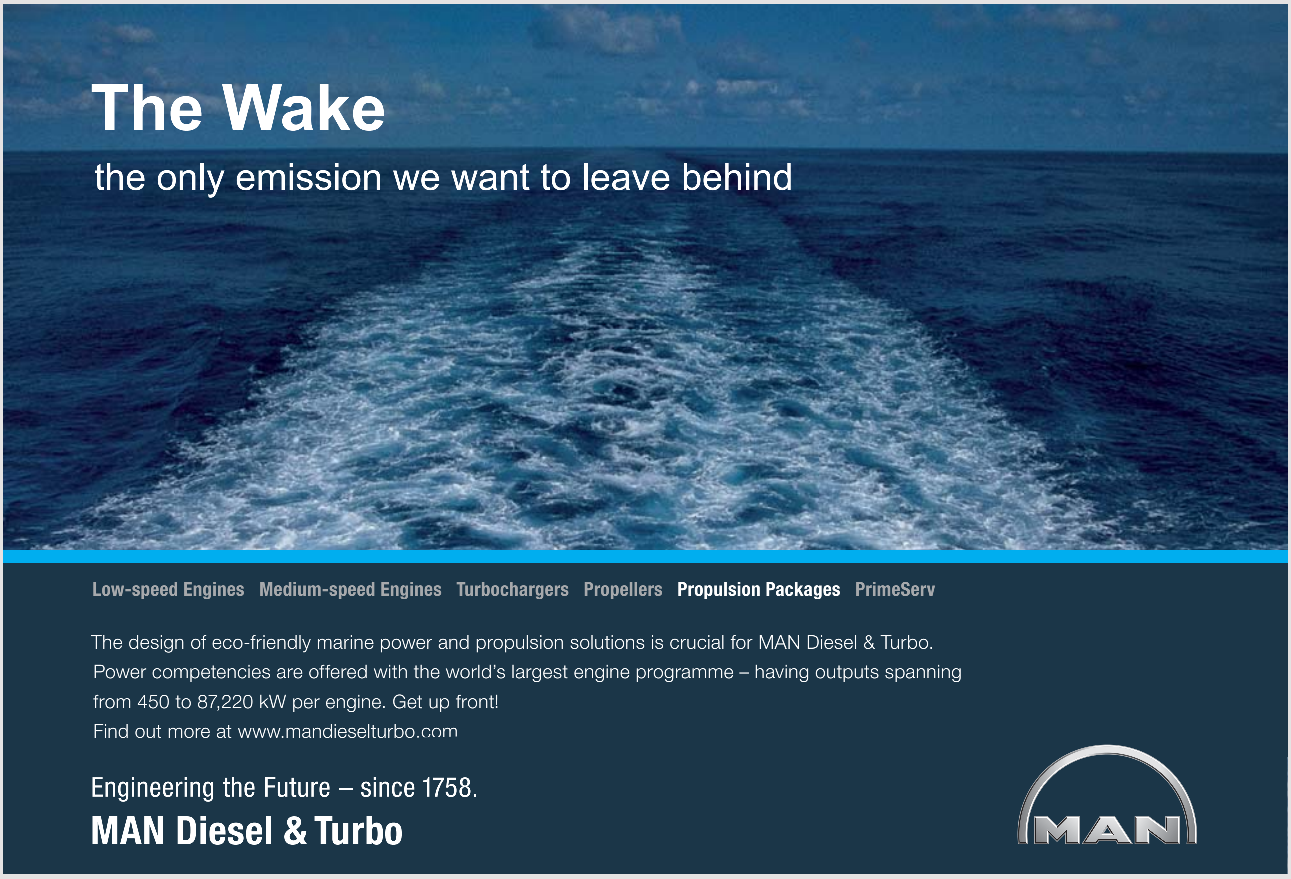


▶ 1..20	1 P3 [kPa]	2 y
Run 1	1,000	0.8653
Run 2	2,000	0.8628
Run 3	3,000	0.8601
Run 4	4,000	0.8572
Run 5	5,000	0.8543
Run 6	6,000	0.8513
Run 7	7,000	0.8483
Run 8	8,000	0.8451
Run 9	9,000	0.8419
Run 10	10,000	0.8386
Run 11	11,000	0.8353
Run 12	12,000	0.8319
Run 13	13,000	0.8285
Run 14	14,000	0.825
Run 15	15,000	0.8215
Run 16	16,000	0.818
Run 17	17,000	0.8144
Run 18	18,000	0.8108
Run 19	19,000	0.8071
Run 20	20,000	0.8034

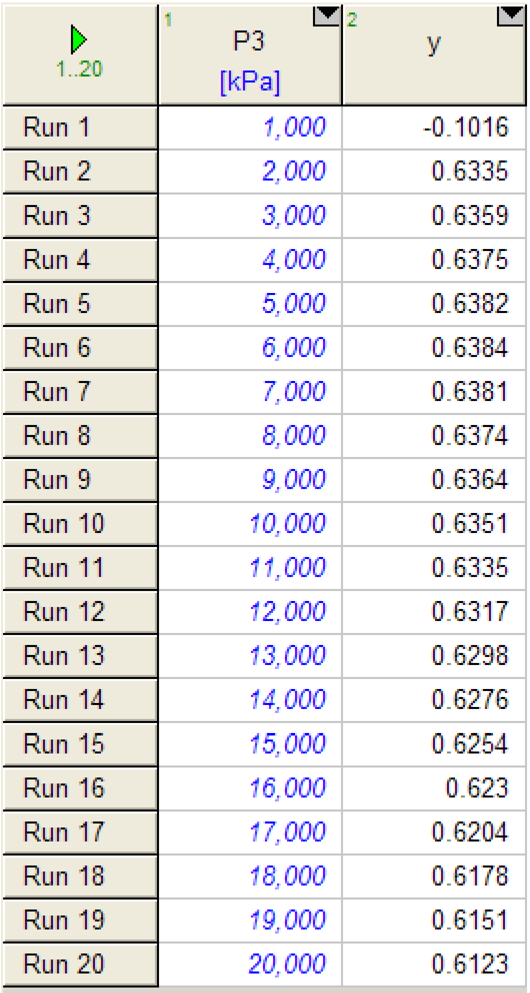
T3 = 100 K



▶ 1..20	1 P3 [kPa]	2 y
Run 1	1,000	0.7549
Run 2	2,000	0.754
Run 3	3,000	0.7528
Run 4	4,000	0.7513
Run 5	5,000	0.7496
Run 6	6,000	0.7476
Run 7	7,000	0.7455
Run 8	8,000	0.7432
Run 9	9,000	0.7408
Run 10	10,000	0.7383
Run 11	11,000	0.7356
Run 12	12,000	0.7329
Run 13	13,000	0.73
Run 14	14,000	0.7271
Run 15	15,000	0.724
Run 16	16,000	0.721
Run 17	17,000	0.7178
Run 18	18,000	0.7146
Run 19	19,000	0.7113
Run 20	20,000	0.708

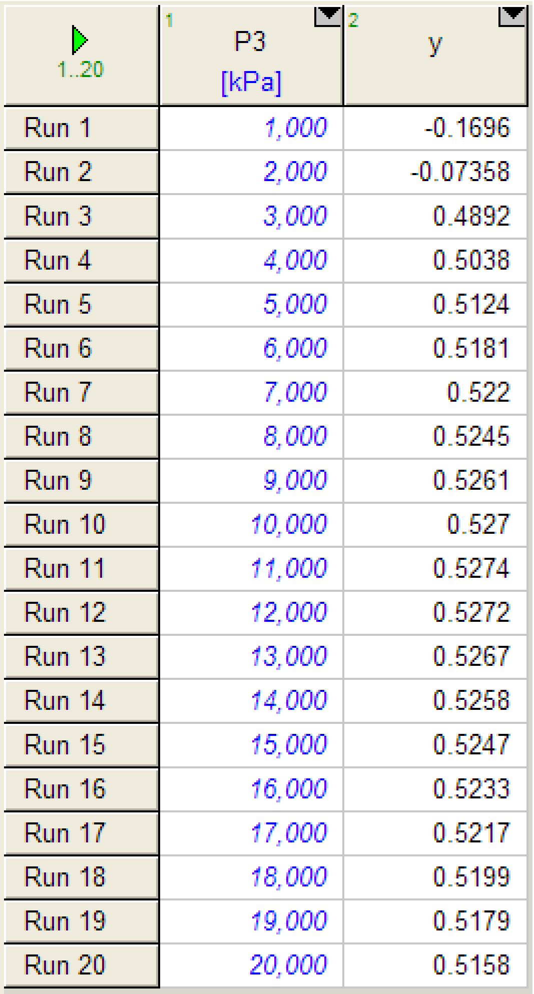


T3 = 110 K



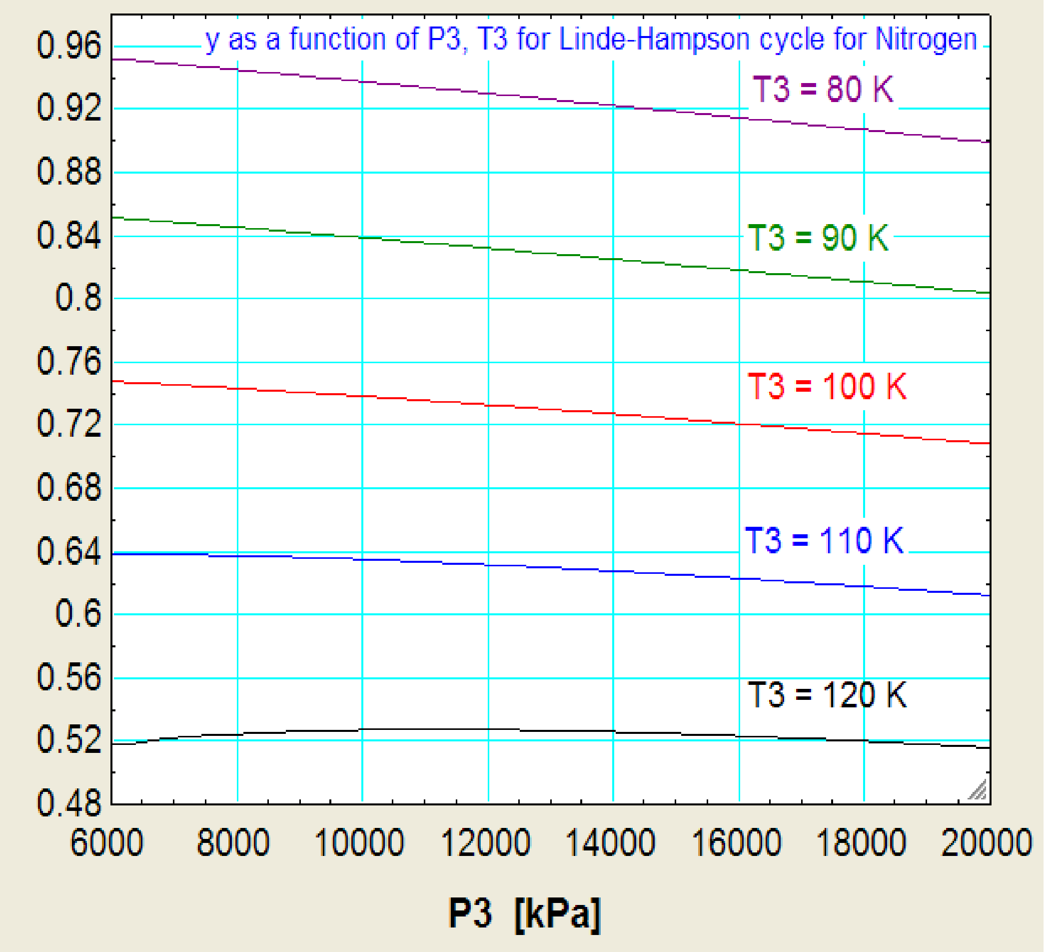
1..20	P3 [kPa]	y
Run 1	1,000	-0.1016
Run 2	2,000	0.6335
Run 3	3,000	0.6359
Run 4	4,000	0.6375
Run 5	5,000	0.6382
Run 6	6,000	0.6384
Run 7	7,000	0.6381
Run 8	8,000	0.6374
Run 9	9,000	0.6364
Run 10	10,000	0.6351
Run 11	11,000	0.6335
Run 12	12,000	0.6317
Run 13	13,000	0.6298
Run 14	14,000	0.6276
Run 15	15,000	0.6254
Run 16	16,000	0.623
Run 17	17,000	0.6204
Run 18	18,000	0.6178
Run 19	19,000	0.6151
Run 20	20,000	0.6123

T3 = 120 K



1..20	P3 [kPa]	y
Run 1	1,000	-0.1696
Run 2	2,000	-0.07358
Run 3	3,000	0.4892
Run 4	4,000	0.5038
Run 5	5,000	0.5124
Run 6	6,000	0.5181
Run 7	7,000	0.522
Run 8	8,000	0.5245
Run 9	9,000	0.5261
Run 10	10,000	0.527
Run 11	11,000	0.5274
Run 12	12,000	0.5272
Run 13	13,000	0.5267
Run 14	14,000	0.5258
Run 15	15,000	0.5247
Run 16	16,000	0.5233
Run 17	17,000	0.5217
Run 18	18,000	0.5199
Run 19	19,000	0.5179
Run 20	20,000	0.5158

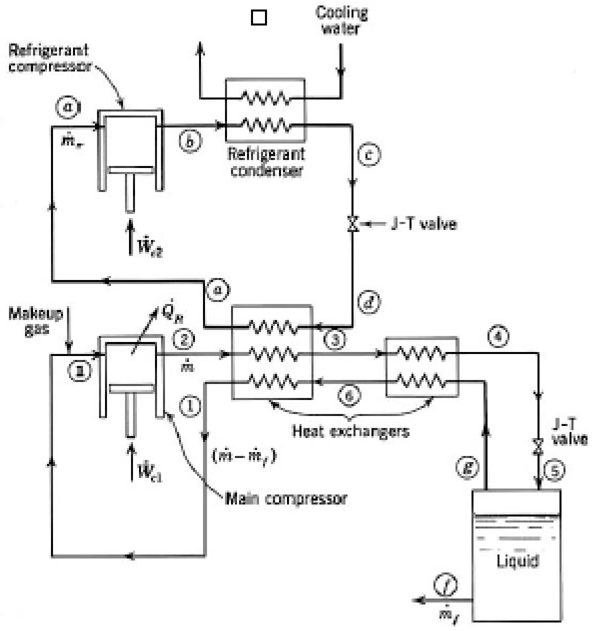
Now, plot the results:



==

"**Prob. 3.2.20** Determine the liquid yield, work per unit mass liquefied, and the work per unit mass compressed for the simple Linde-Hampson system for Nitrogen, with Freon-12 as the refrigerant. The nitrogen portion of the system operates between 1 atm (101.3 kPa) and 300 K and 200 atm. (at point 2). Given: refrigerant flow rate ratio, r = 0.1. See the schematic fig. For the refrigerant portion, point a is at 1 atm, 300K,point 2 is at 7 atm, 375 K, and point c is at 7 atm, 300 K. [Ref.1]

(b) Also, plot y vs r for P2 = 100, 150, 200 and 250 atm."

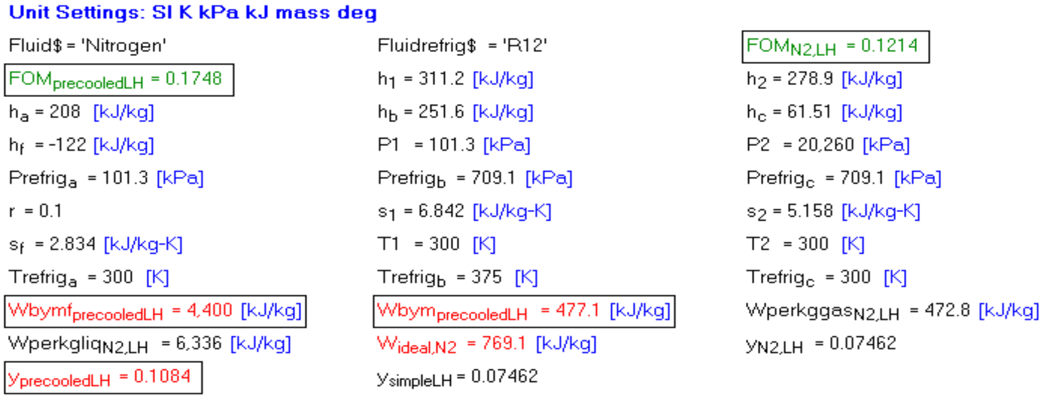

Cooling water
Refrigerant compressor
ⓐ
$\dot{m}_r$
ⓑ
Refrigerant condenser
ⓒ
J-T valve
$\dot{W}_{c2}$
ⓔ
ⓓ
Makeup gas
$\dot{Q}_R$
②
$\dot{m}$
③
④
Ⓑ
①
⑥
Heat exchangers
J-T valve
⑤
$\dot{W}_{c1}$
$(\dot{m} - \dot{m}_f)$
Main compressor
ⓖ
Liquid
ⓕ
$\dot{m}_f$

Solution:

```
"Data:"

Fluid$ = 'Nitrogen'
P1 = 101.3 "kPa"
T1 = 300 "K"
T2 = T1
P2 = 200 * 101.3 "kPa"

"For refrigerant R-12:"

Fluidrefrig$ = 'R12'

Prefrig_a = 101.3 "kPa"
Trefrig_a = 300 "K"
Prefrig_b = 7 * 101.3 "kPa"
Trefrig_b = 375 "K"
Prefrig_c = 7 * 101.3 "kPa"
Trefrig_c = 300 "K"

r = 0.1 "refrigerant flow rate fraction"

"Calculations:"
"Nitrogen system:"

s_1 = Entropy(Fluid$,T=T1,P=P1) "[kJ/kg-K]"

s_2 = Entropy(Fluid$,T=T1,P=P2) "[kJ/kg-K]"

s_f =Entropy(Fluid$,P=P1,x=0) "[kJ/kg-K]"

h_1 = Enthalpy(Fluid$,T=T1,P=P1) "[kJ/kg]"

h_2 = Enthalpy(Fluid$,T=T2,P=P2) "[kJ/kg]"

h_f = Enthalpy(Fluid$, x = 0,P=P1) "[kJ/kg]"

y_simpleLH = (h_1 – h_2) / (h_1 – h_f) "…liquef. fraction for simple L-H system"
```

```
“For refrig system:”

h_a = Enthalpy(Fluidrefrig$,T=Trefrig_a,P=Prefrig_a) “[kJ/kg]”

h_b = Enthalpy(Fluidrefrig$,T=Trefrig_b,P=Prefrig_b) “[kJ/kg]”

h_c = Enthalpy(Fluidrefrig$, T = Trefrig_c,P=Prefrig_c) “[kJ/kg]”

“Liquid yield:”

 y_precooledLH = (h_1 – h_2) / (h_1 – h_f) + r * (h_a – h_c)/(h_1 – h_f) “…liquef. fraction for precooled L-H system”

“Work per unit mass of gas compressed:”

Wbym_precooledLH = T1 * (s_1 – s_2) – (h_1 – h_2) + r * (h_b – h_a) “kJ/kg”

“Work per unit mass of gas liquefied:”

Wbymf_precooledLH = Wbym_precooledLH / y_precooledLH “kJ/kg”

“.............................................................................................................................................................”

“For simple L-H system without pre-cooling:”

“Use the Procedure already written:”

CALL LindeSystem(‘Nitrogen’, P1,P2,T1:y_N2_LH,Wperkggas_N2_LH,Wperkgliq_N2_LH,
W_ideal_N2, FOM_N2_LH)

“.............................................................................................................................................................”

“And, FOM for precooled L-H system:”

FOM_precooledLH = W_ideal_N2 / Wbymf_precooledLH
```

Results:

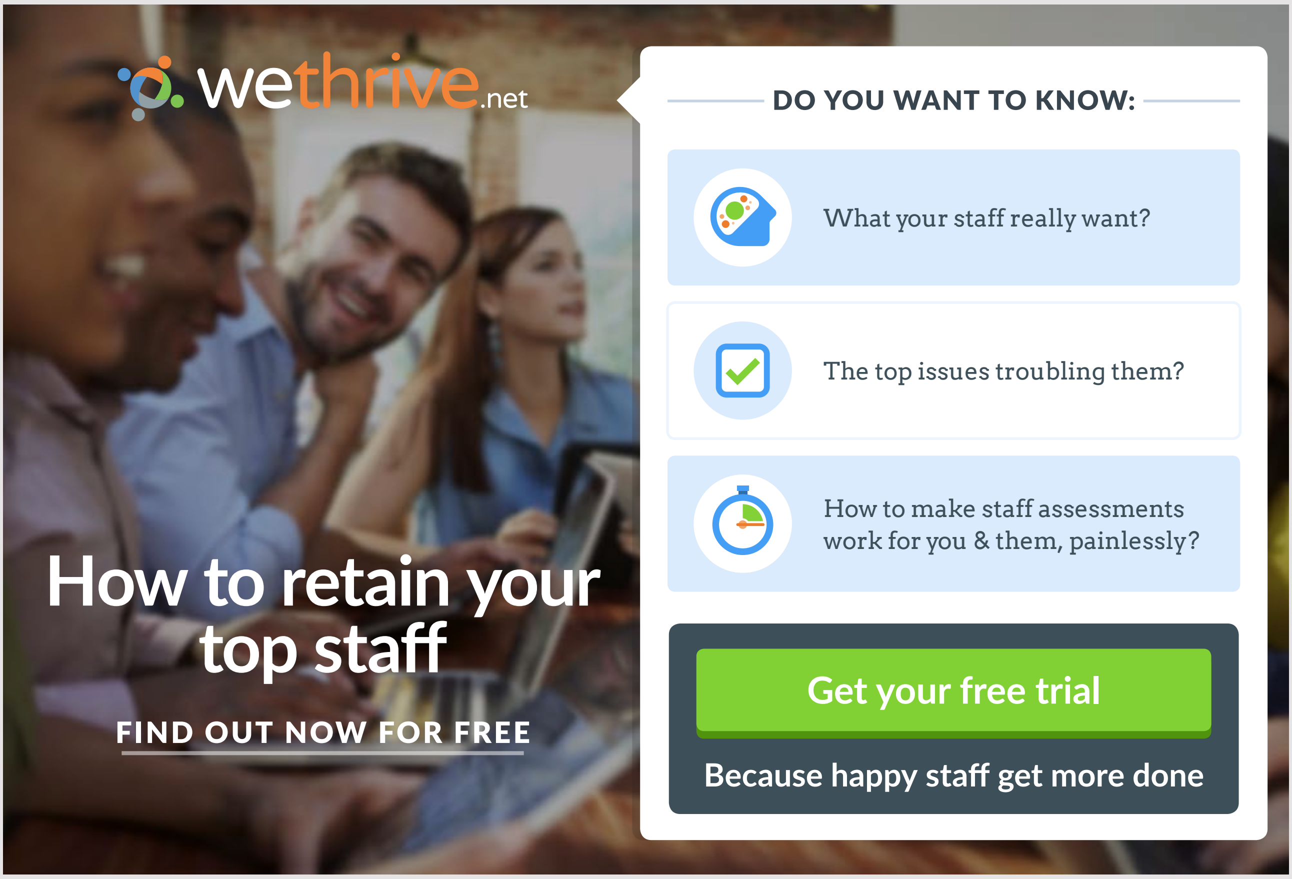


```
Unit Settings: SI K kPa kJ mass deg
Fluid$ = 'Nitrogen'                  Fluidrefrig$  = 'R12'                 FOMN2,LH  = 0.1214
FOMprecooledLH  = 0.1748             h1 = 311.2  [kJ/kg]                   h2 = 278.9  [kJ/kg]
ha = 208  [kJ/kg]                    hb = 251.6  [kJ/kg]                   hc = 61.51  [kJ/kg]
hf  = -122  [kJ/kg]                  P1  = 101.3  [kPa]                    P2  = 20,260  [kPa]
Prefriga  = 101.3  [kPa]             Prefrigb  = 709.1  [kPa]              Prefrigc  = 709.1  [kPa]
r  = 0.1                             s1 = 6.842  [kJ/kg-K]                 s2 = 5.158  [kJ/kg-K]
sf  = 2.834  [kJ/kg-K]               T1  = 300  [K]                        T2  = 300  [K]
Trefriga  = 300  [K]                 Trefrigb  = 375  [K]                  Trefrigc  = 300  [K]
WbymfprecooledLH  = 4,400  [kJ/kg]   WbymprecooledLH  = 477.1  [kJ/kg]     WperkggasN2,LH  = 472.8  [kJ/kg]
WperkgliqN2,LH  = 6,336  [kJ/kg]     Wideal,N2  = 769.1  [kJ/kg]           yN2,LH  = 0.07462
yprecooledLH  = 0.1084               ysimpleLH = 0.07462
```

Thus:

Liquid yield for pre-cooled LH system = y = 0.1084; and for simple LH, y = 0.07462

Work per unit mass of gas compressed for pre-cooled LH system = 477.1 kJ/kg; and for simple LH, = 472.8 kJ/kg

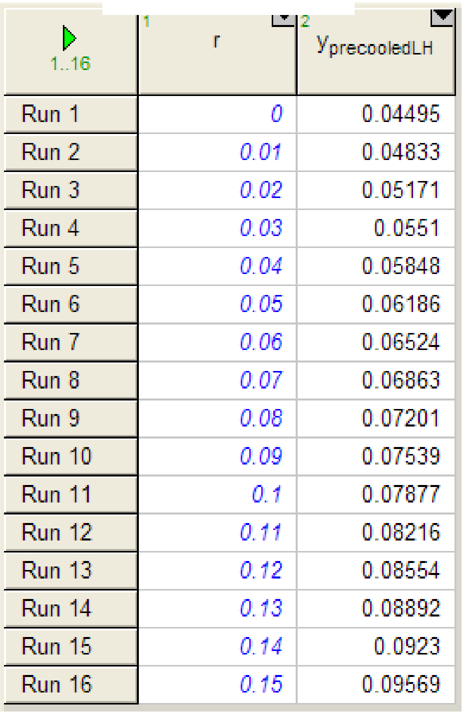


Work per unit mass of gas liquefied for pre-cooled LH system = 4400 kJ/kg; and for simple LH, = 6336 kJ/kg

FOM for pre-cooled LH system = 0.1748; FOM for simple LH = 0.1214

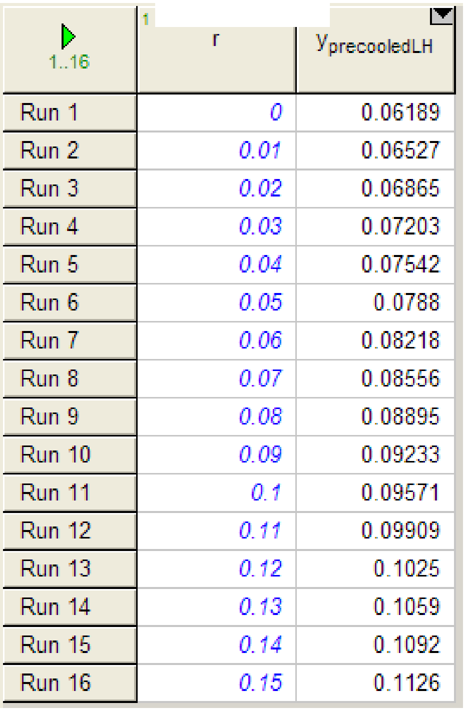
(b) To plot liquid yield (y) vs refrigerant flow rate ratio 'r' for different values of P2:

First, compute the Parametric Table:



P2 = 100 atm.

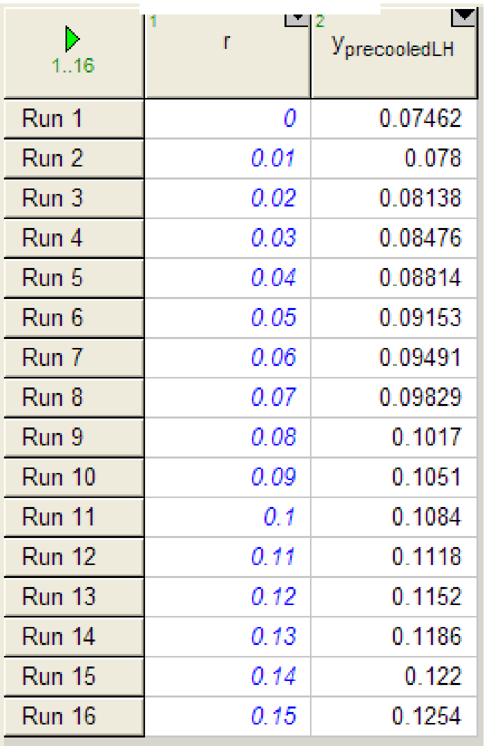
1..16	r	$y_{precooledLH}$
Run 1	0	0.04495
Run 2	0.01	0.04833
Run 3	0.02	0.05171
Run 4	0.03	0.0551
Run 5	0.04	0.05848
Run 6	0.05	0.06186
Run 7	0.06	0.06524
Run 8	0.07	0.06863
Run 9	0.08	0.07201
Run 10	0.09	0.07539
Run 11	0.1	0.07877
Run 12	0.11	0.08216
Run 13	0.12	0.08554
Run 14	0.13	0.08892
Run 15	0.14	0.0923
Run 16	0.15	0.09569



P2 = 150 atm.

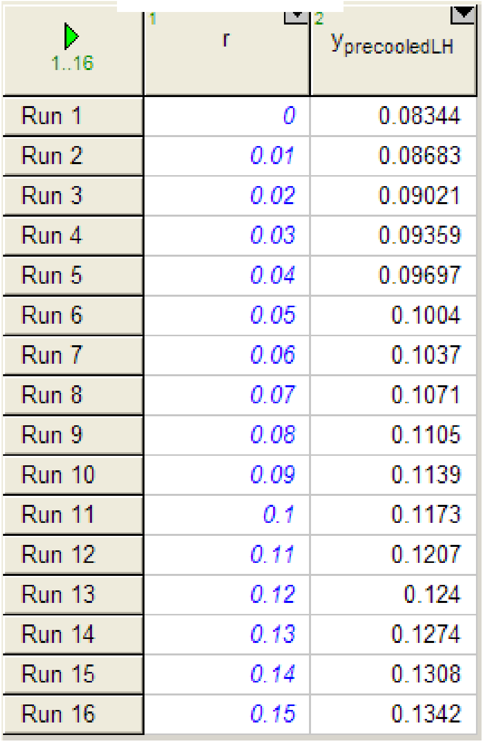
1..16	r	$y_{precooledLH}$
Run 1	0	0.06189
Run 2	0.01	0.06527
Run 3	0.02	0.06865
Run 4	0.03	0.07203
Run 5	0.04	0.07542
Run 6	0.05	0.0788
Run 7	0.06	0.08218
Run 8	0.07	0.08556
Run 9	0.08	0.08895
Run 10	0.09	0.09233
Run 11	0.1	0.09571
Run 12	0.11	0.09909
Run 13	0.12	0.1025
Run 14	0.13	0.1059
Run 15	0.14	0.1092
Run 16	0.15	0.1126

P2 = 200 atm.



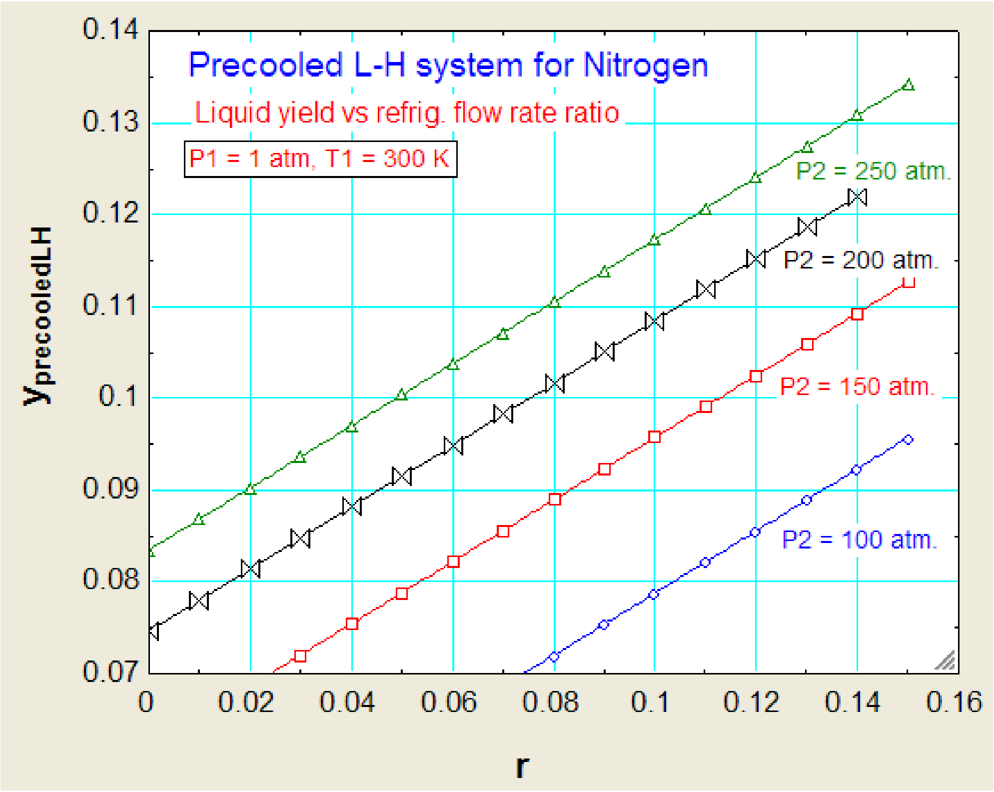
1..16	r	$y_{precooledLH}$
Run 1	0	0.07462
Run 2	0.01	0.078
Run 3	0.02	0.08138
Run 4	0.03	0.08476
Run 5	0.04	0.08814
Run 6	0.05	0.09153
Run 7	0.06	0.09491
Run 8	0.07	0.09829
Run 9	0.08	0.1017
Run 10	0.09	0.1051
Run 11	0.1	0.1084
Run 12	0.11	0.1118
Run 13	0.12	0.1152
Run 14	0.13	0.1186
Run 15	0.14	0.122
Run 16	0.15	0.1254

P2 = 250 atm.



1..16	r	$y_{precooledLH}$
Run 1	0	0.08344
Run 2	0.01	0.08683
Run 3	0.02	0.09021
Run 4	0.03	0.09359
Run 5	0.04	0.09697
Run 6	0.05	0.1004
Run 7	0.06	0.1037
Run 8	0.07	0.1071
Run 9	0.08	0.1105
Run 10	0.09	0.1139
Run 11	0.1	0.1173
Run 12	0.11	0.1207
Run 13	0.12	0.124
Run 14	0.13	0.1274
Run 15	0.14	0.1308
Run 16	0.15	0.1342

Now, plot the results:



"**Prob. 3.2.21** Work out part (a) of the above problem if the fluid being liquefied is Air. All other conditions are the same as in the previous problem."

"Data:"

Fluid$ = 'Air_ha'

P1 = 101.3 "kPa"
T1 = 300 "K"
T2 = T1
P2 = 200 * 101.3 "kPa"

"For refrigerant R-12:"

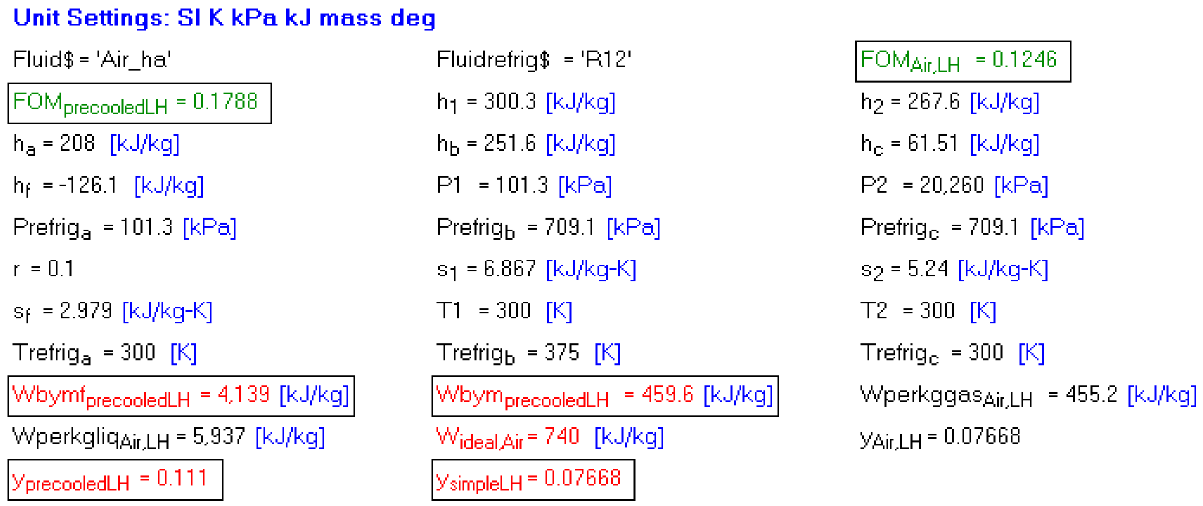
Fluidrefrig$ = 'R12'

Prefrig_a = 101.3 "kPa"
Trefrig_a = 300 "K"
Prefrig_b = 7 * 101.3 "kPa"

```
Trefrig_b = 375 “K”
Prefrig_c = 7 * 101.3 “kPa”
Trefrig_c = 300 “K”

r = 0.1 “refrigerant flow rate fraction”

“Calculations:”
“Nitrogen system:”

s_1 = Entropy(Fluid$,T=T1,P=P1) “[kJ/kg-K]”

s_2 = Entropy(Fluid$,T=T1,P=P2) “[kJ/kg-K]”

s_f =Entropy(Fluid$,P=P1,x=0) “[kJ/kg-K]”

h_1 = Enthalpy(Fluid$,T=T1,P=P1) “[kJ/kg]”

h_2 = Enthalpy(Fluid$,T=T2,P=P2) “[kJ/kg]”

h_f = Enthalpy(Fluid$, x = 0,P=P1) “[kJ/kg]”

 y_simpleLH = (h_1 – h_2) / (h_1 – h_f) “…liquef. fraction for simple L-H system”

“For refrig system:”

h_a = Enthalpy(Fluidrefrig$,T=Trefrig_a,P=Prefrig_a) “[kJ/kg]”

h_b = Enthalpy(Fluidrefrig$,T=Trefrig_b,P=Prefrig_b) “[kJ/kg]”

h_c = Enthalpy(Fluidrefrig$, T = Trefrig_c,P=Prefrig_c) “[kJ/kg]”

“Liquid yield:”

 y_precooledLH = (h_1 – h_2) / (h_1 – h_f) + r * (h_a – h_c)/(h_1 – h_f) “…liquef. fraction for precooled L-H system”
```

```
"Work per unit mass of gas compressed:"

Wbym_precooledLH = T1 * (s_1 – s_2) – (h_1 – h_2) + r * (h_b – h_a) "kJ/kg"

"Work per unit mass of gas liquefied:"

Wbymf_precooledLH = Wbym_precooledLH / y_precooledLH "kJ/kg"

"………………………………………………………………………………………………………………………………"

"For simple L-H system without pre-cooling:"

"Use the Procedure already written:"

CALL LindeSystem('Air_ha', P1,P2,T1:y_Air_LH,Wperkggas_Air_LH,Wperkgliq_Air_LH,
W_ideal_Air, FOM_Air_LH)

"………………………………………………………………………………………………………………………………"

"And, FOM for precooled L-H system:"

FOM_precooledLH = W_ideal_Air / Wbymf_precooledLH
```

Results:



Unit Settings: SI K kPa kJ mass deg

Fluid$ = 'Air_ha'

$FOM_{precooledLH}$ = 0.1788

h_a = 208 [kJ/kg]

h_f = -126.1 [kJ/kg]

$Prefrig_a$ = 101.3 [kPa]

r = 0.1

s_f = 2.979 [kJ/kg-K]

$Trefrig_a$ = 300 [K]

$Wbymf_{precooledLH}$ = 4,139 [kJ/kg]

$Wperkgliq_{Air,LH}$ = 5,937 [kJ/kg]

$y_{precooledLH}$ = 0.111

Fluidrefrig$ = 'R12'

h_1 = 300.3 [kJ/kg]

h_b = 251.6 [kJ/kg]

P1 = 101.3 [kPa]

$Prefrig_b$ = 709.1 [kPa]

s_1 = 6.867 [kJ/kg-K]

T1 = 300 [K]

$Trefrig_b$ = 375 [K]

$Wbym_{precooledLH}$ = 459.6 [kJ/kg]

$W_{ideal,Air}$ = 740 [kJ/kg]

$y_{simpleLH}$ = 0.07668

$FOM_{Air,LH}$ = 0.1246

h_2 = 267.6 [kJ/kg]

h_c = 61.51 [kJ/kg]

P2 = 20,260 [kPa]

$Prefrig_c$ = 709.1 [kPa]

s_2 = 5.24 [kJ/kg-K]

T2 = 300 [K]

$Trefrig_c$ = 300 [K]

$Wperkggas_{Air,LH}$ = 455.2 [kJ/kg]

$y_{Air,LH}$ = 0.07668

Thus:

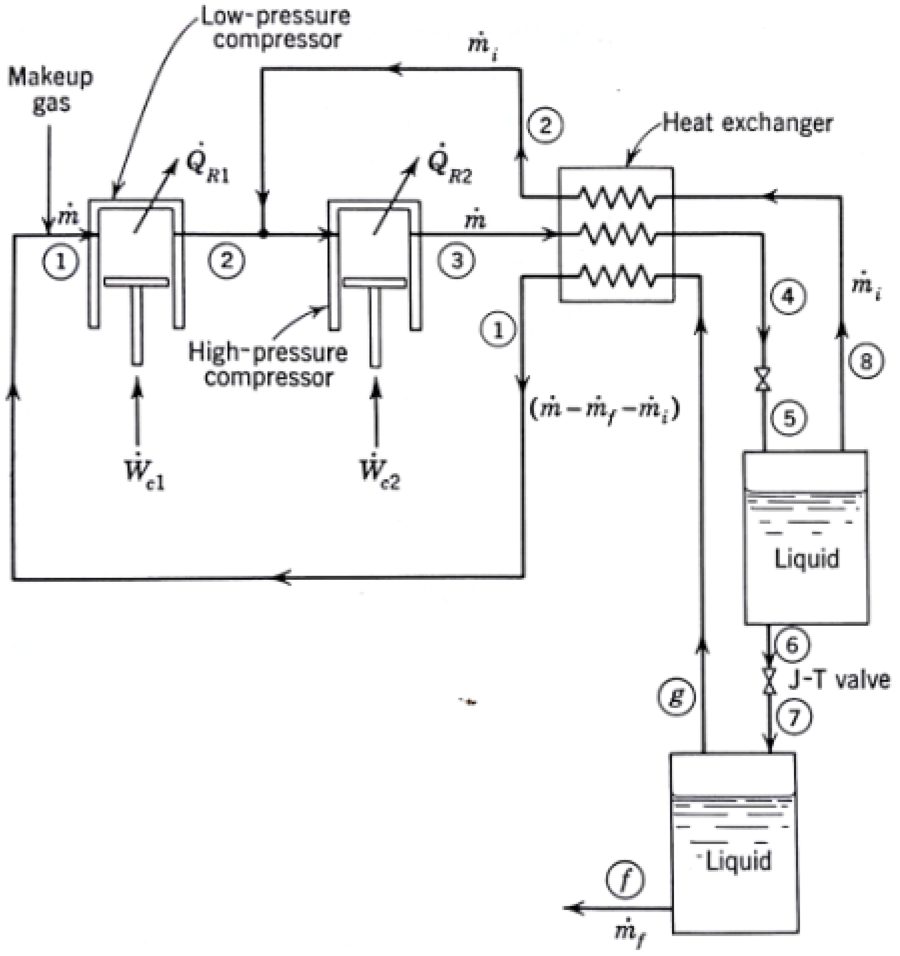
Liquid yield for pre-cooled LH system = y = 0.111; and for simple LH, y = 0.07668

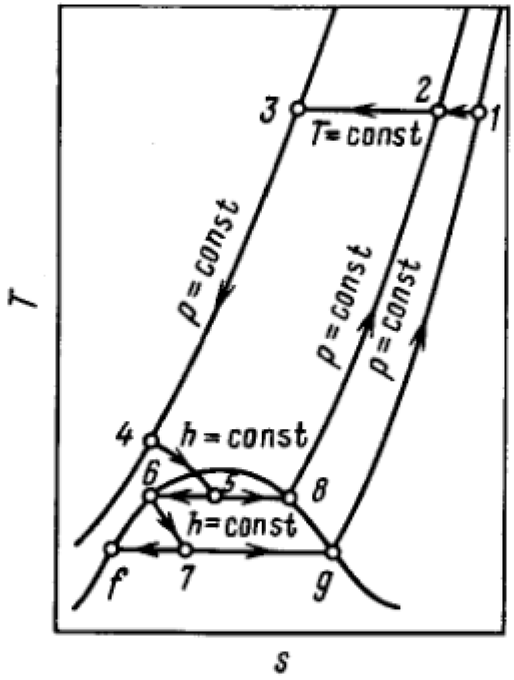
Work per unit mass of gas compressed for pre-cooled LH system = 459.6 kJ/kg; and for simple LH, = 455.2 kJ/kg

Work per unit mass of gas liquefied for pre-cooled LH system = 4139 kJ/kg; and for simple LH, = 5937 kJ/kg

FOM for pre-cooled LH system = 0.1788; FOM for simple LH = 0.1246

"**Prob. 3.2.22** Determine the liquid yield, work per unit mass liquefied, and FOM for a Linde dual pressure system using air as the working fluid. The system operates from 101.3 kPa (1 atm) and 20 C to 20.67 MPa (200 atm). The intermediate pressure is 5.07 MPa (50 atm) and the intermediate pressure flow rate ratio is 0.75. The system is reversible except for expansion through the expansion valve. [1]."

