THE COPPERBELT UNIVERSITY

IN ASSOCIATION WITH

THE TECHNICAL AND VOCATIONAL TEACHERS' COLLEGE

BACHELOR OF SCIENCE IN MATHEMATICS AND SCIENCE WITH EDUCATION

SECOND YEAR - FULLTIME

CLASSICAL MECHANICS (PHY 240)

DECEMBER 2022 PROMOTIONAL EXAMINATION

TOTAL MARKS – 100%

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS AND INFORMATION TO CANDIDATES

1. All questions carry equal marks of 20 marks each.
2. The marks are shown in brackets.
3. This paper contains seven questions. Answer **any five** questions of your choice.
4. Show clearly all the necessary calculations.
5. Marks will be awarded for neat and well-drawn diagrams.
6. Clearly indicate on the answer booklet cover page which questions you have attempted.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

WHEREVER NECESSARY USE:

$g = 9.81 m/s^2$

$c = 3.0 \times 10^8 m/s$

$1eV = 1.60 \times 10^{-19} J$

$G = 6.67 \times 10^{-11} Nm^2/kg^2$

$1Ci = 3.7 \times 10^{10} Bq$

$\mu_0 = 4\pi \times 10^{-7} Tm/A$

$\varepsilon_0 = 8.85 \times 10^{-12} F/m$

$m_e = 9.11 \times 10^{-31} kg$

$k_e = 8.99 \times 10^9 Nm^2/C^2$

$e = 1.60 \times 10^{-19} C$

$\hbar = 1.05 \times 10^{-34} Js$

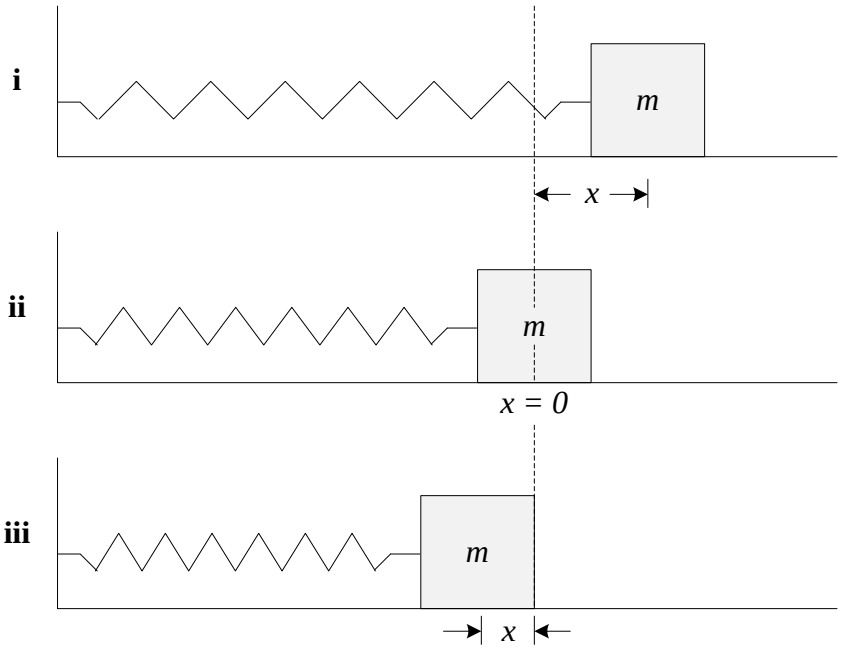
$R_H = 1.097 \times 10^7 m^{-1}$

$\sigma = 5.669 \times 10^{-8} W/m^2 K^4$

$h = 6.626 \times 10^{-34} Js$

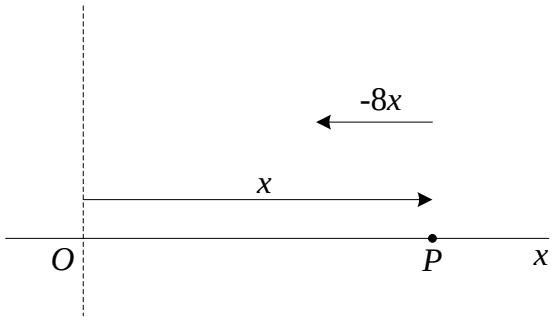
Question One

a. In the figure below, give the state of the spring (compressed, unstretched or stretched) and the restoring force (to the left, zero or to the right) in each of the three cases.



[6 marks]

b. A particle *P* of mass 2 moves along the *x* axis attracted toward origin *O* by a force whose magnitude is numerically equal to 8*x* as shown in the figure below.



If it is initially at rest at $x = 20$, find:

i. an expression for the differential equation describing the motion. [3 marks]

ii. the position of the particle at any time given that the general solution of (a) above is $x = A\cos 2t + B\sin 2t$. [5 marks]

iii. the velocity of the particle at any time. [2 marks]

c. Conservation of energy provides a simple method of deriving an expression for the velocity of an object undergoing periodic motion as a function of position. Starting from the energy interchange equation, derive the expression for the velocity of an object undergoing periodic motion as a function of position. [4 marks]

Question Two

a. Write the translation equations from Cartesian to polar coordinate systems. [2 marks]

b. Convert the following coordinates as indicated

i. $\left(3, \pi/3, -4\right)$ from cylindrical to Cartesian [3 marks]

ii. $\left(-2, 2, 3\right)$ from Cartesian to cylindrical [3 marks]

c. Convert the following coordinates as indicated

i. $\left(8, \pi/4, \pi/6\right)$ from spherical to Cartesian [3 marks]

ii. $\left(2\sqrt{2}, 6, -4\right)$ from Cartesian to spherical [3 marks]

d. Show that the expression for acceleration in polar coordinates is given by

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathbf{r}} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\boldsymbol{\theta}}$$

