THE COPPERBELT UNIVERSITY

IN ASSOCIATION WITH

THE TECHNICAL AND VOCATIONAL TEACHERS' COLLEGE

BACHELOR OF SCIENCE IN MATHEMATICS AND SCIENCE WITH EDUCATION

SECOND YEAR - ODL

CLASSICAL MECHANICS (PHY 240)

DECEMBER 2022 PROMOTIONAL EXAMINATION

TOTAL MARKS – 100%

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS AND INFORMATION TO CANDIDATES

1. All questions carry equal marks of 20 marks each.
2. The marks are shown in brackets.
3. This paper contains seven questions. Answer **any five** questions of your choice.
4. Show clearly all the necessary calculations.
5. Marks will be awarded for neat and well-drawn diagrams.
6. Clearly indicate on the answer booklet cover page which questions you have attempted.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

WHEREVER NECESSARY USE:

$g = 9.81 m/s^2$

$c = 3.0 \times 10^8 m/s$

$1eV = 1.60 \times 10^{-19} J$

$G = 6.67 \times 10^{-11} Nm^2/kg^2$

$1Ci = 3.7 \times 10^{10} Bq$

$\mu_0 = 4\pi \times 10^{-7} Tm/A$

$\varepsilon_0 = 8.85 \times 10^{-12} F/m$

$m_e = 9.11 \times 10^{-31} kg$

$k_e = 8.99 \times 10^9 Nm^2/C^2$

$e = 1.60 \times 10^{-19} C$

$\hbar = 1.05 \times 10^{-34} Js$

$R_H = 1.097 \times 10^7 m^{-1}$

$\sigma = 5.669 \times 10^{-8} W/m^2 K^4$

$h = 6.626 \times 10^{-34} Js$

Question One

a. State the associative law and give its mathematical expression [3 marks]

b. Given two vectors $\mathbf{A} = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$ **and** $\mathbf{B} = (6\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$**,** find**:**

i. $|\mathbf{A}|^2$ [3 marks]

ii. $|\mathbf{B}|^2$ [3 marks]

iii. $(\mathbf{A} \bullet \mathbf{B})^2$ [3 marks]

c. A certain structure is represented by the following vectors in space;

$\mathbf{A} = \left(\frac{\sqrt{3}a}{2}\right)\hat{\mathbf{i}} + \left(\frac{a}{2}\right)\hat{\mathbf{j}}$, $\mathbf{B} = -\left(\frac{\sqrt{3}a}{2}\right)\hat{\mathbf{i}} + \left(\frac{a}{2}\right)\hat{\mathbf{j}}$ and $\mathbf{C} = c\hat{\mathbf{k}}$

Find;

i. $\mathbf{B} \times \mathbf{C}$ [5 marks]

ii. $\mathbf{A} \bullet (\mathbf{B} \times \mathbf{C})$ [3 marks]

Question Two

a. Write the translation equations from Cartesian to polar coordinate system. [2 marks]

b. Draw diagrams of how a point *P* can be described in Cartesian, cylindrical and spherical coordinate systems. [6 marks]

c. Convert the following coordinates as indicated

i. $(3, \pi/3, -4)$ from cylindrical to Cartesian [3 marks]

ii. $(-2, 2, 3)$ from Cartesian to cylindrical [3 marks]

d. Convert the following coordinates as indicated

i. $(8, \pi/4, \pi/6)$ from spherical to Cartesian [3 marks]

ii. $(2\sqrt{2}, 6, -4)$ from Cartesian to spherical [3 marks]

Question Three

a. State what is meant by a scalar quantity and by a vector quantity. [4 marks]

b. The commutative law states that the order of addition of two vectors does not matter. Use the geometrical method to prove the commutative law of the following vectors.