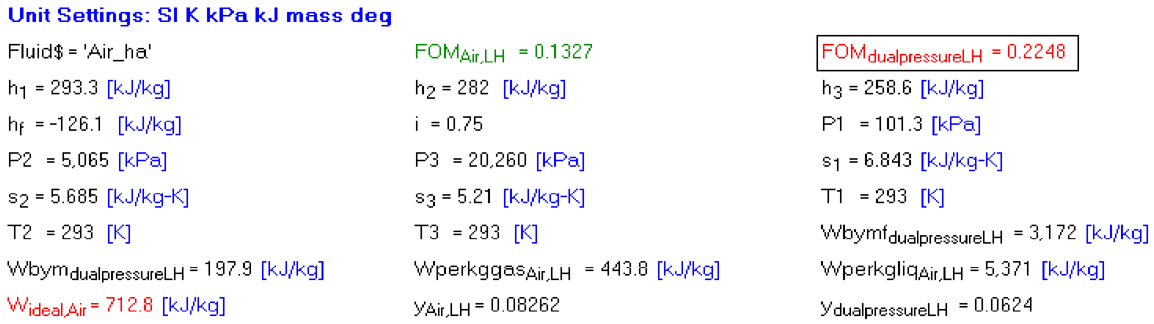

Low-pressure compressor
$\dot{m}_i$
Makeup gas
②
Heat exchanger
$\dot{Q}_{R1}$
$\dot{Q}_{R2}$
$\dot{m}$
$\dot{m}$
①
②
③
①
④
$\dot{m}_i$
⑧
High-pressure compressor
⑤
$(\dot{m}-\dot{m}_f-\dot{m}_i)$
$\dot{W}_{c1}$
$\dot{W}_{c2}$
Liquid
⑥
J-T valve
ⓖ
⑦
Liquid
ⓕ
$\dot{m}_f$


T
s
1
2
3
T=const
p=const
p=const
p=const
4
h=const
6
5
8
h=const
f
7
g

```
"Data:"
Fluid$ = 'Air_ha'

P1 = 101.3 "kPa"
T1 = 293 "K"
T2 = T1
T3 = T1
P2 = 50 * 101.3 "kPa"
P3= 200 * 101.3 "kPa"

i = 0.75 "intermediate pressure flow rate"

"Calculations:"
s_1 = Entropy(Fluid$,T=T1,P=P1) "[kJ/kg-K]"

s_2 = Entropy(Fluid$,T=T1,P=P2) "[kJ/kg-K]"

s_3 =Entropy(Fluid$,P=P3,T=T3) "[kJ/kg-K]"

h_1 = Enthalpy(Fluid$,T=T1,P=P1) "[kJ/kg]"

h_2 = Enthalpy(Fluid$,T=T2,P=P2) "[kJ/kg]"

h_3 = Enthalpy(Fluid$,T=T3,P=P3) "[kJ/kg]"

h_f = Enthalpy(Fluid$, x = 0,P=P1) "[kJ/kg]"

"Liquid yield:"

y_dualpressureLH = ((h_1 – h_3) / (h_1 – h_f))- i * ((h_1 – h_2) /(h_1 – h_f)) "…liquef. fraction for dual pressure L-H system"

"Work per unit mass of gas compressed:"

Wbym_dualpressureLH = T1 * (s_1 – s_3) – (h_1 – h_3) – i * ( T1 * (s_1 – s_2) – (h_1 – h_2)) "kJ/kg"

"Work per unit mass of gas liquefied:"

Wbymf_dualpressureLH = Wbym_dualpressureLH / y_dualpressureLH "kJ/kg"

"………………………………………………………………………………………………………………………………………………………"
```

```
"For simple L-H system without precooling:"

"Use the Procedure already written:"

CALL LindeSystem('Air_ha', P1,P3,T1:y_Air_LH,Wperkggas_Air_LH,Wperkgliq_Air_LH,
W_ideal_Air, FOM_Air_LH)

"..........................................................................................................................................................................................."

"And, FOM for dual pressure L-H system:"

FOM_dualpressureLH = W_ideal_Air / Wbymf_dualpressureLH
```

Results:



```
Unit Settings: SI K kPa kJ mass deg
Fluid$ = 'Air_ha'                 FOMAir,LH  = 0.1327                  FOMdualpressureLH  = 0.2248
h1 = 293.3 [kJ/kg]                h2 = 282  [kJ/kg]                    h3 = 258.6 [kJ/kg]
hf  = -126.1  [kJ/kg]             i  = 0.75                            P1  = 101.3 [kPa]
P2  = 5,065 [kPa]                 P3  = 20,260 [kPa]                   s1 = 6.843 [kJ/kg-K]
s2 = 5.685 [kJ/kg-K]              s3 = 5.21 [kJ/kg-K]                  T1  = 293  [K]
T2  = 293  [K]                    T3  = 293  [K]                       WbymfdualpressureLH  = 3,172 [kJ/kg]
WbymdualpressureLH = 197.9 [kJ/kg]  WperkggasAir,LH  = 443.8 [kJ/kg]   WperkgliqAir,LH = 5,371  [kJ/kg]
Wideal,Air = 712.8 [kJ/kg]        yAir,LH = 0.08262                    ydualpressureLH  = 0.0624
```

Thus:

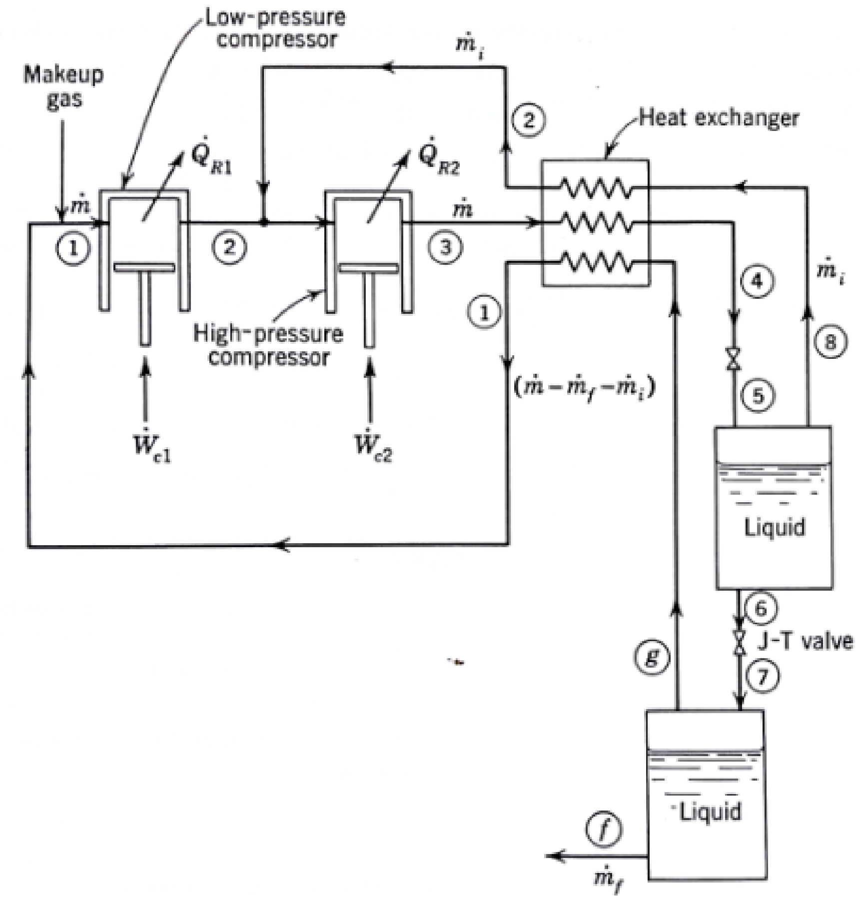
Liquid yield for dual pressure LH system = y = 0.0624; and for simple LH, y = 0.08262

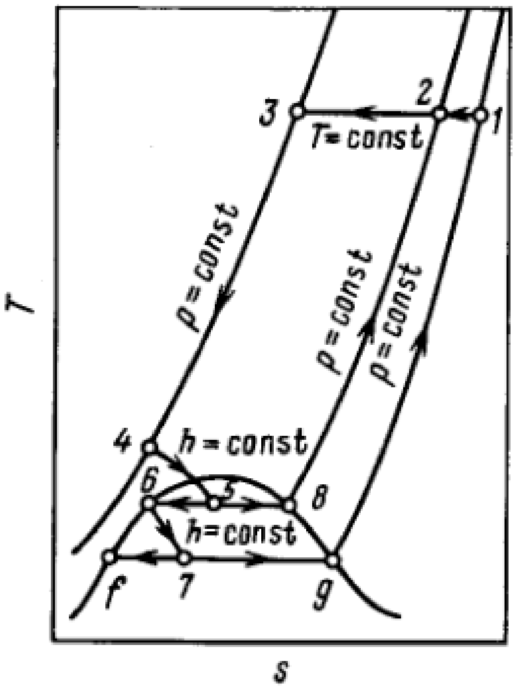
Work per unit mass of gas compressed for dual pressure LH system = 197.9 kJ/kg; and for simple LH = 443.8 kJ/kg

Work per unit mass of gas liquefied for dual pressure LH system = 3172 kJ/kg; and for simple LH, = 5371 kJ/kg

FOM for dual pressure LH system = 0.2248; FOM for simple LH = 0.1327

"**Prob. 3.2.23** In a reversible Linde dual pressure system using air as the working fluid, the system operates from 101.3 kPa (1 atm) and 300 K to 20.67 MPa .The intermediate pressure flow rate ratio is 0.7. The liquid yield is 0.065. Determine the intermediate pressure. [1]."


Low-pressure compressor
Makeup gas
$\dot{m}_i$
$\dot{Q}_{R1}$
$\dot{Q}_{R2}$
Heat exchanger
$\dot{m}$
①
②
③
④
⑤
⑥
⑦
⑧
High-pressure compressor
$(\dot{m}-\dot{m}_f-\dot{m}_i)$
$\dot{W}_{c1}$
$\dot{W}_{c2}$
Liquid
J-T valve
ⓖ
ⓕ
$\dot{m}_f$


T
s
1
2
3
T=const
p=const
p=const
p=const
4
h=const
5
6
8
h=const
f
7
g

"Data:"

Fluid$ = 'Air_ha'

P1 = 101.3 "kPa"
T1 = 300 "K"
T2 = T1
T3 = T1
P3= 20670 "kPa"
"Let P2 be the intermediate pressure."

y_dualpressureLH = 0.065 "…liquid yield…by data"

i = 0.7 "intermediate pressure flow rate "

"Calculations:"

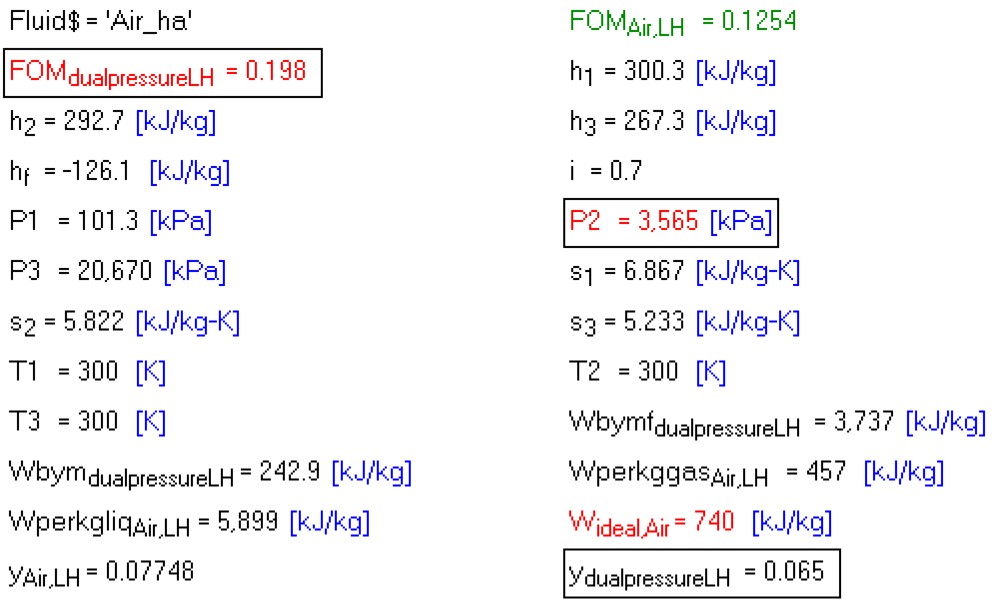
s_1 = Entropy(Fluid$,T=T1,P=P1) "[kJ/kg-K]"

s_2 = Entropy(Fluid$,T=T1,P=P2) "[kJ/kg-K]"

```
s_3 =Entropy(Fluid$,P=P3,T=T3) "[kJ/kg-K]"

h_1 = Enthalpy(Fluid$,T=T1,P=P1) "[kJ/kg]"

h_2 = Enthalpy(Fluid$,T=T2,P=P2) "[kJ/kg]…finds P2 when h_2 is known"

h_3 = Enthalpy(Fluid$,T=T3,P=P3) "[kJ/kg]"

h_f = Enthalpy(Fluid$, x = 0,P=P1) "[kJ/kg]"

"Liquid yield:"

 y_dualpressureLH = ((h_1 – h_3) / (h_1 – h_f))- i * ((h_1 – h_2) /(h_1 – h_f))
"…liquef. fraction for dual pressure L-H system…determines h_2"

"Work per unit mass of gas compressed:"

Wbym_dualpressureLH = T1 * (s_1 – s_3) – (h_1 – h_3) – i * ( T1 * (s_1 – s_2) – (h_1 –
h_2)) "kJ/kg"

"Work per unit mass of gas liquefied:"

Wbymf_dualpressureLH = Wbym_dualpressureLH / y_dualpressureLH "kJ/kg"

"……………………………………………………………………………………………………………………………"

"For simple L-H system without precooling:"

"Use the Procedure already written:"

CALL LindeSystem('Air_ha', P1,P3,T1:y_Air_LH,Wperkggas_Air_LH,Wperkgliq_Air_LH,
W_ideal_Air, FOM_Air_LH)

"……………………………………………………………………………………………………………………………"

"And, FOM for dual pressure L-H system:"

FOM_dualpressureLH = W_ideal_Air / Wbymf_dualpressureLH

"==================================================================="
```

Note: Note that when liquid yield y is given, enthalpy h2 is calculated; and knowing h2, the corresponding intermediate pressure P2 is calculated from the EES Function for enthalpy very easily.

Results:



Fluid$ = 'Air_ha'

$\boxed{FOM_{dualpressureLH} = 0.198}$

h_2 = 292.7 [kJ/kg]

h_f = -126.1 [kJ/kg]

P1 = 101.3 [kPa]

P3 = 20,670 [kPa]

s_2 = 5.822 [kJ/kg-K]

T1 = 300 [K]

T3 = 300 [K]

$Wbym_{dualpressureLH}$ = 242.9 [kJ/kg]

$Wperkgliq_{Air,LH}$ = 5,899 [kJ/kg]

$y_{Air,LH}$ = 0.07748

$FOM_{Air,LH}$ = 0.1254

h_1 = 300.3 [kJ/kg]

h_3 = 267.3 [kJ/kg]

i = 0.7

$\boxed{P2 = 3{,}565 \text{ [kPa]}}$

s_1 = 6.867 [kJ/kg-K]

s_3 = 5.233 [kJ/kg-K]

T2 = 300 [K]

$Wbymf_{dualpressureLH}$ = 3,737 [kJ/kg]

$Wperkggas_{Air,LH}$ = 457 [kJ/kg]

$W_{ideal,Air}$ = 740 [kJ/kg]

$\boxed{y_{dualpressureLH} = 0.065}$

Thus:

Intermediate pressure for y = 0.065: P2 = 3565 kPa…Ans.

In addition:

Work per unit mass of gas compressed for dual pressure LH system = 242.9 kJ/kg; and for simple LH = 457 kJ/kg

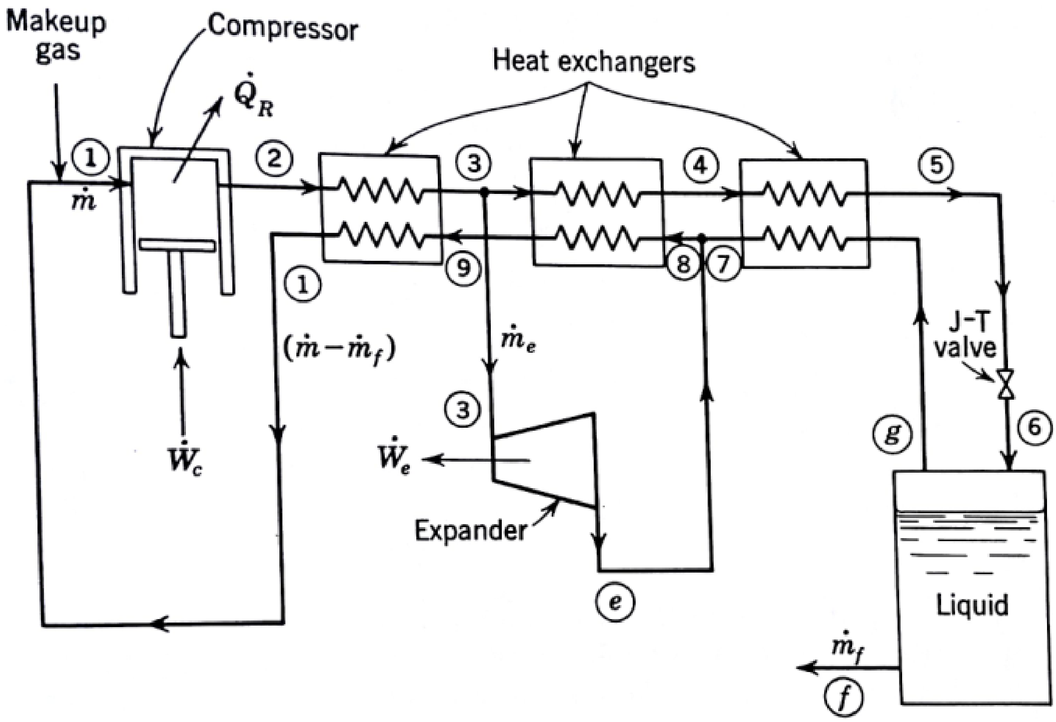
Work per unit mass of gas liquefied for dual pressure LH system = 3737 kJ/kg; and for simple LH, = 5899 kJ/kg

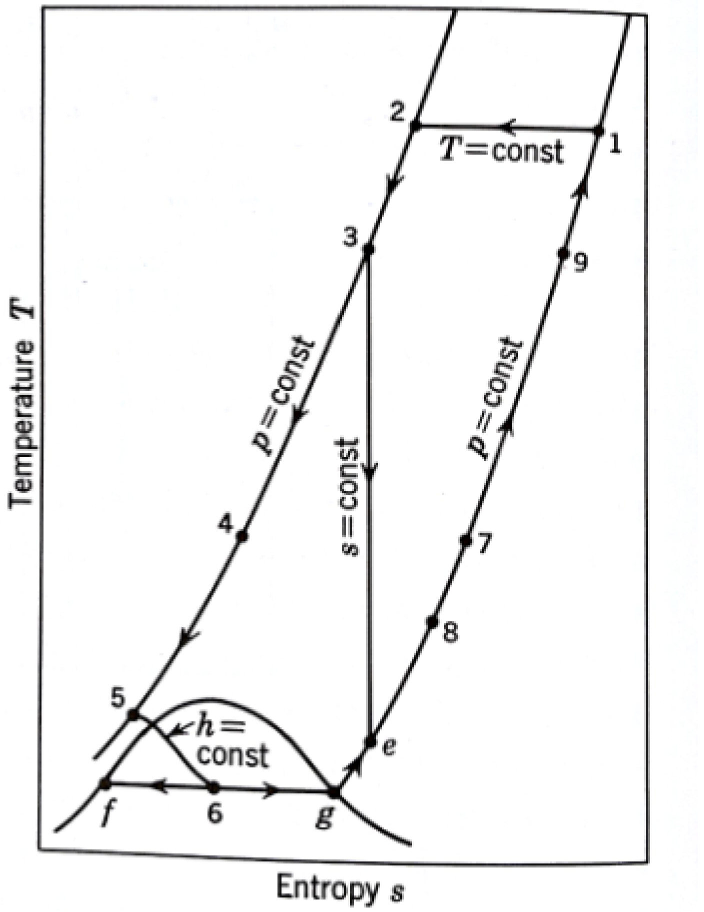
FOM for dual pressure LH system = 0.198; FOM for simple LH = 0.1254

"**Prob. 3.2.24** (a). Write an EES Procedure to calculate liquid yield and FOM of an Ideal Claude system.

(b). Then, determine the liquid yield, work output of expander per unit mass compressed, net work requirement of the system per unit mass liquefied, and the FOM assuming that the expander work is utilized to aid in the compression. Data for the Claude system is given below:
Air is the working fluid, the system operates from 101.3 kPa (1 atm) and 293 K to 10.13 MPa. At 10.13 MPa and 250 K, 40% of the main flow is diverted to the reversible adiabatic expander. The remainder flows through the heat exchangers and expands through the expansion valve to 101.3 kPa. [1].
Also plot the net work required per unit mass liquefied against expander flow rate ratio for P2 = 20, 40, 100 and 140 bar, other parameters remaining the same."

Following figures show the schematic diagram and the T-s diagram for Claude system:





Solution:

EES Procedure:

```
“Ideal Claude for liquefaction of Air and other gases: Liquef. fraction y and compr. work per unit mass liquefied:”

PROCEDURE ClaudeSystem_Ideal(FLUID$, P_1,P_2, P_3, T_1,T_3,x: y,W_net_perkggas,W_net_perkgliq, W_exp_perkggas,W_ideal, FOM)

“Inputs: FLUID$, P_1, P_2, P_3,T_1, T_3:…Pressures in kPa, Temp in K; x = expander flow rate ratio”

“Outputs: y, fraction liquefied, W_net_perkggas (kJ/kg of gas compressed), W_net_perkgliq (kJ/kg of liquid)”

“W_ideal = Ideal work reqd., FOM = Figure of Merit = W_ideal/W_net_perkgliq”

s_1:=Entropy(Fluid$,T=T_1,P=P_1) “[kJ/kg-K]”

s_2:=Entropy(Fluid$,P=P_2,T=T_1) “[kJ/kg-K]”

s_3:=Entropy(Fluid$,P=P_3,T=T_3) “[kJ/kg-K]”

s_e = s_3 “…entropy after isentropic expansion in expander”

h_1:=Enthalpy(Fluid$,T=T_1,P=P_1) “[kJ/kg]”

h_2:=Enthalpy(Fluid$,P=P_2,T=T_1) “[kJ/kg]”

h_3:=Enthalpy(Fluid$,P=P_3,T=T_3) “[kJ/kg]”

h_e = Enthalpy(Fluid$,P=P_1,s= s_e) “[kJ/kg]”

h_f:=Enthalpy(Fluid$,x=0,P=P_1) “[kJ/kg]”

y:= (h_1 – h_2) / (h_1 – h_f) + x * ((h_3 – h_e) / (h_1 – h_f)) “…fraction liquefied”

W_exp_perkggas = x * (h_3 – h_e) “kJ/kg of gas compressed”
```

```
W_net_perkggas:=T_1 * (s_1 – s_2) – (h_1 – h_2) – x * (h_3 – h_e) "[kJ/kg]…net work per kg gas compressed..when the expander work is utilised in compression"

W_net_perkgliq:= W_net_perkggas / y "[kJ/kg]…. net work reqd per kg liq."

W_ideal:= Ideal_Work(FLUID$, P_1,T_1) "kJ/kg …ideal work of liquefaction"

FOM:= W_ideal / W_net_perkgliq "… Figure of Merit"

END

"==========================================================================="
```

(b) Problem:

"Data:"

Fluid\$ = 'Air_ha'

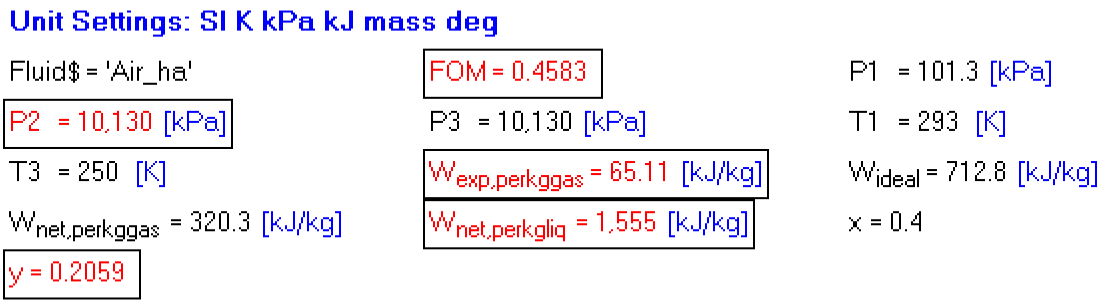
P1 = 101.3 "kPa"
T1 = 293 "K"
P2= 10130 "kPa"
P3 = P2
T3 = 250 "K"

x = 0.4

Solution:

CALL ClaudeSystem_Ideal(FLUID\$, P1,P2, P3, T1,T3,x: y,W_net_perkggas,W_net_perkgliq, W_exp_perkggas,W_ideal, FOM)

Results:



Unit Settings: SI K kPa kJ mass deg

Fluid\$ = 'Air_ha'
P2 = 10,130 [kPa]
T3 = 250 [K]
$W_{net,perkggas}$ = 320.3 [kJ/kg]
y = 0.2059

FOM = 0.4583
P3 = 10,130 [kPa]
$W_{exp,perkggas}$ = 65.11 [kJ/kg]
$W_{net,perkgliq}$ = 1,555 [kJ/kg]

P1 = 101.3 [kPa]
T1 = 293 [K]
W_{ideal} = 712.8 [kJ/kg]
x = 0.4

Thus:

Liquid yield y = 0.2059…Ans.

Work output of expander per unit mass compressed = 65.11 kJ/kg…Ans.

Net work requirement of the system per unit mass liquefied = 1555 kJ/kg…Ans.

FOM = 0.4583…Ans.

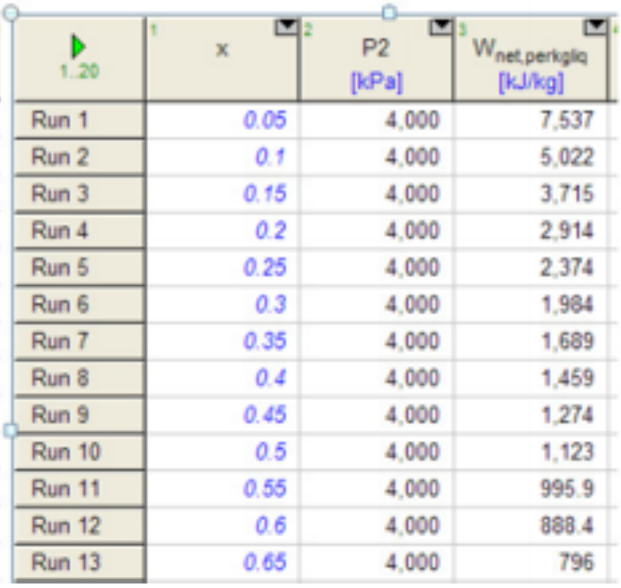
(c) Plot the net work required per unit mass liquefied against expander flow rate ratio for P2 = 20, 40, 100 and 140 bar, other parameters remaining the same:

First, compute the Parametric Tables:

P2 = 20 bar:

1..20	x	P2 [kPa]	$W_{net,perkgliq}$ [kJ/kg]
Run 1	0.05	2,000	8,892
Run 2	0.1	2,000	5,362
Run 3	0.15	2,000	3,773
Run 4	0.2	2,000	2,869
Run 5	0.25	2,000	2,286
Run 6	0.3	2,000	1,878
Run 7	0.35	2,000	1,578
Run 8	0.4	2,000	1,346
Run 9	0.45	2,000	1,163
Run 10	0.5	2,000	1,014
Run 11	0.55	2,000	891.2
Run 12	0.6	2,000	787.5
Run 13	0.65	2,000	699

P2 = 40 bar:

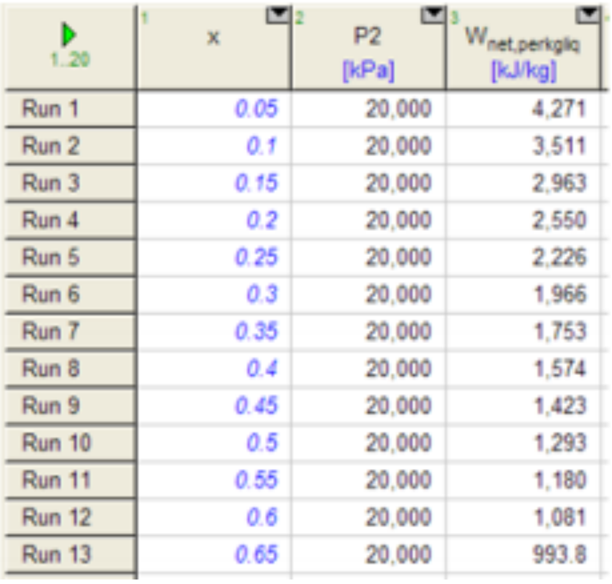


1..20	x	P2 [kPa]	$W_{net,perkgliq}$ [kJ/kg]
Run 1	0.05	4,000	7,537
Run 2	0.1	4,000	5,022
Run 3	0.15	4,000	3,715
Run 4	0.2	4,000	2,914
Run 5	0.25	4,000	2,374
Run 6	0.3	4,000	1,984
Run 7	0.35	4,000	1,689
Run 8	0.4	4,000	1,459
Run 9	0.45	4,000	1,274
Run 10	0.5	4,000	1,123
Run 11	0.55	4,000	995.9
Run 12	0.6	4,000	888.4
Run 13	0.65	4,000	796

P2 = 100 bar:

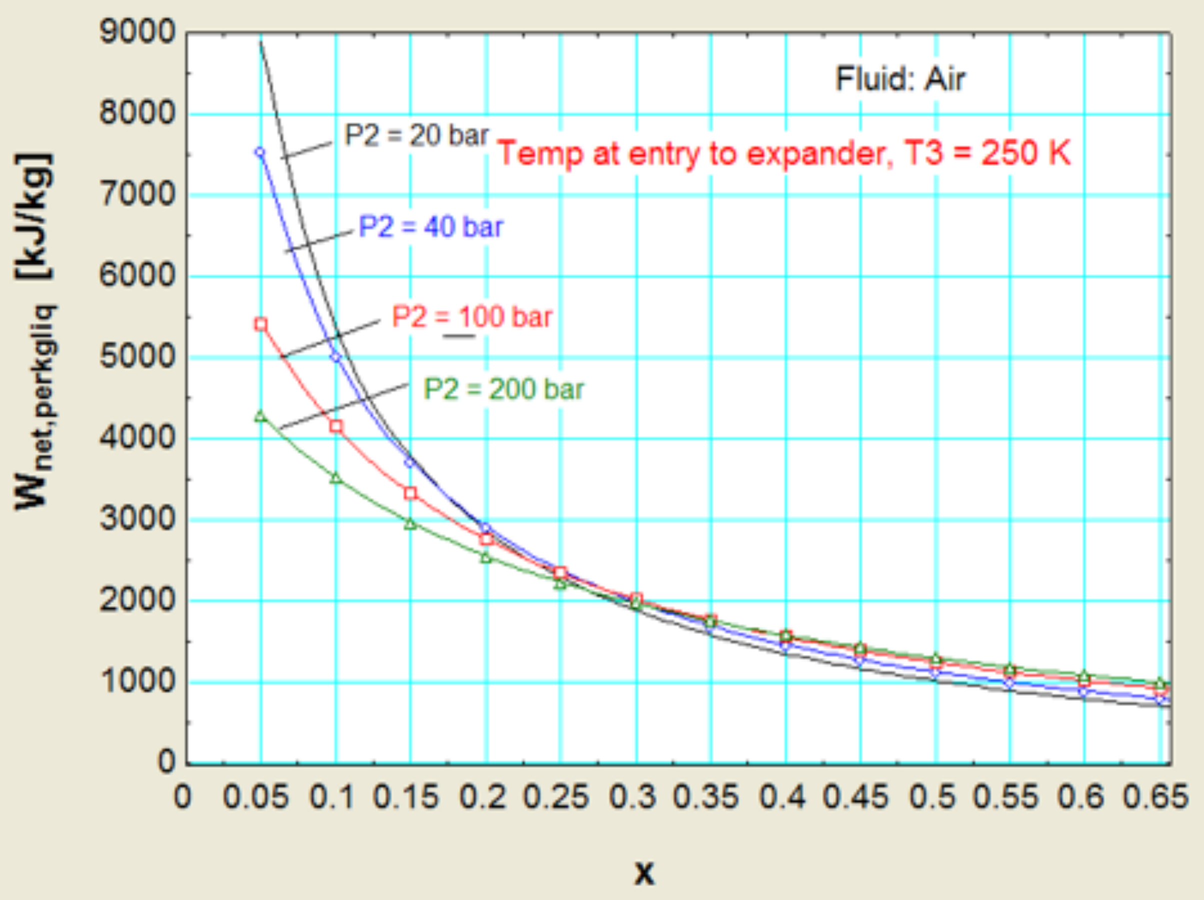
1..20	x	P2 [kPa]	$W_{net,perkgliq}$ [kJ/kg]
Run 1	0.05	10,000	5,413
Run 2	0.1	10,000	4,140
Run 3	0.15	10,000	3,323
Run 4	0.2	10,000	2,755
Run 5	0.25	10,000	2,336
Run 6	0.3	10,000	2,015
Run 7	0.35	10,000	1,761
Run 8	0.4	10,000	1,555
Run 9	0.45	10,000	1,384
Run 10	0.5	10,000	1,241
Run 11	0.55	10,000	1,119
Run 12	0.6	10,000	1,013
Run 13	0.65	10,000	921.3

P2 = 200 bar:

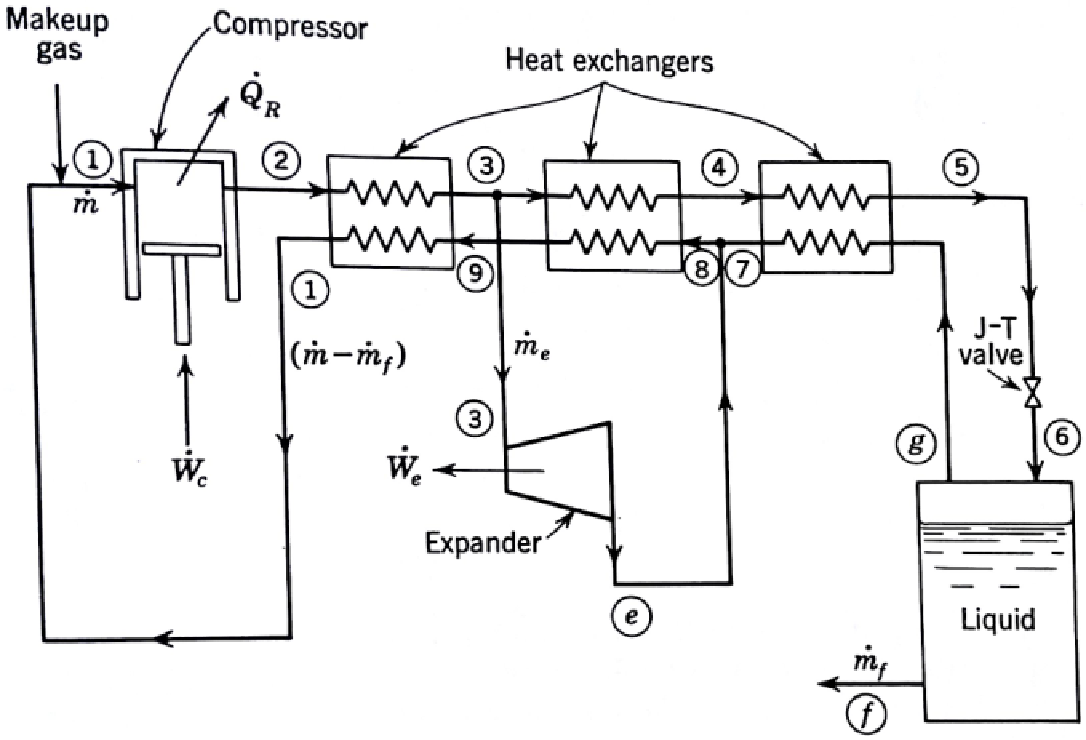


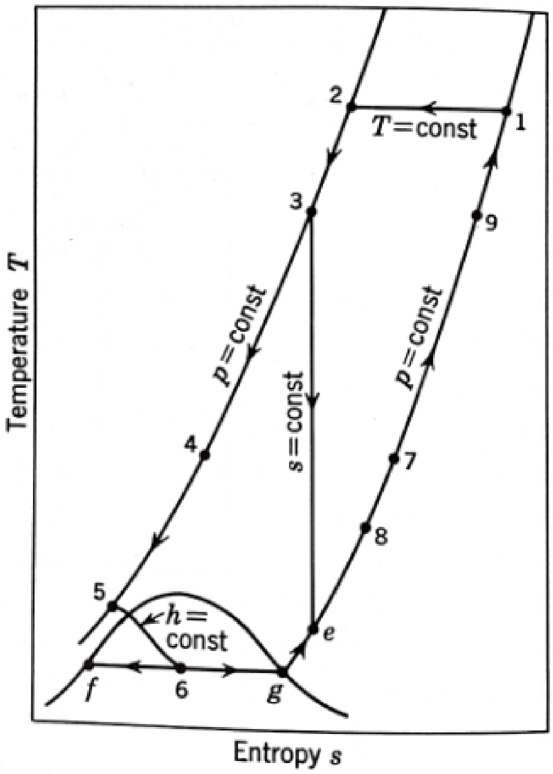
1..20	x	P2 [kPa]	$W_{net,perkgliq}$ [kJ/kg]
Run 1	0.05	20,000	4,271
Run 2	0.1	20,000	3,511
Run 3	0.15	20,000	2,963
Run 4	0.2	20,000	2,550
Run 5	0.25	20,000	2,226
Run 6	0.3	20,000	1,966
Run 7	0.35	20,000	1,753
Run 8	0.4	20,000	1,574
Run 9	0.45	20,000	1,423
Run 10	0.5	20,000	1,293
Run 11	0.55	20,000	1,180
Run 12	0.6	20,000	1,081
Run 13	0.65	20,000	993.8

Now, plot the results:



"**Prob. 3.2.25** In an Ideal Claude system for Nitrogen, the system operates from 101.3 kPa (1 atm) and 293 K to 4.05 MPa. At 4.05 MPa and 240 K, fraction 'x' of the main flow is diverted to the reversible adiabatic expander. Remainder flows through the heat exchangers and expands through the expansion valve to 101.3 kPa. Determine the expander flow rate ratio (x) for a liquid yield of 0.2. [1]."

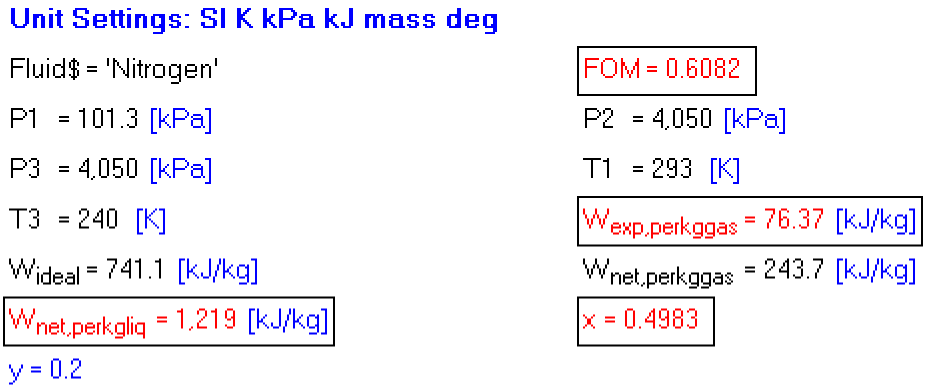

Makeup gas
Compressor
$\dot{Q}_R$
Heat exchangers
①
②
③
④
⑤
$\dot{m}$
⑨
⑧
⑦
①
$(\dot{m}-\dot{m}_f)$
$\dot{m}_e$
③
$\dot{W}_c$
$\dot{W}_e$
Expander
ⓔ
J-T valve
ⓖ
⑥
Liquid
$\dot{m}_f$
ⓕ


2
1
T=const
3
9
p=const
s=const
p=const
4
7
8
5
h= const
e
f
6
g
Temperature T
Entropy s

```
"Data:"
Fluid$ = 'Nitrogen'

P1 = 101.3 "kPa"
T1 = 293 "K"
P2= 4050 "kPa"

P3 = P2
T3 = 240 "K"

y = 0.2 "liquid yield"

"We use the EES Procedure already written. Here, now, y is known, but x is to be determined."

CALL ClaudeSystem_Ideal(FLUID$, P1,P2, P3, T1,T3,x: y,W_net_perkggas,W_net_perkgliq,
W_exp_perkggas,W_ideal, FOM)
```

Note: Note the ease of using the EES. Plug in the known y in the EES Procedure, and the unknown x is calculated immediately.

Results:



Unit Settings: SI K kPa kJ mass deg

Fluid$ = 'Nitrogen'

$FOM = 0.6082$

$P1 = 101.3$ [kPa]

$P2 = 4{,}050$ [kPa]

$P3 = 4{,}050$ [kPa]

$T1 = 293$ [K]

$T3 = 240$ [K]

$W_{exp,perkggas} = 76.37$ [kJ/kg]

$W_{ideal} = 741.1$ [kJ/kg]

$W_{net,perkggas} = 243.7$ [kJ/kg]

$W_{net,perkgliq} = 1{,}219$ [kJ/kg]

$x = 0.4983$

$y = 0.2$

Thus:

Expander flow rate ratio, x = 0.4983, for a Liquid yield y = 0.2…Ans.

In addition:

Work output of expander per unit mass compressed = 76.37 kJ/kg…Ans.

Net work requirement of the system per unit mass liquefied = 1219 kJ/kg…Ans.

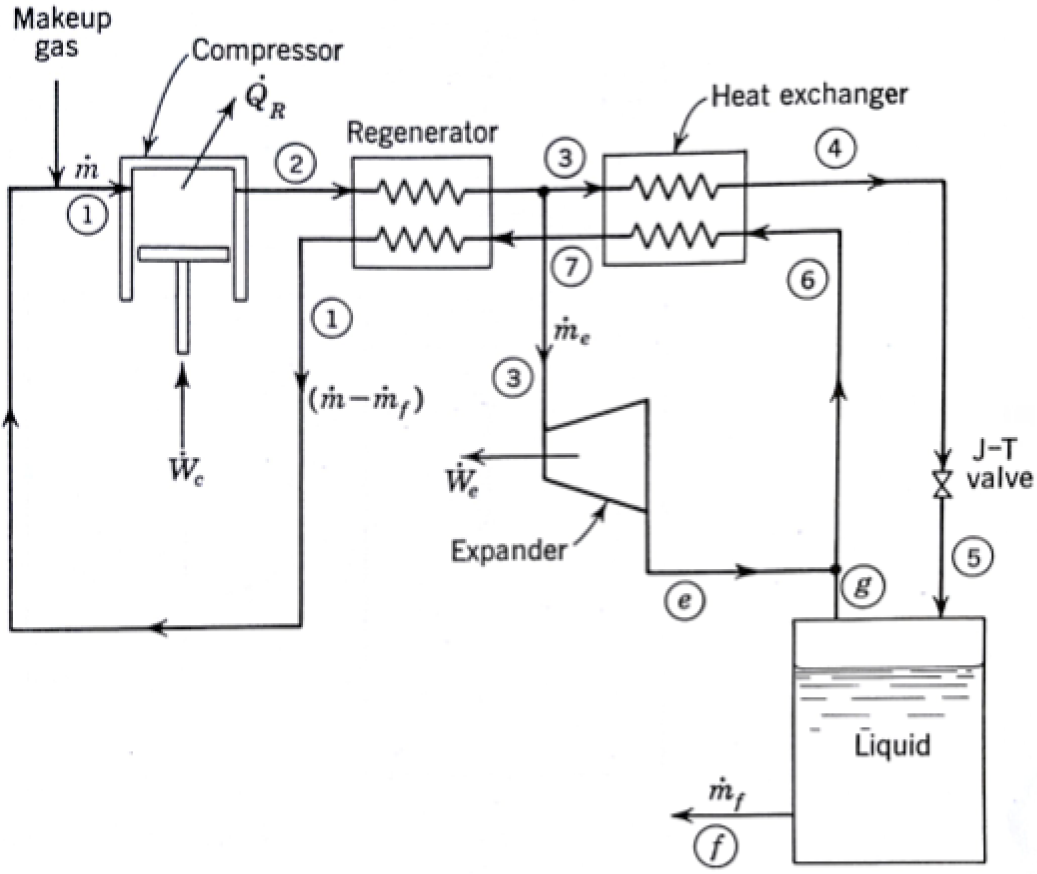
FOM = 0.6082…Ans.