[6 marks]

Question Three

a. State what is meant by a scalar quantity and by a vector quantity. [4 marks]

b. Perform graphically the following vector additions and subtractions, where **A**, **B** and **C** are the vectors shown in the Fig. 4.1 below. [4 marks]

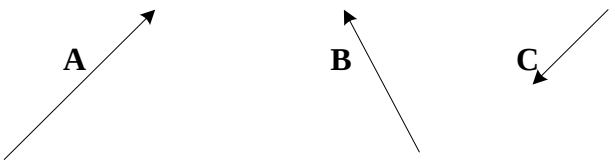


Fig. 4.1

i. **A** + **B**
ii. **A** + **B** + **C**
iii. **A** - **B**
iv. **A** + **B** – **C**

c. Let $A = (3, 5, -7)$ and $\mathbf{B} = (2, 7, 1)$. Find;

i. $\mathbf{A} + \mathbf{B}$ [2 marks]

ii. $\mathbf{A} - \mathbf{B}$ [2 marks]

iii. $\mathbf{A} \bullet \mathbf{B}$ [2 marks]

iv. The angle between **A** and **B** using the scalar product [6 marks]

Question Four

a. Classify each of the following according as they are; scleronomic or rheonomic, holonomic or non-holonomic and conservative or non-conservative.

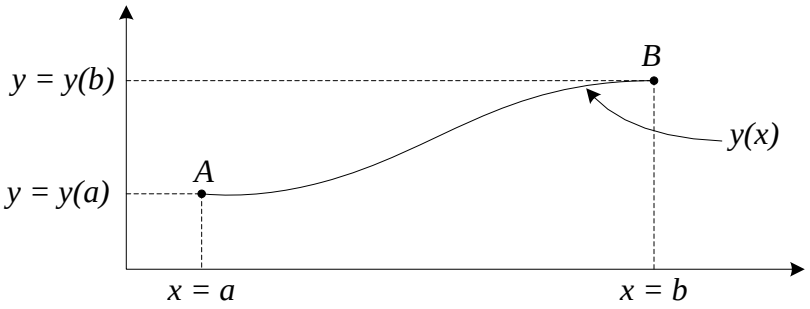
i. A sphere rolling down from the top of a fixed sphere. [2 marks]

ii. A cylinder rolling without slipping down a rough inclined plane of angle θ. [2 marks]

iii. A particle sliding down the inner surface, with coefficient of friction μ, of a paraboloid of revolution having its axis vertical. [2 marks]

iv. A particle moving on a very long frictionless wire which rotates with constant angular speed about a horizontal axis. [2 marks]

b. From the figure below, use calculus of variations to find the function *y(x)* which gives the shortest path between two points *A* and *B* from $x = a$ and $x = b$.

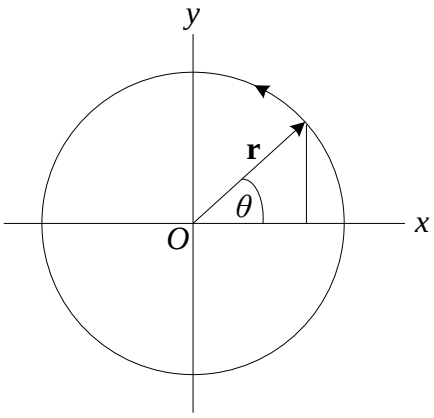


[13 marks]

Question Five

a. State three Newton's laws of motion. [6 marks]

b. In the figure below



Find the work done in moving a particle once around a circle *C* in the *xy* plane, if the circle has center at the origin and radius 3 and if the force field is given by

$$\mathbf{F} = (2x - y + z)\hat{\mathbf{i}} + (x + y - z^2)\hat{\mathbf{j}} + (3x - 2y + 4z)\hat{\mathbf{k}}$$

[9 marks]

c. Show that Newton's second law can be redefined as $F = \dfrac{\Delta p}{\Delta t}$. [5 marks]

Question Six

a. State Hooke's law. [1 mark]

b. A 0.500 kg object connected to a light spring with a spring constant of 20.0 N/m oscillates on a frictionless surface.

i. Calculate the total energy of the system and the maximum speed of the object if the amplitude of the motion is 3.00 cm. [6 marks]

ii. What is the velocity of the object when the displacement is 2.00 cm. [2 marks]

iii. Compute the kinetic and potential energies of the system when the displacement is 2.00 cm. [4 marks]

c. In many real systems, forces, such as friction, retard motion. Consequently, the mechanical energy of the system diminishes in time and the motion is said to be damped. Explain under damping, critical damping and over damping with the aid of a diagram. [7 marks]

Question Seven

a. Define power and write its mathematical expression. [3 marks]

b. Due to a force, a particle of mass 5 units moves along a space curve whose vector is given as a function of time *t* by

$\mathbf{r} = \left(2t^3 + t\right)\hat{\mathbf{i}} + \left(3t^4 - t^2 + 8\right)\hat{\mathbf{j}} + 12t^2\hat{\mathbf{k}}$

Find;

i. the velocity [2 marks]

ii. the momentum [2 marks]

iii. the acceleration [2 marks]

iv. the force at any time *t* [2 marks]

c. If $\mathbf{r}(t) = (t^2 + 2t)\hat{\mathbf{i}} + 3e^{-2t}\hat{\mathbf{j}} + 2\sin 5t\hat{\mathbf{k}}$, find, at $t = 0$;

i. $\dfrac{d\mathbf{r}}{dt}$ [4 marks]

ii. $\left|\dfrac{d\mathbf{r}}{dt}\right|$ [3 marks]

END OF EXAMINATION