[6 marks]

c. Given two vectors $\mathbf{A} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}})$ and $\mathbf{B} = (5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$, find:

i. $\mathbf{A} + \mathbf{B}$ [2 marks]

ii. $\mathbf{A} - \mathbf{B}$ [2 marks]

iii. $|\mathbf{A}|$ [2 marks]

iv. $|\mathbf{B}|$ [2 marks]

v. $\mathbf{A} \bullet \mathbf{B}$ [2 marks]

Question Four

a. $\hat{\mathbf{i}} \bullet \hat{\mathbf{i}} = \hat{\mathbf{j}} \bullet \hat{\mathbf{j}} = \hat{\mathbf{k}} \bullet \hat{\mathbf{k}} = 1$ is a property of base vectors. Write the other four. [4 marks]

b. Circular motion plays an important role in physics. Here we look at the simplest and most important case; uniform circular motion, which is circular motion at constant speed. Consider a particle moving in the $x-y$ plane according to $\mathbf{r} = r(\cos\omega t\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}})$, where r and ω are constants. Find the acceleration of the particle. [3 marks]

c. Suppose that the position of a particle is given by;

$\mathbf{r} = A(e^{\alpha t}\hat{\mathbf{i}} + e^{-\alpha t}\hat{\mathbf{j}})$

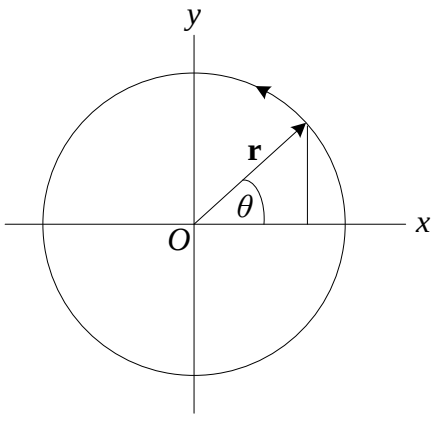
where A and α are constants. Find the velocity and magnitude of the velocity of the particle. [6 marks]

d. A table-tennis ball is released near the surface of the table with velocity $\mathbf{v}_0$ = (0, 5, -3) m/s. It accelerates (downward) with acceleration $\mathbf{a}$ = (0, 0, -1.6) m/s^2. Find the velocity after 5 s. [7 marks]

Question Five

a. State three Newton's laws of motion. [6 marks]

b. In the figure below



Find the work done in moving a particle once around a circle C in the xy plane, if the circle has center at the origin and radius 3 and if the force field is given by

$$\mathbf{F} = (2x - y + z)\hat{\mathbf{i}} + (x + y - z^2)\hat{\mathbf{j}} + (3x - 2y + 4z)\hat{\mathbf{k}}$$

[9 marks]

c. Show that Newton's second law can be redefined as $F = \frac{\Delta p}{\Delta t}$. [5 marks]

Question Six

a. Define power and write its mathematical expression. [3 marks]

b. Due to a force, a particle of mass 5 units moves along a space curve whose vector is given as a function of time *t* by

$\mathbf{r} = \left(2t^3 + t\right)\hat{\mathbf{i}} + \left(3t^4 - t^2 + 8\right)\hat{\mathbf{j}} + 12t^2\hat{\mathbf{k}}$

Find;

i. the velocity [2 marks]

ii. the momentum [2 marks]

iii. the acceleration [2 marks]

iv. the force at any time *t* [2 marks]

c. If $\mathbf{r}(t) = (t^2 + 2t)\hat{\mathbf{i}} + 3e^{-2t}\hat{\mathbf{j}} + 2\sin 5t\hat{\mathbf{k}}$, find, at $t = 0$;

i. $\frac{d\mathbf{r}}{dt}$ [4 marks]

ii. $\left|\frac{d\mathbf{r}}{dt}\right|$ [3 marks]

Question Seven

a. Write the mathematical expressions of the following operators on the specified functions.

i. the gradient of ϕ [2 marks]

ii. the divergence of **F** [2 marks]

iii. the curl of **F** [2 marks]

b. If $\phi(x, y, z) = x^2yz^3$ and $\mathbf{A} = xz\hat{\mathbf{i}} - y^2\hat{\mathbf{j}} + 2x^2y\hat{\mathbf{k}}$, find;

i. $\nabla\phi$ [2 marks]

ii. $\nabla \bullet \mathbf{A}$ [2 marks]

iii. $\nabla \times \mathbf{A}$ [2 marks]

iv. $div\,(\phi\mathbf{A})$ [4 marks]

v. $curl\,(\phi\mathbf{A})$ [4 marks]

END OF EXAMINATION