"**Prob. 3.2.26** In an Ideal Kapitza liquefaction system for Nitrogen, the system operates from 101.3 kPa (1 atm) and 300 K to 5.07 MPa. At 5.07 MPa and 240 K, 50% of the main flow is diverted to the reversible adiabatic expander. Remainder flows through the heat exchanger and expands through the expansion valve to 101.3 kPa. Determine the liquid yield and the work per unit mass liquefied, assuming that the expander work is utilized in the compression process. [1]."

"Solution:"

"Note that Kapitza system is similar to Claude system except that the last heat exchanger (i.e. low temp heat exchanger) is eliminated and the first heat exchanger consists of regenerators.

For calculation purposes, the formulas are the same as for Claude system."



“Data:”

Fluid\$ = ‘Nitrogen’

P1 = 101.3 “kPa”

T1 = 300 “K”

P2= 5070 “kPa”

P3 = P2

T3 = 240 “K”

x = 0.5 “…expander flow rate ratio”

s1 = Entropy(Fluid\$,T=T1,P=P1) “[kJ/kg-K]”

```
s2 = Entropy(Fluid$,P=P2,T=T1) “[kJ/kg-K]”

s3 = Entropy(Fluid$,P=P3,T=T3) “[kJ/kg-K]”

se = s3 “…entropy after isentropic expansion in expander”

h1 = Enthalpy(Fluid$,T=T1,P=P1) “[kJ/kg]”

h2 = Enthalpy(Fluid$,P=P2,T=T1) “[kJ/kg]”

h3 = Enthalpy(Fluid$,P=P3,T=T3) “[kJ/kg]”

he = Enthalpy(Fluid$,P=P1,s= se) “[kJ/kg]”

hf = Enthalpy(Fluid$,x=0,P=P1) “[kJ/kg]”

y = (h1 – h2) / (h1 – hf) + x * ((h3 – he) / (h1 – hf)) “…fraction liquefied”

W_exp_perkggas = x * (h3 – he) “kJ/kg of gas compressed”
```

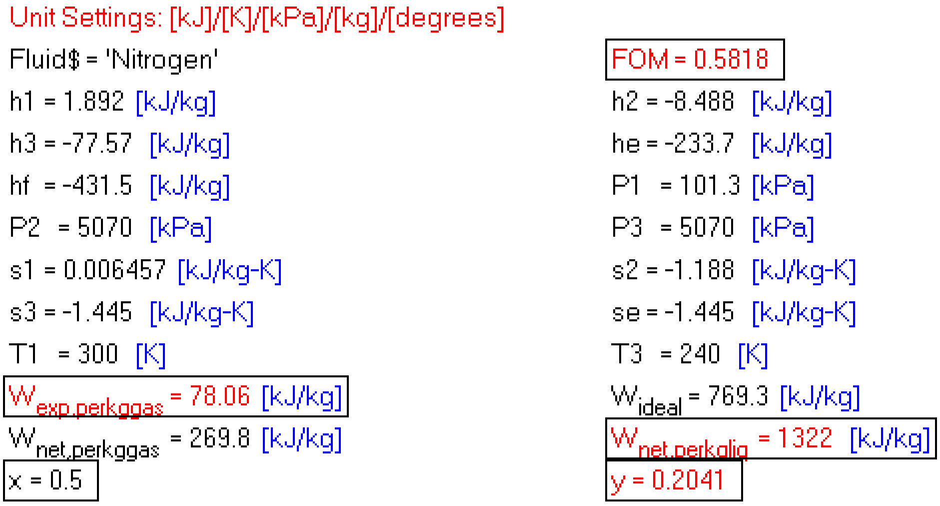
W_net_perkggas =T1 * (s1 – s2) – (h1 – h2) – x * (h3 – he) “[kJ/kg]…net work per kg gas compressed..when the expander work is utilized in compression”

W_net_perkgliq = W_net_perkggas / y “[kJ/kg]…net work reqd per kg liq.”

W_ideal = Ideal_Work(FLUID$, P1,T1) “kJ/kg…ideal work of liquefaction”

FOM = W_ideal / W_net_perkgliq “…Figure of Merit”

Results:



```
Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]
Fluid$ = 'Nitrogen'                            FOM = 0.5818
h1 = 1.892 [kJ/kg]                             h2 = -8.488  [kJ/kg]
h3 = -77.57  [kJ/kg]                           he = -233.7  [kJ/kg]
hf  = -431.5  [kJ/kg]                          P1   = 101.3 [kPa]
P2   = 5070  [kPa]                             P3   = 5070  [kPa]
s1 = 0.006457 [kJ/kg-K]                        s2 = -1.188  [kJ/kg-K]
s3 = -1.445  [kJ/kg-K]                         se = -1.445  [kJ/kg-K]
T1   = 300  [K]                                T3   = 240  [K]
W_exp,perkggas = 78.06 [kJ/kg]                 W_ideal = 769.3 [kJ/kg]
W_net,perkggas = 269.8 [kJ/kg]                 W_net,perkgliq  = 1322  [kJ/kg]
x = 0.5                                        y = 0.2041
```

Thus:

For Expander flow rate ratio, x = 0.5, Liquid yield y = 0.2041…Ans.

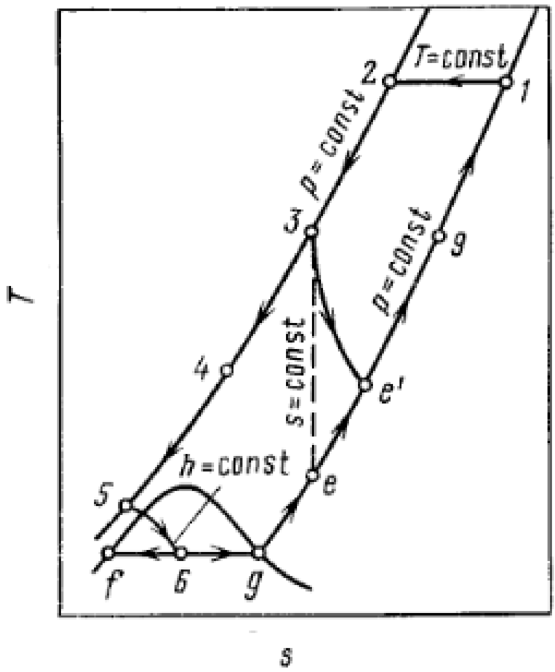
Net work requirement of the system per unit mass liquefied = 1322 kJ/kg…Ans.

Work output of expander per unit mass compressed = 78.06 kJ/kg…Ans.

FOM = 0.5818…Ans.

(b) If the adiabatic efficiency of expander is 80%, mech. effcy of expander = 94% and overall effcy of compressor = 75%, what will be the liquid yield? Plot liquid yield and FOM against adiabatic effcy of expander.

Temp-entropy diagram for this case is:



And, summary of formulas applicable is given below:

Compressor work per unit mass of gas liquefied:

$$-\frac{\dot W_c}{\dot m_f}=\frac{1}{y\eta_{c0}}\left[T_1(s_1-s_2)-(h_1-h_2)\right],$$

Actual enthalpy change across the expander:

$$h_3-h'_e=\eta_{ad}(h_3-h_e).$$

Liquid yield:

$$y=\frac{h_1-h_2}{h_1-h_f}+x\eta_{ad}\frac{h_3-h_e}{h_1-h_f}.$$

Net work requirement per unit mass compressed, when expander work is utilized:

$$\dot W/\dot m=-\dot W_c/\dot m-x\eta_{e,m}(h_3-h'_e)$$

Or,

$$-\frac{\dot{W}}{\dot{m}}=\frac{T_1(s_1-s_2)-(h_1-h_2)}{\eta_{c0}}-x\eta_{e0}(h_3-h_e),$$

EES Program:

“Case 2: For Real Kapitza system (all efficiencies, except HX effectivenesses, less than 100%):”

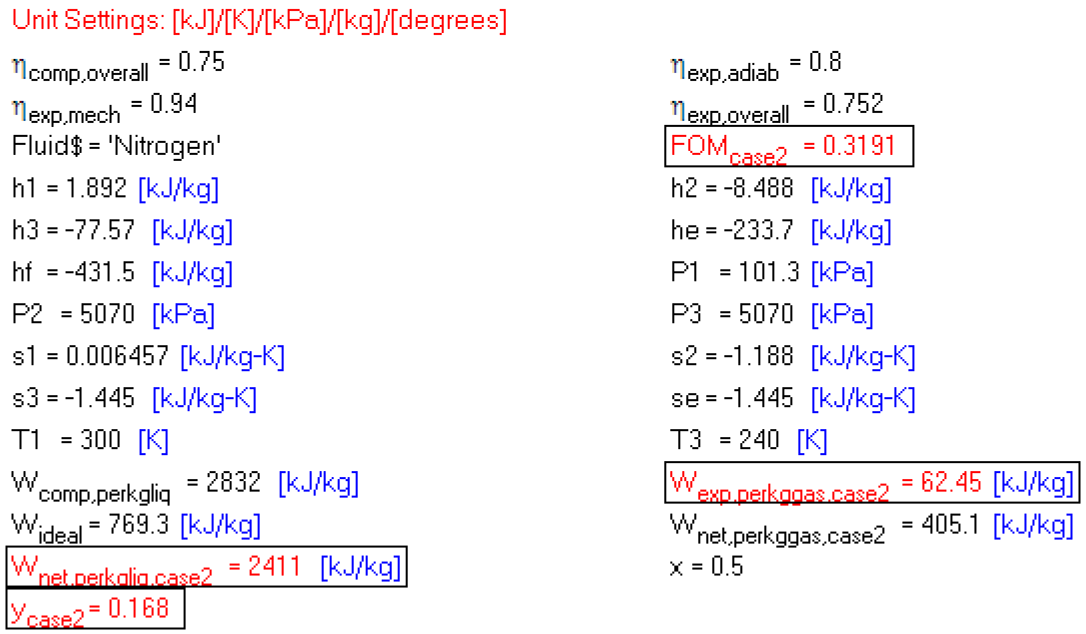
```
Fluid$ = ‘Nitrogen’

P1 = 101.3”kPa”
T1 = 300”K”
P2= 5070”kPa”
P3 = P2
T3 = 240 “K”

x = 0.5 “…expander flow rate ratio”

eta_exp_adiab = 0.8 “…adiab effcy of expander”
```

```
eta_exp_mech = 0.94 “…mech effcy of expander”

eta_comp_overall = 0.75 “…overall effcy of compressor”

eta_exp_overall = eta_exp_adiab * eta_exp_mech “…overall effcy of expander”

s1=Entropy(Fluid$,T=T1,P=P1) “[kJ/kg-K]”

s2=Entropy(Fluid$,P=P2,T=T1) “[kJ/kg-K]”

s3=Entropy(Fluid$,P=P3,T=T3) “[kJ/kg-K]”

se = s3 “…entropy after isentropic expansion in expander”

h1=Enthalpy(Fluid$,T=T1,P=P1) “[kJ/kg]”

h2=Enthalpy(Fluid$,P=P2,T=T1) “[kJ/kg]”

h3=Enthalpy(Fluid$,P=P3,T=T3) “[kJ/kg]”

he = Enthalpy(Fluid$,P=P1,s= se) “[kJ/kg]”

hf=Enthalpy(Fluid$,x=0,P=P1) “[kJ/kg]”

y_case2= (h1 – h2) / (h1 – hf) + x * eta_exp_adiab * ((h3 – he) / (h1 – hf)) “…fraction liquefied”

W_comp_perkgliq = (1 /( y_case2 * eta_comp_overall)) * T1 * (s1 – s2) – (h1 – h2) “[kJ/kg]…compressor work per kg gas liquefied..”

W_net_perkggas_case2= (T1 * (s1 – s2) – (h1 – h2)) / eta_comp_overall – x * eta_exp_overall * (h3 – he) “[kJ/kg]…net work per kg gas compressed…when the expander work is utilised in compression”

W_net_perkgliq_case2= W_net_perkggas_case2 / y_case2 “[kJ/kg]…net work reqd per kg liq.”

W_exp_perkggas_case2 = x * eta_exp_adiab * (h3 – he) “kJ/kg…work output of expander per kg of gas compressed”
```

W_ideal = Ideal_Work(FLUID$, P1,T1) "kJ/kg…ideal work of liquefaction"

FOM_case2= W_ideal / W_net_perkgliq_case2 "…Figure of Merit"

Results:



Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

$\eta_{comp,overall}$ = 0.75
$\eta_{exp,mech}$ = 0.94
Fluid$ = 'Nitrogen'
h1 = 1.892 [kJ/kg]
h3 = -77.57 [kJ/kg]
hf = -431.5 [kJ/kg]
P2 = 5070 [kPa]
s1 = 0.006457 [kJ/kg-K]
s3 = -1.445 [kJ/kg-K]
T1 = 300 [K]
$W_{comp,perkgliq}$ = 2832 [kJ/kg]
W_{ideal} = 769.3 [kJ/kg]
$W_{net,perkgliq,case2}$ = 2411 [kJ/kg]
y_{case2} = 0.168

$\eta_{exp,adiab}$ = 0.8
$\eta_{exp,overall}$ = 0.752
FOM_{case2} = 0.3191
h2 = -8.488 [kJ/kg]
he = -233.7 [kJ/kg]
P1 = 101.3 [kPa]
P3 = 5070 [kPa]
s2 = -1.188 [kJ/kg-K]
se = -1.445 [kJ/kg-K]
T3 = 240 [K]
$W_{exp,perkggas,case2}$ = 62.45 [kJ/kg]
$W_{net,perkggas,case2}$ = 405.1 [kJ/kg]
x = 0.5

Thus:

For Liquid yield y = 0.168…Ans.…when adiabatic effcy of expander = 80%

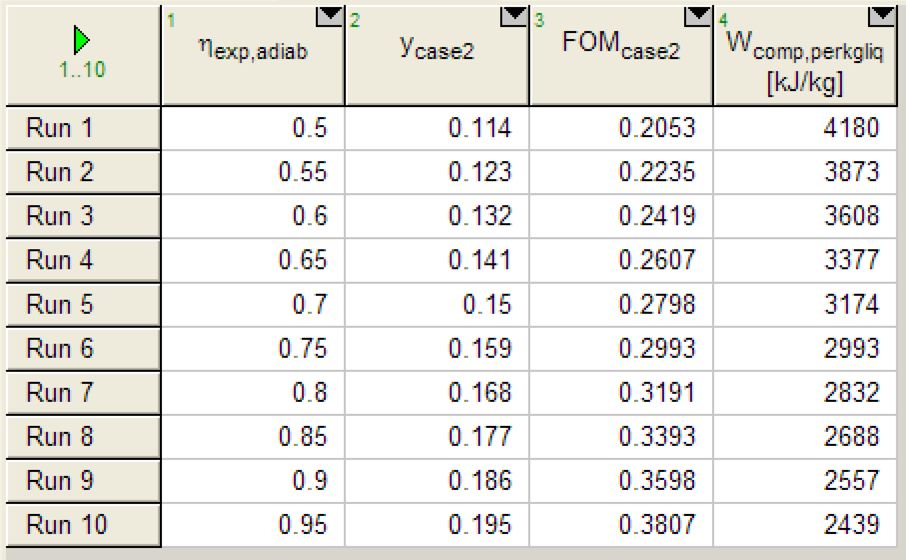
Net work requirement of the system per unit mass liquefied = 2411 kJ/kg…Ans.

Work output of expander per unit mass compressed = 62.45 kJ/kg…Ans.

FOM = 0.3191…Ans.

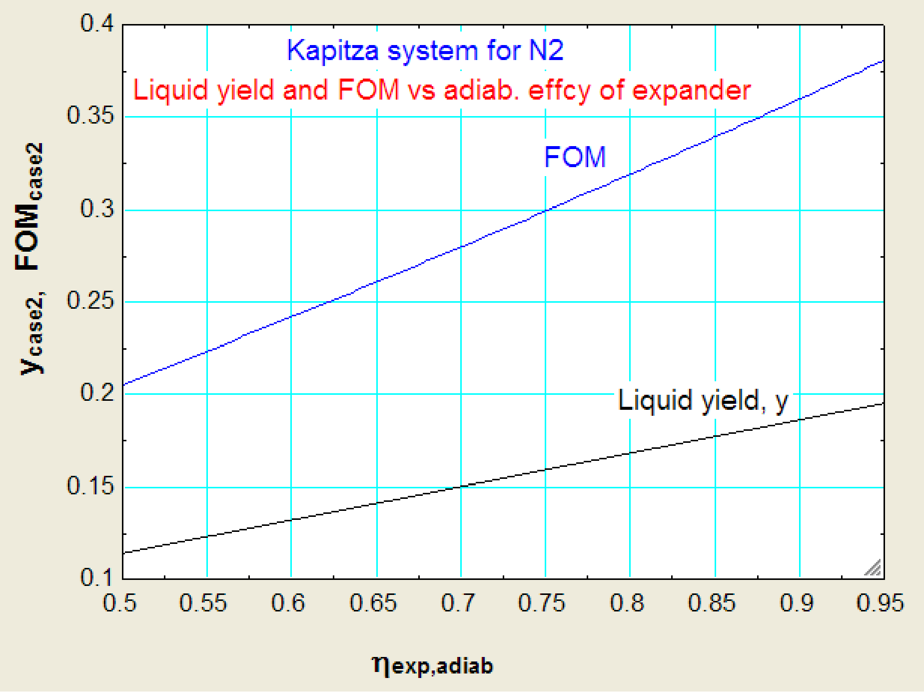
Plot liquid yield and FOM against adiab. effcy of expander:

First compute the parametric table:

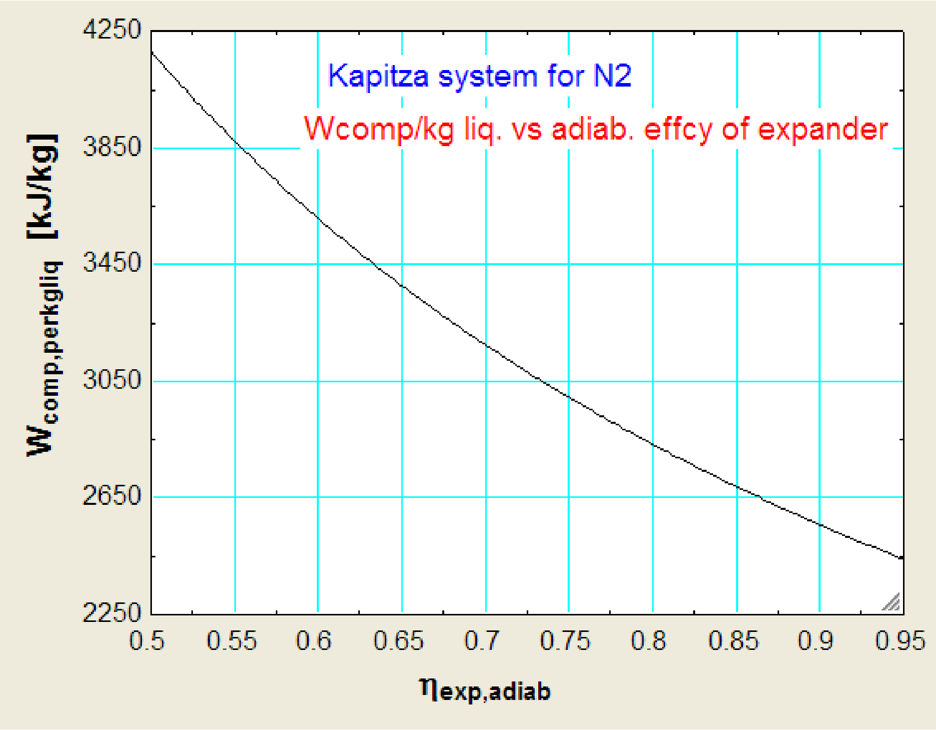


1..10	1 $\eta_{exp,adiab}$	2 y_{case2}	3 FOM_{case2}	4 $W_{comp,perkgliq}$ [kJ/kg]
Run 1	0.5	0.114	0.2053	4180
Run 2	0.55	0.123	0.2235	3873
Run 3	0.6	0.132	0.2419	3608
Run 4	0.65	0.141	0.2607	3377
Run 5	0.7	0.15	0.2798	3174
Run 6	0.75	0.159	0.2993	2993
Run 7	0.8	0.168	0.3191	2832
Run 8	0.85	0.177	0.3393	2688
Run 9	0.9	0.186	0.3598	2557
Run 10	0.95	0.195	0.3807	2439

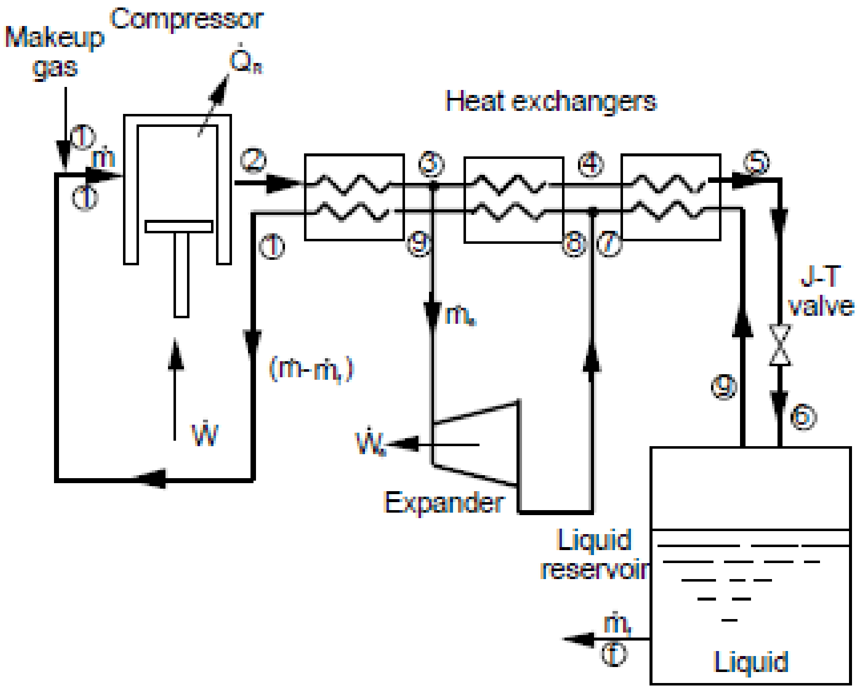
Now, plot the graphs:

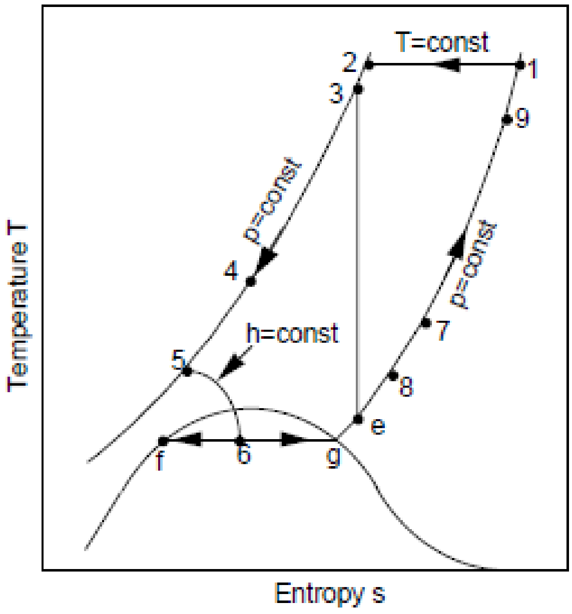


And:



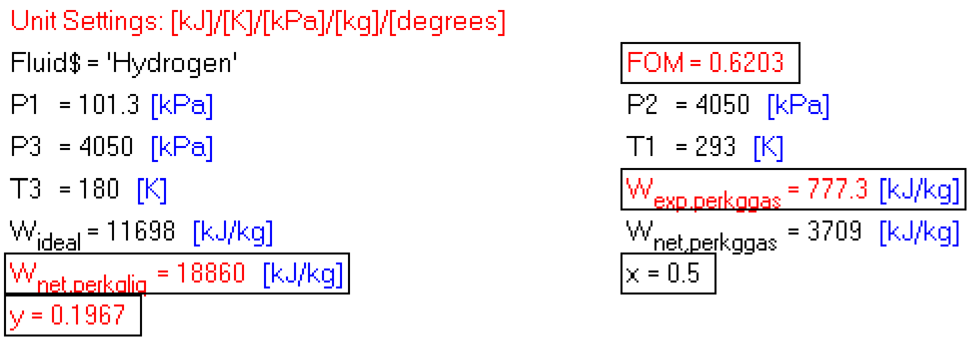
"**Prob. 3.2.27** In an Ideal Claude system for hydrogen, the system operates from 101.3 kPa (1 atm) and 293 K to 4.05 MPa. At 4.05 MPa and 180 K, fraction 'x' = 50% of the main flow is diverted to the reversible adiabatic expander. Remainder flows through the heat exchangers and expands through the expansion valve to 101.3 kPa. Determine the liquid yield, work per unit mass liquefied (assuming that the expander work is utilized in compression), and the FOM of the system. [1]."





```
"Data:"
Fluid$ = 'Hydrogen'

P1 = 101.3 "kPa"
T1 = 293 "K"
P2= 4050 "kPa"
P3 = P2
T3 = 180 "K"
x = 0.5

"We use the EES Procedure already written:"

CALL ClaudeSystem_Ideal(FLUID$, P1,P2, P3,T1,T3,x: y,W_net_perkggas,W_net_perkgliq,
W_exp_perkggas,W_ideal, FOM)
```

Results:



```
Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]
Fluid$ = 'Hydrogen'                              FOM = 0.6203
P1  = 101.3  [kPa]                               P2  = 4050  [kPa]
P3  = 4050  [kPa]                                T1  = 293  [K]
T3  = 180  [K]                                   Wexp,perkggas = 777.3  [kJ/kg]
Wideal = 11698  [kJ/kg]                          Wnet,perkggas = 3709  [kJ/kg]
Wnet,perkgliq = 18860  [kJ/kg]                   x = 0.5
y = 0.1967
```

Thus:

For an Expander flow rate ratio, x = 0.5, we get: Liquid yield y = 0.1967…Ans.

In addition:

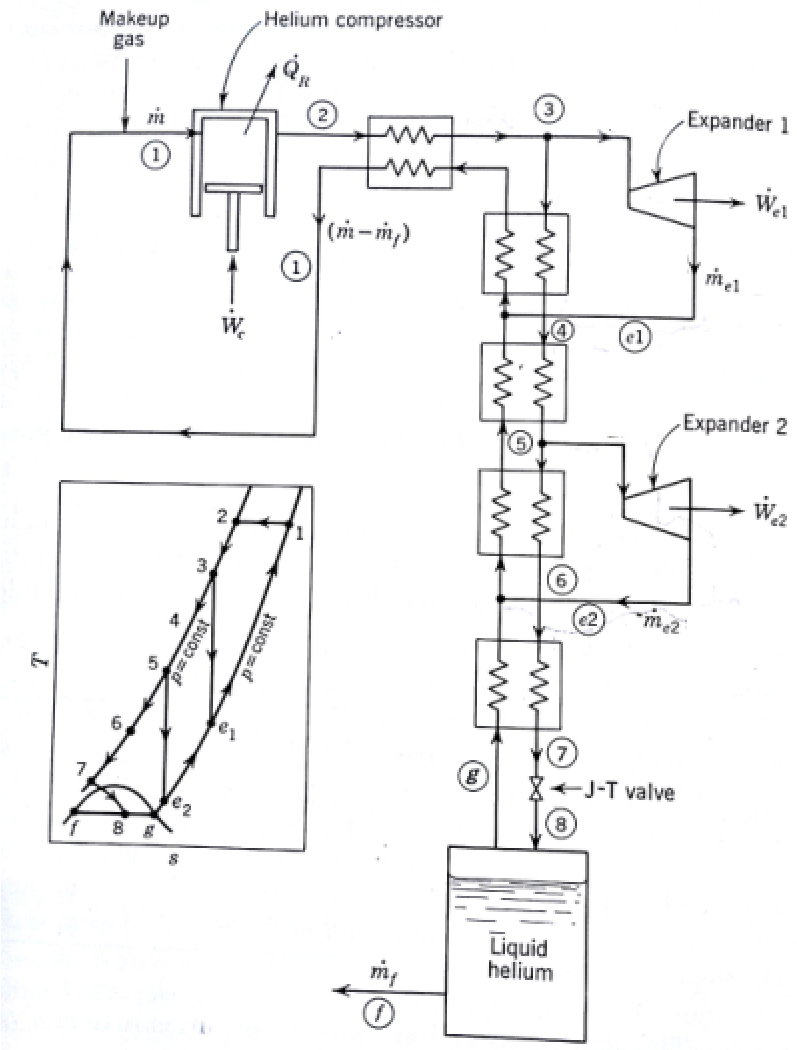
Work output of expander per unit mass compressed = 777.3 kJ/kg…Ans.

Net work requirement of the system per unit mass liquefied = 18860 kJ/kg…Ans.

FOM = 0.6203…Ans.

“**Prob. 3.2.28** Write an EES Procedure to calculate various parameters of interest for an Ideal Collins liquefier with two expanders, with the expander works not being utilized in compression.

(b) An Ideal two-expander Collins helium liquefaction system has expander flow rate ratios of x1 = 0.25 and x2 = 0.5. Helium gas enters the compressor at 101.3 kPa and 300 K and is compressed to 1.42 MPa. The condition of the gas at the inlet to the first expander is 1.42 MPa and 60 K and the condition of the gas at the inlet to the second expander is 15 K and 1.42 MPa. Both expanders are reversible and adiabatic. Determine the liquid yield, work per unit mass liquefied, and the Figure of Merit, assuming that: (a) expander work is not utilized in compression, (b) expander work is utilized in compressing the gas.S [1].”



EES Procedure:

PROCEDURE CollinsSystem_Ideal(FLUID$, P_1,P_2, T_1,T_3, T_5,x1,x2: y, W_ideal, W_perkggas_noexpwork, W_perkggas_withexpwork, W_perkgliq_noexpwork, W_perkgliq_withexpwork, FOM_noexpwork, FOM_withexpwork)

"Inputs: FLUID$, P_1, P_2, T_1, T_3,T_5:…Pressures in kPa, Temp in K;

x1, x2 = expander flow rate ratios for the expander 1 and 2"

"Outputs: y = fraction liquefied"

"W_ideal = Ideal work reqd.,"

“W_perkggas_noexpwork, [kJ/kg]…work per kg gas compressed..when the expander work is not utilised in compression”

“W_perkggas_withexpwork, [kJ/kg]…work per kg gas compressed..when the expander work is utilised in compression”

“W_perkgliq_noexpwork, [kJ/kg]…work reqd per kg liq. when the expander work is not utilised in compression”

“W_perkgliq_withexpwork ,[kJ/kg]…work reqd per kg liq. when the expander work is utilised in compression”

“FOM_noexpwork = W_ideal / W_perkgliq_noexpwork…Figure of Merit…when the expander work is not utilised in compression”

“FOM_withexpwork = W_ideal / W_perkgliq_withexpwork…Figure of Merit…when the expander work is utilised in compression”

P_3: = P_2

```
P_5: = P_2

s_1:=Entropy(Fluid$,T=T_1,P=P_1) “[kJ/kg-K]”

s_2:=Entropy(Fluid$,P=P_2,T=T_1) “[kJ/kg-K]”

s_3:=Entropy(Fluid$,P=P_3,T=T_3) “[kJ/kg-K]”

s_5:=Entropy(Fluid$,P=P_5,T=T_5) “[kJ/kg-K]”

s_e1: = s_3 “…entropy after isentropic expansion in expander-1”

s_e2 = s_5 “…entropy after isentropic expansion in expander-2”

h_1:=Enthalpy(Fluid$,T=T_1,P=P_1) “[kJ/kg]”

h_2: =Enthalpy(Fluid$,P=P_2,T=T_1) “[kJ/kg]”

h_3: =Enthalpy(Fluid$,P=P_3,T=T_3) “[kJ/kg]”

h_5: =Enthalpy(Fluid$,P=P_5,T=T_5) “[kJ/kg]”

h_e1: = Enthalpy(Fluid$,P=P_1,s= s_e1) “[kJ/kg]”

h_e2: = Enthalpy(Fluid$,P=P_1,s= s_e2) “[kJ/kg]”

h_f: =Enthalpy(Fluid$,x=0,P=P_1) “[kJ/kg]”

y: = (h_1 – h_2) / (h_1 – h_f) + x1* ((h_3 – h_e1) / (h_1 – h_f)) + x2* ((h_5 – h_e2)
/ (h_1 – h_f)) “…fraction liquefied”

W_exp1_perkggas: = x1 * (h_3 – h_e1) “kJ/kg of gas compressed”

W_exp2_perkggas: = x2* (h_5 – h_e2) “kJ/kg of gas compressed”

W_perkggas_noexpwork: =T_1 * (s_1 – s_2) – (h_1 – h_2) “[kJ/kg]…work per kg gas
compressed..when the expander work is not utilised in compression”

W_perkgliq_noexpwork: = W_perkggas_noexpwork / y “[kJ/kg]…work reqd per kg liq.
when the expander work is not utilised in compression”
```

```
W_ideal:= Ideal_Work(FLUID$, P_1,T_1) “kJ/kg…ideal work of liquefaction”

FOM_noexpwork: = W_ideal / W_perkgliq_noexpwork “…Figure of Merit… when the expander work is not utilised in compression”

W_perkggas_withexpwork: =T_1 * (s_1 – s_2) – (h_1 – h_2) – x1 * (h_3 – h_e1) – x2 * (h_5 – h_e2) “[kJ/kg]…work per kg gas compressed…when the expander work is utilised in compression”

W_perkgliq_withexpwork: = W_perkggas_withexpwork / y “[kJ/kg]…work reqd per kg liq. when the expander work is utilised in compression”

FOM_withexpwork: = W_ideal / W_perkgliq_withexpwork “…Figure of Merit…when the expander work is utilised in compression”

END

“=====================================================================”
```

(b) Problem:

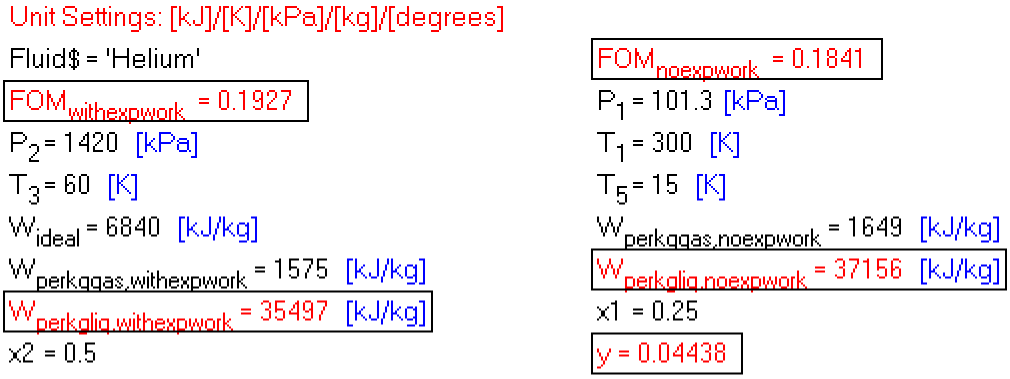
“Data:”

```
Fluid$ = ‘Helium’

x1 = 0.25
x2 = 0.5
P_1 = 101.3 “kPa”
P_2 = 1420 “kPa”
T_1 = 300 “K”
T_3 = 60 “K”
T_5 = 15 “K”
```

Now, use the EES Procedure already written:

```
CALL CollinsSystem_Ideal(FLUID$, P_1,P_2,T_1,T_3,T_5,x1,x2: y, W_ideal, W_perkggas_noexpwork, W_perkggas_withexpwork, W_perkgliq_noexpwork, W_perkgliq_withexpwork, FOM_noexpwork, FOM_withexpwork)
```

Results:



Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

Fluid$ = 'Helium'

$FOM_{withexpwork}$ = 0.1927

P_2 = 1420 [kPa]

T_3 = 60 [K]

W_{ideal} = 6840 [kJ/kg]

$W_{perkggas,withexpwork}$ = 1575 [kJ/kg]

$W_{perkgliq,withexpwork}$ = 35497 [kJ/kg]

x2 = 0.5

$FOM_{noexpwork}$ = 0.1841

P_1 = 101.3 [kPa]

T_1 = 300 [K]

T_5 = 15 [K]

$W_{perkggas,noexpwork}$ = 1649 [kJ/kg]

$W_{perkgliq,noexpwork}$ = 37156 [kJ/kg]

x1 = 0.25

y = 0.04438

Thus:

a) **When Expander work is utilized in compression:**
Liquid yield = y = 0.04438 = 4.438%…Ans.
Work per unit mass liquefied = 35497 kJ/kg…Ans.
Figure of Merit = 0.1927…Ans.

b) **When Expander work is *not utilized* in compression:**
Liquid yield = y = 0.04438 = 4.438%…Ans.
Work per unit mass liquefied = 37156 kJ/kg…Ans.
Figure of Merit = 0.1841…Ans.

"**Prob. 3.2.29** Data for a Claude system is given below:
Air is the working fluid, the system operates from 101.3 kPa (1 atm) and 293 K to 10.13 MPa. At 10.13 MPa and 250 K, 40% of the main flow is diverted to the reversible adiabatic expander. Remainder flows through the heat exchangers and expands through the expansion valve to 101.3 kPa. Assume that the expander has an adiabatic effcy of 80% and a mech. effcy of 90%. Compressor has an overall effcy of 75% and all heat exchangers are 100% effective. After compression, air is after-cooled to 293 K. Determine the liquid yield, work output of expander per unit mass compressed and the net work requirement of the system per unit mass liquefied, assuming that the expander work is utilized to aid in the compression.[1]."

"Data:"

Fluid$ = 'Air_ha'

P1 = 101.3 "kPa"
T1 = 293 "K"
P2= 10130 "kPa"
P3 = P2
T3 = 250 "K"

x = 0.4

eta_exp_adiab = 0.8 "…adiab effcy of expander"

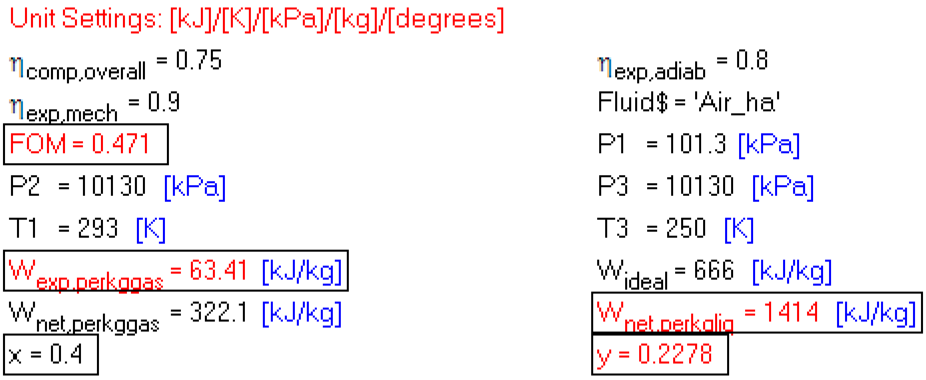
eta_exp_mech = 0.9 "…mech effcy of expander"

eta_comp_overall = 0.75 "…overall effcy of compressor"

“Case 1: For Ideal Claude system (all efficiencies = 100%):”

CALL ClaudeSystem_Ideal(FLUID$, P1,P2, P3, T1,T3,x: y,W_net_perkggas,W_net_perkgliq, W_exp_perkggas,W_ideal, FOM)

Results:



Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

$\eta_{comp,overall}$ = 0.75
$\eta_{exp,adiab}$ = 0.8
$\eta_{exp,mech}$ = 0.9
Fluid$ = 'Air_ha'
FOM = 0.471
P1 = 101.3 [kPa]
P2 = 10130 [kPa]
P3 = 10130 [kPa]
T1 = 293 [K]
T3 = 250 [K]
$W_{exp,perkggas}$ = 63.41 [kJ/kg]
W_{ideal} = 666 [kJ/kg]
$W_{net,perkggas}$ = 322.1 [kJ/kg]
$W_{net,perkgliq}$ = 1414 [kJ/kg]
x = 0.4
y = 0.2278

Thus:

Liquid yield = y = 0.2278 = 22.78%…Ans.

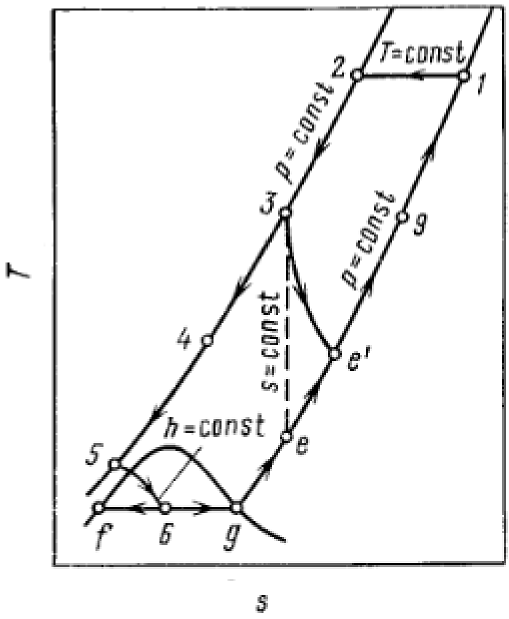
Work output of expander per unit mass compressed = 63.41 kJ/kg…Ans.

Net work requirement per unit mass liquefied = 1414 kJ/kg…Ans.

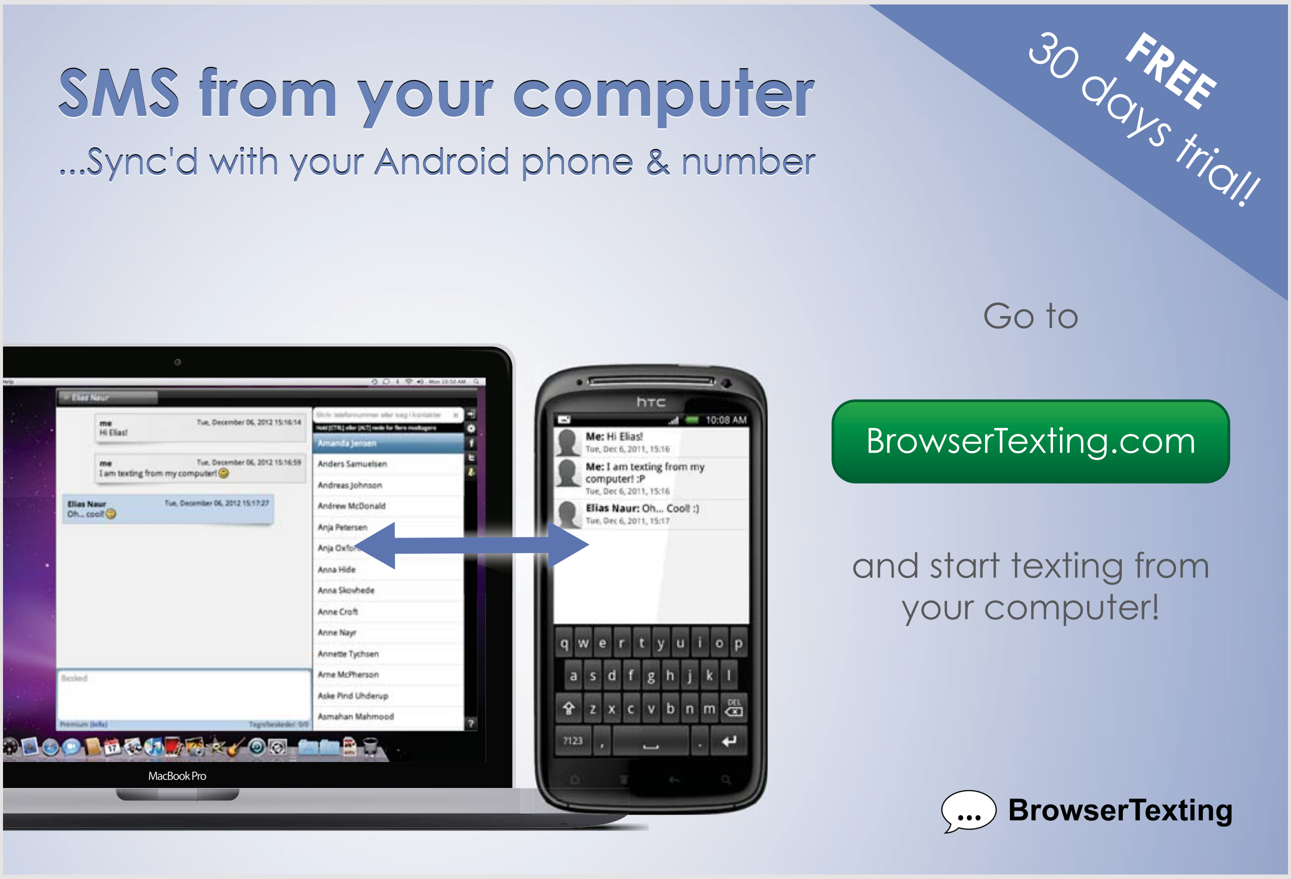
Figure of Merit = 0.471…Ans.

“Case 2: When the efficiencies of compressor and expander are less than 100%, with the effectiveness of heat exchangers remaining 100%:”

Temp-entropy diagram for this case is:



And, summary of formulas applicable is given below:



Compressor work per unit mass of gas liquefied:

$$-\frac{\dot{W}_c}{\dot{m}_f}=\frac{1}{y\eta_{c0}}\left[T_1(s_1-s_2)-(h_1-h_2)\right],$$

Actual enthalpy change across the expander:

$$h_3-h'_e=\eta_{ad}(h_3-h_e).$$

Liquid yield:

$$y=\frac{h_1-h_2}{h_1-h_f}+x\eta_{ad}\frac{h_3-h_e}{h_1-h_f}.$$

Net work requirement per unit mass compressed, when expander work is utilized:

$$\dot{W}/\dot{m}=-\dot{W}_c/\dot{m}-x\eta_{e,m}(h_3-h'_e)$$

Or,

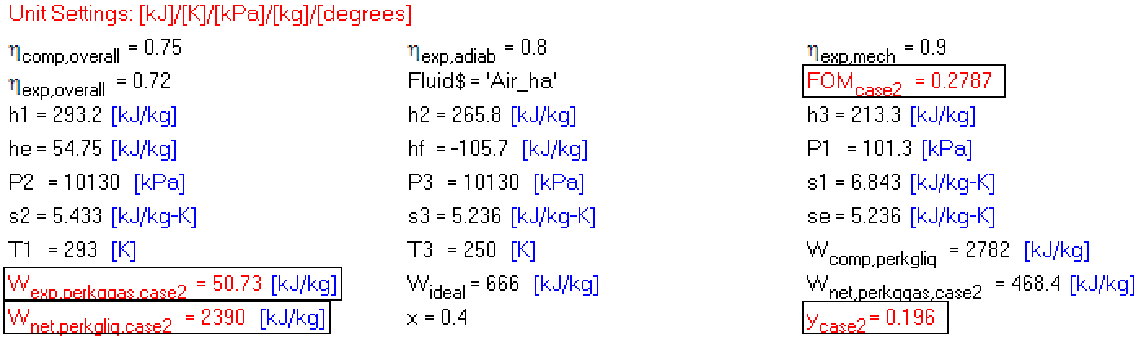
$$-\frac{\dot{W}}{\dot{m}}=\frac{T_1(s_1-s_2)-(h_1-h_2)}{\eta_{c0}}-x\eta_{e0}(h_3-h_e),$$

EES Program:

```
eta_exp_adiab = 0.8 “…adiab effcy of expander”

eta_exp_mech = 0.9 “…mech effcy of expander”

eta_comp_overall = 0.75 “…overall effcy of compressor”

eta_exp_overall = eta_exp_adiab * eta_exp_mech “…overall effcy of expander”

s1=Entropy(Fluid$,T=T1,P=P1) “[kJ/kg-K]”

s2=Entropy(Fluid$,P=P2,T=T1) “[kJ/kg-K]”

s3=Entropy(Fluid$,P=P3,T=T3) “[kJ/kg-K]”
```

```
se = s3 “…entropy after isentropic expansion in expander”

h1=Enthalpy(Fluid$,T=T1,P=P1) “[kJ/kg]”

h2=Enthalpy(Fluid$,P=P2,T=T1) “[kJ/kg]”

h3=Enthalpy(Fluid$,P=P3,T=T3) “[kJ/kg]”

he = Enthalpy(Fluid$,P=P1,s= se) “[kJ/kg]”

hf=Enthalpy(Fluid$,x=0,P=P1) “[kJ/kg]”

y_case2= (h1 – h2) / (h1 – hf) + x * eta_exp_adiab * ((h3 – he) / (h1 – hf)) “…fraction liquefied”

W_comp_perkgliq = (1 /( y_case2 * eta_comp_overall)) * T1 * (s1 – s2) – (h1 – h2) “[kJ/kg]…compressor work per kg gas liquefied”

W_net_perkggas_case2= (T1 * (s1 – s2) – (h1 – h2)) / eta_comp_overall – x * eta_exp_overall * (h3 – he) “[kJ/kg]…net work per kg gas compressed..when the expander work is utilised in compression”

W_net_perkgliq_case2= W_net_perkggas_case2 / y_case2 “[kJ/kg]…net work reqd per kg liq.”

W_exp_perkggas_case2: = x * eta_exp_adiab * (h_3 – h_e) “kJ/kg…work output of expander per kg of gas compressed”

W_ideal = Ideal_Work(FLUID$, P1,T1) “kJ/kg …ideal work of liquefaction”

FOM_case2= W_ideal / W_net_perkgliq_case2 “…Figure of Merit”
```

Results:



Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

$\eta_{comp,overall}$ = 0.75
$\eta_{exp,overall}$ = 0.72
h1 = 293.2 [kJ/kg]
he = 54.75 [kJ/kg]
P2 = 10130 [kPa]
s2 = 5.433 [kJ/kg-K]
T1 = 293 [K]
$W_{exp,perkggas,case2}$ = 50.73 [kJ/kg]
$W_{net,perkgliq,case2}$ = 2390 [kJ/kg]

$\eta_{exp,adiab}$ = 0.8
Fluid$ = 'Air_ha'
h2 = 265.8 [kJ/kg]
hf = -105.7 [kJ/kg]
P3 = 10130 [kPa]
s3 = 5.236 [kJ/kg-K]
T3 = 250 [K]
W_{ideal} = 666 [kJ/kg]
x = 0.4

$\eta_{exp,mech}$ = 0.9
FOM_{case2} = 0.2787
h3 = 213.3 [kJ/kg]
P1 = 101.3 [kPa]
s1 = 6.843 [kJ/kg-K]
se = 5.236 [kJ/kg-K]
$W_{comp,perkgliq}$ = 2782 [kJ/kg]
$W_{net,perkggas,case2}$ = 468.4 [kJ/kg]
y_{case2} = 0.196

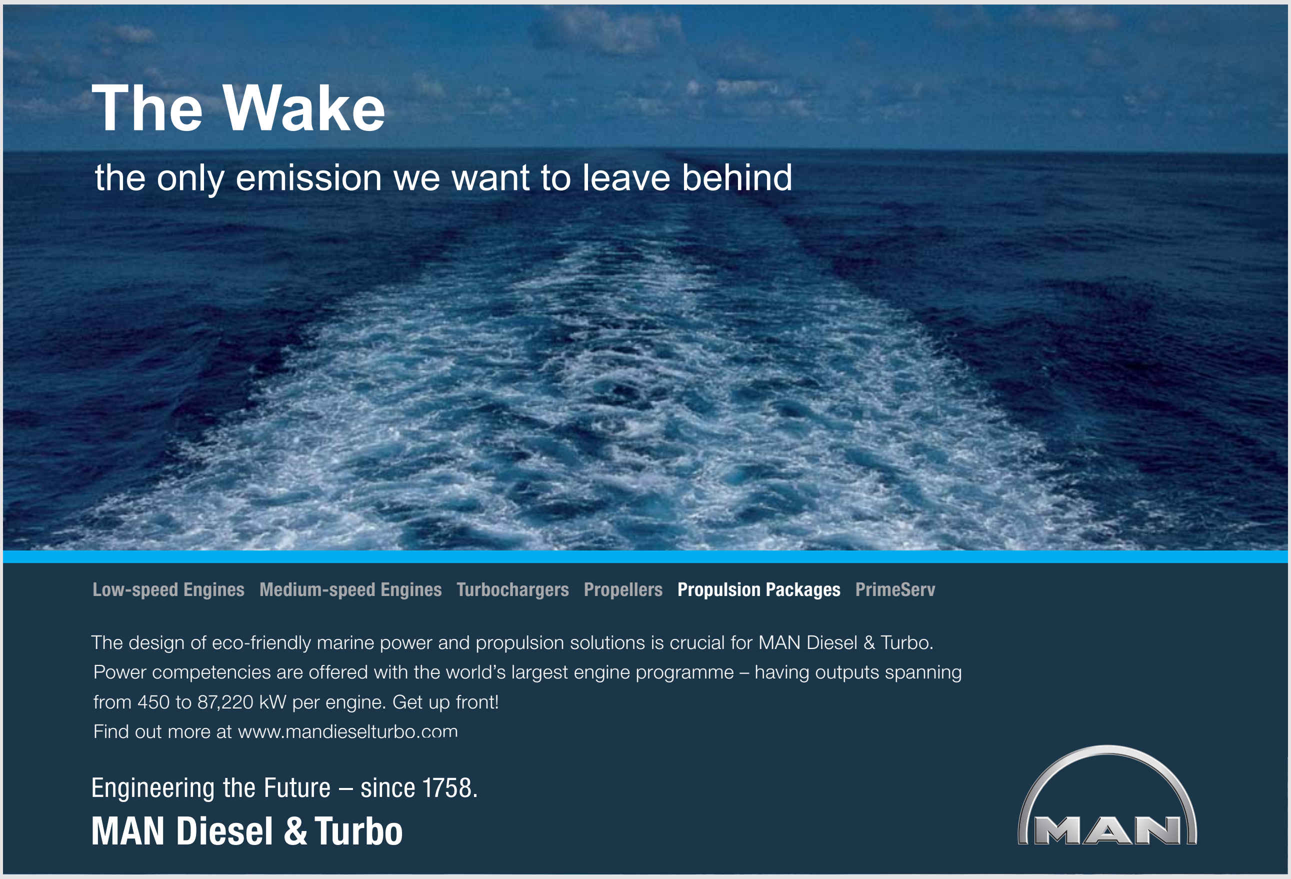
Thus:

Liquid yield = y = 0.196 = 19.6%…Ans.

Work output of expander per unit mass compressed = 50.73 kJ/kg…Ans.

Net work requirement per unit mass liquefied = 2390 kJ/kg…Ans.

Figure of Merit = 0.2787…Ans.



"**Prob. 3.2.30** Data for a Claude system is given below:
Neon is the working fluid, the system operates from 101.3 kPa (1 atm) and 296 K to 6.08 MPa. At 6.08 MPa and 210 K, 40% of the main flow is diverted to the reversible adiabatic expander. Remainder flows through the heat exchangers and expands through the expansion valve to 101.3 kPa. Assume that the expander has an adiabatic effcy of 82% and a mech. effcy of 94%. Compressor has an overall effcy of 75% and all heat exchangers are 100% effective. After compression, neon is after-cooled to 296 K. Determine the liquid yield, work output of expander per unit mass compressed and the net work requirement of the system per unit mass liquefied, assuming that the expander work is utilized to aid in the compression.[1]."

"Data:"

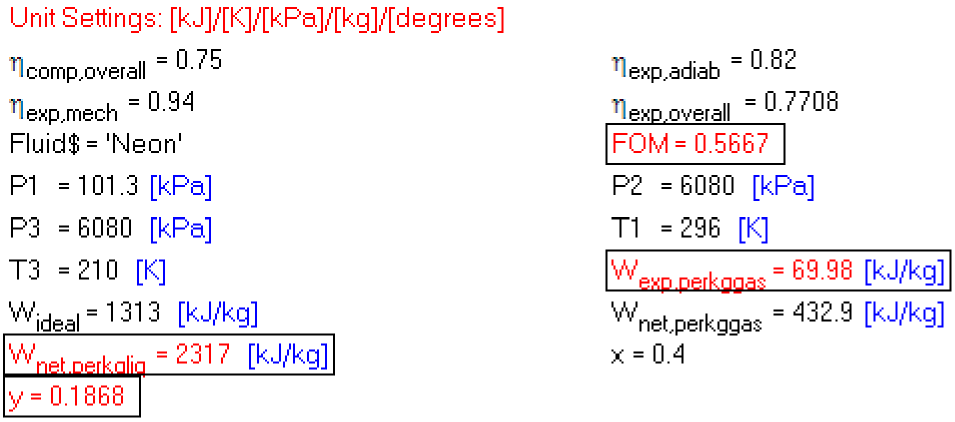
```
Fluid$ = 'Neon'

P1 = 101.3 "kPa"
T1 = 296 "K"
P2= 6080 "kPa"
P3 = P2
T3 = 210 "K"

x = 0.4

eta_exp_adiab = 0.82 "…adiab effcy of expander"

eta_exp_mech = 0.94 "…mech effcy of expander"

eta_comp_overall = 0.75 "…overall effcy of compressor"

eta_exp_overall = eta_exp_adiab * eta_exp_mech "…overall effcy of expander"
```

"Case 1: For Ideal Claude system (all efficiencies = 100%):"

```
CALL ClaudeSystem_Ideal(FLUID$, P1,P2, P3,T1,T3,x: y,W_net_perkggas,W_net_perkgliq,
W_exp_perkggas,W_ideal, FOM)
```

……

Results:



Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

$\eta_{comp,overall} = 0.75$

$\eta_{exp,mech} = 0.94$

Fluid$ = 'Neon'

P1 = 101.3 [kPa]

P3 = 6080 [kPa]

T3 = 210 [K]

$W_{ideal} = 1313$ [kJ/kg]

$W_{net,perkgliq} = 2317$ [kJ/kg]

y = 0.1868

$\eta_{exp,adiab} = 0.82$

$\eta_{exp,overall} = 0.7708$

FOM = 0.5667

P2 = 6080 [kPa]

T1 = 296 [K]

$W_{exp,perkggas} = 69.98$ [kJ/kg]

$W_{net,perkggas} = 432.9$ [kJ/kg]

x = 0.4

Thus:

Liquid yield = y = 0.1868 = 18.68%…Ans.

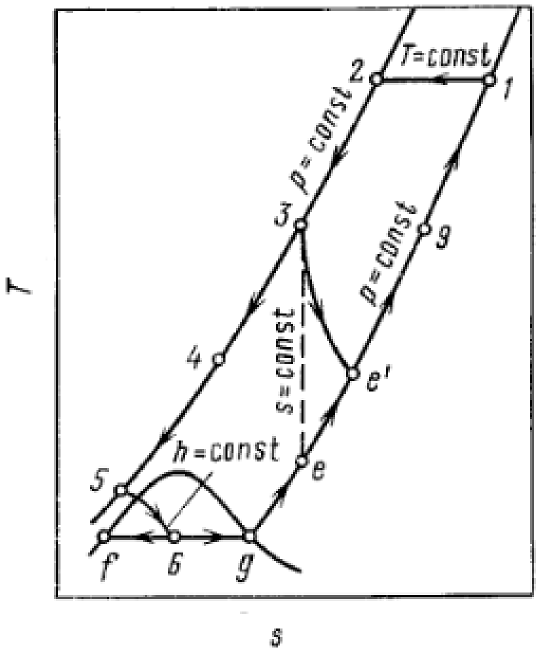
Work output of expander per unit mass compressed = 69.98 kJ/kg…Ans.

Net work requirement per unit mass liquefied = 2317 kJ/kg…Ans.

Figure of Merit = 0.5667…Ans.

"Case 2: When the efficiencies of compressor and expander are less than 100%, with the effectiveness of heat exchangers remaining 100%:"

Temp-entropy diagram for this case is:



And, summary of formulas applicable is given below:

Compressor work per unit mass of gas liquefied:

$$-\frac{\dot{W}_c}{\dot{m}_f}=\frac{1}{y\eta_{c0}}\left[T_1(s_1-s_2)-(h_1-h_2)\right].$$

Actual enthalpy change across the expander:

$$h_3-h'_e=\eta_{ad}(h_3-h_e).$$

Liquid yield:

$$y=\frac{h_1-h_2}{h_1-h_f}+x\eta_{ad}\frac{h_3-h_e}{h_1-h_f}.$$

Net work requirement per unit mass compressed, when expander work is utilized:

$$\dot{W}/\dot{m}=-\dot{W}_c/\dot{m}-x\eta_{e,m}(h_3-h'_e)$$

Or,

$$-\frac{\dot{W}}{\dot{m}}=\frac{T_1(s_1-s_2)-(h_1-h_2)}{\eta_{c0}}-x\eta_{e0}(h_3-h_e),$$

EES Program:

```
s1=Entropy(Fluid$,T=T1,P=P1) “[kJ/kg-K]”

s2=Entropy(Fluid$,P=P2,T=T1) “[kJ/kg-K]”

s3=Entropy(Fluid$,P=P3,T=T3) “[kJ/kg-K]”

se = s3 “…entropy after isentropic expansion in expander”

h1=Enthalpy(Fluid$,T=T1,P=P1) “[kJ/kg]”

h2=Enthalpy(Fluid$,P=P2,T=T1) “[kJ/kg]”

h3=Enthalpy(Fluid$,P=P3,T=T3) “[kJ/kg]”

he = Enthalpy(Fluid$,P=P1,s= se) “[kJ/kg]”
```

hf=Enthalpy(Fluid$,x=0,P=P1) "[kJ/kg]"

y_case2= (h1 – h2) / (h1 – hf) + x * eta_exp_adiab * ((h3 – he) / (h1 – hf)) "…fraction liquefied"

W_comp_perkgliq = (1 /(y_case2 * eta_comp_overall)) * T1 * (s1 – s2) – (h1 – h2) "[kJ/kg]…compressor work per kg gas liquefied"

W_net_perkggas_case2= (T1 * (s1 – s2) – (h1 – h2)) / eta_comp_overall – x * eta_exp_overall * (h3 – he) "[kJ/kg]…net work per kg gas compressed…when the expander work is utilised in compression"

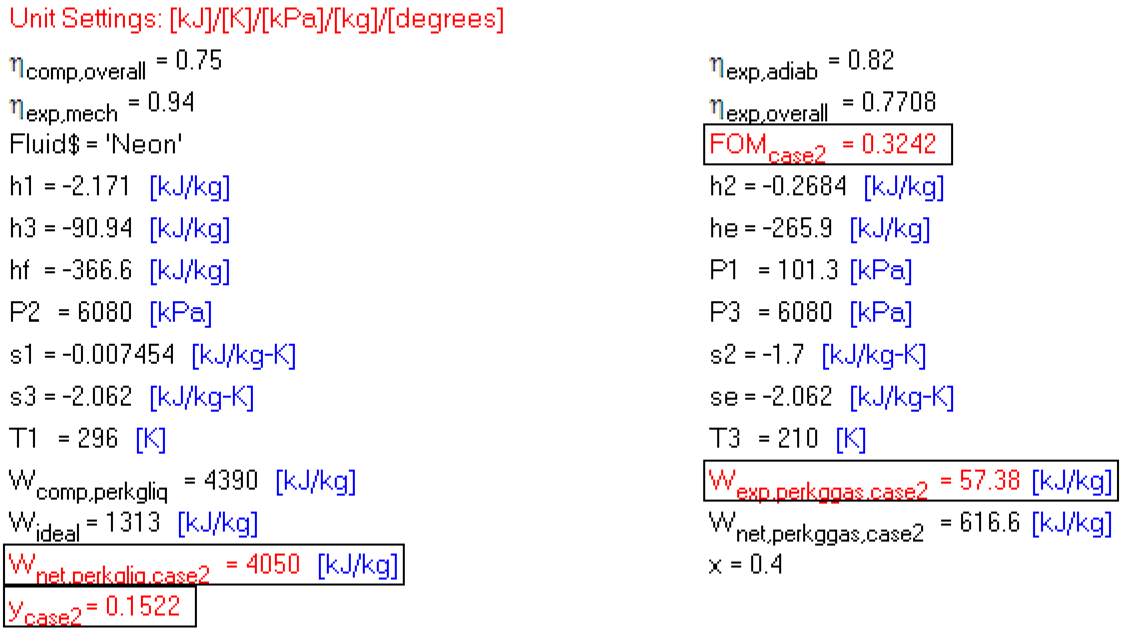
W_net_perkgliq_case2= W_net_perkggas_case2 / y_case2 "[kJ/kg]…net work reqd per kg liq."

W_exp_perkggas_case2: = x * eta_exp_adiab * (h_3 – h_e) "kJ/kg…work output of expander per kg of gas compressed"

W_ideal = Ideal_Work(FLUID$, P1,T1) "kJ/kg…ideal work of liquefaction"

FOM_case2= W_ideal / W_net_perkgliq_case2 "…Figure of Merit"

Results:



```
Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]
ηcomp,overall = 0.75                       ηexp,adiab = 0.82
ηexp,mech = 0.94                           ηexp,overall = 0.7708
Fluid$ = 'Neon'                            FOMcase2 = 0.3242
h1 = -2.171  [kJ/kg]                       h2 = -0.2684  [kJ/kg]
h3 = -90.94  [kJ/kg]                       he = -265.9  [kJ/kg]
hf = -366.6  [kJ/kg]                       P1 = 101.3 [kPa]
P2 = 6080  [kPa]                           P3 = 6080  [kPa]
s1 = -0.007454  [kJ/kg-K]                  s2 = -1.7  [kJ/kg-K]
s3 = -2.062  [kJ/kg-K]                     se = -2.062  [kJ/kg-K]
T1 = 296  [K]                              T3 = 210  [K]
Wcomp,perkgliq = 4390  [kJ/kg]             Wexp,perkggas,case2 = 57.38 [kJ/kg]
Wideal = 1313  [kJ/kg]                     Wnet,perkggas,case2 = 616.6 [kJ/kg]
Wnet,perkgliq,case2 = 4050  [kJ/kg]        x = 0.4
ycase2 = 0.1522
```

Thus:

Liquid yield = y = 0.1522 = 15.22%…Ans.

Work output of expander per unit mass compressed = 53.78 kJ/kg…Ans.

Net work requirement per unit mass liquefied = 4050 kJ/kg…Ans.

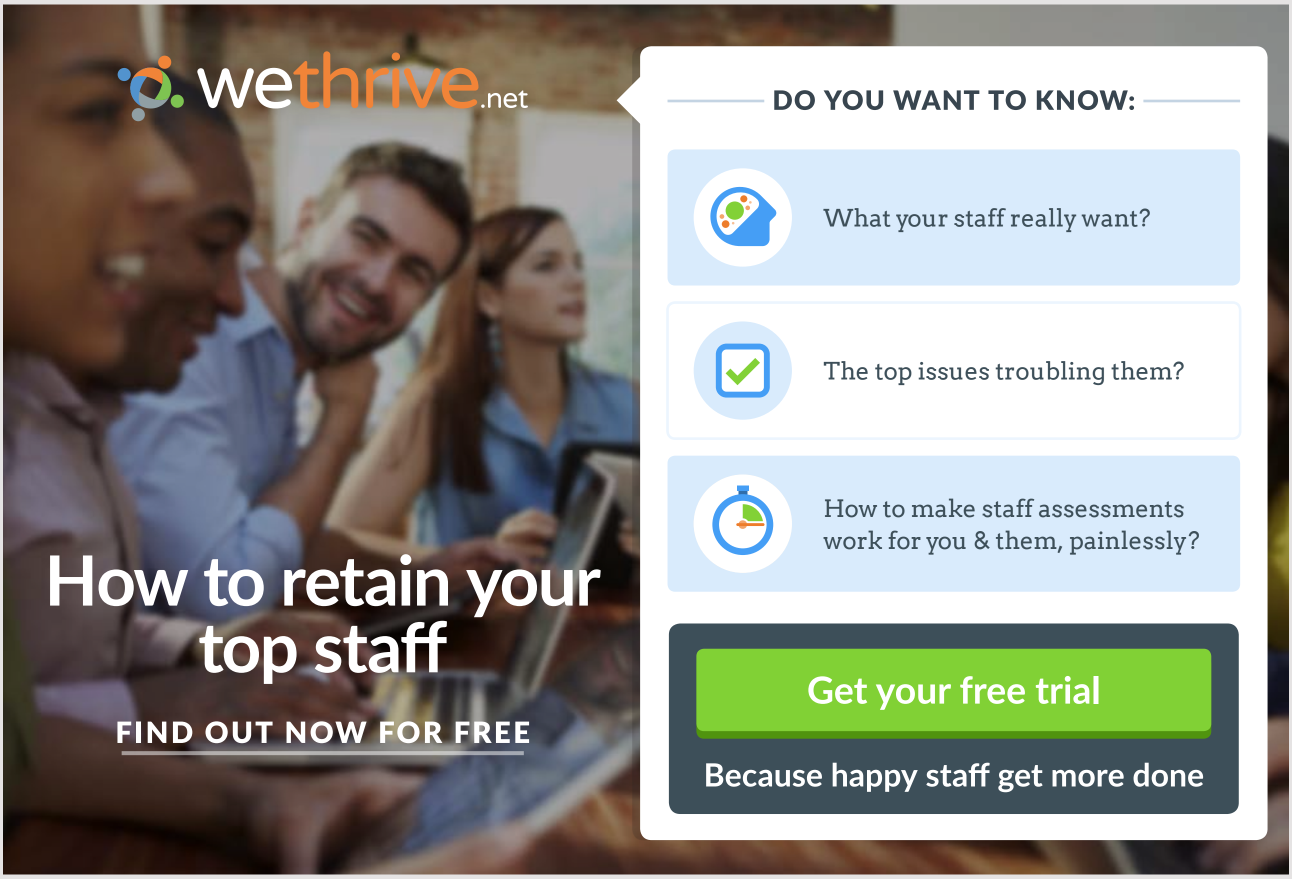
Figure of Merit = 0.3242…Ans.

"**Prob. 3.2.31** Determine the inside heat transfer coeff (hc) and friction factor (f) for flow of nitrogen gas at 150 K and 101.3 kPa inside a 12 mm inside dia smooth tube that is coiled in a 600 mm dia helix. The tube wall has a temp of 160 K and the mass flow rate of nitrogen gas is 30 g/s.
Also, plot the variation of hc and f as m_dot varies from 10 g/s to 70 g/s [1]."

EES Solution:

"Data:"

T1 = 150 "K…bulk temp of nitrogen gas"



P1 = 101.3 “kPa…pressure of nitrogen gas”

m_dot = 0.03 “kg/s…mass flow rate of nitrogen gas”

D = 0.012 “m…inside dia of tube”

D_h = 0.6 “m…dia of helix”

T_w = 160 “K…wall temp.”

L = 50 “m…length of tube”

“Calculations”

“Properties of Nitrogen”

mu = VISCOSITY(Nitrogen,T=T1,P=P1) “kg/m.s…viscosity of nitrogen gas”

A_c = pi * D^2 /4 “m^2…cross-sectional area of tube”

G = m_dot / A_c “kg/s.m^2…mass velocity”

“Reynolds No.”

N_Re = G * D / mu “ = 310865…Reynolds No… > 3000…therefore, turb. flow”

“For turb. flow calculations, fluid props should be evaluated at mean film temp Tm = (Tb + Tw) / 2”

T_m = (T1 + T_w) / 2 “ K…mean film temp”

“Then, we have:”

“Properties of Nitrogen”

k_t1 = CONDUCTIVITY(Nitrogen,T=T_m,P=P1) “W/m.K…thermal conductivity of nitrogen”

mu1 = VISCOSITY(Nitrogen,T=T_m,P=P1) “kg/m.s…viscosity of nitrogen gas”

cp1 = SPECHEAT(Nitrogen,T=T_m,P=P1) “kJ/kg.K…sp. heat of nitrogen gas”

rho1 = DENSITY(Nitrogen,T=T_m,P=P1) “kg/m^3…density of nitrogen gas”

N_Pr1 = mu1 * cp1 * 10^3 / k_t1 “…Prandtl No.”

“And:”

N_Re1 = G * D / mu1

“Colburn J factor:”

j_H = 0.023 * (N_Re1^ (-0.2)) * (1 + 3.5 * D / D_h) “…finds Colburn j factor“

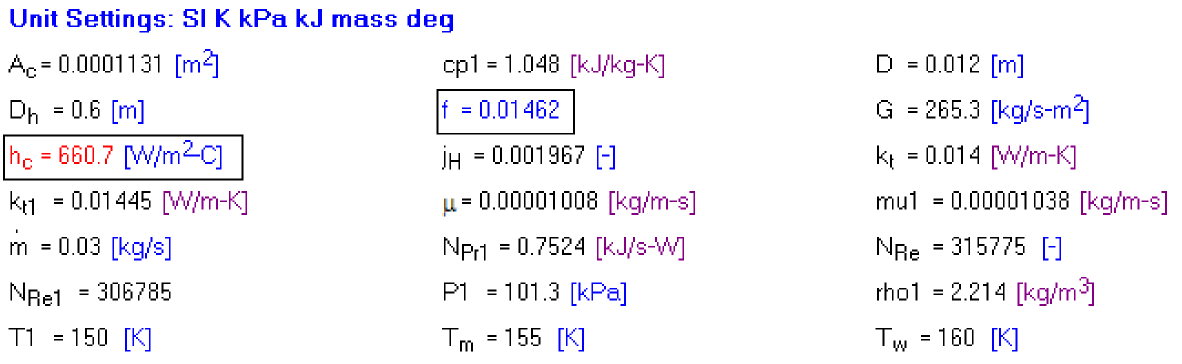
j_H = (h_c /(cp1 * 10^3 * G)) * N_Pr1^(2/3) “…finds h_c, the film coeff of heat transfer, W/m^2.K”

“Friction factor:”

“Note that Reynolds no. is > 5000. So, the friction factor relation is:”

f = 0.184 * N_Re^(-0.2) “…friction factor”

Results:



```
Unit Settings: SI K kPa kJ mass deg
A_c = 0.0001131 [m^2]     cp1 = 1.048 [kJ/kg-K]          D = 0.012 [m]
D_h = 0.6 [m]             f = 0.01462                    G = 265.3 [kg/s-m^2]
h_c = 660.7 [W/m^2-C]     j_H = 0.001967 [-]             k_t = 0.014 [W/m-K]
k_t1 = 0.01445 [W/m-K]    μ = 0.00001008 [kg/m-s]        mu1 = 0.00001038 [kg/m-s]
m = 0.03 [kg/s]           N_Pr1 = 0.7524 [kJ/s-W]        N_Re = 315775 [-]
N_Re1 = 306785            P1 = 101.3 [kPa]               rho1 = 2.214 [kg/m^3]
T1 = 150 [K]              T_m = 155 [K]                  T_w = 160 [K]
```

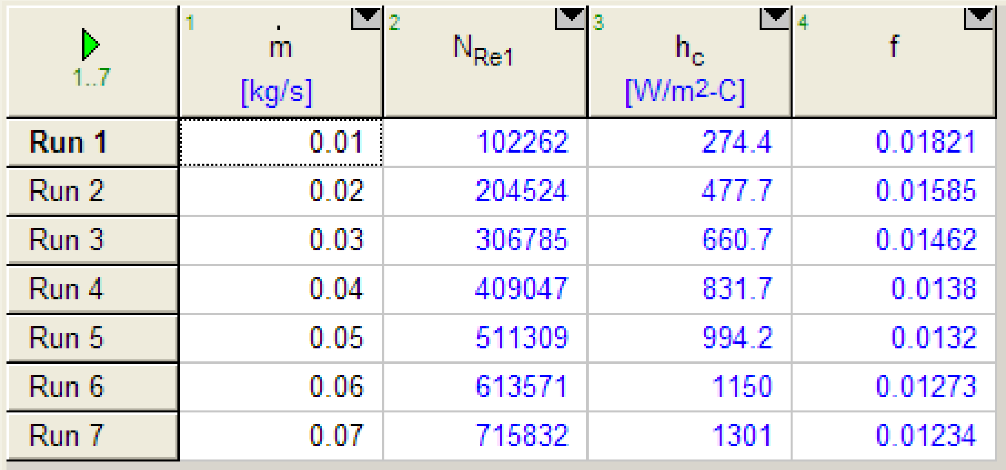
Thus:

Heat transfer coeff. h_c = 660.7 W/m^2.K…Ans.

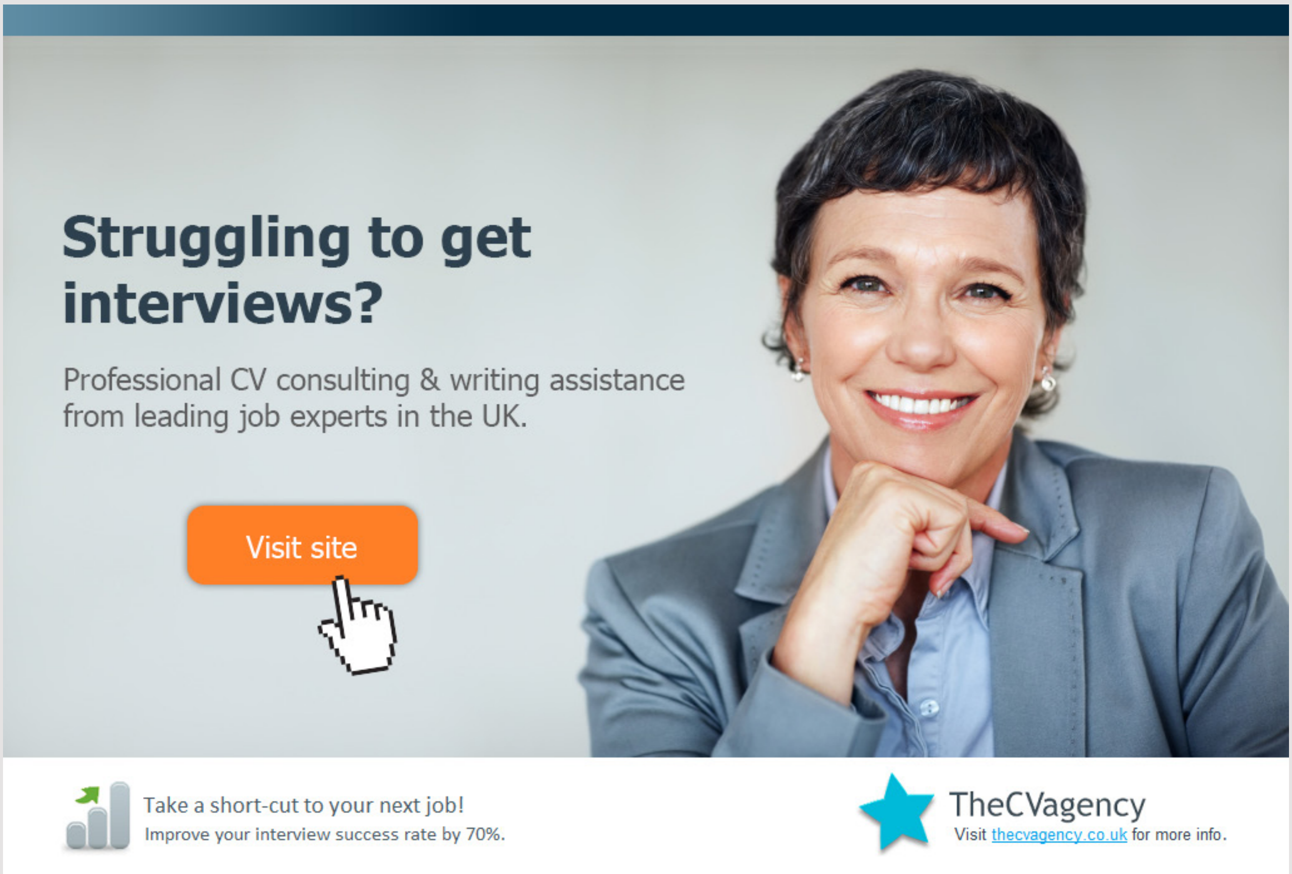
Friction factor, f = 0.01462…Ans.

Also, plot the variation of hc and f as m_dot varies from 10 g/s to 70 g/s:

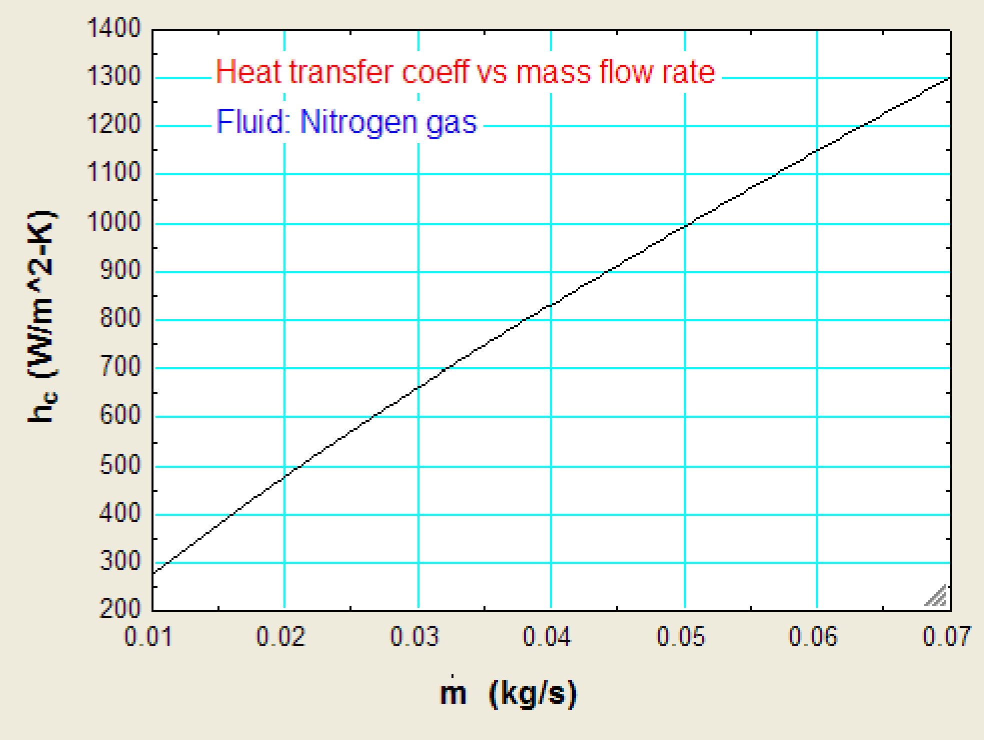
First compute the parametric table:

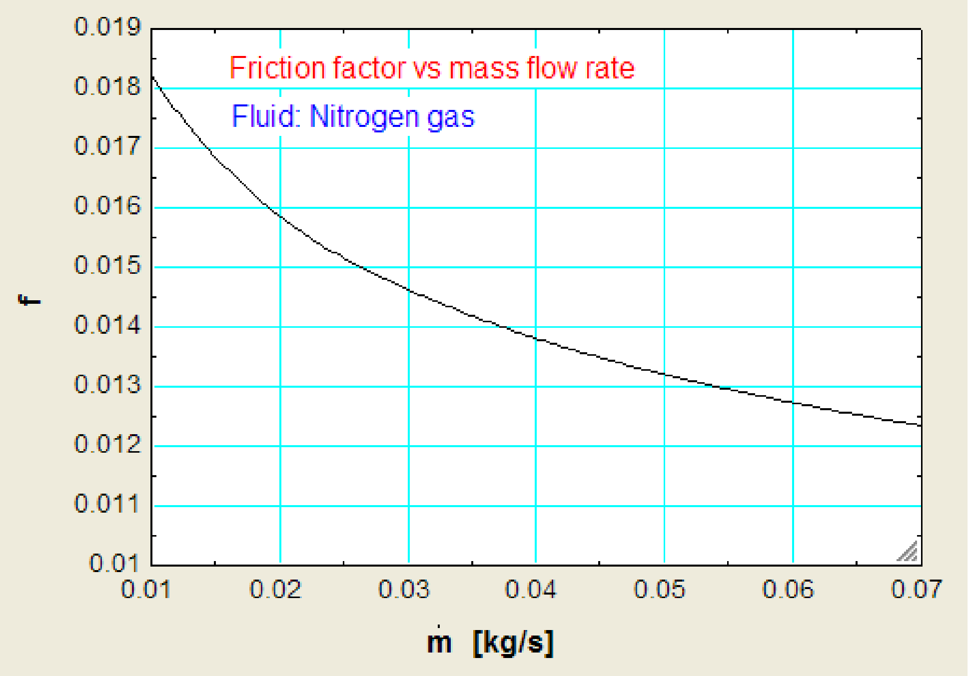


1..7	1 $\dot{m}$ [kg/s]	2 N_{Re1}	3 h_c [W/m2-C]	4 f
Run 1	0.01	102262	274.4	0.01821
Run 2	0.02	204524	477.7	0.01585
Run 3	0.03	306785	660.7	0.01462
Run 4	0.04	409047	831.7	0.0138
Run 5	0.05	511309	994.2	0.0132
Run 6	0.06	613571	1150	0.01273
Run 7	0.07	715832	1301	0.01234



Now, plot the results:





"**Prob. 3.2.32** A circular tube is constructed of copper (kt = 50 W/m.K), has circular pin fins protruding from its outer surface. The circular tube has an outer dia of 25 mm. The pin fins are 1.25 mm in dia and 12.5 mm long. There are 2000 fins/m length of tube, and the fins are also of copper. The outside heat transfer coeff is 35 W/m^2.K and the tube temp is 100 K. Determine the fin effectiveness and the overall surface effectiveness.[1]. "

EES Solution:

"Data:"

D_o = 0.025 "m…OD of tube"

L = 1 "m…length of tube"

d_f = 0.00125 "m…dia of fin"

L_f = 0.0125 "m…length of fin"

N_f = 2000 "…no. of fins per meter length of tube"

k_f = 50 "W/m-K…thermal cond. of fin material, copper"

h_o = 35 "W/m^2-K…outside heat transfer coeff."

"Calculations:"

A_c = pi * d_f^2 / 4 "m^2…cross-sectional area of fin"

V_f = A_c * L_f "m^3…volume of fin"

A_sf = pi * d_f * L_f "m^2…surface area of one fin"

"Therefore, to find fin parameter M"

delta = V_f / A_sf "m… parameter delta to calculate fin parameter M"

M = sqrt(h_o / (k_f * delta)) " 1/m…fin parameter"

"Fin effectiveness eta_f:"

eta_f = tanh(M * L_f) / (M * L_f) "… fin effcy."

{

"Note: M may also be determined as:

M = sqrt (h_o * pi * d_f / (k_f * A_c))" i.e M = sqrt(hc * P / (kf * Ac)) where P is the perimeter of fin"

}

"To calculate Overall surface area effectiveness:"

"Total fin surface area:"

A_f = A_sf * N_f "m^2…"

"Total prime area:"

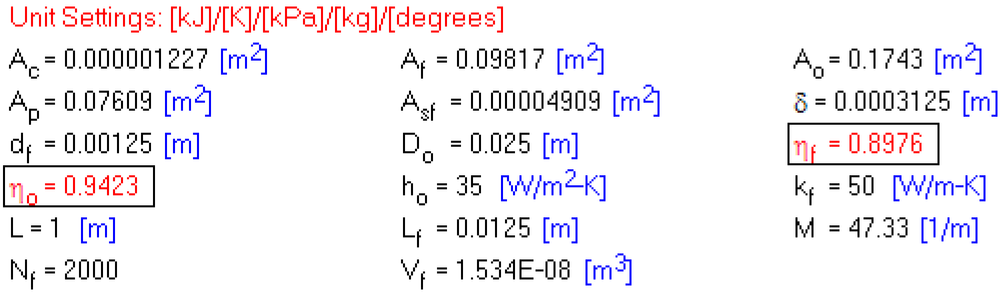
A_p = L * pi * D_o – N_f * pi * d_f^2 / 4 "m^2…prime area"

"Then, Overall surface area effectiveness:"

A_o = A_f + A_p "m^2…Sum of total fin area and prime area"

eta_o = 1 – (A_f / A_o) * (1 – eta_f) "…overall surface area effectiveness"

Results:



```
Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]
Ac = 0.000001227 [m2]      Af  = 0.09817 [m2]        Ao = 0.1743 [m2]
Ap = 0.07609 [m2]          Asf = 0.00004909 [m2]     δ = 0.0003125 [m]
df = 0.00125 [m]           Do  = 0.025 [m]           ηf = 0.8976
ηo = 0.9423                ho = 35  [W/m2-K]         kf = 50  [W/m-K]
L = 1  [m]                 Lf  = 0.0125 [m]          M  = 47.33 [1/m]
Nf = 2000                  Vf  = 1.534E-08  [m3]
```

Thus:

Fin effectiveness = eta_f = 0.8976…Ans.

Overall surface area effectiveness = eta_o = 0.9423…Ans.

"**Prob. 3.2.33** Determine the overall heat transfer coeff in the previous problem if the inside dia of the tube is 22 mm and is plain. Base the overall heat transfer coeff on the total outside surface area (i.e. fin area plus the prime area). Determine the total heat transfer rate if the tube is 15 m long. The inside fluid temp is 140 K and the outside fluid temp is 106 K. The inside convective heat transfer coeff is 225 W/m^2.K. [1]."

EES Solution:

"Data:"

D_o = 0.025 "m…OD of tube"

D_i = 0.022 "m…ID if tube"

L = 15 "m…length of tube"

d_f = 0.00125 “m…dia of fin”

L_f = 0.0125 “m…length of fin”

N_f = 2000 “…no. of fins per meter length of tube”

k_f = 50 “W/m-K…thermal cond. of fin material, copper”

h_o = 35 “W/m^2-K…outside heat transfer coeff.”

h_i = 225 “W/m^2-K…inside heat transfer coeff.”

T_i = 140 “K…inside fluid temp”

T_o = 106 “K…outside fluid temp”

eta_o = 0.9423 “…overall surface area effectiveness on the outside…calculated earlier”

“Calculations:”

A_sf = pi * d_f * L_f “m^2…surface area of one fin”

“Total fin surface area:”

A_f = A_sf * N_f “m^2”

“Total prime area:”

A_p = L * pi * D_o – N_f * pi * d_f^2 / 4 “m^2…prime area”

“Total heat transfer area on the outside:”

A_oh = A_f + A_p “ m^2…Sum of total fin area and prime area = total heat transfer area on the outside”

“Total heat transfer area on the inside:”

A_oc = pi * D_i * L “m^2…inside area of heat transfer”

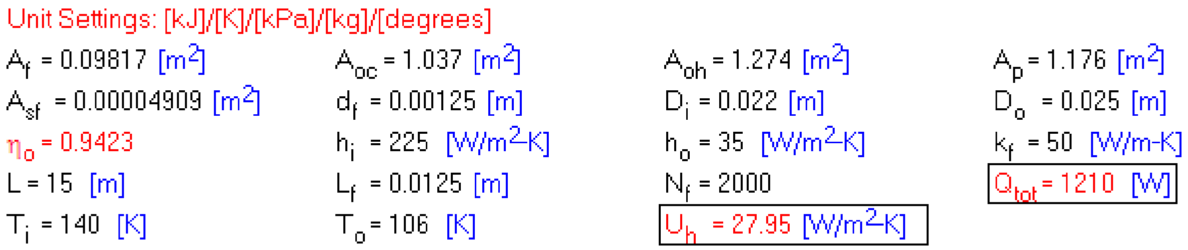
"Overall heat transfer coeff. U:"

1/U_h = (1 / (eta_o * h_o)) + (A_oh / A_oc) / h_i "…W/m^2.K…overall heat transfer coeff. based on total outer area"

"Total heat transfer rate:"

Q_tot = U_h * A_oh * (T_i – T_o) "W…total heat transfer"

Results:



Unit Settings: [kJ]/[K]/[kPa]/[kg]/[degrees]

A_f = 0.09817 [m^2] A_{oc} = 1.037 [m^2] A_{oh} = 1.274 [m^2] A_p = 1.176 [m^2]

A_{sf} = 0.00004909 [m^2] d_f = 0.00125 [m] D_i = 0.022 [m] D_o = 0.025 [m]

η_o = 0.9423 h_i = 225 [W/m^2-K] h_o = 35 [W/m^2-K] k_f = 50 [W/m-K]

L = 15 [m] L_f = 0.0125 [m] N_f = 2000 $\boxed{Q_{tot} = 1210 \text{ [W]}}$

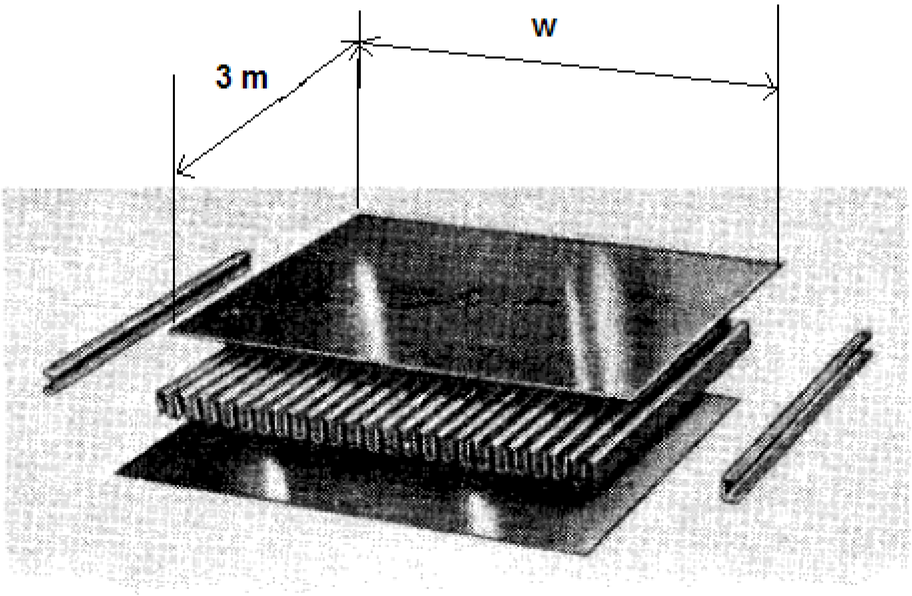
T_i = 140 [K] T_o = 106 [K] $\boxed{U_h = 27.95 \text{ [W/m}^2\text{-K]}}$

Thus:

Overall heat transfer coeff. based on outside area = U_h = 27.95 W/m^2.K…Ans.

Total heat transfer rate = Q_tot = 1210 W…Ans.

"**Prob. 3.2.34** A plate – fin heat exchanger is constructed using straight fins, 7.87 mm (0.31 in) high, 0.15 mm (0.006 in) thick, with 492 fins/m (12.5 fins/in) width. The equivalent dia for the flow passage is 3 mm (0.1181 in), the free-flow area for the exchanger is 0.07 m^2 (0.7535 ft^2), and the exchanger length is 3 m (9.84 ft). The fluid flowing is helium gas, which enters at 300 K and 3.04 MPa and leaves at 100 K. The mass flow rate of helium gas is 0.35 kg/s. The heat exchanger is constructed of aluminium (k_t = 150 W/m.K). Determine the convective heat transfer coeff. for this surface and the fin effectiveness. [1]."



EES Solution:

"Data:"

L_f = 0.00787 / 2 "m…fin length, divided by 2 since fin is fixed at both ends"

t_f = 0.00015 "m…thickness of fin"

D_e = 0.003 "m…equiv. dia of flow passage"

N_f = 492 "…no. of fins per meter"

A_ff = 0.07 "m^2…free flow area for the exchanger"

L = 3 "m…exchanger length"

```
"Fluid: Helium"

P1 = 3040 "kPa…pressure of helium gas"

T_i = 300 "K… inlet temp of helium gas"

T_o = 100 "K…exit temp of helium gas"

m_dot = 0.35 "kg/s…mass flow rate of helium gas"

k_f = 150 "W/m.K…thermal cond. of fin material, i.e. Aluminium"

"Calculations:"
A_c_f = L * t_f "m^2…cross-sectional area of fin"

G = m_dot / A_ff "kg/s.m^2…mass velocity"

T_b = (T_i + T_o ) / 2 "K…mean bulk temp of fluid"

"Properties of helium gas at T_b:"

rho=Density(Helium,T=T_b,P=P1) "kg/ m^3…density of helium gas"

cp=Cp(Helium,T=T_b,P=P1) "kJ/kg.K…sp. heat of helium gas"

mu=Viscosity(Helium,T=T_b,P=P1) "kg/m.s…viscosity of helium gas"

k=Conductivity(Helium,T=T_b,P=P1) "W/m.K…thermal cond. of helium gas"

"Reynolds No…etc.:"

N_Re = G * D_e /mu "…Reynolds No."

N_Pr = mu * cp * 1000/ k "…Prandtl No."

N_St = h_c / (G * cp * 1000) "…Stanton No."
```

"Colburn j factor and heat transfer coeff.:"

j_H = 0.0291 * N_Re^(-0.24) "…Colburn j factor…for 500 < N_Re < 10^4…**From Ref.[1]**"
j_H = N_St * N_Pr^(2/3) "…finds convective heat tr coeff, W/m^2.K"

"Fin efficiency:"

A_c = L * t_f "m^2…cross-sectional area of fin"

V_f = A_c * 2 * L_f "m^3…volume of fin"

A_sf = L * 2 * L_f * 2 "m^2…surface area of one fin"

"Therefore, to find fin parameter M:"

delta = V_f / A_sf "m… parameter delta to calculate fin parameter M"

M = sqrt(h_c / (k_f * delta)) "1/m …fin parameter"

{

"Note: M may also be determined as:"

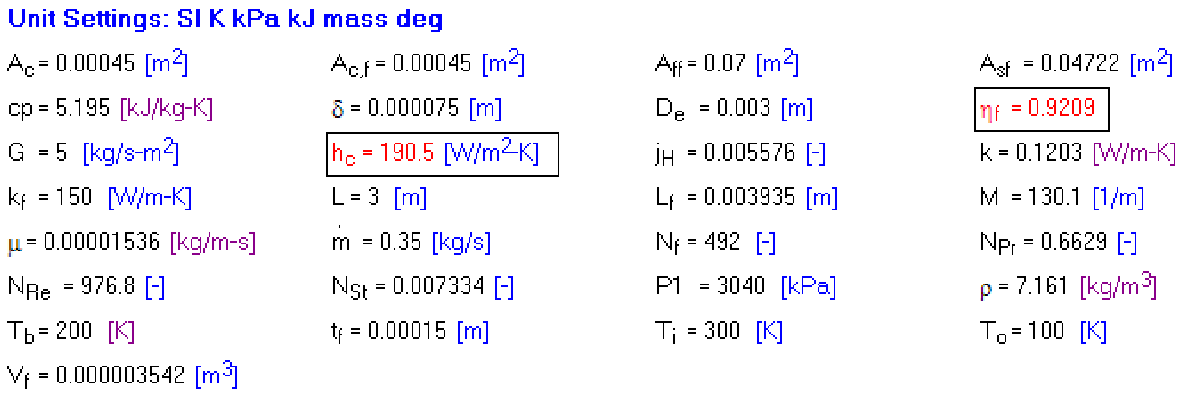
M = sqrt (h_c * 2 * (L + t_f) / (k_f * A_c))" i.e M = sqrt(hc * P / (kf * Ac)) where P is the perimeter of fin"

}

"Fin effectiveness eta_f:"

eta_f = tanh(M * L_f) / (M * L_f) "…fin effcy."

Results:



```
Unit Settings: SI K kPa kJ mass deg
A_c = 0.00045 [m^2]          A_c,f = 0.00045 [m^2]       A_ff = 0.07 [m^2]          A_sf = 0.04722 [m^2]
cp = 5.195 [kJ/kg-K]         δ = 0.000075 [m]            D_e = 0.003 [m]            η_f = 0.9209
G = 5 [kg/s-m^2]             h_c = 190.5 [W/m^2-K]       j_H = 0.005576 [-]         k = 0.1203 [W/m-K]
k_f = 150 [W/m-K]            L = 3 [m]                   L_f = 0.003935 [m]         M = 130.1 [1/m]
μ = 0.00001536 [kg/m-s]      ṁ = 0.35 [kg/s]             N_f = 492 [-]              N_Pr = 0.6629 [-]
N_Re = 976.8 [-]             N_St = 0.007334 [-]         P1 = 3040 [kPa]            ρ = 7.161 [kg/m^3]
T_b = 200 [K]                t_f = 0.00015 [m]           T_i = 300 [K]              T_o = 100 [K]
V_f = 0.000003542 [m^3]
```

Therefore:

Heat transfer coeff. = h_c = 190.5 W/m^2.K…Ans.

Fin effcy. = eta_f = 0.9209…Ans.

"**Prob. 3.2.35** (a) Write an EES Procedure to calculate the NTU, effectiveness, heat transferred etc for a Counterflow heat exchanger.

(b) Gaseous nitrogen enters the warm end of a counterflow HX at 20.27 MPa and 295 K at a mass flow rate of 1.25 kg/s. Gaseous nitrogen enters at the cold end of the HX at 80 K. For the warm stream, the sp. heat is 1.296 J/g.K and the sp. heat for the cold stream is 1.08 J/g.K. The mass flow rate of cold stream is 1.125 kg/s. The overall heat transfer coeff is 150 W/m^2.K and the heat transfer area on which U is based is 72.9 m^2. Determine the heat exchanger effectiveness, exit temp of cold stream and the heat transfer rate.[1]."

EES Procedure:

```
PROCEDURE CounterflowHX (m_c, m_h, cp_c, cp_h, Th_1,Tc_1,A, U: C_h, C_c, C_R, NTU, epsilon,Th_2, Tc_2, Q)

"Finds the NTU, Effectiveness, heat transferred etc when the inlet temps and mass flow rates and sp. heats of hot and cold streams are known."

"Inputs:: m_c, m_h, cp_c, cp_h, Th_1,Tc_1,A, U…mass in kg/s, cp in J/kg.KTemp in K; A in m^2, U in W/m^2.K"

"Outputs: C_h, C_c, …in W/K, C_R = C_min/C_max, NTU…is a number"

C_h: =m_h * cp_h "…capacity rate of hot fluid, W/K"

C_c: =m_c * cp_c "…capacity rate of cold fluid, W/K"

IF C_c < C_h THEN

    C_min: = C_c

    C_max: = C_h

    NTU: = (U * A) /C_min "…finds NTU"

    C_R: = C_min / C_max "…finds capacity ratio"

    epsilon: = (1 – exp ( – NTU * (1 – C_R))) / (1 – C_R * exp( – NTU * (1 – C_R)))
    "Effectiveness of Counter-flow HX."

    Tc_2: = epsilon * (Th_1 – Tc_1) + Tc_1"…finds Tc_2, exit temp of cold fluid"
```

```
      Q: = C_c * (Tc_2 – Tc_1) “W…heat transfer rate in the HX”

      Th_2: = Th_1 – Q / C_h “…finds Th_2, exit temp of hot fluid”

    ENDIF

IF C_c > C_h THEN

      C_min: = C_h

      C_max: = C_c

      NTU: = (U * A) /C_min “…finds NTU”

      C_R: = C_min / C_max “…finds capacity ratio”

      epsilon: = (1 – exp ( – NTU * (1 – C_R))) / (1 – C_R * exp( – NTU * (1 – C_R)))
      “Effectiveness of Counter-flow HX.”
```

```
        Th_2: = Th_1 – epsilon * (Th_1 – Tc_1) "…finds Th_2, exit temp of hot fluid"

        Q: = C_h * (Th_1 – Th_2) "W…heat transfer rate in the HX"

        Tc_2: = Tc_1 + Q / C_c "…finds Tc_2, exit temp of cold fluid"

ENDIF

IF C_c = C_h THEN

        C_R: = 1 "…capacity ratio"

        NTU: = (U * A) /C_c "… finds NTU"

        epsilon: = NTU / (1 + NTU) "…finds effectiveness"

        Th_2: = Th_1 – epsilon * (Th_1 – Tc_1) "…finds Th_2, exit temp of hot fluid"

        Q: = C_h * (Th_1 – Th_2) "W…heat transfer rate in the HX"

        Tc_2: = Tc_1 + Q / C_c "…finds Tc_2, exit temp of cold fluid"

ENDIF

END

"=================================================================================="
```

EES Solution to Problem:

We use the EES Procedure written above to solve the problem:

```
"Data:"

Th_1 = 295 "K"

Tc_1 = 80 "K"

m_h = 1.25 "kg/s"
```

m_c = 1.125 "kg/s"

cp_h = 1296 "J/kg.K"

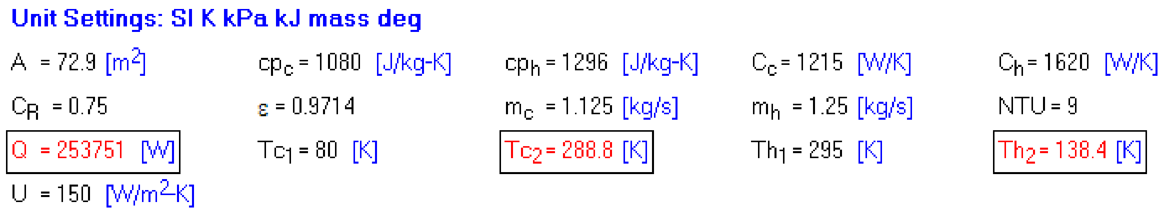
cp_c = 1080 "J/kg.K"

U = 150 "W/m^2.K"

A = 72.9 "m^2"

CALL CounterflowHX (m_c, m_h, cp_c, cp_h, Th_1,Tc_1,A, U: C_h, C_c, C_R, NTU, epsilon,Th_2, Tc_2, Q)

Results:



Unit Settings: SI K kPa kJ mass deg

A = 72.9 [m^2]
cp_c = 1080 [J/kg-K]
cp_h = 1296 [J/kg-K]
C_c = 1215 [W/K]
C_h = 1620 [W/K]
C_R = 0.75
ε = 0.9714
m_c = 1.125 [kg/s]
m_h = 1.25 [kg/s]
NTU = 9
Q = 253751 [W]
Tc_1 = 80 [K]
Tc_2 = 288.8 [K]
Th_1 = 295 [K]
Th_2 = 138.4 [K]
U = 150 [W/m^2-K]

Thus:

Effectiveness of HX = ε = 0.9714…Ans.

NTU = 9…Ans.

Exit temp of cold fluid = Tc_2 = 288.8 K…Ans.

Exit temp of hot fluid = Th_2 = 138.4 K…Ans.

Heat transfer = Q = 253751 W…Ans.

"**Prob. 3.2.36** (a) Write an EES Procedure to calculate the NTU, effectiveness, heat transferred etc for a Parallel flow heat exchanger.

(b) Determine the heat exchanger effectiveness, exit temp of cold stream and the heat transfer rate for the Problem 3.2.35 if the HX is parallel flow type.[1]."

EES Procedure:

```
PROCEDURE ParallelflowHX (m_c, m_h, cp_c, cp_h, Th_1,Tc_1,A, U: C_h, C_c, C_R, NTU, epsilon,Th_2, Tc_2, Q)

"Finds the NTU, Effectiveness, heat transferred etc when the inlet temps and mass flow rates and sp. heats of hot and cold streams are known."

"Inputs: m_c, m_h, cp_c, cp_h, Th_1,Tc_1,A, U…mass in kg/s, cp in J/kg.KTemp in K; A in m^2, U in W/m^2.K"

"Outputs: C_h, C_c, …in W/K, C_R = C_min/C_max, NTU…is a number"

C_h: =m_h * cp_h "…capacity rate of hot fluid, W/K"

C_c: =m_c * cp_c "…capacity rate of coldt fluid, W/K"

IF C_c < C_h THEN
```

```
    C_min: = C_c

    C_max: = C_h

    NTU: = (U * A) /C_min “… finds NTU”

    C_R: = C_min / C_max “…finds capacity ratio”

    epsilon = (1 – exp ( – NTU * (1 + C_R))) / (1 + C_R) “Effectiveness of parallel-flow
    HX….Finds epsilon”

    Tc_2: = epsilon * (Th_1 – Tc_1) + Tc_1 “…finds Tc_2, exit temp of cold fluid”

    Q: = C_c * (Tc_2 – Tc_1) “W…heat ransfer rate in the HX”

    Th_2: = Th_1 – Q / C_h “…finds Th_2, exit temp of hot fluid”

ENDIF

IF C_c > C_h THEN

    C_min: = C_h

    C_max: = C_c

    NTU: = (U * A) /C_min “… finds NTU”

    C_R: = C_min / C_max “…finds capacity ratio”

    epsilon = (1 – exp ( – NTU * (1 + C_R))) / (1 + C_R) “Effectiveness of parallel-flow
    HX….Finds epsilon”

    Th_2: = Th_1 – epsilon * (Th_1 – Tc_1) “…finds Th_2, exit temp of hot fluid”

    Q: = C_h * (Th_1 – Th_2) “W…heat ransfer rate in the HX”

    Tc_2: = Tc_1 + Q / C_c “…finds Tc_2, exit temp of coldt fluid”

ENDIF
```

```
IF C_c = C_h THEN

          C_R: = 1 "…capacity ratio"

          NTU: = (U * A) /C_c "…finds NTU"

          epsilon: = (1 / 2) * (1 – exp (-2 * NTU)) "…finds effectiveness"

          Th_2: = Th_1 – epsilon * (Th_1 – Tc_1) "…finds Th_2, exit temp of hot fluid"

          Q: = C_h * (Th_1 – Th_2) "W…heat ransfer rate in the HX"

          Tc_2: = Tc_1 + Q / C_c "…finds Tc_2, exit temp of cold fluid"

ENDIF

END

"======================================================================"
```

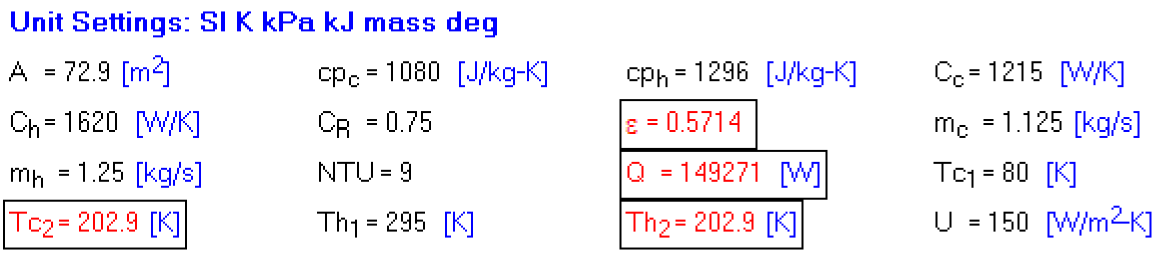
Problem:

Use the EES Procedure written above to solve the problem easily:

```
"Data:"
Th_1 = 295 "K"

Tc_1 = 80 "K"

m_h = 1.25 "kg/s"

m_c = 1.125 "kg/s"

cp_h = 1296 "J/kg.K"

cp_c = 1080 "J/kg.K"

U = 150 "W/m^2.K"

A = 72.9 "m^2"
```

CALL ParallelflowHX (m_c, m_h, cp_c, cp_h, Th_1,Tc_1,A, U: C_h, C_c, C_R, NTU, epsilon,Th_2, Tc_2, Q)

Results:



```
Unit Settings: SI K kPa kJ mass deg
A   = 72.9 [m2]          cpc = 1080  [J/kg-K]     cph = 1296  [J/kg-K]     Cc = 1215  [W/K]
Ch = 1620  [W/K]         CR  = 0.75               ε = 0.5714               mc  = 1.125 [kg/s]
mh  = 1.25 [kg/s]        NTU = 9                  Q  = 149271  [W]         Tc1 = 80  [K]
Tc2 = 202.9 [K]          Th1 = 295  [K]           Th2 = 202.9 [K]          U  = 150  [W/m2-K]
```

Thus:

Effectiveness of HX = ε = 0.5714…Ans. NTU = 9…Ans.

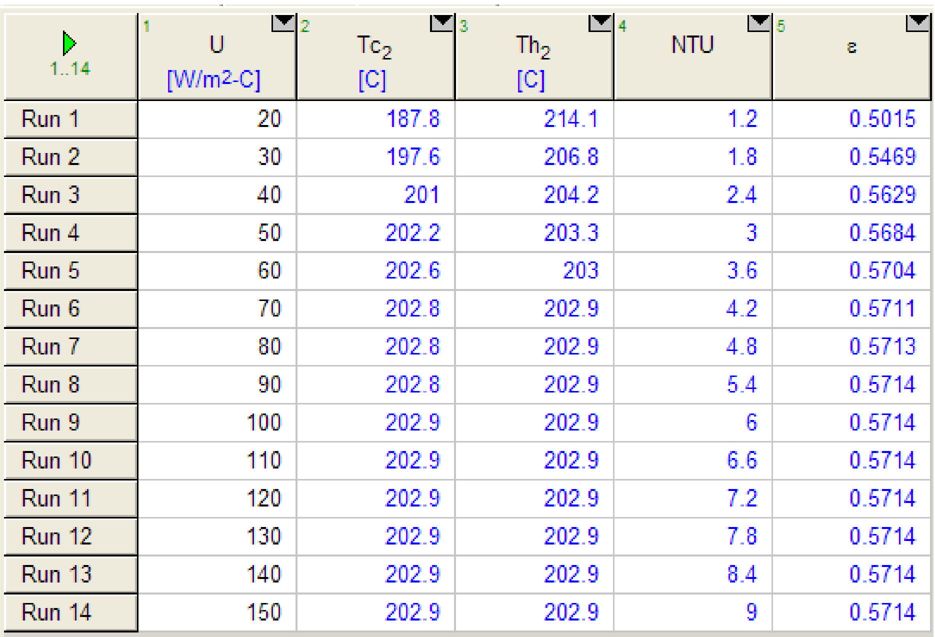
Exit temp of cold fluid = Tc_2 = 202.9 K; Exit temp of hot fluid = Th_2 = 202.9 K…Ans.

Heat transfer = Q = 149271 W…Ans.

Note: Effectiveness and heat transfer are more for the counter-flow HX.

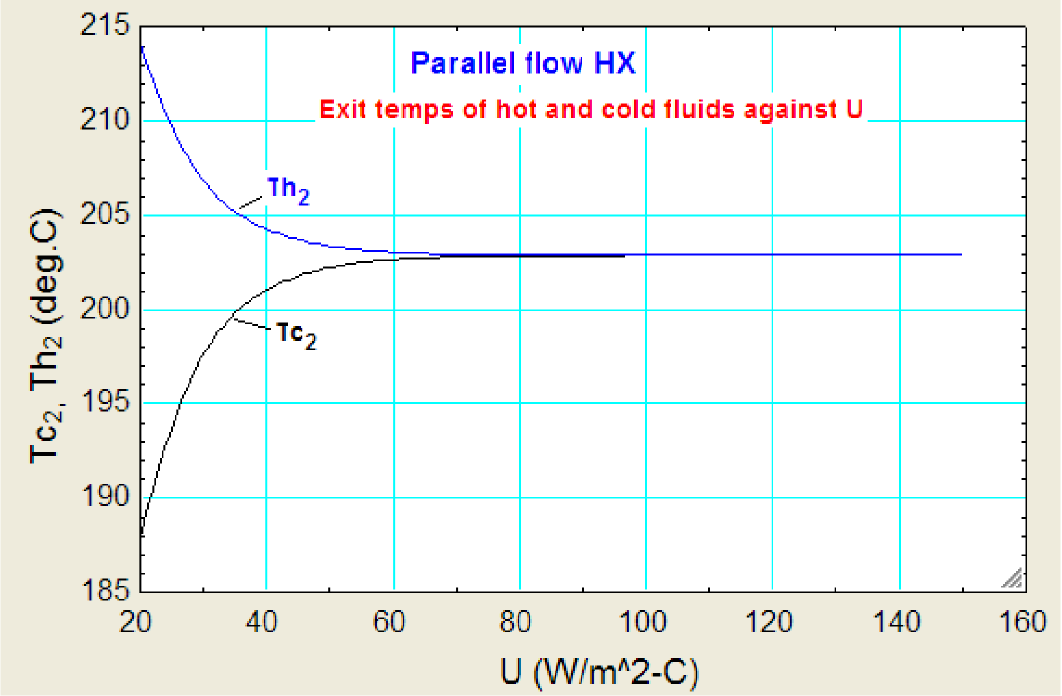
Plot the exit temps of hot and cold fluids as the overall heat transfer coeff varies from 20 to 150 W/m^2.K:

Parametric Table is computed first:



1..14	U [W/m2-C]	Tc_2 [C]	Th_2 [C]	NTU	ε
Run 1	20	187.8	214.1	1.2	0.5015
Run 2	30	197.6	206.8	1.8	0.5469
Run 3	40	201	204.2	2.4	0.5629
Run 4	50	202.2	203.3	3	0.5684
Run 5	60	202.6	203	3.6	0.5704
Run 6	70	202.8	202.9	4.2	0.5711
Run 7	80	202.8	202.9	4.8	0.5713
Run 8	90	202.8	202.9	5.4	0.5714
Run 9	100	202.9	202.9	6	0.5714
Run 10	110	202.9	202.9	6.6	0.5714
Run 11	120	202.9	202.9	7.2	0.5714
Run 12	130	202.9	202.9	7.8	0.5714
Run 13	140	202.9	202.9	8.4	0.5714
Run 14	150	202.9	202.9	9	0.5714

Now, plot the results:



Prob. 3.2.37. A parallel flow HX has following data: m_h = 0.17 kg/s, m_c = 0.417 kg/s, cp_h = cp_c = 4180 J/kg.C, Th_1 = 70 C, Th_2 = 50 C, Tc_1 = 25 C. Individual heat transfer coeff on hot and cold side are both equal to 60 W/m^2.C. Find the area of the HX.

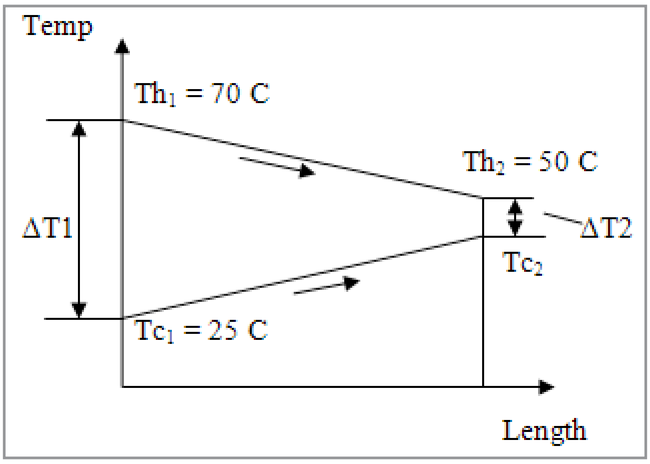


Fig. Parallel flow arrangement

EES Solution:

"Data:"

Th_1 = 70 "C"

Th_2 = 50 "C"

Tc_1 = 25 "C"

m_h = 0.17 "kg/s"

m_c = 0.417 "kg/s"

cp_h = 4180 "J/kg.K"

cp_c = 4180 "J/kg.K"

h_c = 60 "W/m^2.C"

h_h = 60 "W/m^2.C"

"Calculations:"

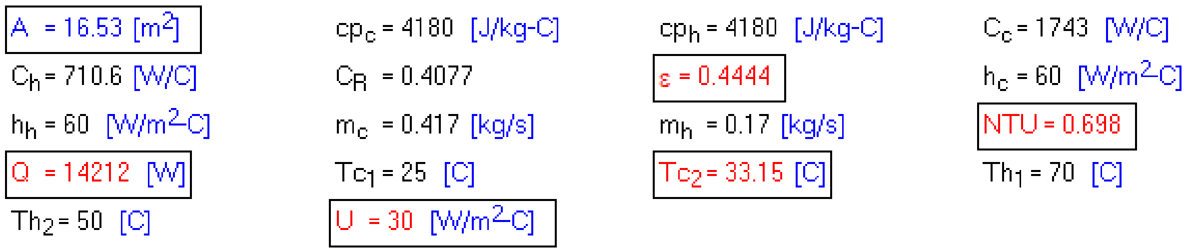
"Overall heat transfer coeff. U:"

1/U = 1/h_c + 1/h_h "…overall heat transfer coeff. is calculated"

"Now, use the EES Procedure written earlier:"

CALL ParallelflowHX (m_c, m_h, cp_c, cp_h, Th_1,Tc_1,A, U: C_h, C_c, C_R, NTU, epsilon,Th_2, Tc_2, Q)

Results:



```
A  = 16.53 [m2]        cpc = 4180  [J/kg-C]    cph = 4180  [J/kg-C]    Cc = 1743  [W/C]
Ch = 710.6 [W/C]       CR  = 0.4077            ε = 0.4444              hc = 60  [W/m2-C]
hh = 60  [W/m2-C]      mc  = 0.417 [kg/s]      mh  = 0.17 [kg/s]       NTU = 0.698
Q  = 14212  [W]        Tc1 = 25  [C]           Tc2 = 33.15 [C]         Th1 = 70  [C]
Th2 = 50  [C]          U  = 30  [W/m2-C]
```

Thus:

Exit temp of cold fluid = Tc_2 = 33.15 C…Ans.

NTU = 0.698; Effectiveness = 0.4444; U = 30 W/m^2.C; Q = 14212 W…Ans.

A = area of heat exchanger = 16.53 m^2…Ans.

Note: Here, A was not given, but Th2 was given. Still, same EES Procedure calculates the NTU, effectiveness and the area required.

(b) **If the mass flow rate of hot fluid is doubled, find the exit temp of both the fluids.**

Note: Now, h_h will change since heat transfer coeff is proportional to the Re^0.8 where Re is the Reynolds No. i.e. proportional to m_h^0.8.

"Data:"

Th_1 = 70 "C"

A = 16.53 "m^2…calculated in the part (a) of this problem"

Tc_1 = 25 "C"

m_h = 0.17 * 2 "kg/s…Mass flow of hot fluid is doubled."

m_c = 0.417 "kg/s"

cp_h = 4180 "J/kg.K"

cp_c = 4180 "J/kg.K"

h_c = 60 "W/m^2.C"

h_h = 60 * (m_h / 0.17)^0.8 "W/m^2.C…note that h_h varies as the mass flow rate to the power of 0.8."

"Calculations:"

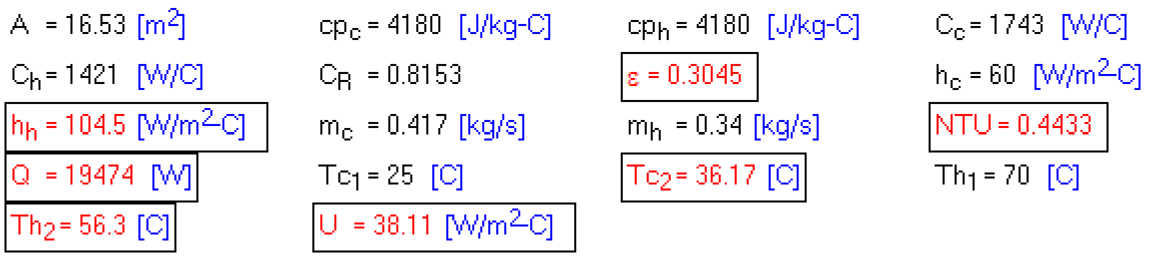
"Overall heat transfer coeff. U:"

1/U = 1/h_c + 1/h_h "…overall heat transfer coeff. is calculated"

"Now, use the EES Procedure written earlier:"

CALL ParallelflowHX (m_c, m_h, cp_c, cp_h, Th_1,Tc_1,A, U: C_h, C_c, C_R, NTU, epsilon,Th_2, Tc_2, Q)

Results:



A = 16.53 [m2]
C_h = 1421 [W/C]
h_h = 104.5 [W/m2-C]
Q = 19474 [W]
Th_2 = 56.3 [C]
cp_c = 4180 [J/kg-C]
C_R = 0.8153
m_c = 0.417 [kg/s]
Tc_1 = 25 [C]
U = 38.11 [W/m2-C]
cp_h = 4180 [J/kg-C]
ε = 0.3045
m_h = 0.34 [kg/s]
Tc_2 = 36.17 [C]
C_c = 1743 [W/C]
h_c = 60 [W/m2-C]
NTU = 0.4433
Th_1 = 70 [C]

Thus:

Exit temp of cold fluid = Tc_2 = 36.17 C; Exit temp of hot fluid = Th_2 = 56.3 C…Ans.

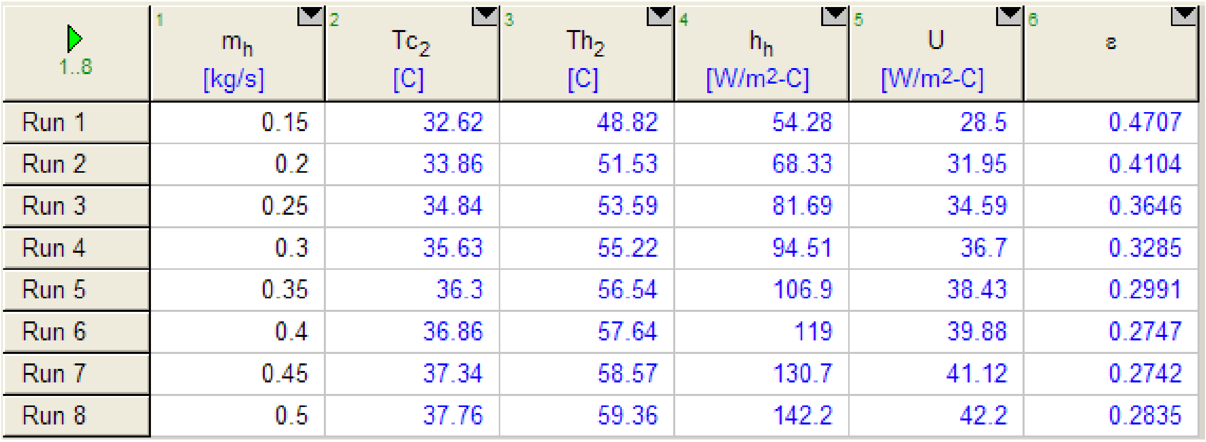
NTU = 0.4433; Effectiveness = 0.3045; U = 38.11 W/m^2.C; Q = 19474 W…Ans.

h_h = 104.5 W/m^2.C…Ans.

Note: Here, also the same EES Procedure calculates the NTU, effectiveness and the exit temps of fluids.

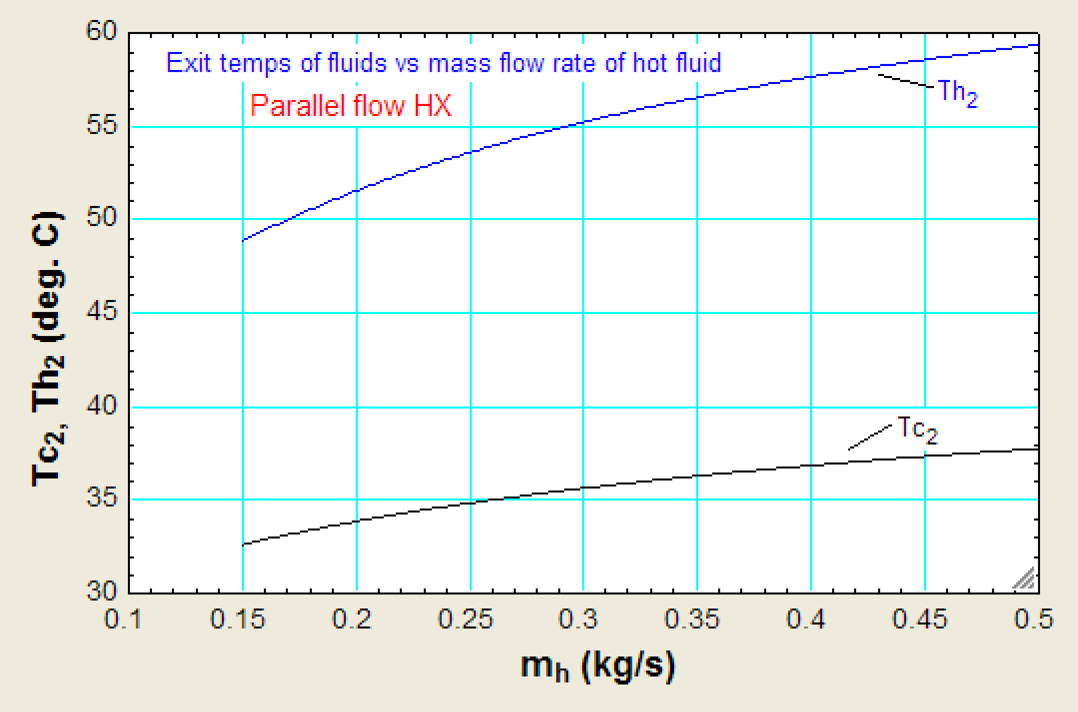
Plot the variation of exit temps of hot and cold fluids, and effectiveness of HX as the mass flow rate of hot fluid varies from 0.15 kg/s to 0.5 kg/s:

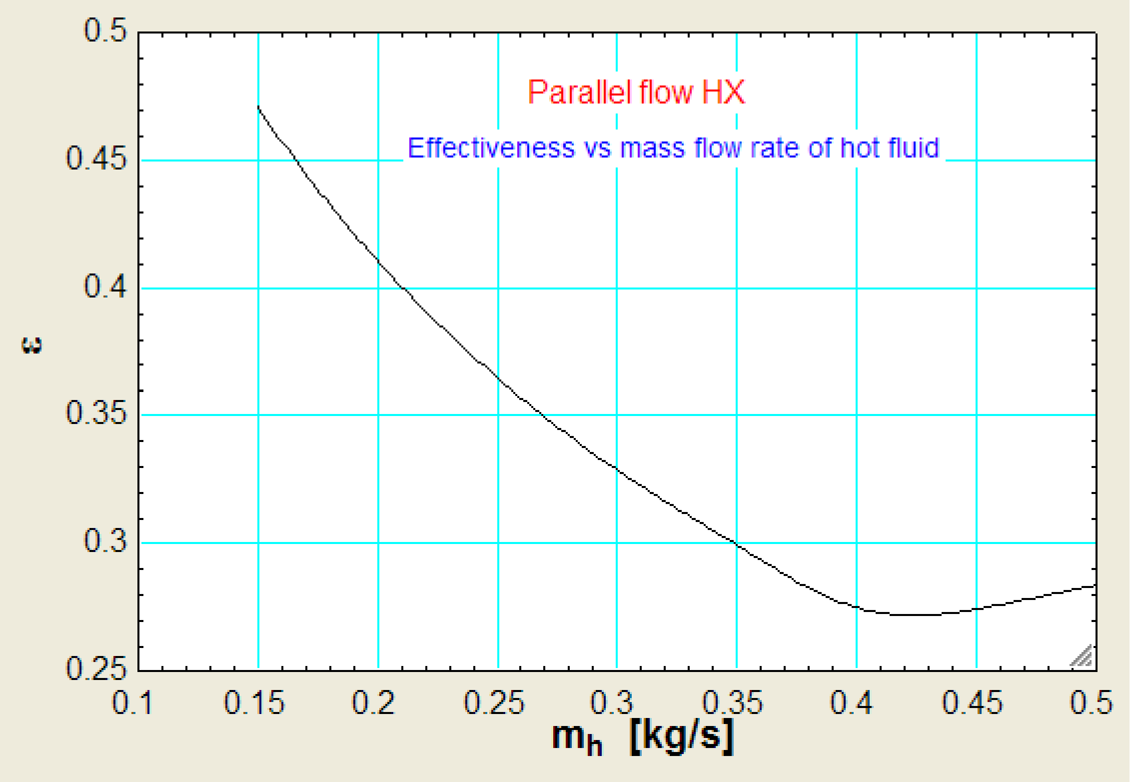
First, compute the Parametric Table:



1..8	1 m_h [kg/s]	2 Tc_2 [C]	3 Th_2 [C]	4 h_h [W/m2-C]	5 U [W/m2-C]	6 ε
Run 1	0.15	32.62	48.82	54.28	28.5	0.4707
Run 2	0.2	33.86	51.53	68.33	31.95	0.4104
Run 3	0.25	34.84	53.59	81.69	34.59	0.3646
Run 4	0.3	35.63	55.22	94.51	36.7	0.3285
Run 5	0.35	36.3	56.54	106.9	38.43	0.2991
Run 6	0.4	36.86	57.64	119	39.88	0.2747
Run 7	0.45	37.34	58.57	130.7	41.12	0.2742
Run 8	0.5	37.76	59.36	142.2	42.2	0.2835

Now, plot the results:





"**Prob. 3.2.38**. A Linde concentric tube HX is to be constructed from a 25 mm ID tube having a 2.5 mm wall thickness concentric to a 62.5 mm inside dia tube. Air at 10.13 MPa and 300 K enters the smaller tube, and air at 101.3 kPa and 85 K enters the larger tube and leaves at 289 K. The mass flow rate of high pressure stream is 0.25 kg/s, and the mass flow rate of low pressure stream is 0.233 kg/s. The mean sp. heat of high pressure stream is 1.56 kJ/kg.K, and the mean sp. heat of low pressure stream is 1.005 kJ/kg.K. If the concentric tube is wound in a 450 mm dia helix and heavily insulated, determine the length of concentric tubing required for this heat exchanger.[1]"

EES Solution:

"First, let us solve this problem by LMTD method:"

"Data:"

d_i_h = 0.025 "m…inside dia of high temp stream flow tube"

d_o_h = 0.03 "m…outside dia of high temp stream flow tube"

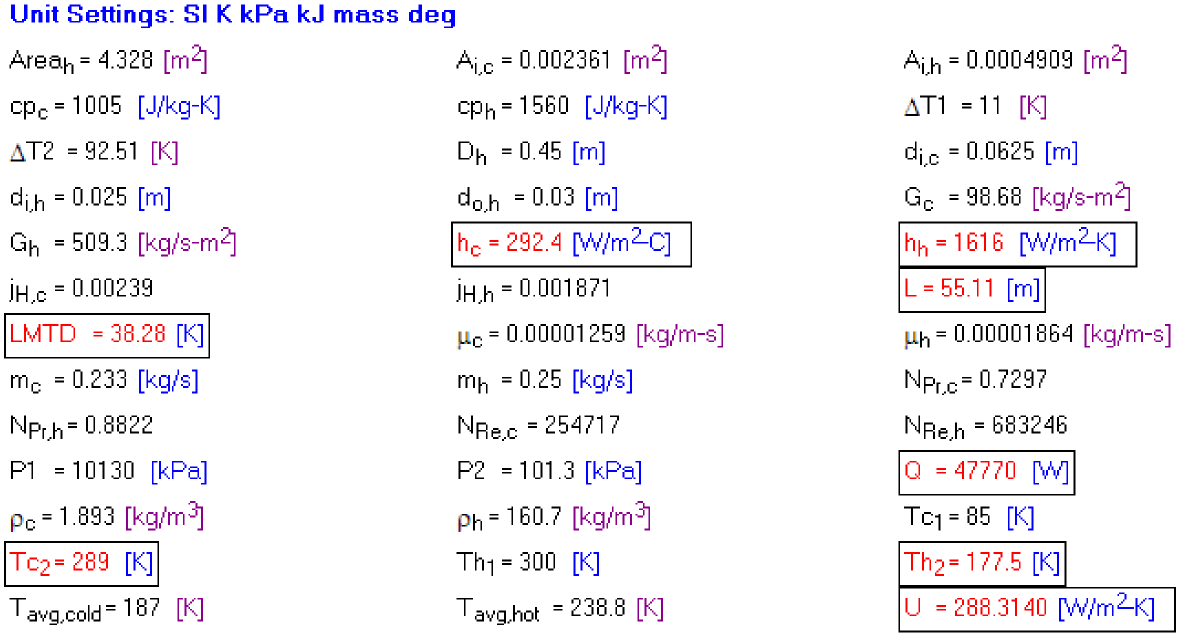
d_i_c = 0.0625 "m…inside dia of low temp stream flow tube"

```
P1 = 10130 “kPa…high pressure flow”

P2 = 101.3 “kPa…Low pressure flow”

Th_1 = 300 “K…high pressure stream entry temp”

Tc_1 = 85 “K…low pressure stream entry temp”

Tc_2 = 289 “K…low pressure stream exit temp”

m_h = 0.25 “kg/s…mass flow rate of high pressure stream”

m_c = 0.233 “kg/s…mass flow rate of low pressure stream”

cp_h = 1560 “J/kg.K…sp. heat of high pressure stream”

cp_c = 1005 “J/kg.K…sp. heat of low pressure stream”

D_h = 0.45 “m…helix dia”

“Calculations:”
Q = m_c * cp_c * (Tc_2 – Tc_1) “W…heat transfer“

Q = m_h * cp_h * (Th_1 – Th_2) “K…determines exit temp of high pressure stream, Th_2”

T_avg_hot = (Th_1 + Th_2) / 2 “K…avg temp of hot flow”

T_avg_cold = (Tc_1 + Tc_2) / 2 “K…avg temp of cold flow”

“To find heat transfer coeff. h_h for high pressure stream”

A_i_h = (pi * d_i_h^2) / 4 “m^2…cross-sectional area of high pressure flow”

G_h = m_h / A_i_h “kg/s.m^2…mass velocity for high pressure flow”

mu_h = Viscosity(Air_ha,T=T_avg_hot,P=P1) “kg/m.s…viscosity of high pressure flow”

rho_h = Density(Air_ha,T=T_avg_hot,P=P1) “kg/m^3…density of high pressure flow”
```

```
N_Re_h = G_h * d_i_h / mu_h "…Reynolds No. of high pressure stream = 683246"

N_Pr_h = Prandtl(Air_ha,T=T_avg_hot,P=P1) "…Prandtl No. for high pressure stream"

"Then, since Reynolds No. is > 3000, we have:"

j_H_h = (h_h / (cp_h * G_h)) * N_Pr_h^(2/3) "…Colburn j-factor"

j_H_h = 0.023 * N_Re_h^(-0.2) * (1 + 3.5 * d_i_h / D_h) "…determines h_h = W/m^2.K"

"To find heat transfer coeff. h_c for low pressure stream"

A_i_c = (pi / 4) * (d_i_c^2 – d_o_h^2) "m^2…cross-sectional area of low pressure flow…the annulus"

G_c = m_c / A_i_c "kg/s.m^2…mass velocity for low pressure flow"

mu_c = Viscosity(Air_ha,T=T_avg_cold,P=P2) "kg/m.s…viscosity of high pressure flow"
```

```
rho_c = Density(Air_ha,T=T_avg_cold,P=P2) “kg/m^3…density of low pressure flow”

N_Re_c = G_c * (d_i_c – d_o_h) / mu_c “…Reynolds No. of low pressure stream = 172104”

N_Pr_c = Prandtl(Air_ha,T=T_avg_cold,P=P2) “…Prandtl No. for low pressure stream”

“Then, since Reynolds No. is > 3000, we have:”

j_H_c = (h_c / (cp_c * G_c)) * N_Pr_c^(2/3) “…Colburn j-factor”

j_H_c = 0.023 * N_Re_c^(-0.2) * (1 + 3.5 * (d_i_c – d_o_h) / D_h) “…determines h_c = W/m^2.K”

“Overall heat transfer coeff. U:”

1/U = 1 / h_h + d_i_h / (h_c * d_o_h) “…U based on heat transfer area on the high temp side”

“To find LMTD:”

DELTAT1 = (Th_1 – Tc_2 ) “K…temp diff at the beginning of HX”

DELTAT2 = (Th_2 – Tc_1 ) “K…temp diff at the end of HX”

LMTD = (DELTAT1 – DELTAT2) / ln(DELTAT1 / DELTAT2) “K…calculates LMTD”

“To find the area and Length of HX:”

Q = U * Area_h * LMTD “…finds area of heat transfer on the high temp side, m^2”

Area_h = pi * d_i_h * L “m^2…area of heat transfer on the high (temp) pressure flow…finds length of tube, L”
```

Results:



Unit Settings: SI K kPa kJ mass deg

$Area_h = 4.328$ [m^2]
$A_{i,c} = 0.002361$ [m^2]
$A_{i,h} = 0.0004909$ [m^2]
$cp_c = 1005$ [J/kg-K]
$cp_h = 1560$ [J/kg-K]
$\Delta T1 = 11$ [K]
$\Delta T2 = 92.51$ [K]
$D_h = 0.45$ [m]
$d_{i,c} = 0.0625$ [m]
$d_{i,h} = 0.025$ [m]
$d_{o,h} = 0.03$ [m]
$G_c = 98.68$ [kg/s-m^2]
$G_h = 509.3$ [kg/s-m^2]
$h_c = 292.4$ [W/m^2-C]
$h_h = 1616$ [W/m^2-K]
$j_{H,c} = 0.00239$
$j_{H,h} = 0.001871$
$L = 55.11$ [m]
$LMTD = 38.28$ [K]
$\mu_c = 0.00001259$ [kg/m-s]
$\mu_h = 0.00001864$ [kg/m-s]
$m_c = 0.233$ [kg/s]
$m_h = 0.25$ [kg/s]
$N_{Pr,c} = 0.7297$
$N_{Pr,h} = 0.8822$
$N_{Re,c} = 254717$
$N_{Re,h} = 683246$
$P1 = 10130$ [kPa]
$P2 = 101.3$ [kPa]
$Q = 47770$ [W]
$\rho_c = 1.893$ [kg/m^3]
$\rho_h = 160.7$ [kg/m^3]
$Tc_1 = 85$ [K]
$Tc_2 = 289$ [K]
$Th_1 = 300$ [K]
$Th_2 = 177.5$ [K]
$T_{avg,cold} = 187$ [K]
$T_{avg,hot} = 238.8$ [K]
$U = 288.3140$ [W/m^2-K]

Thus:

Overall heat transfer coeff. = U = 288.314 W/m^2.K; LMTD = 38.28 K…Ans.

Length of tube in HX = L = 55.11 m; Heat transfer in HX = Q = 47770 W…Ans.

"Now, let us solve this problem by epsilon-NTU method. We shall use the EES Procedure already written for the Counter-flow HX:"

"Data:"

Th_1 = 300 "K…high pressure stream entry temp"

Tc_1 = 85 "K…low pressure stream entry temp"

Tc_2 = 289 "K…low pressure stream exit temp"

m_h = 0.25 "kg/s…mass flow rate of high pressure stream"

m_c = 0.233 "kg/s…mass flow rate of low pressure stream"

cp_h = 1560 "J/kg.K…sp. heat of high pressure stream"

cp_c = 1005 "J/kg.K…sp. heat of low pressure stream"

U = 288.314 “W/m^2.K”

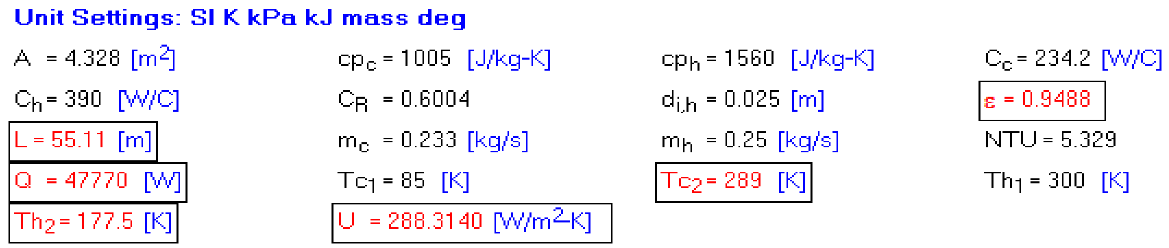
CALL CounterflowHX (m_c, m_h, cp_c, cp_h, Th_1,Tc_1,A, U: C_h, C_c, C_R, NTU, epsilon,Th_2, Tc_2, Q)

“To find the area and Length of HX:”

d_i_h = 0.025 “m…inside dia of high temp stream flow tube”

A = pi * d_i_h * L “m^2…area of heat transfer on the high (temp) pressure flow…finds length of tube, L”

Results:



```
Unit Settings: SI K kPa kJ mass deg
A  = 4.328 [m2]       cpc = 1005  [J/kg-K]    cph = 1560  [J/kg-K]   Cc = 234.2 [W/C]
Ch = 390  [W/C]       CR  = 0.6004            di,h = 0.025 [m]       ε = 0.9488
L = 55.11  [m]        mc  = 0.233 [kg/s]      mh  = 0.25 [kg/s]      NTU = 5.329
Q  = 47770  [W]       Tc1 = 85   [K]          Tc2 = 289   [K]        Th1 = 300   [K]
Th2 = 177.5 [K]       U  = 288.3140 [W/m2-K]
```

Thus:

Overall heat transfer coeff. = U = 288.314 W/m^2.K…Ans.

NTU = 5.329; epsilon = 0.9488…Ans.

Length of tube in HX = L = 55.11 m ; Heat transfer in HX = Q = 47770 W…Ans.

……

"**Prob. 3.2.39**. Write an EES Function to find the effectiveness of a Counter-flow HX as a function of NTU and Capacity Ratio, C_R. Then, plot the Effectiveness vs NTU graphs for different values of Capacity ratios."

EES Function:

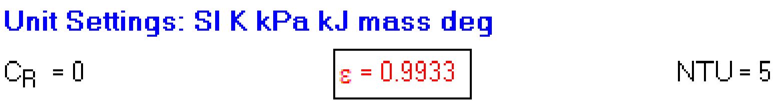
```
FUNCTION CounterflowHX_epsilon(NTU, C_R)

"Inputs: NTU…No. of Transfer Units = U A /C_min; Capacity ratio = C_R = C_min / C_max)"

"Output: Effectiveness = epsilon"

IF (C_R = 1) THEN

CounterflowHX_epsilon: = NTU / (1 + NTU)

ENDIF

IF (C_R = 0) THEN

CounterflowHX_epsilon: = 1 – exp(-NTU)

ENDIF

IF (C_R <>1) AND (C_R <> 0) THEN

A: = 1 – exp(-NTU * (1 – C_R))
```

```
B: = 1 – C_R * exp(-NTU * (1 – C_R))

CounterflowHX_epsilon: = A / B

ENDIF

END

"==================================================================="
```

To plot the Effectiveness-NTU graphs:

EES Program:

```
NTU = 5; C_R = 0

epsilon = CounterflowHX_epsilon(NTU, C_R)
```



Unit Settings: SI K kPa kJ mass deg

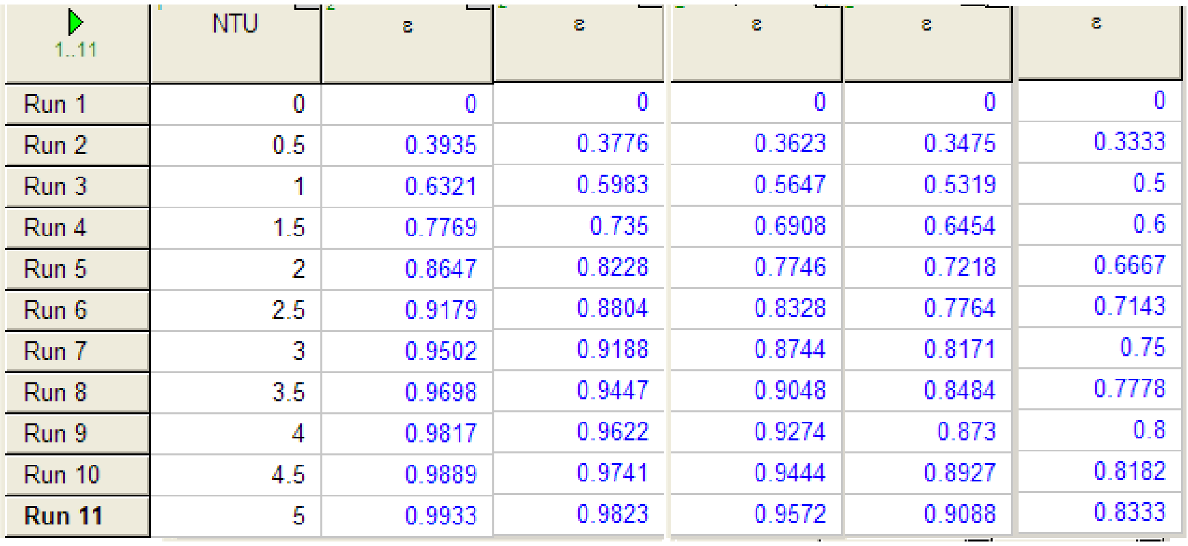
C_R = 0

ε = 0.9933

NTU = 5

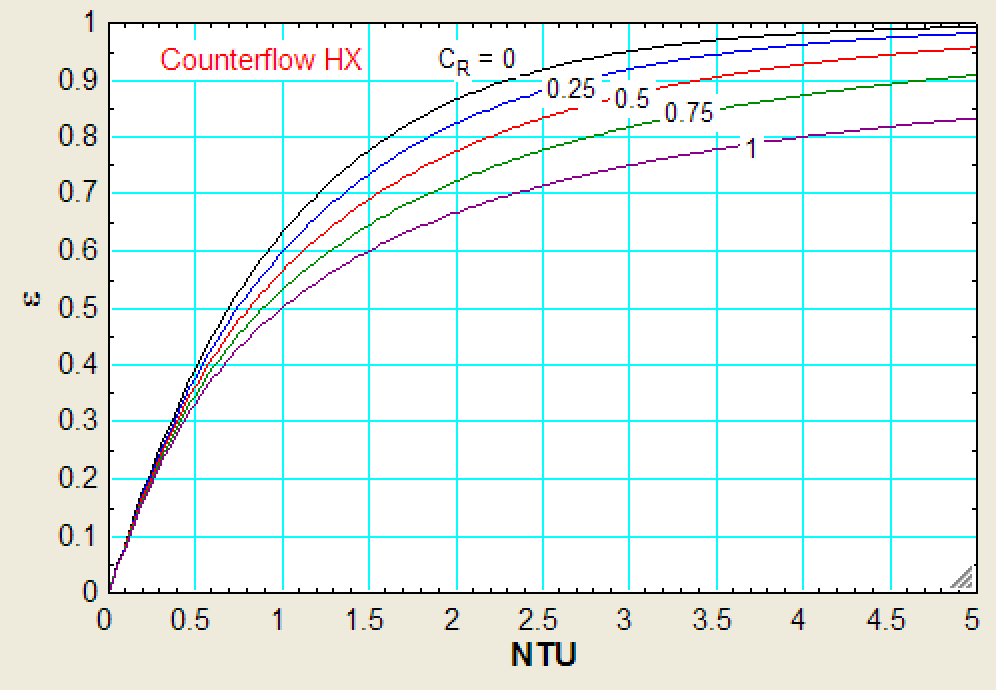
Compute the Parametric Table:

C_R = 0 C_R = 0.25 C_R = 0.5 C_R = 0.75 C_R = 1



1..11	NTU	ε	ε	ε	ε	ε
Run 1	0	0	0	0	0	0
Run 2	0.5	0.3935	0.3776	0.3623	0.3475	0.3333
Run 3	1	0.6321	0.5983	0.5647	0.5319	0.5
Run 4	1.5	0.7769	0.735	0.6908	0.6454	0.6
Run 5	2	0.8647	0.8228	0.7746	0.7218	0.6667
Run 6	2.5	0.9179	0.8804	0.8328	0.7764	0.7143
Run 7	3	0.9502	0.9188	0.8744	0.8171	0.75
Run 8	3.5	0.9698	0.9447	0.9048	0.8484	0.7778
Run 9	4	0.9817	0.9622	0.9274	0.873	0.8
Run 10	4.5	0.9889	0.9741	0.9444	0.8927	0.8182
Run 11	5	0.9933	0.9823	0.9572	0.9088	0.8333

Now, plot the graphs:



“**Prob. 3.2.40**. Write an EES Function to find the effectiveness of a Parallel-flow HX as a function of NTU and Capacity Ratio, C_R. Then, plot the Effectiveness vs NTU graphs for different values of Capacity ratios.”

EES Function:

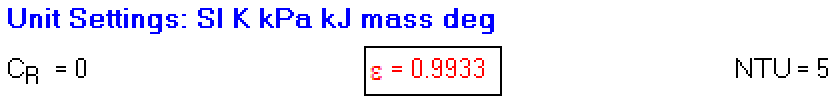
```
FUNCTION ParallelflowHX_epsilon(NTU, C_R)

“Inputs: NTU…No. of Transfer Units = U A /C_min; Capacity ratio + C_R = C_min / C_max)”

“Output: Effectiveness = epsilon”

IF (C_R = 1) THEN

ParallelflowHX_epsilon: = (1 / 2) * (1 – exp(-2*NTU))

ENDIF

IF (C_R = 0) THEN

ParallelflowHX_epsilon: = 1 – exp(-NTU)

ENDIF

IF (C_R <>1) AND (C_R <> 0) THEN

A: = 1 – exp(-NTU * (1 + C_R))

B: = 1 + C_R

ParallelflowHX_epsilon: = A / B

ENDIF

END

“==================================================================”
```

To plot the Effectiveness-NTU graphs:

EES Program:

NTU = 5; C_R = 0

epsilon = CounterflowHX_epsilon(NTU, C_R)

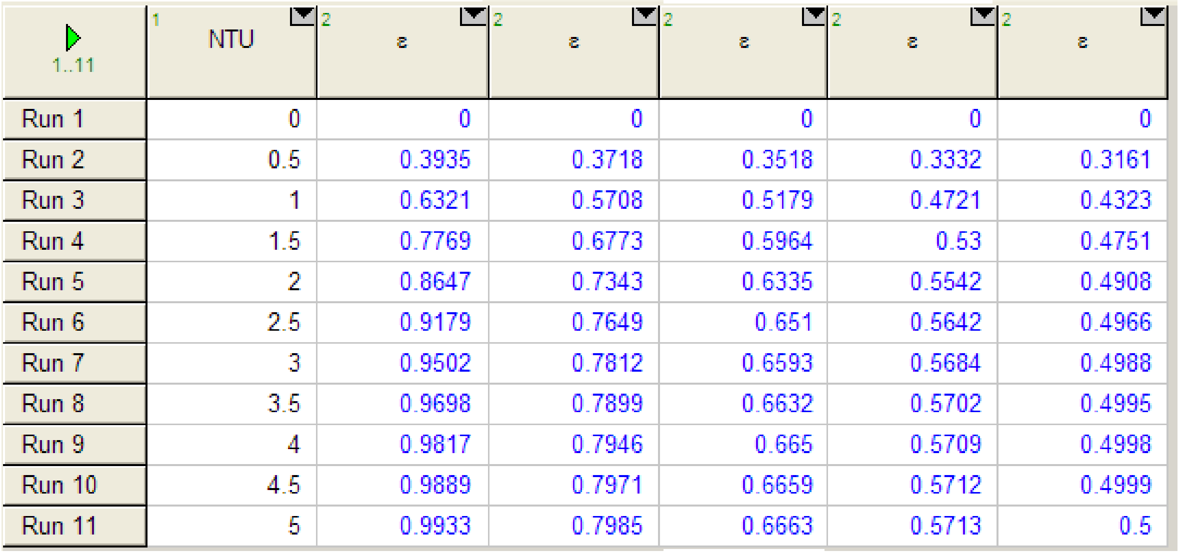


Unit Settings: SI K kPa kJ mass deg

C_R = 0 | ε = 0.9933 | NTU = 5

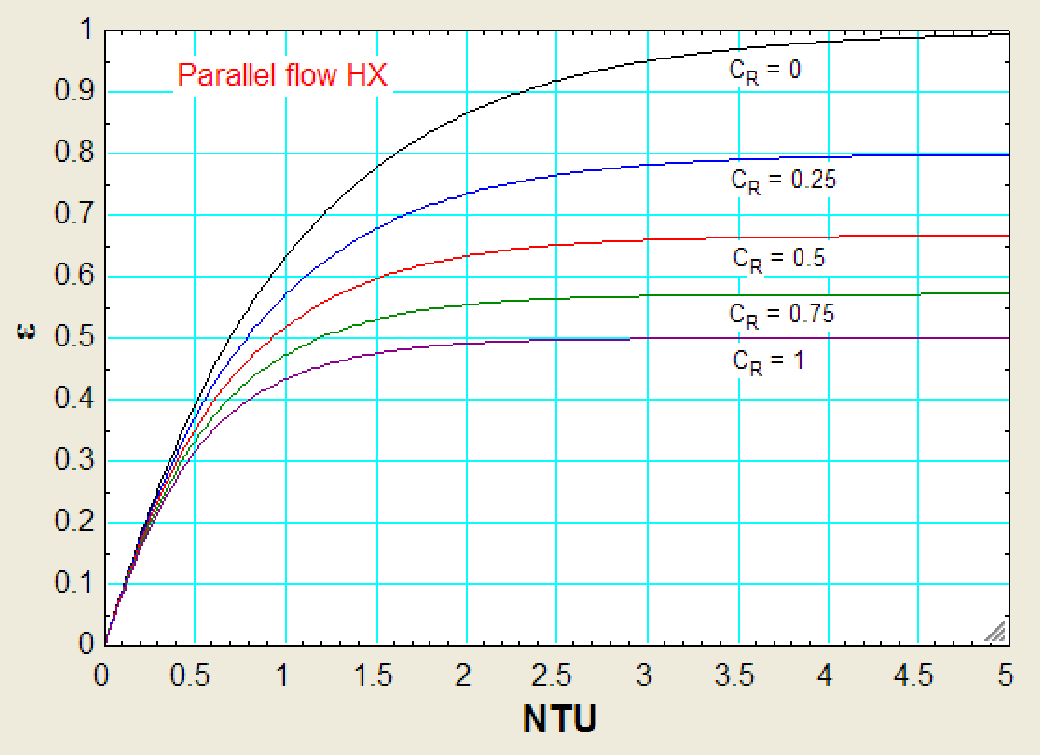
Compute the Parametric Table:

C_R = 0   C_R = 0.25   C_R = 0.5   C_R = 0.75   C_R = 1



1..11	1 NTU	2 ε	2 ε	2 ε	2 ε	2 ε
Run 1	0	0	0	0	0	0
Run 2	0.5	0.3935	0.3718	0.3518	0.3332	0.3161
Run 3	1	0.6321	0.5708	0.5179	0.4721	0.4323
Run 4	1.5	0.7769	0.6773	0.5964	0.53	0.4751
Run 5	2	0.8647	0.7343	0.6335	0.5542	0.4908
Run 6	2.5	0.9179	0.7649	0.651	0.5642	0.4966
Run 7	3	0.9502	0.7812	0.6593	0.5684	0.4988
Run 8	3.5	0.9698	0.7899	0.6632	0.5702	0.4995
Run 9	4	0.9817	0.7946	0.665	0.5709	0.4998
Run 10	4.5	0.9889	0.7971	0.6659	0.5712	0.4999
Run 11	5	0.9933	0.7985	0.6663	0.5713	0.5

Now. Plot the results:



==

"**Prob. 3.2.41**. Write an EES Function to find the effectiveness of a Cross-flow HX, with both the fluids un-mixed, as a function of NTU and Capacity Ratio, C_R. Then, plot the Effectiveness vs NTU graphs for different values of Capacity ratios."

EES Function:

```
FUNCTION CrossflowHX_bothUnmixed(NTU, C_R)

"Inputs: NTU…No. of Transfer Units = U A /C_min; Capacity ratio + C_R = C_min / C_max)"

"Output: Effectiveness = epsilon"

IF (C_R = 0) THEN

CrossflowHX_bothUnmixed: = 1 – exp(-NTU)

ENDIF

IF (C_R <> 0) THEN

A: = (NTU^0.22 / C_R) * (exp(-C_R * NTU^0.78) – 1)

CrossflowHX_bothUnmixed: = 1 – exp(A)

ENDIF

END

"========================================================================"
```

To plot the Effectiveness-NTU graphs:

EES Program:

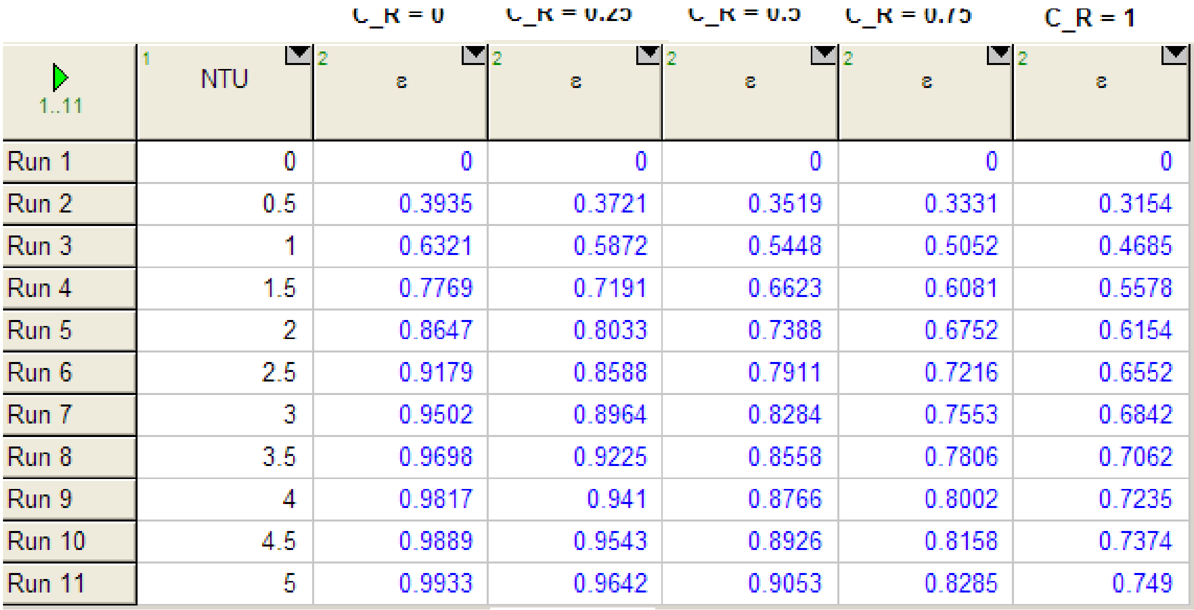
```
NTU = 5; C_R = 0
```

epsilon = CrossflowHX_ One bothmixed(NTU, C_R)

Unit Settings: SI K kPa kJ mass deg

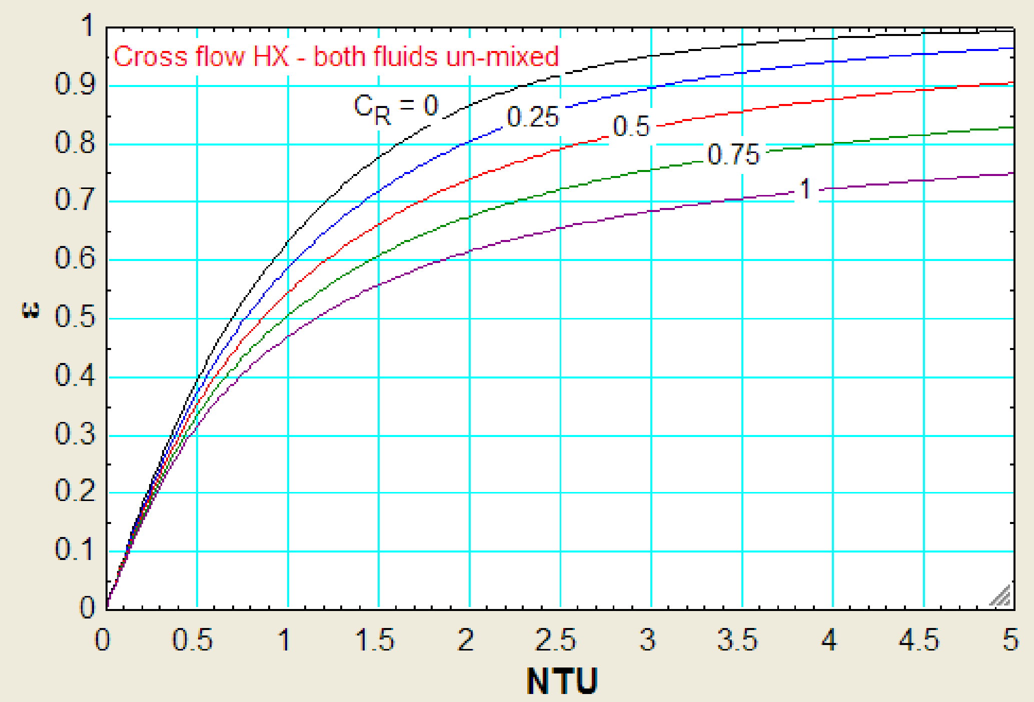
C_R = 0   ε = 0.9933   NTU = 5

Compute the Parametric Table:



		C_R = 0	C_R = 0.25	C_R = 0.5	C_R = 0.75	C_R = 1
1..11	NTU	ε	ε	ε	ε	ε
Run 1	0	0	0	0	0	0
Run 2	0.5	0.3935	0.3721	0.3519	0.3331	0.3154
Run 3	1	0.6321	0.5872	0.5448	0.5052	0.4685
Run 4	1.5	0.7769	0.7191	0.6623	0.6081	0.5578
Run 5	2	0.8647	0.8033	0.7388	0.6752	0.6154
Run 6	2.5	0.9179	0.8588	0.7911	0.7216	0.6552
Run 7	3	0.9502	0.8964	0.8284	0.7553	0.6842
Run 8	3.5	0.9698	0.9225	0.8558	0.7806	0.7062
Run 9	4	0.9817	0.941	0.8766	0.8002	0.7235
Run 10	4.5	0.9889	0.9543	0.8926	0.8158	0.7374
Run 11	5	0.9933	0.9642	0.9053	0.8285	0.749

Now, plot the results:



"**Prob. 3.2.42**. Write an EES Function to find the effectiveness of a Cross-flow HX, with one fluid mixed, and the other un-mixed, as a function of NTU and Capacity Ratio, C_R. Then, plot the Effectiveness vs NTU graphs for different values of Capacity ratios."

EES Function:

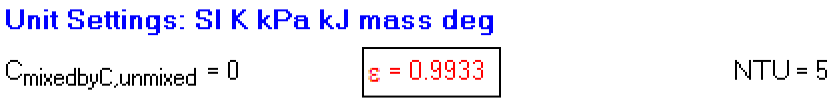
```
FUNCTION CrossflowHX_OneMixed(NTU, C_mixedbyC_unmixed)

"Inputs: NTU…No. of Transfer Units = U A /C_min; Capacity rates: C_mixed, C_unmixed =
(W/K)"

"Output: Effectiveness = epsilon"

IF (C_mixedbyC_unmixed = 0) OR (C_mixedbyC_unmixed >= 10) THEN

CrossflowHX_OneMixed: = 1 – exp(-NTU)

ENDIF
```

```
IF (C_mixedbyC_unmixed <= 1) AND (C_mixedbyC_unmixed > 0) THEN

C_R: = C_mixedbyC_unmixed

CrossflowHX_OneMixed: = 1 – exp((-1 / C_R) * (1 – exp( – C_R * NTU)))

ENDIF

IF ((C_mixedbyC_unmixed > 1) AND (C_mixedbyC_unmixed < 10 )) THEN

C_R: = 1/ C_mixedbyC_unmixed

CrossflowHX_OneMixed: = (1 / C_R) * (1 – exp( – C_R * (1 – exp( – NTU))))

ENDIF

END

“===========================================================================”
```

To plot the Effectiveness-NTU graphs:

EES Program:

```
NTU = 5; C_R = 0

epsilon = CrossflowHX_OneMixed(NTU, C_mixedbyC_unmixed)
```



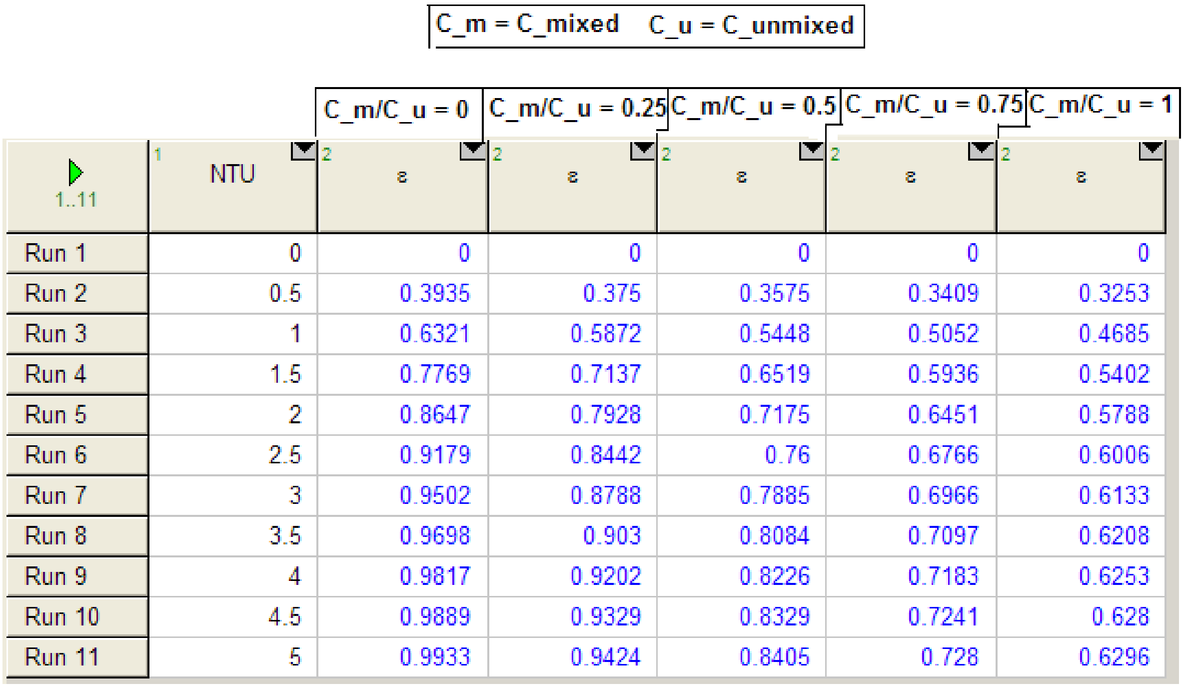
Unit Settings: SI K kPa kJ mass deg

$C_{mixedbyC,unmixed}$ = 0

ε = 0.9933

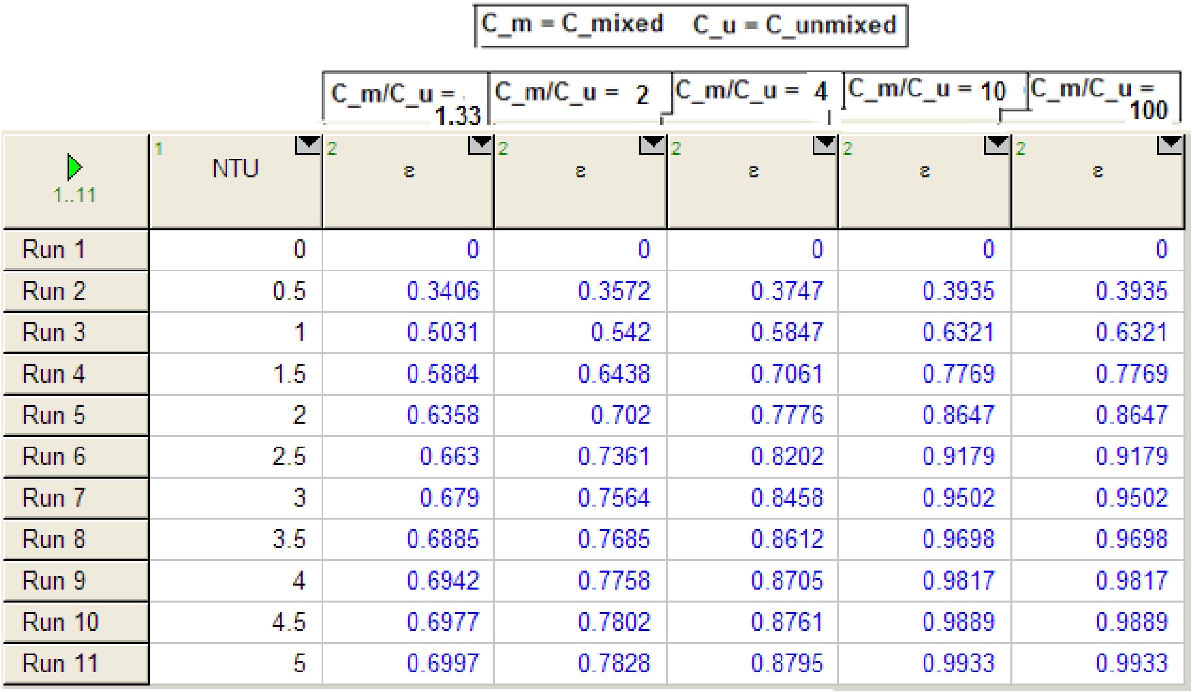
NTU = 5

Compute the Parametric Table:



C_m = C_mixed C_u = C_unmixed

		C_m/C_u = 0	C_m/C_u = 0.25	C_m/C_u = 0.5	C_m/C_u = 0.75	C_m/C_u = 1
1..11	NTU	ε	ε	ε	ε	ε
Run 1	0	0	0	0	0	0
Run 2	0.5	0.3935	0.375	0.3575	0.3409	0.3253
Run 3	1	0.6321	0.5872	0.5448	0.5052	0.4685
Run 4	1.5	0.7769	0.7137	0.6519	0.5936	0.5402
Run 5	2	0.8647	0.7928	0.7175	0.6451	0.5788
Run 6	2.5	0.9179	0.8442	0.76	0.6766	0.6006
Run 7	3	0.9502	0.8788	0.7885	0.6966	0.6133
Run 8	3.5	0.9698	0.903	0.8084	0.7097	0.6208
Run 9	4	0.9817	0.9202	0.8226	0.7183	0.6253
Run 10	4.5	0.9889	0.9329	0.8329	0.7241	0.628
Run 11	5	0.9933	0.9424	0.8405	0.728	0.6296

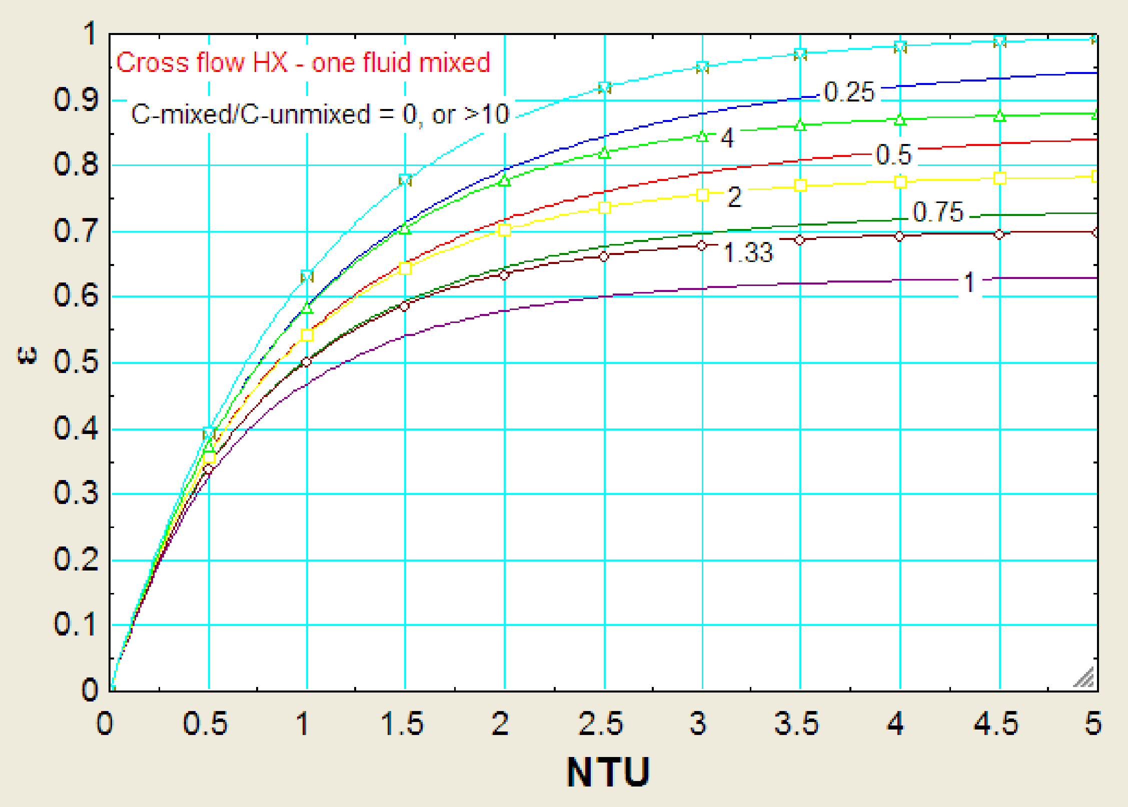


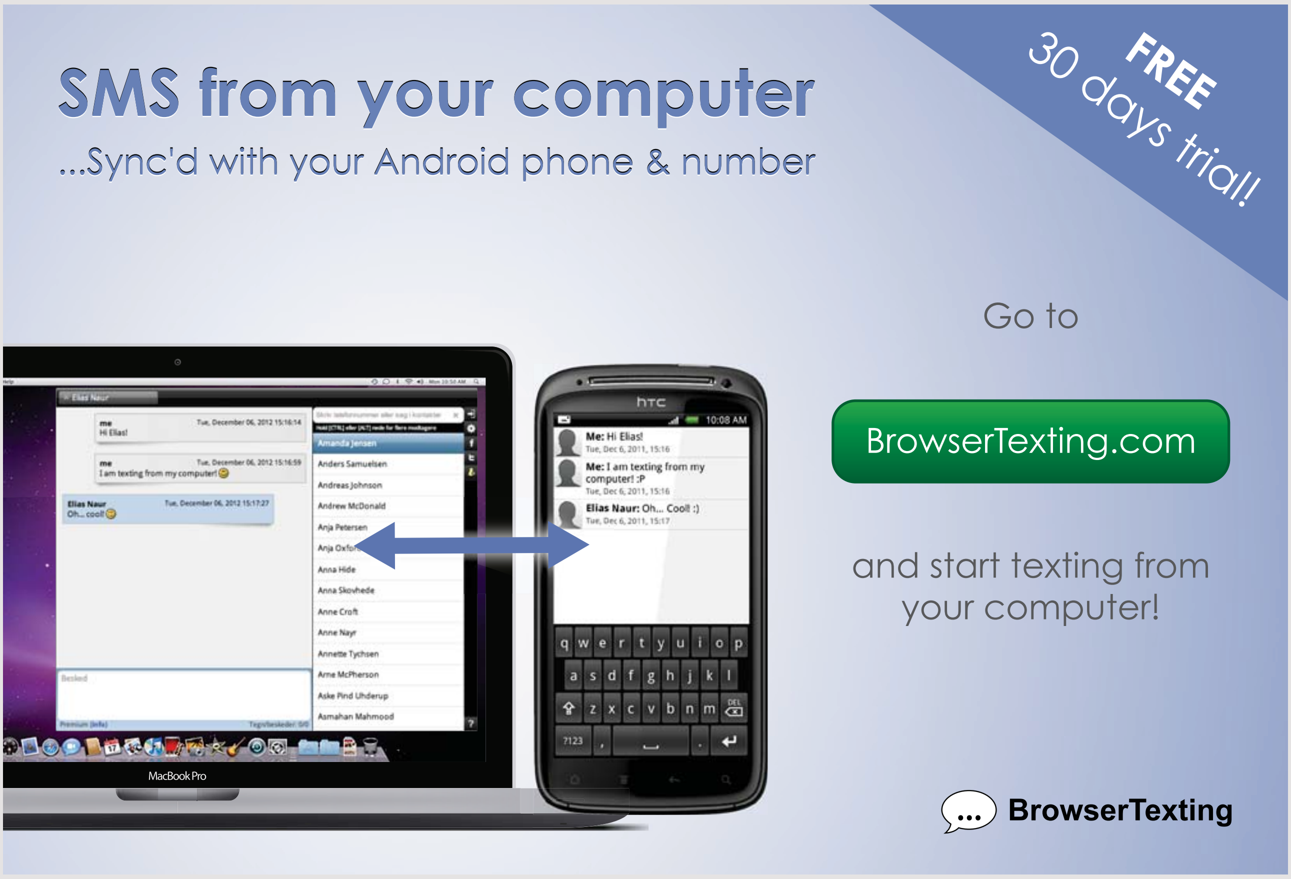
C_m = C_mixed C_u = C_unmixed

		C_m/C_u = 1.33	C_m/C_u = 2	C_m/C_u = 4	C_m/C_u = 10	C_m/C_u = 100
1..11	NTU	ε	ε	ε	ε	ε
Run 1	0	0	0	0	0	0
Run 2	0.5	0.3406	0.3572	0.3747	0.3935	0.3935
Run 3	1	0.5031	0.542	0.5847	0.6321	0.6321
Run 4	1.5	0.5884	0.6438	0.7061	0.7769	0.7769
Run 5	2	0.6358	0.702	0.7776	0.8647	0.8647
Run 6	2.5	0.663	0.7361	0.8202	0.9179	0.9179
Run 7	3	0.679	0.7564	0.8458	0.9502	0.9502
Run 8	3.5	0.6885	0.7685	0.8612	0.9698	0.9698
Run 9	4	0.6942	0.7758	0.8705	0.9817	0.9817
Run 10	4.5	0.6977	0.7802	0.8761	0.9889	0.9889
Run 11	5	0.6997	0.7828	0.8795	0.9933	0.9933

Note that for (C_mixed/C_unmixed) values greater than 10, the effectiveness values remain the same.

Now, plot the results:





“**Prob. 3.2.43**. Write EES Functions to find the exergy of heat supplied at constant temp, exergy of heat supplied at varying temp and at constant specific heat (cp), and the exergy of mass flow.”

EES Functions:

```
FUNCTION ExergyofHeat_ConstTemp(Q,T,T0)

{$ExergyofHeat_constTemp
This function returns the specific availability of a fluid in kJ/kg as a function of

        Q[kJ], T [K], T0 (K)

        }

ExergyofHeat_ConstTemp =Q * (1 – T0 / T) "…kJ/kg"

END

"======================================================================"

FUNCTION ExergyofHeat_ConstPressure(cp,T,T0)

{$ExergyofHeat_constPressure
This function returns the specific availability of a fluid in kJ/kg as a function of

        cp[kJ/kg.K], T [K], T0 (K)

        }

ExergyofHeat_ConstPressure = cp * ( (T – T0) – T0 * ln(T / T0 )) "…kJ/kg"

END

"======================================================================"

FUNCTION Exergy_massflow_IdealGas(IdealGas$,T, P, V, Z,T0, P0)

{$Exergy_massflow_IdealGas
```

```
This function returns the specific availability of IdealGas$ in kJ/kg as a function of

          T [K], P [kPa], V [m/sec], Z [m], and 'dead state' P0 (kPa), T0 (K)

Ideal gases: Air, Ar, CO, CO2, N2, O2, H2, He, H2O, CH4 etc.

See in EES: Options-Function Info-Fluid Props-Ideal gases.

          }

IF ((T = T0) AND (P = P0)) THEN

Exergy_massflow_IdealGas: = 0

ELSE

g: = 9.81 "m/s^2"

 h: = Enthalpy(IdealGas$, T=T) "…kJ/kg"

 s: = Entropy(IdealGas$, T=T, P=P) "…kJ/kg.K"

h0: = Enthalpy(IdealGas$, T=T0) "…kJ/kg"

s0: = Entropy(IdealGas$, T=T0, P=P0) "…kJ/kg.K"

Exergy_massflow_IdealGas: = (h – h0) – T0 * (s – s0) + (V^2 / 2) / 1000 + (g * Z) / 1000
"…kJ/kg"

ENDIF

END

"=========================================================================="
```

Example:

IdealGas$ = ‘N2’

T = 400 “K”

P = 20000 “kPa”

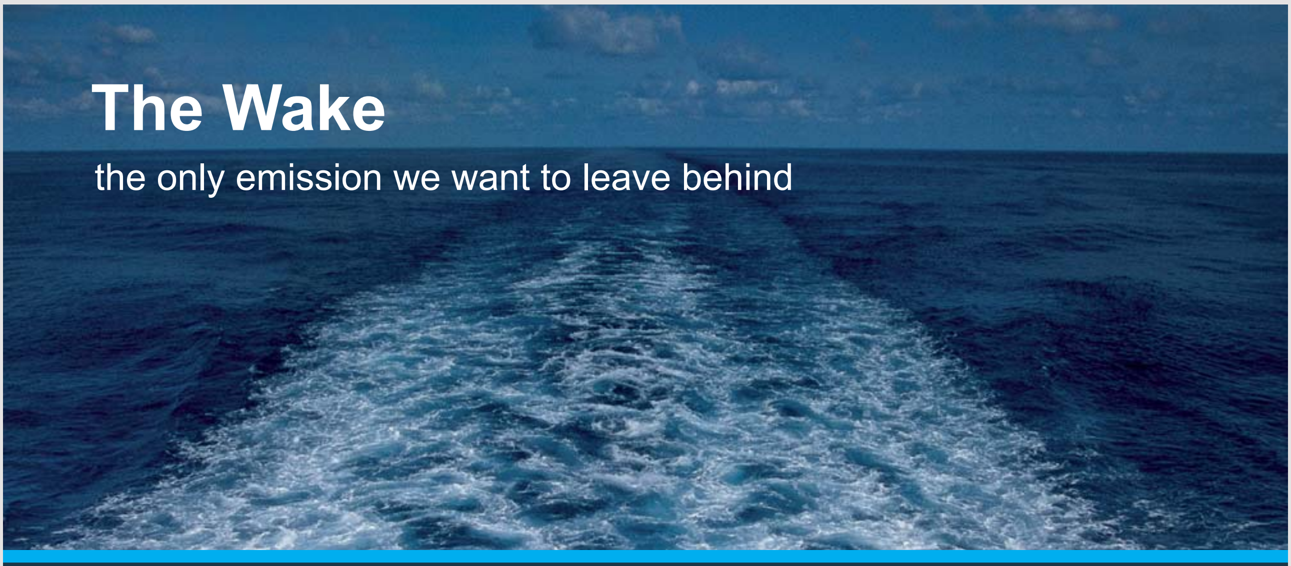
T0 = 300 “K”

P0 = 101.3 “kPa”

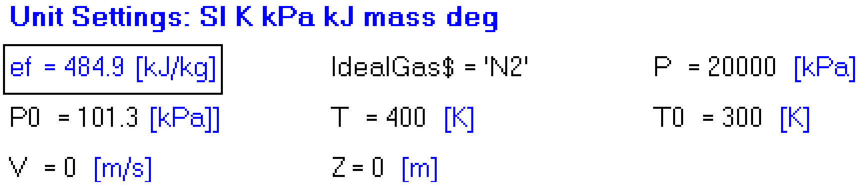
V = 0 “m/s”

Z = 0 “m”

ef = Exergy_massflow_IdealGas(IdealGas$,T, P, V, Z,T0, P0)



Result:



```
Unit Settings: SI K kPa kJ mass deg
ef = 484.9 [kJ/kg]      IdealGas$ = 'N2'      P  = 20000  [kPa]
P0  = 101.3 [kPa]]      T  = 400  [K]         T0  = 300  [K]
V  = 0  [m/s]           Z = 0  [m]
```

i.e. exergy = ef = 484.9 kJ/ kg…Ans. when Nitrogen is treated as an *ideal fluid*, i.e. cp is a function of T only.

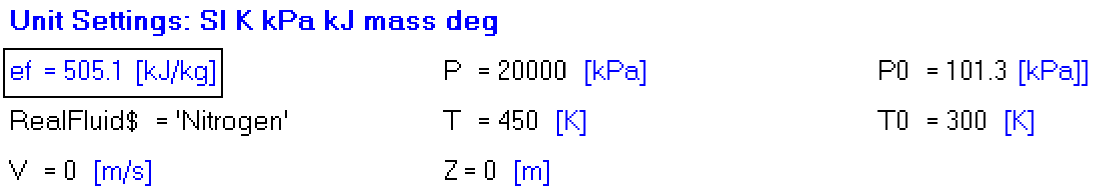
```
==============================================================

FUNCTION Exergy_massflow_RealFluid(RealFluid$,T, P, V, Z,T0, P0)

{$Exergy_massflow_RealFluid

This function returns the specific availability of RealFluid$ in kJ/kg as a function of

          T [K], P [kPa], V [m/sec], Z [m], and 'dead state' P0 (kPa), T0 (K)

RealFluids: Air_ha, Acetone, Ammonia, Argon, R12, R12, R124, R125, R134a, R23, R13,
R22, Steam, Steam_NBS, Steam_IAPWS, Sulphur dioxide, Water, Xenon…etc.

See in EES: Options-Function Info-Fluid Props-Real Fluids.

          }

IF ((T = T0) AND (P = P0)) THEN

Exergy_massflow_RealFluid: = 0

ELSE

g: = 9.81 "m/s^2"

 h: = Enthalpy(RealFluid$, T=T, P=P) "…kJ/kg"

 s: = Entropy(RealFluid$, T=T, P=P) "…kJ/kg.K"

 h0: = Enthalpy(RealFluid$, T=T0, P=P0) "…kJ/kg"
```

```
 s0: = Entropy(RealFluid$, T=T0, P=P0) "…kJ/kg.K"

Exergy_massflow_RealFluid: = (h – h0) – T0 * (s – s0) +( V^2 / 2) / 1000 + (g * Z) / 1000 "…kJ/kg"

ENDIF

END
```

Example:

```
RealFluid$ = 'Nitrogen'

T = 450 "K"

P = 20000 "kPa"

T0 = 300 "K"

P0 = 101.3 "kPa"

V = 0 "m/s"

Z = 0 "m"

ef = Exergy_massflow_RealFluid(RealFluid$,T, P, V, Z,T0, P0)
```

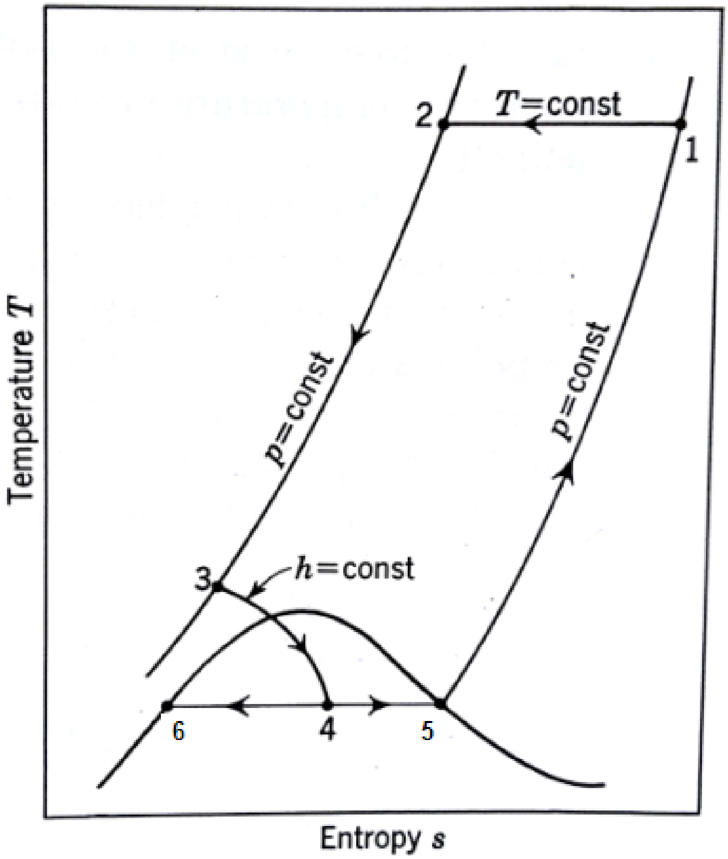
Result:



```
Unit Settings: SI K kPa kJ mass deg
ef = 505.1  [kJ/kg]            P  = 20000  [kPa]          P0  = 101.3 [kPa]]
RealFluid$  = 'Nitrogen'       T  = 450  [K]              T0  = 300  [K]
V  = 0  [m/s]                  Z = 0  [m]
```

i.e. exergy = ef = 505.1 kJ/ kg…. Ans. when Nitrogen is treated as a *real fluid*, i.e. cp is a function of both P and T.

"**Prob.3.2.44.** Make an exergy analysis for a simple Linde-Hampson system using Nitrogen as the working fluid. The system operates between 101.3 kPa and 300 K at point 1 and 20 MPa at point 2. The system may be assumed reversible, except for the expansion through the expansion valve."


T=const
2
1
p=const
p=const
3
h=const
6
4
5
Temperature T
Entropy s

Solution:

To determine the liquid yield, the work per unit mass compressed, the work per unit mass liquefied, and the figure of merit for a simple Linde-Hampson system. Then, determine the exergetic effcy of the cycle. Also, determine the exergy losses in the compressor (assuming the isothermal effcy of compressor as 70%), expansion valve and the heat exchanger.

"Data:"

Fluid$ = 'Nitrogen'

T_1=300 "[K]"

T_2 = T_1 "K"

T0 = 300 "K…dead state"

P_1=101.3 "[kPa]"

P0 =101.3 "kPa…dead state"

V = 0 "m/s"

Z = 0 "m"

P_2= 20000 "[kPa]"

P_3 = P_2 "kPa"

P_4 = P_1 "kPa"

P_5 = P_1 "kPa"

P_6 = P_1 "kPa"

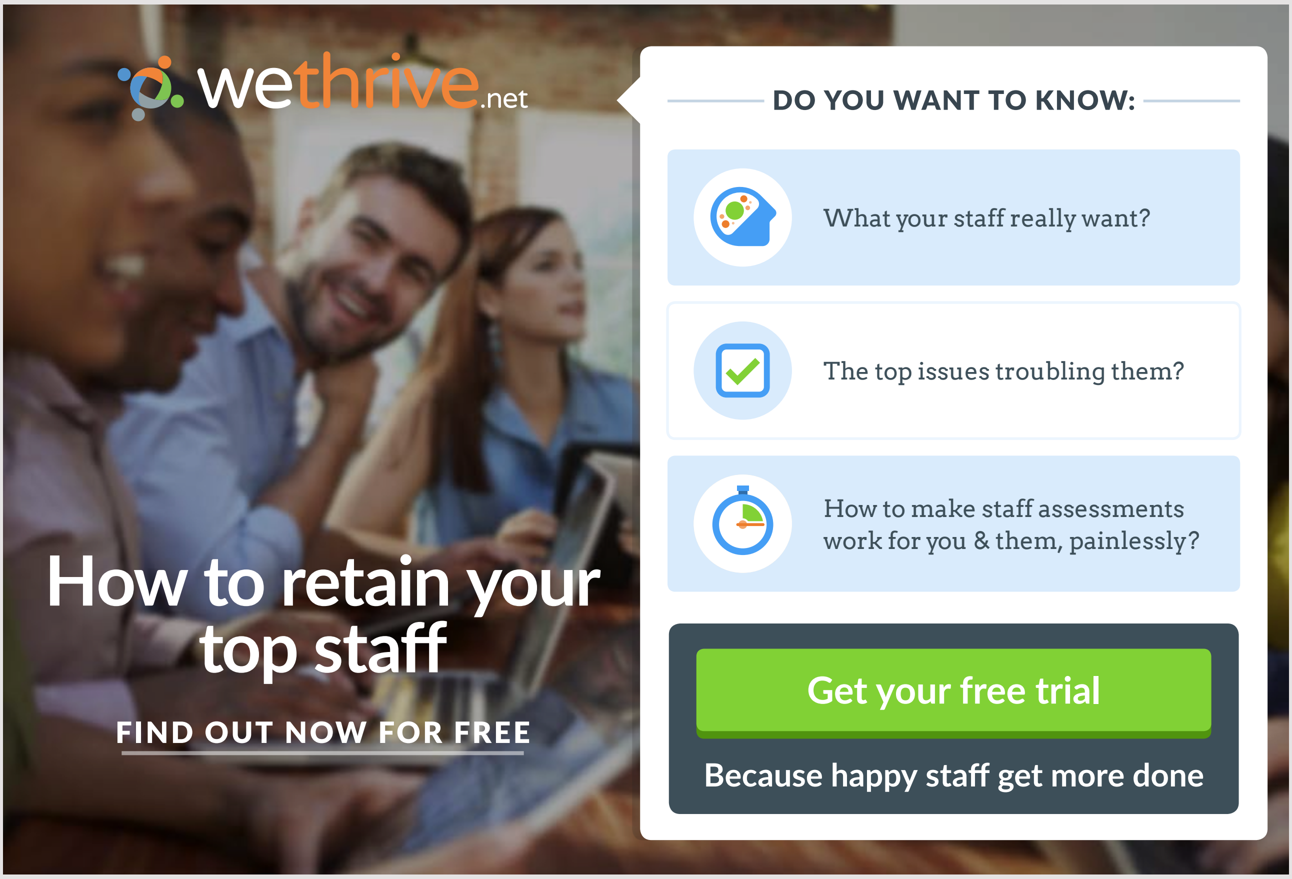
"Calculations:"

s_1= Entropy(Fluid$,T=T_1,P=P_1) "[kJ/kg-K]"

s_2= Entropy(Fluid$,P=P_2,T=T_2) "[kJ/kg-K]"

```
h_1= Enthalpy(Fluid$,P=P_1,T=T_1) “[kJ/kg]”
h_2= Enthalpy(Fluid$,P=P_2,T=T_2) “[kJ/kg]”
h_f = Enthalpy(Fluid$,x=0,P=P_1) “[kJ/kg]”
h_6 = h_f “…sat. liquid enthalpy”
y = (h_1 – h_2) / (h_1 – h_f) “…fraction liquefied”
(h_2 – h_3) = (1 – y) * (h_1 – h_5) “…finds h_3”
h_5 = Enthalpy(Fluid$, x=1,P=P_5) “[kJ/kg]”
h_4 = h_3 “…since process 3–4 in the J-T valve is isenthalpic”
s_3 = Entropy(Fluid$,P=P_3,h=h_3) “[kJ/kg-K]”
s_4 = Entropy(Fluid$,P=P_4,h=h_4) “[kJ/kg-K]”
s_5= Entropy(Fluid$,P=P_5,h=h_5) “[kJ/kg-K]”
s_6 = Entropy(Fluid$,P=P_6,h=h_6) “[kJ/kg-K]”
T_3=Temperature(Fluid$,P=P_3,s=s_3) “K…temp at point 3”
T_6=Temperature(Fluid$,P=P_6,x = 0) “K…temp at point 6”
T_4=T_6 “K…temp at point 4”
T_5=T_6 “K…temp at point 5”
Wperkggas =T_1 * (s_1 – s_2) – (h_1 – h_2) “[kJ/kg]”
Wperkgliq = (T_1 * (s_1 – s_2 )- (h_1 – h_2)) / y “[kJ/kg]”
W_ideal = Ideal_Work(FLUID$, P_1,T_1) “kJ/kg…ideal work of liquefaction”
FOM = W_ideal / Wperkgliq “…Figure of Merit”
COP = (h_1 – h_2) / Wperkggas “…Coeff of Performance”
```

```
"Exergies:"

h0 = Enthalpy(Fluid$, T=T0, P=P0) "…kJ/kg"

 s0 = Entropy(Fluid$, T=T0, P=P0) "…kJ/kg.K"

ef_1 = (h_1 – h0) – T0 * (s_1 – s0) "kJ/kg…exergy of flow at point 1"

ef_2 = (h_2 – h0) – T0 * (s_2 – s0) "kJ/kg…exergy of flow at point 2"

ef_3 = (h_3 – h0) – T0 * (s_3 – s0) "kJ/kg…exergy of flow at point 3"

ef_4 = (h_4 – h0) – T0 * (s_4 – s0) "kJ/kg…exergy of flow at point 4"

ef_5 = (h_5 – h0) – T0 * (s_5 – s0) "kJ/kg…exergy of flow at point 5"

ef_6 = (h_6 – h0) – T0 * (s_6 – s0) "kJ/kg…exergy of flow at point 6"
```



"Exergetic effcy.:"

W_rev = ef_6 – ef_1 "…reversible work, kJ/kg"

W_actual = Wperkgliq "…kJ/kg liq…actual work input"

W_actual = W_rev + exergy_destroyed "…finds exergy destroyed, kJ/kg of liq."

eta_II_cycle = W_rev / W_actual "…Second Law effcy of cycle = exergetic effcy."

"Writing exergy balance to determine exergy losses in each component:"

"Exergy loss in Isothermal compressor:"

"Note that in an isothermal compressor, heat is rejected; however, in this case its exergy is zero since heat is rejected at ambient conditions."

"Writing an exergy balance for a compressor of isothermal effcy = 70%:"

ef_1 + Wperkggas/0.7 = ef_2 + DELTAe_comp "…finds exergy loss in compressor, DELTAe_comp"

"Exergy loss in Heat exchanger:"

"Writing an exergy balance:"

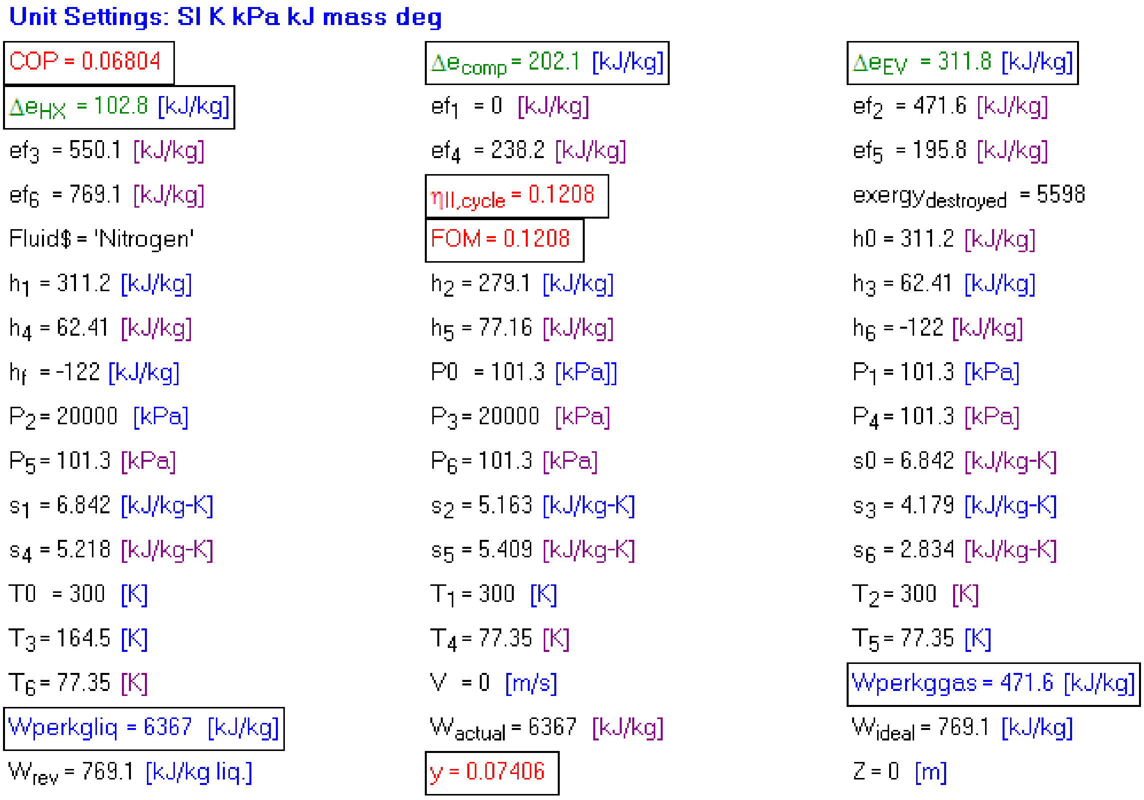
ef_2 +(1 – y) * ef_5 = ef_3 + (1 – y) * ef_1 + DELTAe_HX "…finds exergy loss in HX, DELTAe_HX"

"Exergy loss in Expansion valve:"

"Writing an exergy balance:"

ef_3 = ef_4 + DELTAe_EV "…finds exergy loss in Expn valve, DELTAe_EV"

Results:



Unit Settings: SI K kPa kJ mass deg

COP = 0.06804
Δe_{comp} = 202.1 [kJ/kg]
Δe_{EV} = 311.8 [kJ/kg]
Δe_{HX} = 102.8 [kJ/kg]
ef_1 = 0 [kJ/kg]
ef_2 = 471.6 [kJ/kg]
ef_3 = 550.1 [kJ/kg]
ef_4 = 238.2 [kJ/kg]
ef_5 = 195.8 [kJ/kg]
ef_6 = 769.1 [kJ/kg]
$\eta_{II,cycle}$ = 0.1208
$exergy_{destroyed}$ = 5598
Fluid$ = 'Nitrogen'
FOM = 0.1208
h0 = 311.2 [kJ/kg]
h_1 = 311.2 [kJ/kg]
h_2 = 279.1 [kJ/kg]
h_3 = 62.41 [kJ/kg]
h_4 = 62.41 [kJ/kg]
h_5 = 77.16 [kJ/kg]
h_6 = -122 [kJ/kg]
h_f = -122 [kJ/kg]
P0 = 101.3 [kPa]]
P_1 = 101.3 [kPa]
P_2 = 20000 [kPa]
P_3 = 20000 [kPa]
P_4 = 101.3 [kPa]
P_5 = 101.3 [kPa]
P_6 = 101.3 [kPa]
s0 = 6.842 [kJ/kg-K]
s_1 = 6.842 [kJ/kg-K]
s_2 = 5.163 [kJ/kg-K]
s_3 = 4.179 [kJ/kg-K]
s_4 = 5.218 [kJ/kg-K]
s_5 = 5.409 [kJ/kg-K]
s_6 = 2.834 [kJ/kg-K]
T0 = 300 [K]
T_1 = 300 [K]
T_2 = 300 [K]
T_3 = 164.5 [K]
T_4 = 77.35 [K]
T_5 = 77.35 [K]
T_6 = 77.35 [K]
V = 0 [m/s]
Wperkggas = 471.6 [kJ/kg]
Wperkgliq = 6367 [kJ/kg]
W_{actual} = 6367 [kJ/kg]
W_{ideal} = 769.1 [kJ/kg]
W_{rev} = 769.1 [kJ/kg liq.]
y = 0.07406
Z = 0 [m]

Thus:

Liquid yield, y = 0.0746 kg liq. / kg gas compressed…Ans.

Work per kg gas compressed = Wperkggas = 471.6 kJ/kg…Ans.

Work per kg liquid = Wperkgliq = 6367 kJ/kg…Ans.

Figure of Merit = FOM = 0.1208…Ans.

Exergetic effcy (or, second law effcy) of cycle = ηII = 0.1208 (= FOM)…Ans.

Exergy losses in compressor (assuming its isoth. effcy is 70%) = Δecomp = 202.1 kJ/kg…Ans.

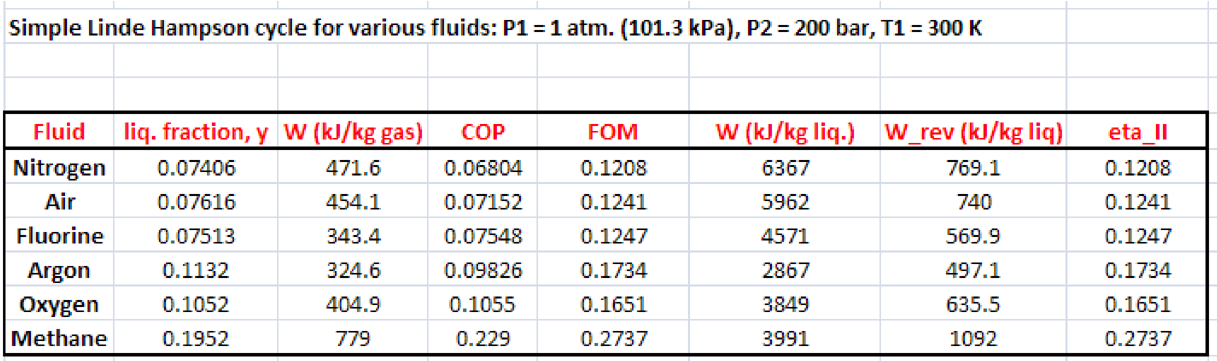
Exergy losses in Heat Exchanger = ΔeHX = 102.8 kJ/kg…Ans.

Exergy losses in Expansion Valve = ΔeEV = 311.8 kJ/kg…Ans.

B. **Tabulate the salient results for Nitrogen, Air, Fluorine, Argon, Oxygen and Methane:**

Run the above EES program for the different fluids.

Following Table summarizes the results:

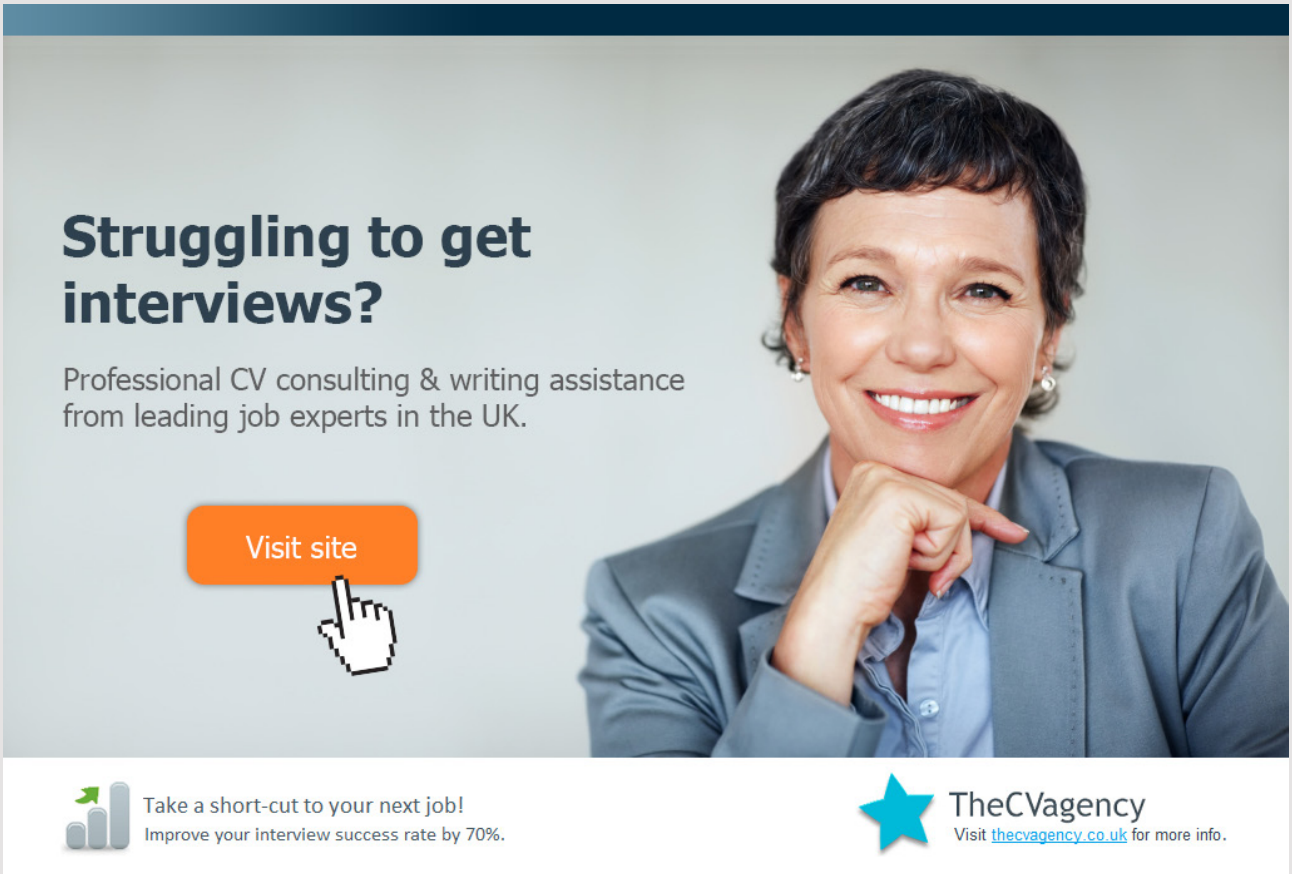


Simple Linde Hampson cycle for various fluids: P1 = 1 atm. (101.3 kPa), P2 = 200 bar, T1 = 300 K

Fluid	liq. fraction, y	W (kJ/kg gas)	COP	FOM	W (kJ/kg liq.)	W_rev (kJ/kg liq)	eta_II
Nitrogen	0.07406	471.6	0.06804	0.1208	6367	769.1	0.1208
Air	0.07616	454.1	0.07152	0.1241	5962	740	0.1241
Fluorine	0.07513	343.4	0.07548	0.1247	4571	569.9	0.1247
Argon	0.1132	324.6	0.09826	0.1734	2867	497.1	0.1734
Oxygen	0.1052	404.9	0.1055	0.1651	3849	635.5	0.1651
Methane	0.1952	779	0.229	0.2737	3991	1092	0.2737

C. **Plot y, Wperkgas, COP and η_{II} against P2, other parameters remaining constant:**

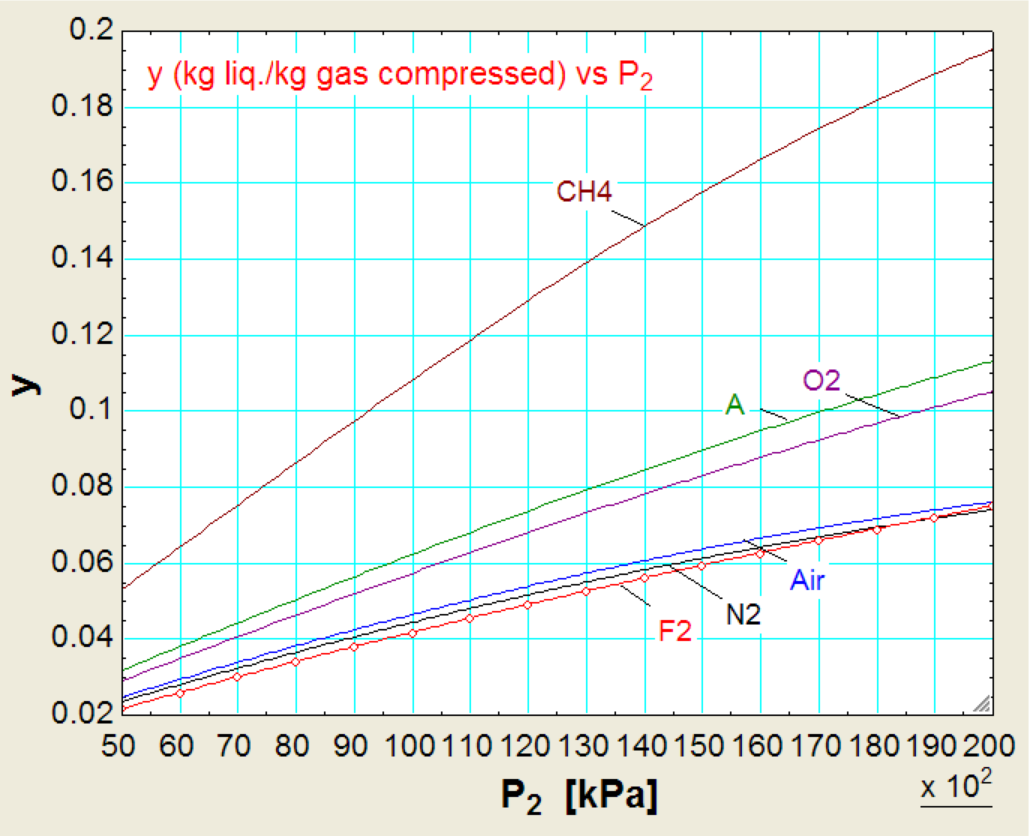
First, compute the Parametric Tables.

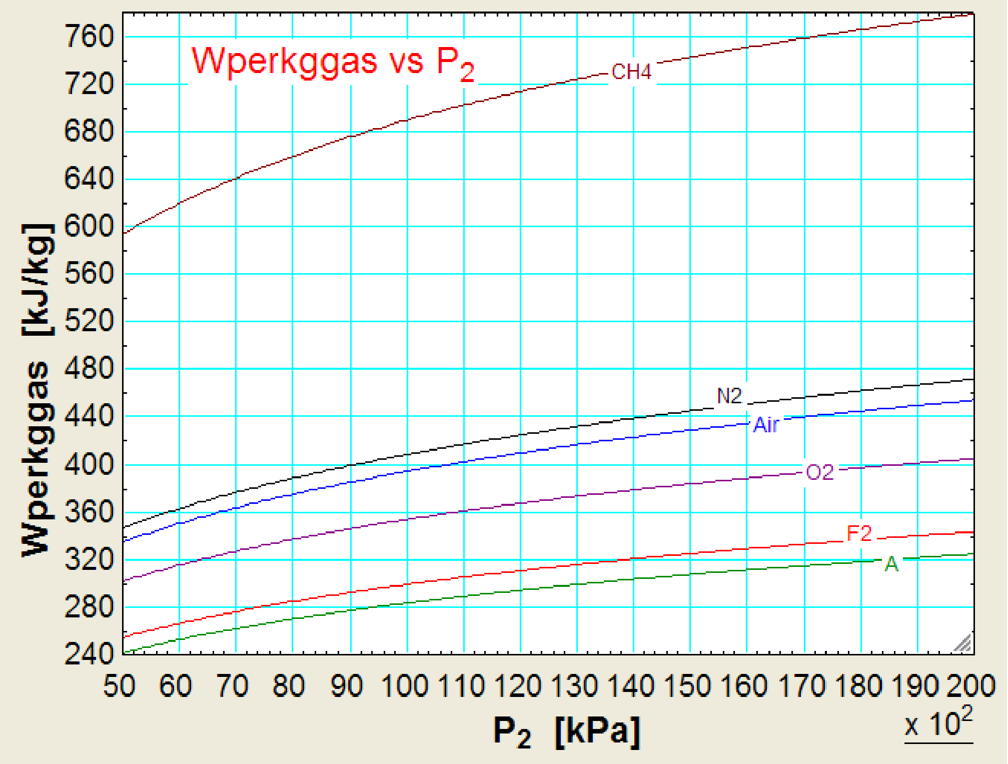


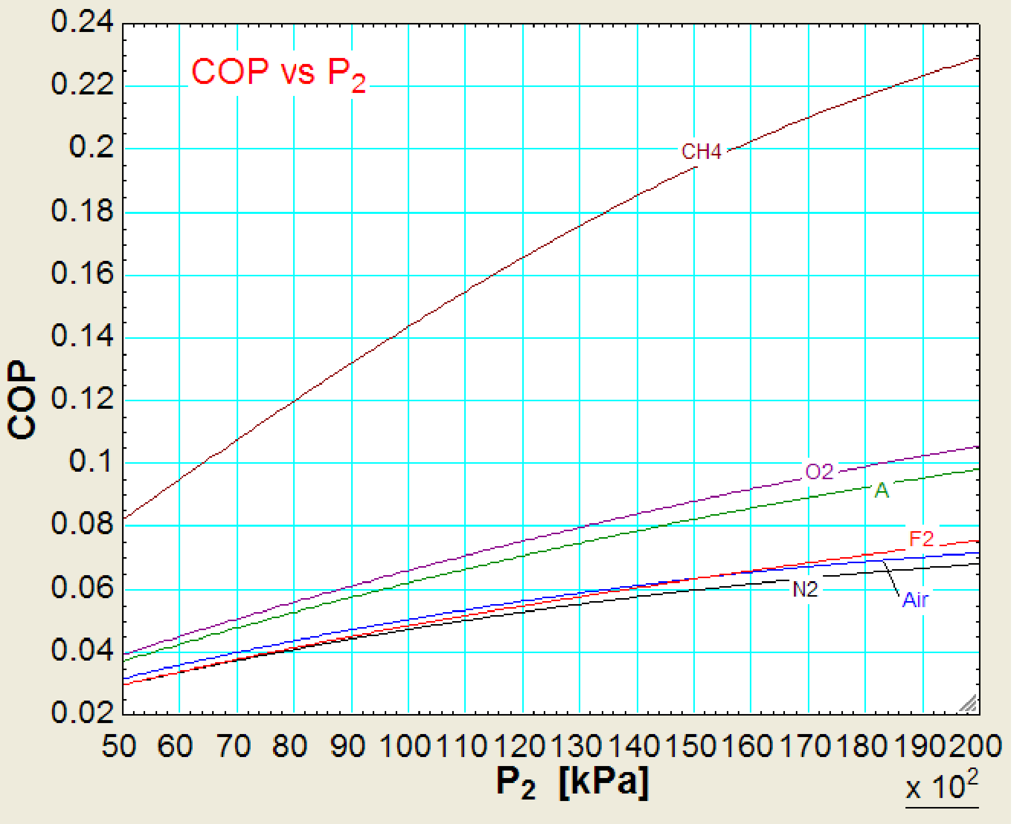
One example of the Table for Nitrogen is shown below:

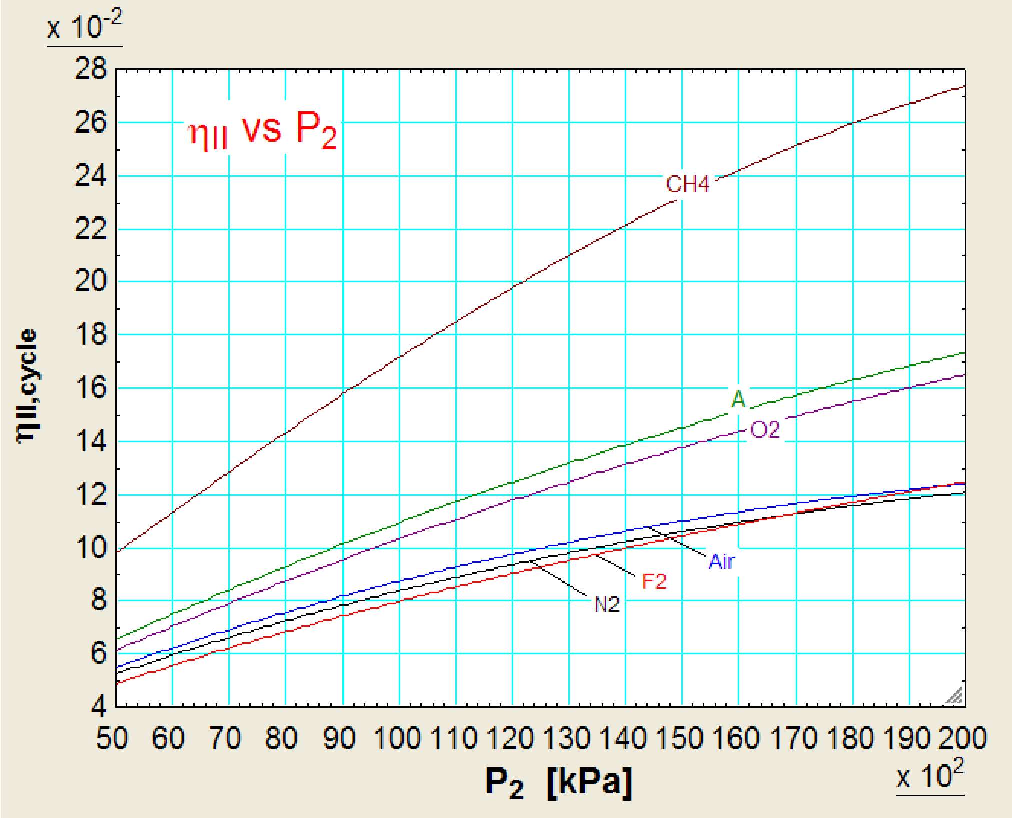
1..16	P_2 [kPa]	y	Wperkggas [kJ/kg]	COP	FOM	Wperkgliq [kJ/kg]	W_{rev} [kJ/kg liq.]	$\eta_{II,cycle}$
Run 1	5000	0.02364	346.6	0.02954	0.05245	14663	769.1	0.05245
Run 2	6000	0.0281	362.8	0.03355	0.05956	12913	769.1	0.05956
Run 3	7000	0.03241	376.5	0.03729	0.06621	11616	769.1	0.06621
Run 4	8000	0.03658	388.4	0.0408	0.07244	10617	769.1	0.07244
Run 5	9000	0.0406	398.9	0.04409	0.07828	9825	769.1	0.07828
Run 6	10000	0.04446	408.3	0.04717	0.08374	9184	769.1	0.08374
Run 7	11000	0.04816	416.9	0.05005	0.08885	8656	769.1	0.08885
Run 8	12000	0.0517	424.7	0.05273	0.09362	8215	769.1	0.09362
Run 9	13000	0.05507	431.9	0.05523	0.09806	7843	769.1	0.09806
Run 10	14000	0.05827	438.6	0.05755	0.1022	7527	769.1	0.1022
Run 11	15000	0.06131	444.9	0.0597	0.106	7257	769.1	0.106
Run 12	16000	0.06419	450.8	0.06168	0.1095	7024	769.1	0.1095
Run 13	17000	0.0669	456.4	0.0635	0.1127	6823	769.1	0.1127
Run 14	18000	0.06944	461.7	0.06516	0.1157	6649	769.1	0.1157
Run 15	19000	0.07183	466.7	0.06667	0.1184	6498	769.1	0.1184
Run 16	20000	0.07406	471.6	0.06804	0.1208	6367	769.1	0.1208

Similar Tables for Air, Fluorine, Argon, Oxygen and Methane are also computed. Then, results are drawn in plots as shown below:



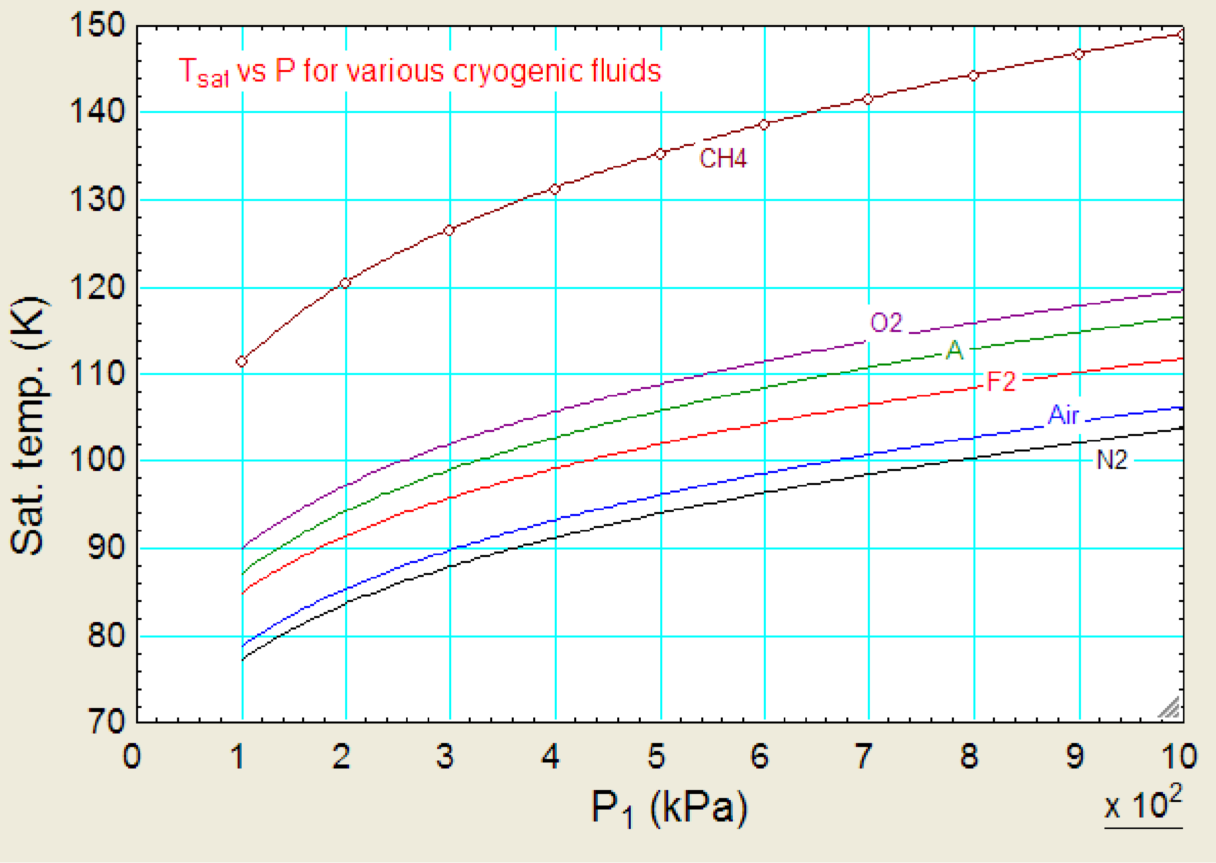

Wperkggas vs P$_2$
CH4
N2
Air
O2
F2
A
Wperkggas [kJ/kg]
760
720
680
640
600
560
520
480
440
400
360
320
280
240
50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
P$_2$ [kPa]
x 10^2


COP vs P$_2$
CH4
O2
A
F2
N2
Air
COP
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
P$_2$ [kPa]
x 10^2


η_{II} vs P_2
x 10^{-2}
28
26
24
22
20
18
16
14
12
10
8
6
4
ηII,cycle
CH4
A
O2
Air
F2
N2
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
P_2 [kPa]
x 10^2

D. **Plot y, COP and η_{II} against P1, other parameters remaining constant:**

P1 is the inlet pressure. As P1 varies, the liquefaction temp (i.e. the boiling point) of the fluid also varies. This variation for various fluids is shown first:

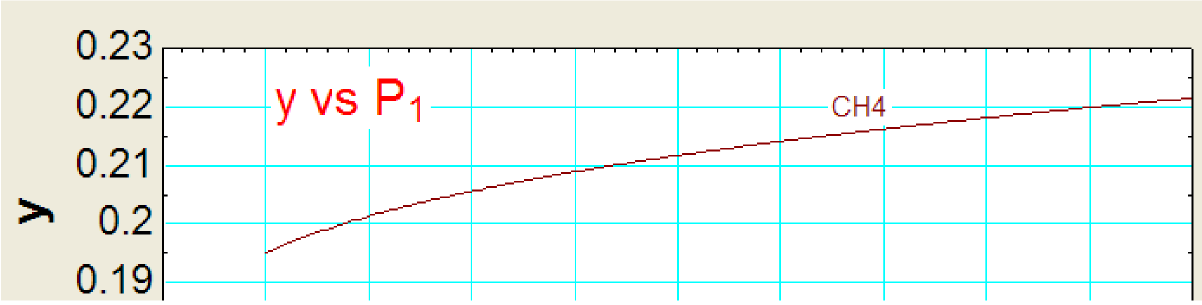


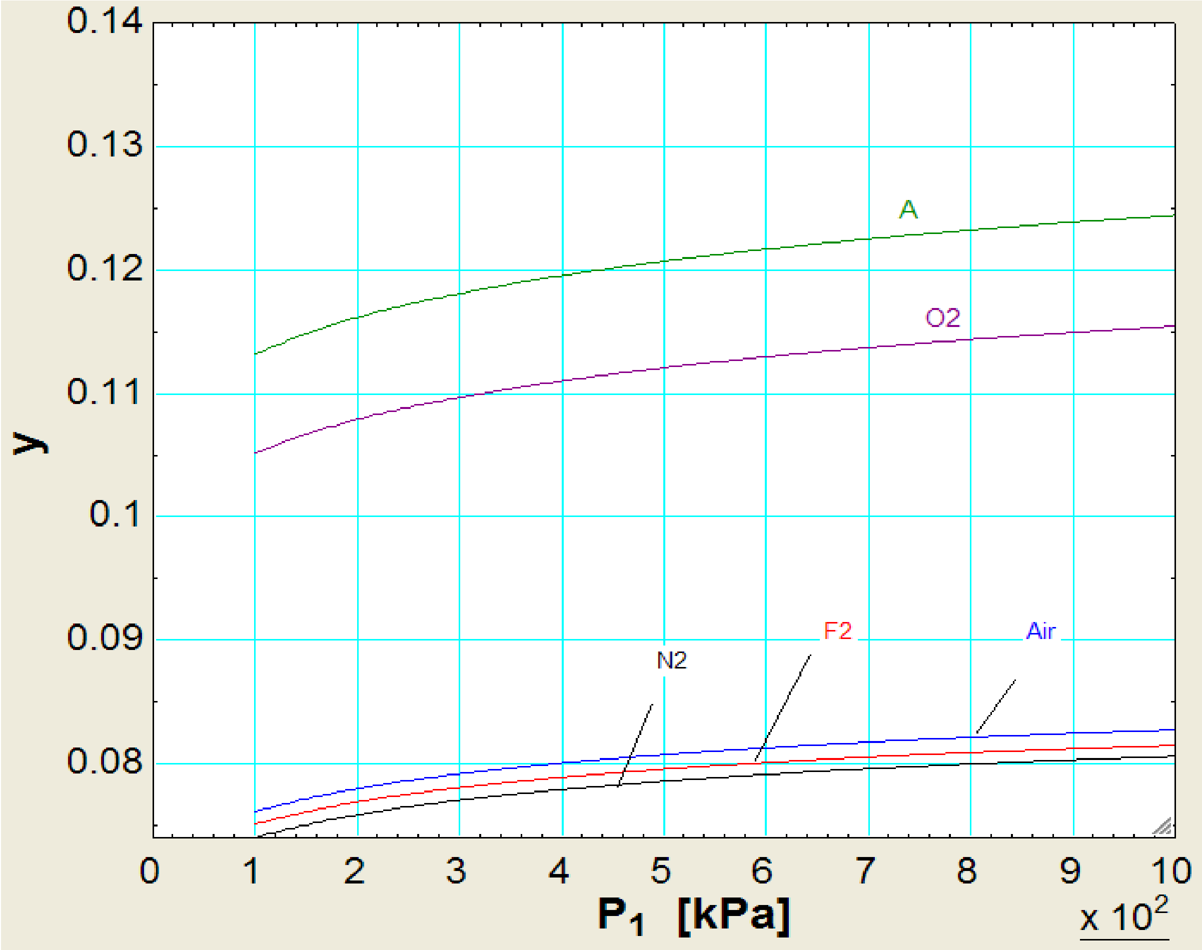
Now, compute the Parametric Tables of variables as P1 varies from 1 to 10 bar. An example Table for Nitrogen is shown below:

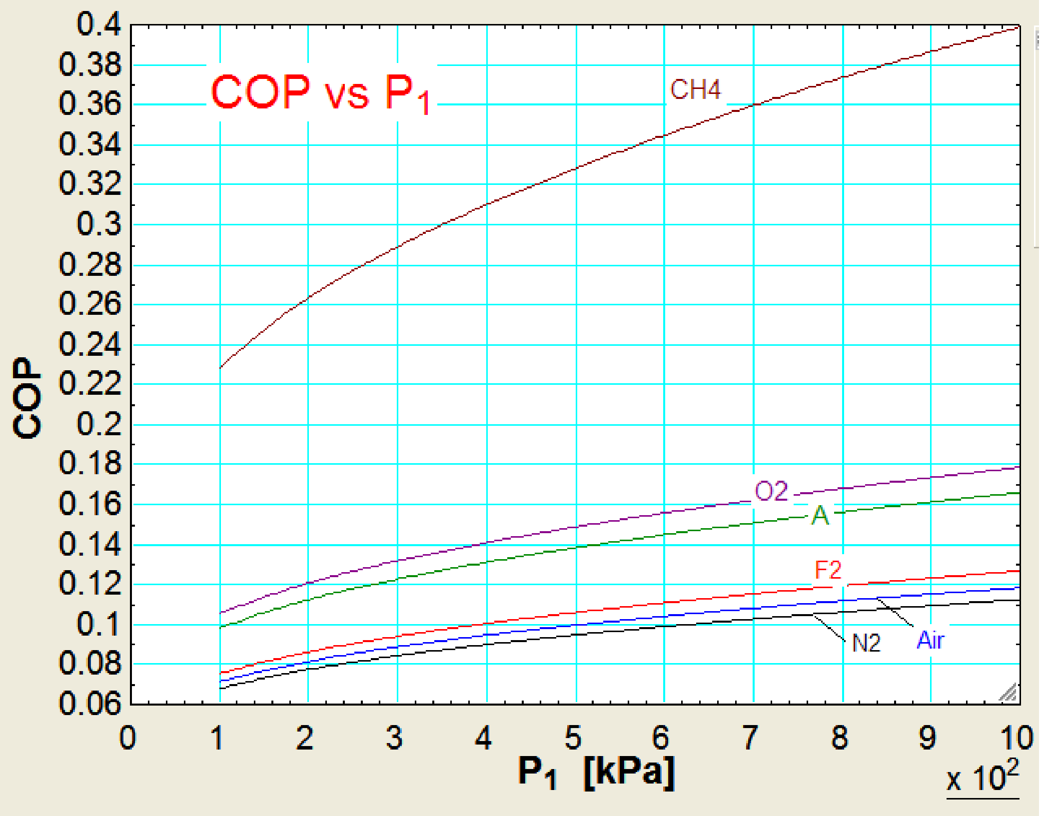
1..10	P_1 [kPa]	T_4	y	Wperkggas [kJ/kg]	COP	FOM	Wperkgliq [kJ/kg]	W_{rev} [kJ/kg liq.]	$\eta_{II,cycle}$
Run 1	100	77.24	0.07403	472.7	0.06788	0.1207	6385	770.9	0.1207
Run 2	200	83.63	0.07587	411	0.07753	0.1243	5417	673.6	0.1243
Run 3	300	87.91	0.07704	374.9	0.08441	0.1265	4866	615.4	0.1265
Run 4	400	91.23	0.07791	349.3	0.08997	0.1279	4484	573.4	0.1279
Run 5	500	93.99	0.07859	329.5	0.09472	0.1289	4192	540.5	0.1289
Run 6	600	96.38	0.07914	313.2	0.09893	0.1297	3958	513.2	0.1297
Run 7	700	98.49	0.0796	299.5	0.1027	0.1302	3763	490	0.1302
Run 8	800	100.4	0.07999	287.7	0.1062	0.1306	3596	469.7	0.1306
Run 9	900	102.1	0.08032	277.2	0.1094	0.1309	3451	451.7	0.1309
Run 10	1000	103.7	0.0806	267.8	0.1125	0.1311	3323	435.5	0.1311

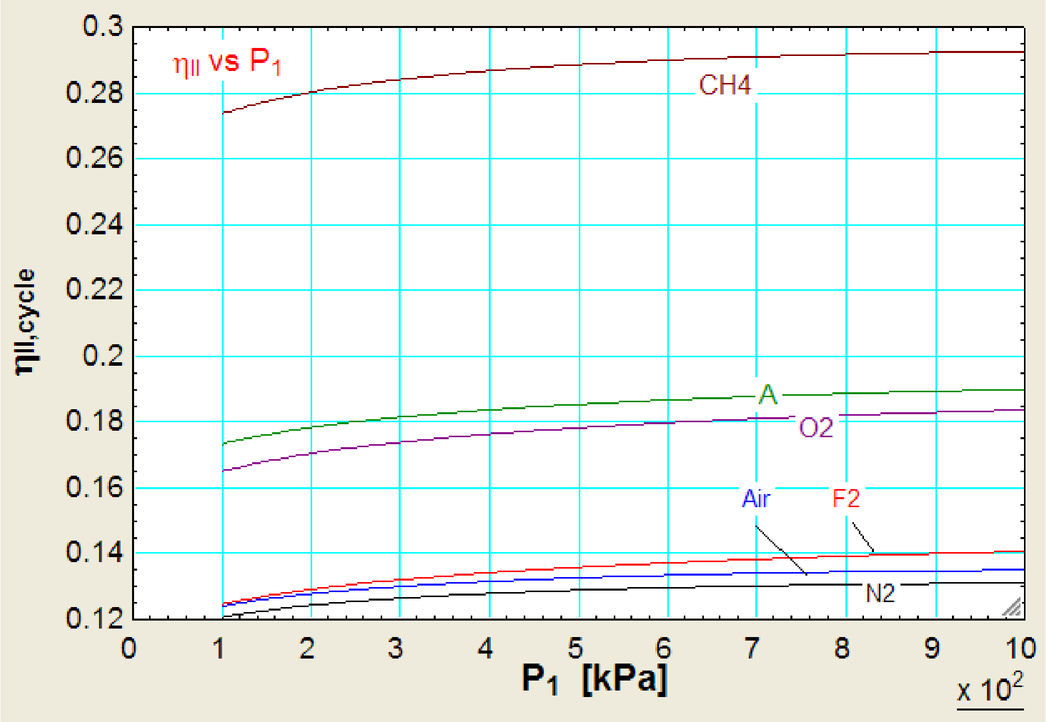
Similar Tables for Air, Fluorine, Argon, Oxygen and Methane are also computed.

Then, results are drawn in plots as shown below:





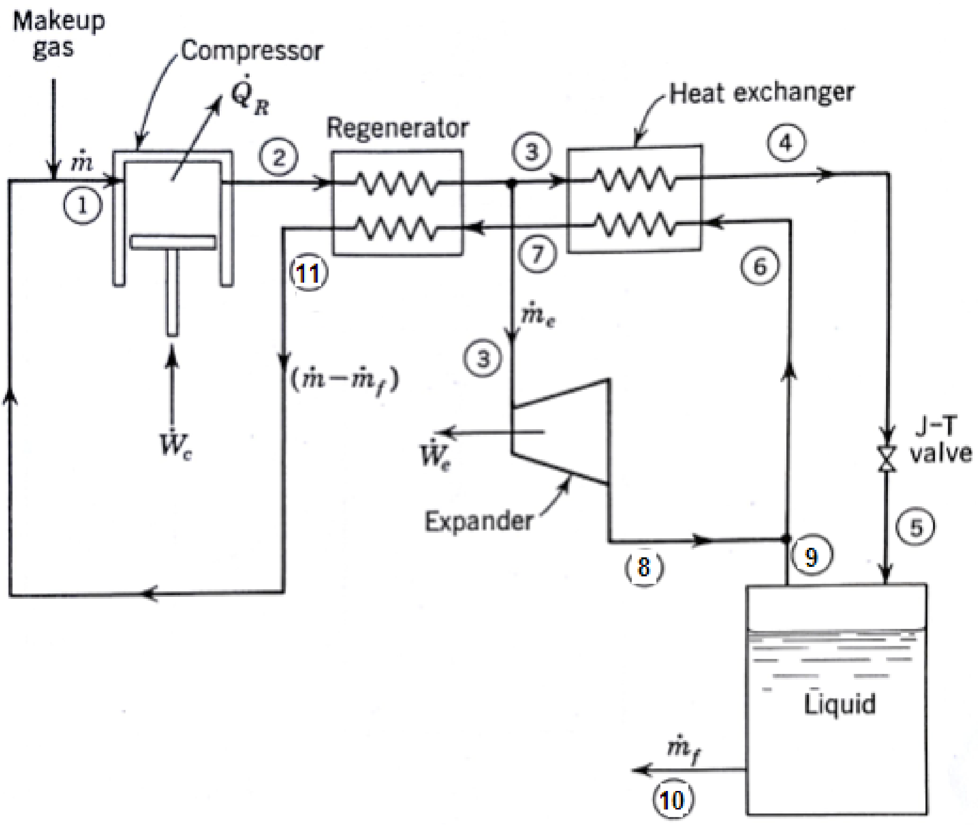

COP vs P_1
CH4
O2
A
F2
N2
Air
COP
0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0
1
2
3
4
5
6
7
8
9
10
P_1 [kPa]
x 10^2

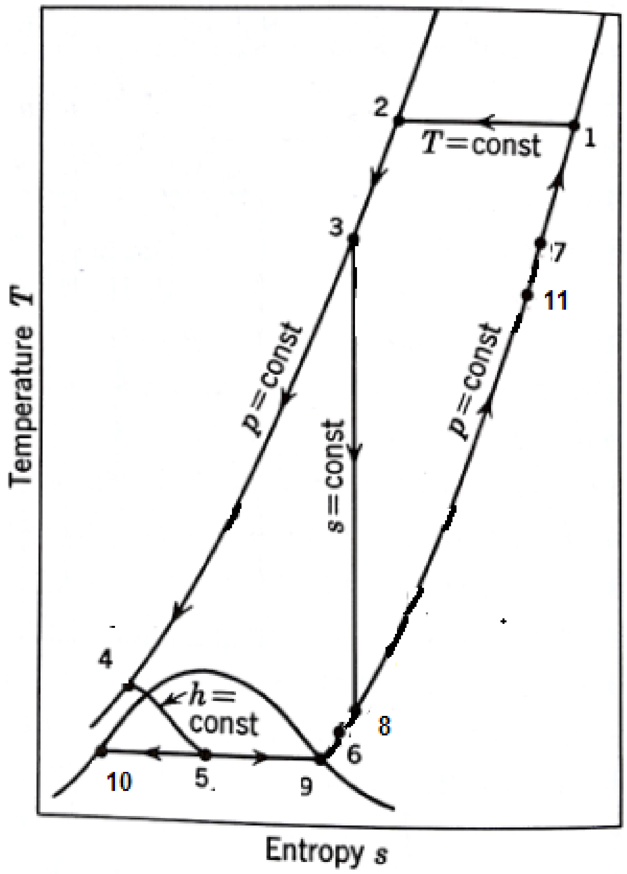


Prob. 3.2.45. Data for an ideal Kapitza system is given below:
Nitrogen is the working fluid, the system operates from 101.3 kPa (1 atm) and 300 K to 5.07 MPa (i.e. 50 atm.). At 5.07 MPa and 255 K, 40% of the main flow is diverted to the reversible adiabatic expander. Remainder flows through the heat exchanger and expands through the expansion valve to 101.3 kPa. Determine the liquid yield and the work per unit mass liquefied, assuming that the expander work is utilized in the compression process. Make a second law analysis of the system.

Solution:

"Note that Kapitza system is similar to Claude system except that the last heat exchanger (i.e. low temp heat exchanger) is eliminated and the first heat exchanger consists of regenerators. For calculation purposes, the formulas are the same as for Claude system."


Makeup gas
Compressor
$\dot{Q}_R$
Regenerator
Heat exchanger
$\dot{m}$
①
②
③
④
⑦
⑥
⑪
$\dot{m}_e$
③
$(\dot{m}-\dot{m}_f)$
$\dot{W}_c$
$\dot{W}_e$
Expander
J-T valve
⑤
⑧
⑨
Liquid
$\dot{m}_f$
⑩


2
1
T=const
3
7
11
p=const
s=const
p=const
4
h= const
8
6
10
5
9
Temperature T
Entropy s

```
"Data:"
Fluid$ = 'Nitrogen'

P[1] = 101.3 "kPa"

P[2] = 5070 "kPa"

P[3] = P[2]; P[4] = P[2] "kPa"

P[5] = P[1]; P[6] = P[1];P[7] = P[1]; P[8] = P[1]; P[9] = P[1]; P[10] = P[1]; P[11] = P[1]
"kPa"

T[1] = 300 "K"

T[2] = 300 "K"

T[3] = 255 "K"
```

```
P0 = 101.3 “kPa…Ambient or dead state”
T0 = 300 “K…Ambient or dead state”
V = 0 “m/s”
Z = 0 “m”

x = 0.4 “…expander flow rate ratio”

s[1]=Entropy(Fluid$,T=T[1],P=P[1]) “[kJ/kg-K]”

s[2]=Entropy(Fluid$,P=P[2],T=T[2]) “[kJ/kg-K]”

s[3]=Entropy(Fluid$,P=P[3],T=T[3]) “[kJ/kg-K]”

s[8] = s[3] “…entropy after isentropic expansion in expander”

h[1]=Enthalpy(Fluid$,T=T[1],P=P[1]) “[kJ/kg]”

h[2]=Enthalpy(Fluid$,P=P[2],T=T[2]) “[kJ/kg]”

h[3]=Enthalpy(Fluid$,P=P[3],T=T[3]) “[kJ/kg]”

h[8] = Enthalpy(Fluid$,P=P[8],s= s[8]) “[kJ/kg]”

h[9]=Enthalpy(Fluid$,x=1,P=P[9]) “[kJ/kg]”

h[10] = Enthalpy(Fluid$,x=0,P=P[10]) “[kJ/kg]”

“Calculations:”
y = (h[1] – h[2]) / (h[1] – h[10]) + x * ((h[3] – h[8]) / (h[1] – h[10])) “…fraction liquefied”

W_exp_perkggas = x * (h[3] – h[8]) “kJ/kg of gas compressed”

W_net_perkggas =T[1] * (s[1] – s[2]) – (h[1] – h[2]) – x * (h[3] – h[8]) “[kJ/kg]…net work per kg gas compressed..when the expander work is utilised in compression”

W_net_perkgliq = W_net_perkggas / y “[kJ/kg]…net work reqd per kg liq.”

W_ideal = Ideal_Work(FLUID$, P[1],T[1]) “kJ/kg…ideal work of liquefaction”
```

```
FOM = W_ideal / W_net_perkgliq "…Figure of Merit"

COP = (h[1] – h[2]) / W_net_perkggas "…Coeff of Performance"

"………………………………………………………………………………………………………………………………………………………………………………………………………"

T[9]=Temperature(Fluid$,P=P[9],x = 1) "K…temp at point 9…sat. liq."
T[5] = T[10]
T[9] = T[10]

(1 – x) * h[5] = y * h[10] + (1 – y) * h[9] "…finds h[5]… kJ/kg"

h[4] = h[5] "…for expansion in the Expansion Valve"

T[4] = Temperature(Fluid$,P=P[4],h = h[4]) "K…temp at point 4"

T[8] = Temperature(Fluid$,P=P[8],h = h[8]) "K…temp at point 8"

T[6] = Temperature(Fluid$,P=P[6],h = h[6]) "K…temp at point 6"

x * h[8] + (1 – x – y) * h[9] = (1 – y) * h[6] "…finds h[6]…by energy balance for mixing"

(1 – x) * (h[3] – h[4]) = (1 – y) * (h[7] – h[6]) "…finds h[7]… by energy balance on HX2"

T[7] = Temperature(Fluid$,P=P[7],h = h[7]) "K…temp at point 7"

h[2] – h[3] = (1 – y) * (h[11] – h[7]) "…finds h[11]…by energy balance on HX1"

T[11] = Temperature(Fluid$,P=P[11],h = h[11]) "K…temp at point 11"

s[4]=Entropy(Fluid$,T=T[4],P=P[4]) "[kJ/kg-K]"

s[5]=Entropy(Fluid$,P=P[5],x= (1 – y)) "[kJ/kg-K]"

s[6]=Entropy(Fluid$,P=P[6],T=T[6]) "[kJ/kg-K]"

s[7]=Entropy(Fluid$,T=T[7],P=P[7]) "[kJ/kg-K]"

s[9]=Entropy(Fluid$,P=P[9], x = 1) "[kJ/kg-K]"
```

```
s[10]=Entropy(Fluid$,P=P[10], x = 0) "[kJ/kg-K]"

s[11]=Entropy(Fluid$,T=T[11],P=P[11]) "[kJ/kg-K]"

"To find exergies:"

h0 = Enthalpy(Fluid$, T=T0, P=P0) "…kJ/kg"

s0 = Entropy(Fluid$, T=T0, P=P0) "…kJ/kg.K"

duplicate k = 1,11

ex[k] = (h[k] – h0) – T0 * (s[k] – s0) +( V^2 / 2) / 1000 + (9.81 * Z) / 1000 "….kJ/kg"

End

"Exergetic or Second Law effcy.:"

W_rev = ex[10] – ex[1] "…reversible work, kJ/kg"
```

W_actual =W_net_perkgliq "…kJ/kg liq…actual work input"

W_actual = W_rev + exergy_destroyed "…finds exergy destroyed, kJ/kg of liq"

eta_II_cycle = W_rev / W_actual "…Second Law effcy of cycle = exergetic effcy."

"Writing exergy balance to determine exergy losses in each component:"

"Exergy loss in Isothermal compressor:"

"Note that in an isothermal compressor, heat is rejected; however, in this case its exergy is zero since heat is rejected at ambient conditions."

"Writing an exergy balance for a compressor of isothermal effcy = 70%:"

ex[1] + (T[1] * (s[1] – s[2]) – (h[1] – h[2]))/0.7 = ex[2] + DELTAe_comp "…finds exergy loss in compressor, DELTAe_comp"

"Exergy loss in Heat exchanger – 1:"

"Writing an exergy balance:"

ex[2] +(1 – y) * ex[7] = ex[3] + (1 – y) * ex[11] + DELTAe_HX1 "…finds exergy loss in HX-1…DELTAe_HX1"

"Exergy loss in Heat exchanger – 2:"

"Writing an exergy balance:"

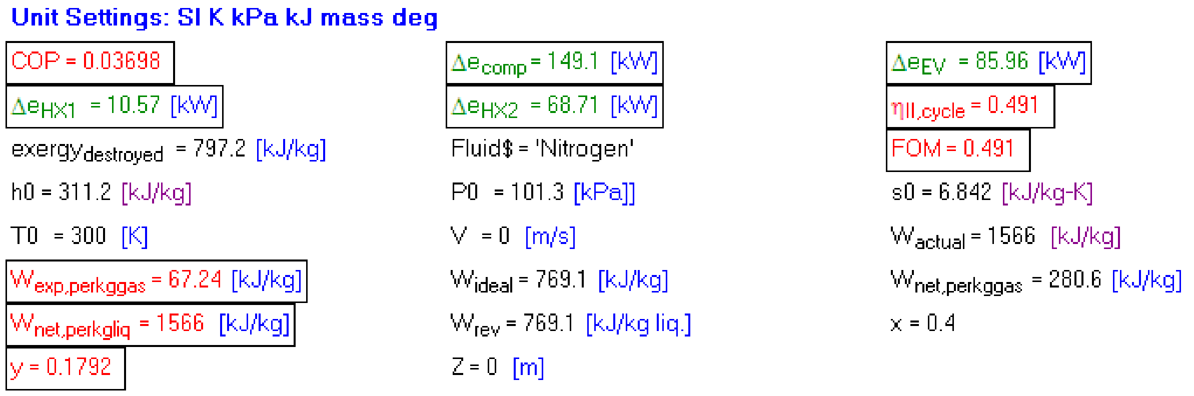
(1 – x) * ex[3] + (1 – y) * ex[6]= (1 – x) * ex[4] + (1 – y) * ex[7] + DELTAe_HX2 "…finds exergy loss in HX-2…DELTAe_HX2"

"Exergy loss in Expansion valve:"

"Writing an exergy balance:"

(1 – x) * ex[4] = (1 – x) * ex[5]+ DELTAe_EV "…finds exergy loss in Expn valve, DELTAe_EV"

Results:



Unit Settings: SI K kPa kJ mass deg

COP = 0.03698

Δe_{comp} = 149.1 [kW]

Δe_{EV} = 85.96 [kW]

Δe_{HX1} = 10.57 [kW]

Δe_{HX2} = 68.71 [kW]

$\eta_{II,cycle}$ = 0.491

$exergy_{destroyed}$ = 797.2 [kJ/kg]

Fluid$ = 'Nitrogen'

FOM = 0.491

h0 = 311.2 [kJ/kg]

P0 = 101.3 [kPa]]

s0 = 6.842 [kJ/kg-K]

T0 = 300 [K]

V = 0 [m/s]

W_{actual} = 1566 [kJ/kg]

$W_{exp,perkggas}$ = 67.24 [kJ/kg]

W_{ideal} = 769.1 [kJ/kg]

$W_{net,perkggas}$ = 280.6 [kJ/kg]

$W_{net,perkgliq}$ = 1566 [kJ/kg]

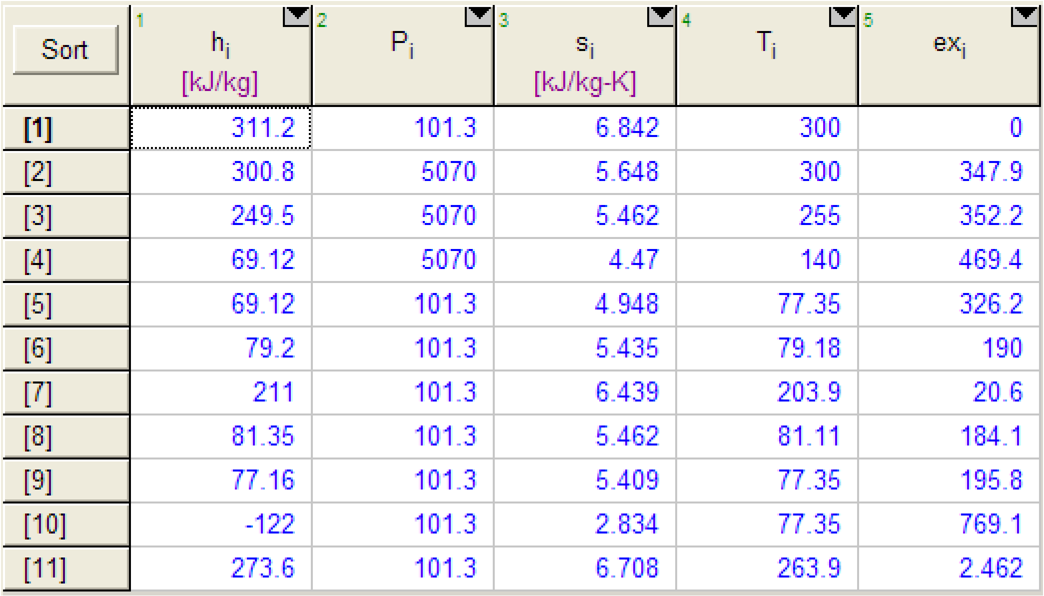
W_{rev} = 769.1 [kJ/kg liq.]

x = 0.4

y = 0.1792

Z = 0 [m]

And, parameters at various state points are:



Sort	1 h_i [kJ/kg]	2 P_i	3 s_i [kJ/kg-K]	4 T_i	5 ex_i
[1]	311.2	101.3	6.842	300	0
[2]	300.8	5070	5.648	300	347.9
[3]	249.5	5070	5.462	255	352.2
[4]	69.12	5070	4.47	140	469.4
[5]	69.12	101.3	4.948	77.35	326.2
[6]	79.2	101.3	5.435	79.18	190
[7]	211	101.3	6.439	203.9	20.6
[8]	81.35	101.3	5.462	81.11	184.1
[9]	77.16	101.3	5.409	77.35	195.8
[10]	-122	101.3	2.834	77.35	769.1
[11]	273.6	101.3	6.708	263.9	2.462

Thus:

Liquid yield, y = 0.1792 kg liq. / kg gas compressed…Ans.

Net Work per kg gas compressed = Wnetperkggas = 280.6 kJ/kg…Ans.

Net Work per kg liquid = Wnetperkgliq = 1566 kJ/kg…Ans.

Figure of Merit = FOM = 0.491…Ans.

Coeff of performance = COP = 0.03698…Ans.

Exergetic effcy (or, second law effcy) of cycle = ηII = 0.491 (= FOM)…Ans.

Exergy losses in compressor (assuming its isoth. effcy is 70%) = Δecomp = 149.1 kW… Ans.

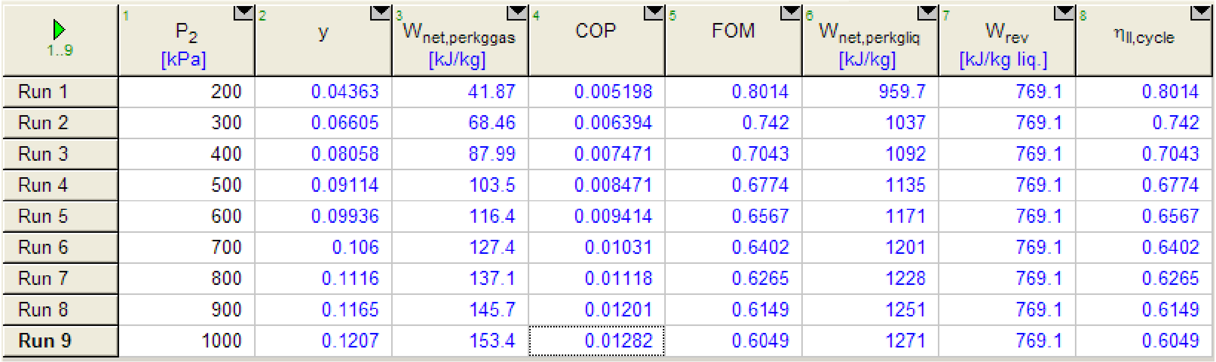
Exergy losses in Heat Exchanger-1 = ΔeHX-1 = 10.57 kW…Ans.

Exergy losses in Heat Exchanger-2 = ΔeHX-2 = 68.71 kW…Ans.

Exergy losses in Expansion Valve = ΔeEV = 85.96 kW…Ans.

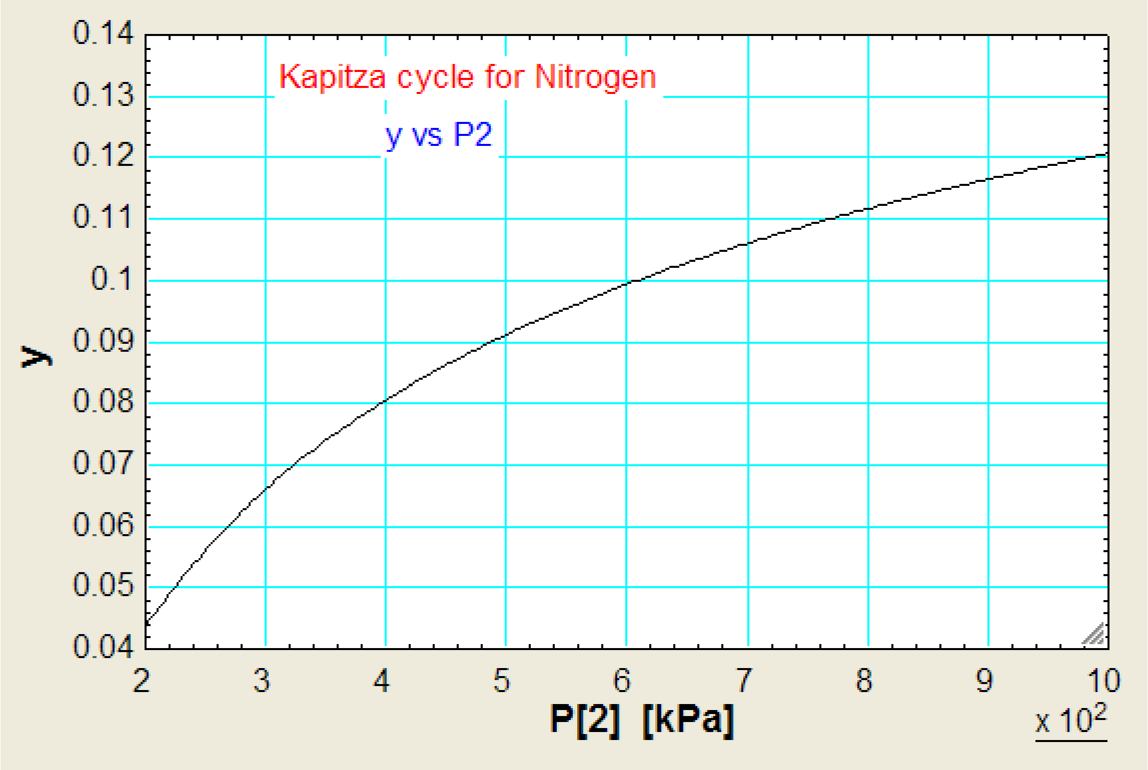
B. **Plot y, Wnetperkgas, COP and η_{II} against P2, other parameters remaining constant:**

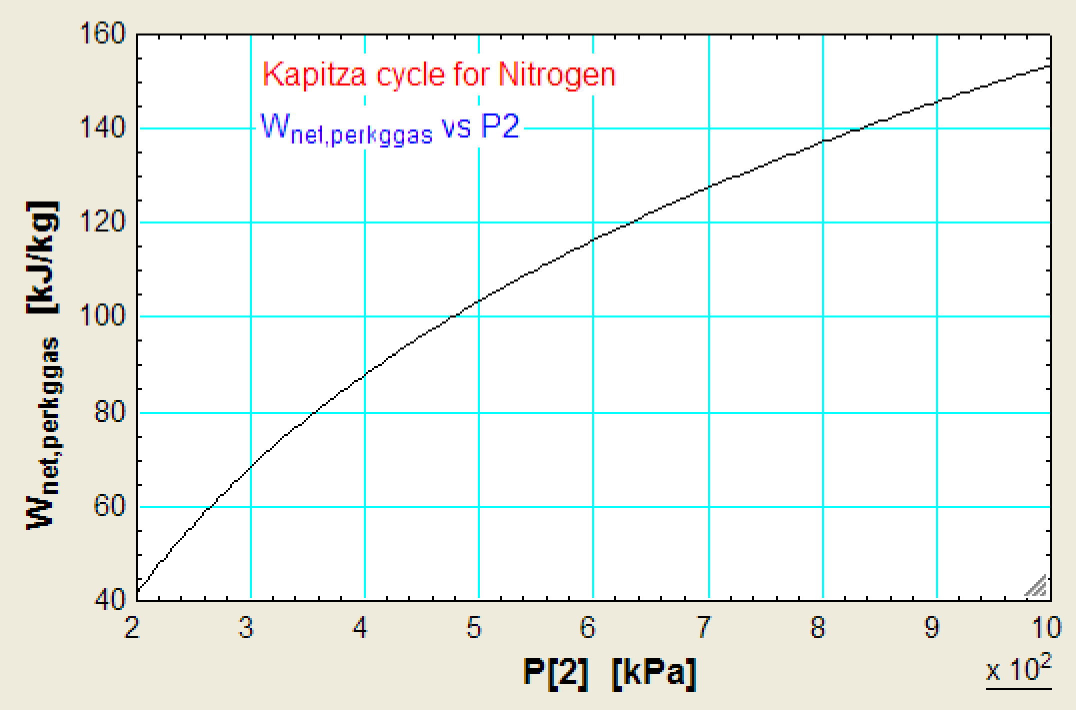
First, compute the Parametric Table:

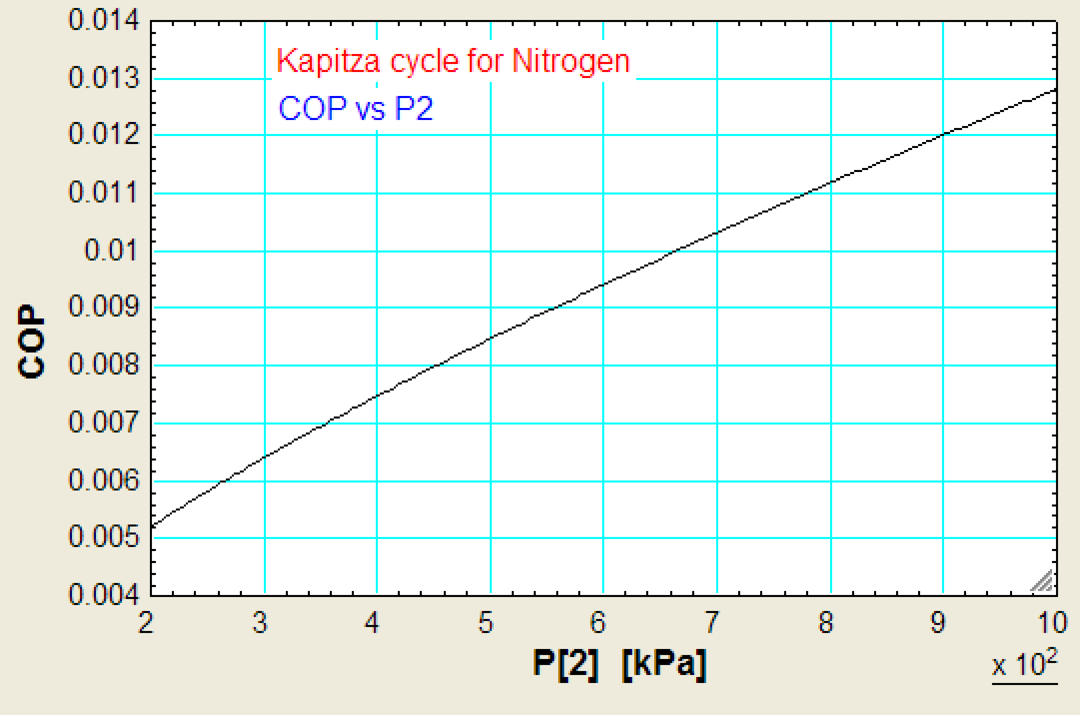


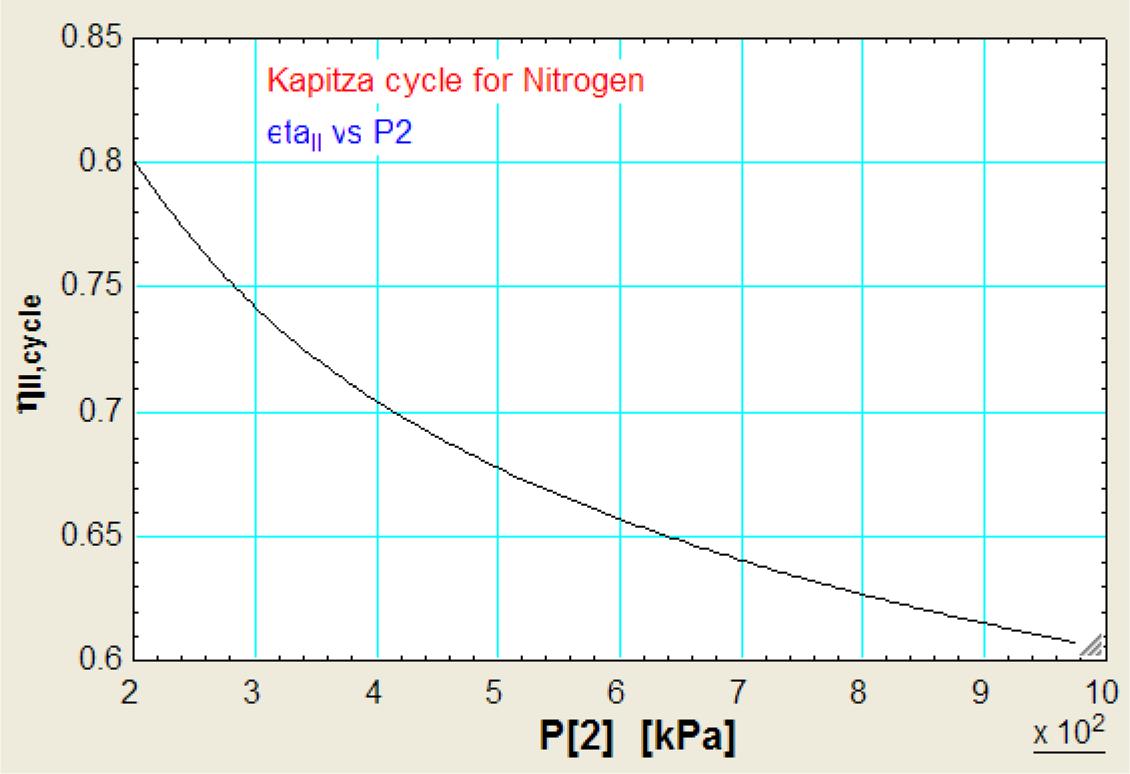
1..9	1 P_2 [kPa]	2 y	3 $W_{net,perkggas}$ [kJ/kg]	4 COP	5 FOM	6 $W_{net,perkgliq}$ [kJ/kg]	7 W_{rev} [kJ/kg liq.]	8 $\eta_{II,cycle}$
Run 1	200	0.04363	41.87	0.005198	0.8014	959.7	769.1	0.8014
Run 2	300	0.06605	68.46	0.006394	0.742	1037	769.1	0.742
Run 3	400	0.08058	87.99	0.007471	0.7043	1092	769.1	0.7043
Run 4	500	0.09114	103.5	0.008471	0.6774	1135	769.1	0.6774
Run 5	600	0.09936	116.4	0.009414	0.6567	1171	769.1	0.6567
Run 6	700	0.106	127.4	0.01031	0.6402	1201	769.1	0.6402
Run 7	800	0.1116	137.1	0.01118	0.6265	1228	769.1	0.6265
Run 8	900	0.1165	145.7	0.01201	0.6149	1251	769.1	0.6149
Run 9	1000	0.1207	153.4	0.01282	0.6049	1271	769.1	0.6049

Now, plot the results:




Kapitza cycle for Nitrogen
Wnet,perkggas vs P2
Wnet,perkggas [kJ/kg]
160
140
120
100
80
60
40
2
3
4
5
6
7
8
9
10
P[2] [kPa]
x 10^2


Kapitza cycle for Nitrogen
COP vs P2
COP
0.014
0.013
0.012
0.011
0.01
0.009
0.008
0.007
0.006
0.005
0.004
2
3
4
5
6
7
8
9
10
P[2] [kPa]
x 10^2


Kapitza cycle for Nitrogen
eta$_{II}$ vs P2
ηII,cycle
0.85
0.8
0.75
0.7
0.65
0.6
2
3
4
5
6
7
8
9
10
x 10^2
P[2] [kPa]

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